

Miller application.

I am applying for a Miller professorship because, after a period of administrative work as Director of the Mathematical Sciences Research Institute, a position I occupied for most of the years 1997-2022 (in which I felt that I accomplished my major goals, extending and greatly improving the physical facility and raising a very significant endowment) I have discovered a novel realm of phenomena in an area of mathematics that has been studied from a different point of view for about 70 years. I can guess at far more of the picture than I can establish mathematically, and I would like to have a period in which I can pursue this new line of thought with the rigor and intensity that I think it deserves.

Here is some background on my project. My fundamental area of study is algebraic geometry: the qualitative study of geometric forms defined by polynomial equations. Examples of such forms that are familiar to every high school student are the curves in the plane that we call circle, ellipse, parabola, and hyperbola – they are called the “conic sections” because they can all be constructed by slicing a circular cone. These are all part of one family algebraically too, defined by equations involving only linear and quadratic terms. They were intensely studied by mathematicians from the time of Euclid until the first third of the 19th century. Nowadays algebraic geometry is concerned with a far wider class of objects, often embedded in high-dimensional spaces, and this has led to wide field of applications from String theory to artificial vision to robotic motion planning.

Underpinning much work of this kind is the need to understand solutions of systems of linear equations of a certain kind. Again high-school students typically learn to solve two linear equations in two unknowns, say $ax+by = c$ and $dx+ey = f$, and what they learn is actually more sophisticated than what was known to mathematicians before about 1700. But the modern theory that can be used in algebraic geometry requires the solution of such equations when the coefficients a,b,c,d,e,f are themselves varying (perhaps as polynomials in further variables) and the solutions to be found are also systems of polynomials. Such equations were first seriously studied by David Hilbert, the greatest mathematician of his day, around 1890, and became the center of a great deal of research starting in the 1950s. From the 1960s on, people began to understand the need to study situations where the coefficients and the solutions come from even more exotic domains, rings of functions on other algebraic varieties. In this context the complete resolution of one system of equations leads inevitably to an infinite sequence of further equations and solutions called in total an *infinite free resolution*.

It is in the context of these infinite free resolutions that my new observations, conjecture and theorems fall. Over the last 70 years, most of the work done in this area (and there has been a lot) has focused only on the numbers of independent solutions of each successive system of equations. Modern computational algebra and fast computers allow one to look much more deeply at the successive systems. Using that power my collaborators and I have seen many unexpected phenomena. In some cases, we have already been able to prove that what we are observing are general phenomena; but our observations greatly outstrip our current proofs. This is the area on which I would like to focus during a year as Miller professor.

