

## chapter 7

1 message

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To: Joe Harris <harris@math.harvard.edu>, David Eisenbud <de@berkeley.edu>

Dear Joe and David,

Here are some thoughts on Chapter 7. Do you want me to continue reading the rest of the chapters? I can read what you have now and can read them again if you make substantial revisions.

Best, Izzet

page 151, first paragraph, did you define coarse moduli space already?

page 151, paragraph above 7.1.3, incomplete reference

page 152, line 3 of proof of 7.1.4, 'given as in 7.1' is a little ambiguous since there is section 7.1, theorem 7.1.x, etc. I think you mean formula (7.1)

page 153, line1 of second paragraph of 7.1.3, overfull box

page 154, line -7, 'For example, for example'

page 156, Theorem 7.2.1 part (2), at this stage do you mean  $\overline{M}_g$  or should that be  $M_g$ ?

page 156, 3 lines below 7.2.1, should it be  $\overline{M}_1 = P^1$ ?

page 156, paragraph below 7.2.1, Should the last  $M_g$  be  $M_1$ ?

page 156, line 4 of last paragraph, "boundary" has the wrong "

page 157, line 2, is ad divisor should be is a divisor

page 157, 2nd sentence of 2nd paragraph of 7.2.1, you need some commas (or maybe reword the sentence)  
However, it is possible to lift the action of  $G$  to an action on  $A$  of a slightly larger group, in this case  $SL_{N+1}$  a process called linearization.

Maybe better to say, However, it is possible to lift the action of  $G$  to an action on  $A$  of a slightly larger group  $SL_{N+1}$ , a process called linearization.

Your GIT quotients look a little off. It feels like there is too much space between  $//$  in  $X//G$ . This could be a function of my computer.

You can easily give examples of stable and unstable points for actions of  $C^*$  on  $P^n$ , maybe even as exercises. Consider the two  $C^*$  actions on  $P^3$ , given by  $(a x, a y, 1/a z, 1/a w)$  and  $(ax, ay, az, 1/a^3 w)$ . In the first the quotient is  $P^1 \times P^1$ . The unstable locus is the union of the two lines  $x=y=0$  and  $z=w=0$ . The invariants are easy to write down, they are  $xz, xw, yz, yw$ , these satisfy the relation  $xz yw = xw yz$ , so you explicitly see the quadric. The orbits are either points on the unstable lines or given a point  $p$  not on these lines the unique line through  $p$  meeting the two unstable lines (minus the intersection points with the lines). You again see  $P^1 \times P^1$  explicitly as the quotient. In the second example, the quotient is  $P^2$ . The unstable locus is the union of the plane  $w=0$  and the point  $x=y=z=0$ . The orbits are the points in the unstable locus and the lines joining  $x=y=z=0$  to the plane  $w=0$  (minus the intersection with the unstable locus). The invariants are  $F(x,y,z)w$  where  $F$  is a homogeneous cubic, so you explicitly see the 3uple Veronese embedding of  $P^2$  as the quotient. You can get more intricate examples by varying the weights of the action. The advantage is that it is easier for students to study  $C^*$  actions than  $PGL_n$  actions.

page 159, I am a little confused about Theorem 7.3.2. Do you mean Theorem 7.3.2. to be about  $M_g$  or  $\overline{M}_g$ ? What is the difference between 7.3.2 and 7.3.3. If 7.3.2 becomes only about  $M_g$ , how will it compare to 7.2.1?

page 160, line 2 of Proposition 7.3.5, it should be  $1 + \lfloor (g-1)/2 \rfloor$

Broken reference right after 7.3.5

page 162, first sentence of second paragraph. Either put a period after 23 or do not capitalize Surprisingly

Do you want to mention/give a reference to  $g=21, 22$  and Gabi's work after 7.4.1?

page 163, last line of 1st paragraph, should it be 'historical importance'?

page 164, 2nd paragraph of 7.5.2, I sort of know what you mean by 'getting inside the geometry of abstract moduli spaces', but what do you really mean? Might be better to replace 'getting inside' with 'understanding'

page 166, last line, do you mean to continue the expansion?

page 167, line 11, do you want a space 'singularity) implies'?

Broken reference in the first line of proof of 7.6.5

proof of 7.6.6 'discriminant will consist of' -> 'discriminant consists of'

I assume you will make up some exercises for this section?