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Chapter 11

1 message

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To: Joe Harris <harris@math.harvard.edu>, David Eisenbud <de@berkeley.edu>

Dear Joe and David,

Here are some thoughts on Chapter 11.

Best, Izzet

page 207, statement of Theorem 11.1.1, should it be 'a' line bundle?

page 207, sentence after Theorem 11.1.1, suggestion: when there can be a g_d^r that is birationally very ample on a curve of genus $g \rightarrow$ when does there exist a curve of genus g with a birationally very ample g_d^r

page 208, statement of Theorem 11.2.1, the period after the BN number should be a comma ','

page 208, suggestion: It is interesting to compare the values of d, r that are possible on special and general curves, and see how many fewer are possible for birationally very ample series and for general curves, as in the graphs in Figure 11.1. \rightarrow It is interesting to compare the values of d, r that are possible on special and general curves (see the graphs in Figure 11.1).

page 208, line 13, is the inequality symbol for the Castelnuovo's Theorem the one you intended?

page 208, sentence before 11.2.1, you have a broken reference

page 210, sentence before 11.2.2, is the reference to chapter 13 correct?

page 211, Theorem 11.2.3 (4), will the students know that you are talking about the universal W_d^r over the moduli space M_g ?

page 211, Theorem 11.2.3 (5), do you want a tensor product in $H^0(\omega_C L^{-1})$ page 211, line before Corollary 11.2.4, there is an x page 211, proof of Corollary 11.2.4, when do you put a tensor product sign in $L \otimes M$ and when do you just write LM ?page 211, last sentence of proof of Corollary 11.2.4, should it be 'contradicting specialness of L^m '?

page 211, Remark 11.2.5, maybe give the expression $C_{d-1} = 1/d \{2d-2 \text{ choose } d-1\}$. Students these days are unlikely to know about Catalan numbers.

page 212, Remark 2, The capitalization of Part is not consistent

page 212, Theorem 11.2.6. Capitalize 'if'. It might be better to say 'Let $C \dots$ and $|D|$ be a general g_d^r on C .' 1) If $r \geq 3, \dots$ etc.

page 213, expression of h_C between 11.2.7 and 11.2.8, might want to use $\left($ and $\right)$

page 213, you might want to give a reference to 11.2.8

page 213, Suggestion: 'But it doesn't tell us what a minimal set of generators for the homogeneous ideal of C might look like.' \rightarrow 'However, it doesn't tell us the degrees of a minimal set of generators of I_C '

page 213, Suggestion: 'This can't always be the case, since it's easy enough to come up with examples where, for example, the smallest nonzero graded piece of $I(C)$ has dimension 1.' \rightarrow This can't always be the case, since there are examples where the smallest nonzero graded piece of $I(C)$ has dimension 1.'

page 214, you might want to mention that Hannah, Isabel and Eric proved a version of the BN Theorem for the general k -gonal curve.

page 214, do you mean Exercise 11.2.9 to be here instead of the end of the chapter?

page 214, last sentence of paragraph 1, we need to use 'Brill-Noether Theory' or 'the Brill-Noether Theorem'

page 214, sentence before 11.3.1, needs some revision. I am not sure what it is supposed to say.

page 214, statement of 11.3.1, you need a period after \emptyset

page 215, last few sentences of paragraph 4. If all the 4 nodes are collinear, then the sextic would be reducible by Bezout. You can rule out this case from the beginning. This distraction also obscures your argument that no three of the nodes are collinear by the reducedness of W_4^1 .

page 216, paragraph 3 of 11.3.1, why not $P^1 \times P^1$ embedded in P^8 by $(2,2)$?

page 218, line 3, it should be capitalized. Might be better to say, 'It is the intersection of a general linear P^5 with $G(2,5)$...'

page 219, line 2 of 11.4.1, the $g_6^2[D]$ doesn't look great, maybe omit the g_6^2 ?

page 220, line 1, the image $\phi_D(C)$ is smooth

page 220, line 2, C is isomorphic to a plane quintic

page 220, last line, swap 16 and 17 in the Chapter numbers.

page 220 and 221, you could remove the word 'necessarily' from these pages without losing much

page 221, line 8 of Case 1, But the genus formula tells us \rightarrow By the genus formula

page 222, line 1 of 11.4.3, the cubic curve E necessarily smooth \rightarrow The cubic curve E 'is' smooth

page 223, 2nd sentence of 11.4.4. If C_0 has a quadruple point, why would C_0 be rational? Isn't the geometric genus of a sextic with a quadruple point typically 4? Such a curve is hyperelliptic by projecting from the quadruple point.

page 224, first paragraph has several facts in future tense

The stuff about $g=6$ curves needs some organization and editing. Would it make more sense to state the whole picture as a theorem? Then discuss the individual cases. I agree that in this chapter it might make more sense to have the exercises interspersed with the text, but this is different than the other chapters. Then there are a couple of exercises that are not directly related to genus 6 curves at the end. The last page or two of this section has the quality of being more for the amusement of the authors and will be less appreciated by the students. You could help by stating the explicit configurations of 4 points needed to blow up in order to get the corresponding del Pezzo surfaces.