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david eisenbud PhD <de@berkeley.edu>

Re: quartic with 3 real cusps??

1 message

Frank Sottile <sottile@tamu.edu> Mon, May 8, 2023 at 4:53 AM
To: David Eisenbud <de@berkeley.edu>

Dear David,

The existence is trivial. There is a unique parametrized quartic with cusps specified at any three points. When the points are real, the curve must be. More generally, Shapiro's conjecture implies that any rational curve with specified ramification at real points is necessarily real (and all ramification is real). I have an old paper that predates the proof of the Shapiro conjecture

Yes I have a pictures and equations.

Parametrization:

```
X:=-(-2*s^4+4*t*s^3+t^4-2*t^3*s):
Y:=3^(1/2)*t^3*(t-2*s):
Z:=(3*t^2*s^2+s^4-2*t*s^3+t^4-2*t^3*s):
#
# Defining equation
#
Eq:= 3*x^4+6*x^2*y^2+3*y^4-16*x^3+48*x*y^2+24*x^2+24*y^2-16:
```

Attached is a maple file I used to compute these, as well as three images.

Here is reference to a paper I wrote before the Shapiro conjecture was proven on this topic:

Maximally inflected real rational curves, with Viatcheslav Kharlamov. Moscow Mathematics Journal **3** (2003), no. 3, 947--987, 1199--1200.

A Webpage with more pictures.

-Frank

On 5/7/23 21:36, David Eisenbud wrote:

> Dear Frank,
>
> I hope you are well! Joe Harris and I are writing a book about algebraic curves, and in that connection we had a question: A complex plane quartic can have 3 cusps. The usual real pictures have 1 real, the other two conjugate complex. Do you know whether all three can be real? If so, do you have a picture, or an equation?

thanks,

David

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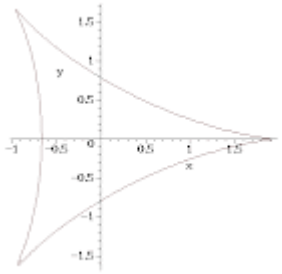
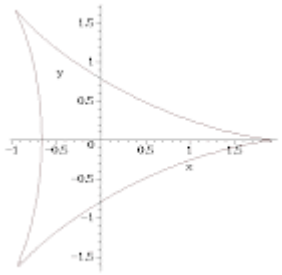
4 attachments



3_Cusps.gif
12K



3_Cusps_implicit.gif
12K

3_cusps.maple
2K

3_Cusps.eps
13K