ECEN321 : Engineering Statistics
Assignment 7 Submission

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Central Limit Theorem

1. (Navidi 4.11.14) $\lambda = 30$ particles.ml$^{-1} \times 2$ml $= 60$

   (a) $\lambda > 10,\ X \sim \mathcal{N}(\lambda, \lambda) = \mathcal{N}(60, 60)$
   $z = \frac{x-\mu}{\sigma} = \frac{50-60}{\sqrt{60}} = -1.290994$
   From lookup table:
   $P(X > 50) = P(Z > -1.29) = 90.15\%$

   (b) $p = P(X > 50) = 0.9015,\ n = 10$
   $P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$

   $P(X \geq 9) = P(X = 9) + P(X = 10)$
   $P(X = 9) = \frac{10!}{9!(10-9)!} 0.9015^9 (1 - 0.9015)^{10-9} = 0.387372$
   $P(X = 10) = \frac{10!}{10!(10-10)!} 0.9015^{10} (1 - 0.9015)^{10-10} = 0.354534$
   $P(X \geq 9) = 0.741906$

   (c) $p = P(X > 50) = 0.9015,\ n = 100$
   $P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$

   $P(X \geq 90) = P(X = 90) + P(X = 91) + \cdots + P(X = 100)$
   $P(X = 90) = \frac{100!}{90!(100-90)!} 0.9015^{90} (1 - 0.9015)^{100-90} = 0.131699$
   $P(X = 91) = \frac{100!}{91!(100-91)!} 0.9015^{91} (1 - 0.9015)^{100-91} = 0.132456$
   $\ldots$
   $P(X = 92) = 0.118592, P(X = 93) = 0.093367, P(X = 94) = 0.063634, P(X = 95) = 0.036783, P(X = 96) = 0.017534, P(X = 97) = 0.006618, P(X = 98) = 0.001854, P(X = 99)$ $=0.000343, P(X = 100) = 0.000031$
   $P(X \geq 90) = 0.6029$

2. (Navidi 4.11.16)

   (a) $z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma/\sqrt{n}} = (36.7 - 40)/(5/\sqrt{100}) = -6.6$
   $P(z < -6.6) = 0$

   (b) The probability is less than 0.05, ie small. So a sample mean of 36.7 **is** short.

   (c) As the it is very small the claim seems implausible.

   (d) $z = (39.8 - 40)/(5/\sqrt{100}) = -0.4$
   $P(z < -0.4) = 0.3446$

   (e) The probability is greater than 0.05, ie not small, so its is not unusually short.

   (f) Therefore the claim seems plausible.

Confidence Intervals

3. (Navidi 5.1.2)

   (a) $P(-1.96 < Z < 1.96) = P(Z < 1.96) - P(Z < -1.96) = 0.975 - 0.025 = 0.95 = 95\%$

   (b) $P(-2.17 < Z < 2.17) = P(Z < 2.17) - P(Z < -2.17) = 0.985 - 0.015 = 0.97 = 97\%$

   (c) $P(-1.28 < Z < 1.28) = P(Z < 1.28) - P(Z < -1.28) = 0.8997 - 0.1003 = 0.7994 = 79.94\%$

   (d) $P(-3.28 < Z < 3.28) = P(Z < 3.28) - P(Z < -3.28) = 0.9995 - 0.0005 = 0.999 = 99.9\%$

4. (Navidi 5.1.4) $\quad n = 50 \quad \bar{x} = 654.1 \quad s = 311.7$

   (a) $1 - \alpha = 0.95, \alpha/2 = 0.05/2 = 0.025, Z_{\alpha/2} = 1.96$
   $E \approx 1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{311.7}{\sqrt{50}} = 86.398832$
   $\bar{x} \pm E = 654.1 \pm 86.4 \rightarrow (558.7, 731.5)$

   (b) $1 - \alpha = 0.98, \alpha/2 = 0.02/2 = 0.01, Z_{\alpha/2} \approx 2.33$
   $E \approx 2.33 \frac{s}{\sqrt{n}} = 2.33 \frac{311.7}{\sqrt{50}} = 102.708815602$
   $\bar{x} \pm E = 654.1 \pm 102.71 \rightarrow (551.39, 756.81)$

   (c) $CI = (581.6, 726.6)$
   $z = \frac{\bar{x}-\mu_{\bar{x}}}{s/\sqrt{n}}$
   $\frac{581.6-654.1}{311.7/\sqrt{50}} = -1.644698 \qquad \frac{726.6-654.1}{311.7/\sqrt{50}} = 1.644698$
   $P(-1.64 < Z < 1.64) = 0.9495 - 0.0505 = 0.899 \approx 90\%$

   (d) $1.96 \frac{311.7}{\sqrt{n}} = 50$
   $\left(1.96 \frac{311.7}{50}\right)^2 = n = 149.295 \rightarrow 150$

   (e) $\left(2.33 \frac{311.7}{50}\right)^2 = 210.98 \rightarrow 211$

5. (Navidi 5.1.6) $\quad n = 123 \quad \mu = 136.9 \quad \sigma = 22.6$

   (a) For 95%, $Z_{\alpha/2} = 1.96$
   $E \ = \ 1.96 \cdot \frac{22.6}{\sqrt{123}} = 3.994$
   $\bar{x} \pm E = 136.9 \pm 3.994 \rightarrow (132.906, 140.894)$

   (b) $1 - \alpha = 0.995, \alpha/2 = 0.005/2 = 0.0025, Z_{\alpha/2} = 2.81$
   $E \ = \ 2.81 \cdot \frac{22.6}{\sqrt{123}} = 5.726$
   $\bar{x} \pm E = 136.9 \pm 5.726 \rightarrow (131.174, 142.626)$

   (c) $CI = (133.9, 139.9)$
   $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
   $\frac{133.9-\mu}{\sigma/\sqrt{n}} - 1.47219511147 \qquad \frac{139.9-\mu}{\sigma/\sqrt{n}} = 1.47219511147$
   $P(-1.47 < Z < 1.47) = 0.92922 - 0.07078 = 0.85844 \approx 86\%$

   (d) $1.96 \frac{22.6}{\sqrt{n}} = 3$
   $\left(1.96 \frac{22.6}{3}\right)^2 = n = 218.015068444 \rightarrow 219$

   (e) $\left(2.575 \frac{22.6}{3}\right)^2 = 376.295336111 \rightarrow 377$

   (f) $\mu - 1.645 \frac{\sigma}{\sqrt{n}} = 133.55$

   (g) $\frac{\mu - 134.3}{\frac{\sigma}{\sqrt{n}}} = Z_{\alpha/2} = 1.27590242994 \rightarrow 89.8\%$ confidence