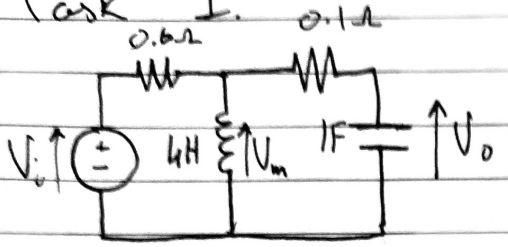
315 Class 1 Exercises

Task 1.

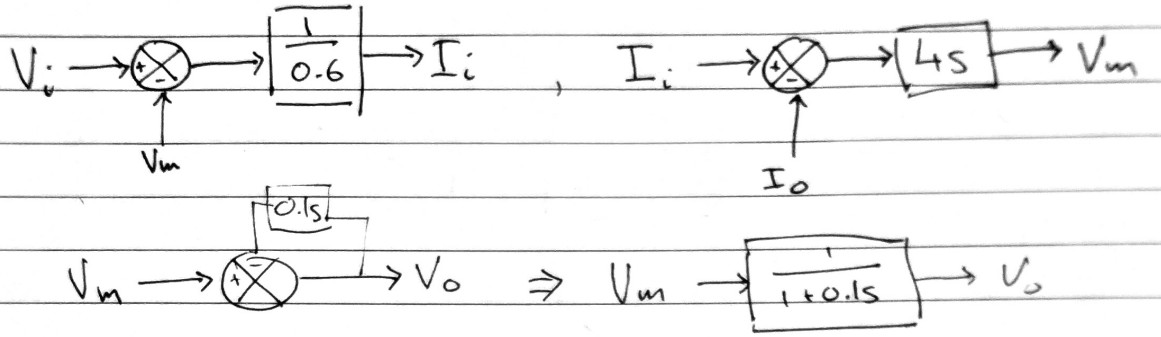



$V_i = V_m = 0.6 I_i$

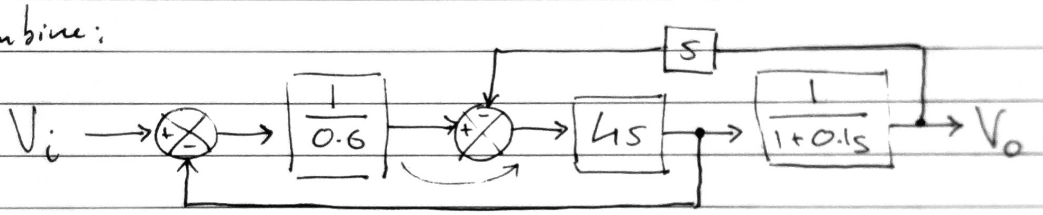
$I_m = I_i - I_o$

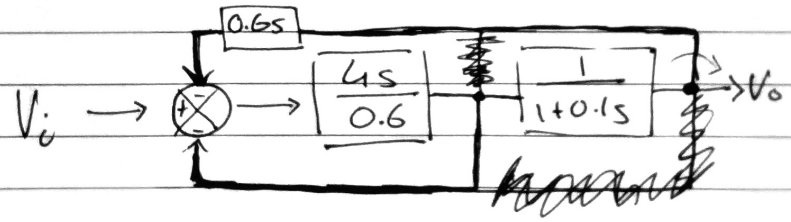
$V_m = 4s I_m$, $V_m - V_o = 0.1 I_o$

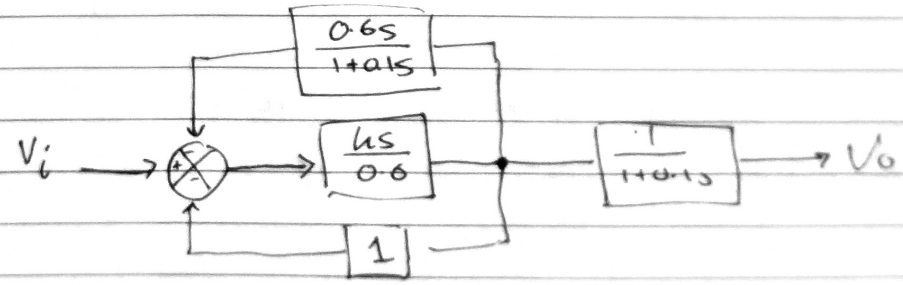
$V_o = \frac{1}{s} I_o$

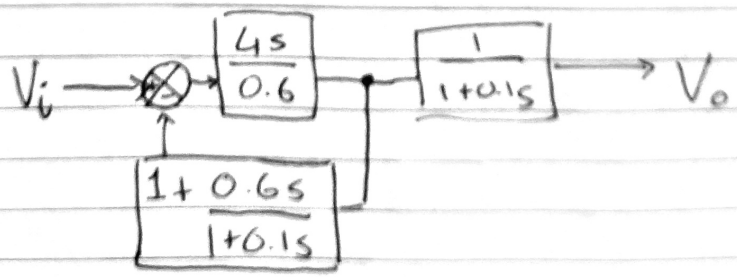



Combine:



$$\frac{\text{forward}}{1-\text{loop}} \Rightarrow \frac{\frac{4s}{0.6}}{1+\left(1+\frac{0.6s}{1+0.1s}\cdot\frac{4s}{0.6}\right)}$$

$$= \frac{4s}{0.6\left(1+\frac{4s}{0.6}+\frac{4s^2}{1.01s}\right)} = \frac{4s}{0.6\left(\frac{2.8s^2+4.06s+0.6}{0.6(1+0.1s}\right)}$$

$$\Rightarrow V_i \rightarrow \boxed{\frac{4s(1+0.1s)}{2.8s^2+4.06s+0.6}} \rightarrow \boxed{\frac{1}{1+0.1s}} \rightarrow V_o$$

$$\therefore \frac{V_o}{V_i} = \frac{4s}{2.8s^2+4.06s+0.6}$$

$$V_o(t) = \mathcal{L}^{-1}\left(\frac{1}{s}\cdot\frac{4s}{2.8s^2+4.06s+0.6}\right)$$
(step input)

$$\Rightarrow \frac{4}{2.8}\cdot\left[\frac{1}{s^2+1.45s+0.2143}\right]$$

via quadratic eq. $\Rightarrow$ $\frac{1}{(s+0.167)(s+1.283)} = \frac{A}{s+0.167} + \frac{B}{s+1.283}$

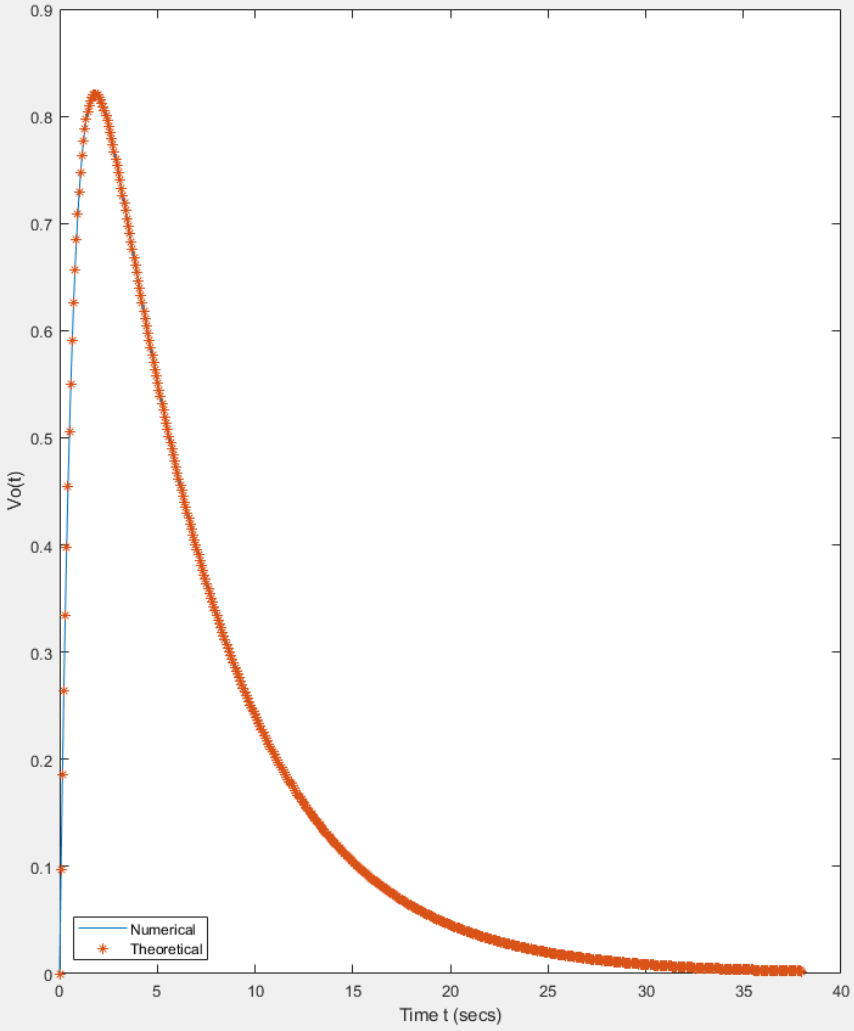
$$1 = A(s+1.283) + B(s+0.167)$$

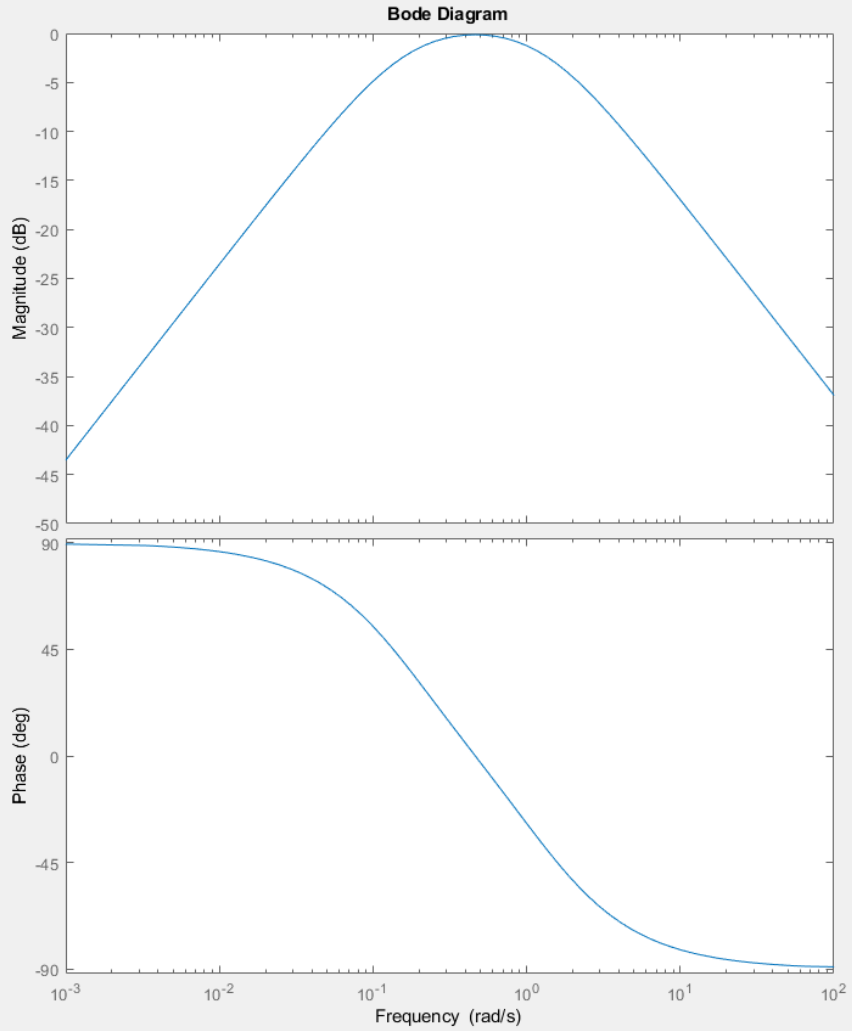
$$A = 0.896 \qquad B = -0.896$$

$$\therefore \frac{4}{2.8}\mathcal{L}^{-1}\left\{\frac{0.896}{s+0.167} - \frac{0.896}{s+1.283}\right.$$

$$= \frac{4}{2.8}\left[0.896e^{-0.167t} - 0.896e^{-1.283t}\right]$$

$$V_o(t) = 1.28\left(e^{-0.167t} - e^{-1.283t}\right)$$


Vo(t)
Time t (secs)
Numerical
Theoretical


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

# Task 2

Model: $T_1 \longrightarrow T$

input — output

Assumptions: $T_0 = 25°C$

$m = 10\,kg,\ 10{,}000\,g$

$A \triangleq$ cylinder $= 2\pi rh + 2\pi r^2$

$\simeq 2\pi(0.15)(0.2) + 2\pi(0.15)^2$

$= 0.329\,m^2$

$A \simeq \frac{1}{3}\,m^2$

Given: $C = 0.45\ J/g \cdot K \qquad h = 100\ W/m^2 K$

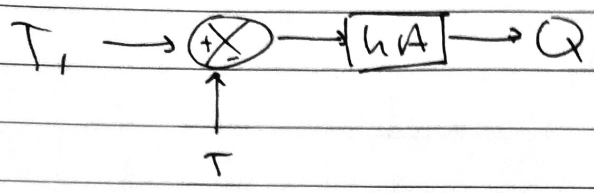
$q' = hA(T_1 - T)$

$q' \propto \Delta T$

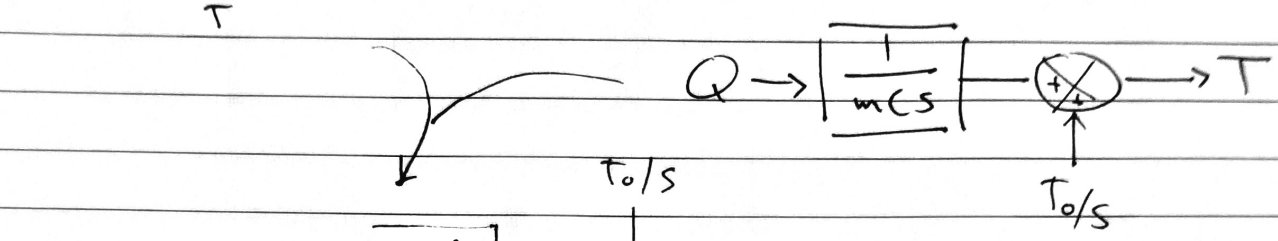
$q' = C_{1kg} \Delta T$

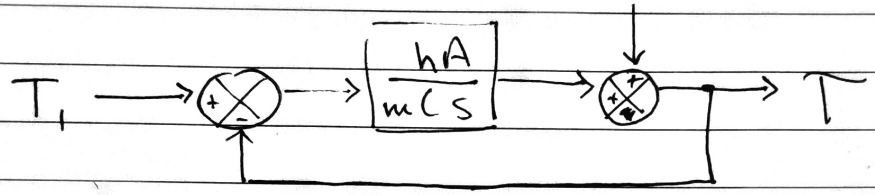
$\therefore q' = mC\Delta T \Rightarrow q' = mC\frac{d}{dt}T$

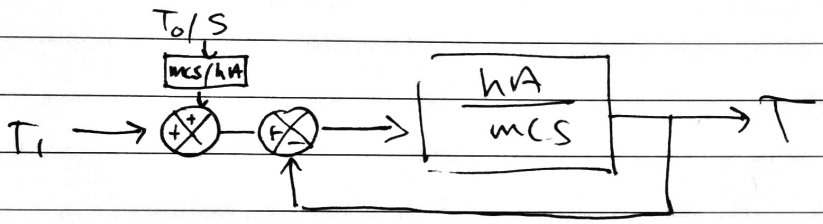
$Q = hA(T_1 - T)$ $\quad Qs = mC[sT - T_0]$

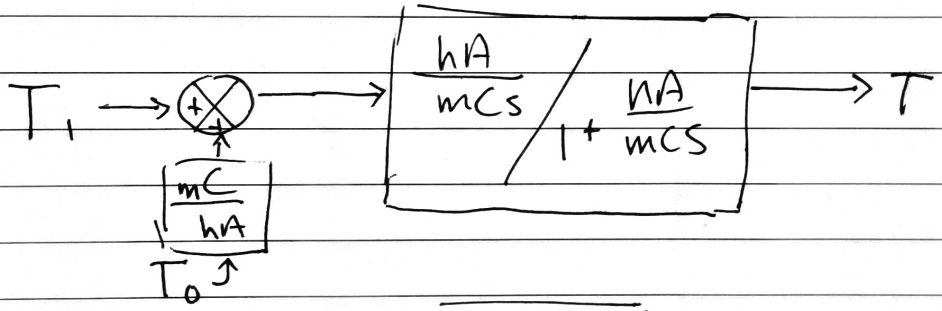


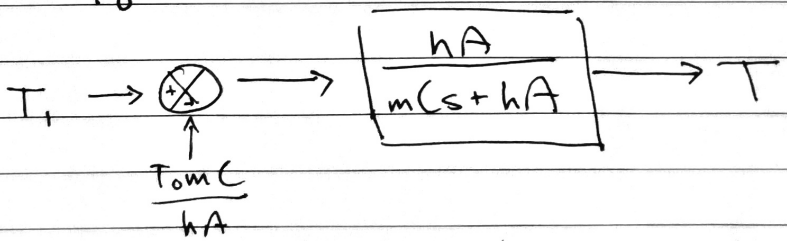

$\therefore T = \frac{Q}{mCs} + \frac{T_0}{s}$



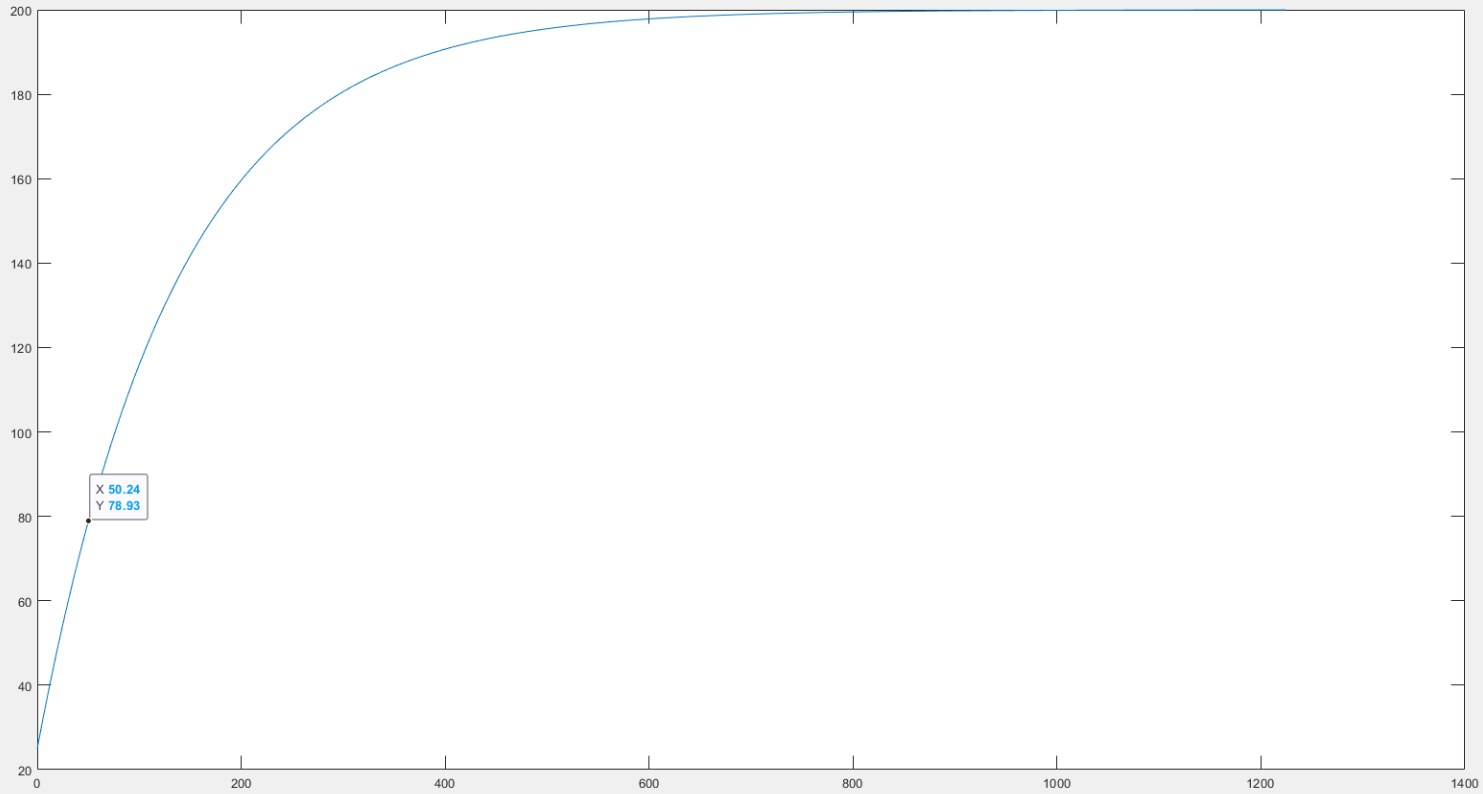

time response of initial Conditions

$\mathcal{L}^{-1}\left\{\frac{T_0 mC}{mCs + hA}\right\} \Rightarrow T_0 e^{-\frac{hA}{mC}t}$

See plots - e Code

Time estimate ~~60 - 70 seconds to reach 80°C~~

~ 51 secs




```
clear;

s = tf('s');

T0 = 25;
T1 = 200;
m=10000;
A=0.33;

C=0.45;
h=100;

sys = (h*A)/(m*C*s + h*A);

opt = stepDataOptions('StepAmplitude', T1);

figure(2);
[x,t] = step(sys,opt);

init = T0*exp((-h*A*t)/(m*C));

plot(t,x+init)
```