REVIEW OF "PRACTICAL CURVES" BY EISENBUD AND HARRIS

1. Reviewer Questions

1.1. **Content selection.** The book starts with two chapters of basics needed for curve theory and then continues on to in-depth discussions of the simplest kinds of curves: curves of low genus, and hyperelliptic curves. The authors devote two chapters to an overview of moduli spaces of curves. Later chapters of the book include much about the extrinsic geometry of curves in projective space: their hyperplane sections and secants, curves in the plane, linkage, and free resolutions. It also contains a proof of the dimension statement in the foundational Brill–Noether theorem. The final chapter on the Hilbert scheme ties together ideas from the entire book and introduces some open questions.

Overall, the topics are well-selected and comprehensive. While it is impossible to cover everything, I think the authors have struck a nice balance and should not make the book any longer. The authors might consider adding a "dependence diagram" indicating which chapters should be read before others, since it seems a linear reading of the chapters is not required (e.g. it seems to me Chapters 14 – 16 should be read in order but could be read without reading 10 – 13 beforehand).

1.2. **Audience and prerequisites.** The stated prerequisites (a standard course in graduate algebraic geometry) are appropriate. The first several chapters include refreshers on necessary background as well. It helps that references to standard texts are given when important theorems (e.g. cohomology and base change) are used.

1.3. **Exposition.** The exposition is very nice. The proofs are well-written and readable. There is ample motivating, conversational text throughout (e.g. I especially liked the discussion of moduli problems at the start of Chapter 6). The authors take the time to explain things that might confuse readers who are new to the subject (e.g. what it means for "a general X to have property Y"). Later chapters give a sense of things "coming full circle": for example, Chapter 3 was about curves of genus 0, and then in Chapter 13, we see that by understanding genus 0 curves very well we can understand the maps to projective space of general curves of any genus. Overall, the book accomplishes the goal of a clear and rich introduction to curves.

1.4. **Examples.** Wonderful – the book is full of examples. The authors often present the simplest case or cases of concepts before making general statements (e.g. the twisted cubic is a running example of many concepts throughout, which also gives a sense of cohesion). Examples are given after new definitions, at the start of a section as motivation for the next topic of discussion, and in some cases, entire chapters are devoted to examples.

1.5. **Exercises.** The exercises include problems that range in difficulty, some easier some harder. I think it might help if there were some system to set apart the easiest and hardest exercises (e.g. star the hardest exercises and/or put a checkmark by easier "reality check" questions, say like 3.7.8). The interspersed exercises in Chapter 11 are cool.

1.6. **Graphics.** The graphics are appropriate. I say the more pictures the better, and there are lots of them!

1.7. **References.** For important statements not proven in the text, references are always given. The historical references are interesting (and sometimes amusing) and add to the narrative. They also include many relevant current references on the cutting edge of research.

A few specific additions come to mind:

- Section 7.5.2 number 2 should also include a reference to Mullane 2020, which has proved the Picard rank conjecture in the range $d > g - 1$.
- Mukai's papers on general curves of genus $7, 8, 9$ seem like relevant references that I don't see anywhere (perhaps they belong in Section 7.4, or perhaps somewhere else, e.g. before or after the genus 6 discussion in Section 11.3).
- A paper of Coppens and Kato from 1991 proves an analogue of Exercise 14.4.3 when δ is bounded by roughly $d^2/4$. I believe it is an open problem to determine the gonality plane curves whose number of nodes is larger than this.

1.8. **Course use.** I think the book would make a great resource for teaching a graduate-level topics course on algebraic curves.

1.9. **Title.** The title is good. It conveys the topic (curves) and what sets it apart from other texts (its practical, concrete nature).

1.10. **Existing literature.** There are many references on curves — what sets this book apart is its focus on curves of low genus, lots of examples, and conversational tone. The focus is really on the geometry of curves in projective spaces, so there is minimal overlap with books whose focus is on the geometry of the moduli space of curves. The proof of the Brill–Noether theorem presented in Chapter 13 is also different from what I have seen presented in other standard texts (e.g. using limit linear series in Harris–Morrison).

1.11. **Series.** Yes, I think it is suitable for the Graduate Studies in Mathematics series.

1.12. **Recommendation.** I enthusiastically recommend the book.

One small specific comment: I recommend that the authors include a note on their convection for projectivization and Grassmannians. I gather that $\mathbb{P}V$ is 1-dimensional quotients of V and $G(k, V)$ is k-dimensional subspaces of V. So after noting the conventions, it would be helpful to write that $\mathbb{P}V = G(1, V^\vee)$.