B+/A-

ECEN315 LABORATORY REPORT ONE

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ABSTRACTS ARE USEFUL.

1 Introduction

This project has the goal of creating a system for controlling a pendulum arm driven by a propeller. This necessitates the modelling of system, breaking it into sub-blocks/sub-systems and evaluating these models. The work outlined in this report covers the modelling of the Motor/Propeller and the Pendulum subsystems, the measured values for the physical system constants, deriving those that couldn't be directly measured, and how these systems combine in relation to the full open loop model of the system. *WORTH NOTING THE ULTIMATE GOAL.*
ALSO CONSIDER LTI ASSUMPTIONS.

2 Background

Control

Control theory deals with the control of dynamic systems with the objective to develop a model for controlling such systems by driving this system to result in an optimum output without delay or overshoot and stability. *OTHER CRITERIA APPLY*

ODD FORMAT

Delay refers to the speed of the systems response to an input.
Overshoot is the degree of which the response becomes greater than the desired value.
Stability is defined as the final value of the system being bounded and finite.

Extensive use of the block diagram is use to model these systems. In it is the transfer function, this is the model of relation between the input and output based on the differential equations describing the system, this is usually represented in the s domain. *s~*

DC Motors

A DC motor consists of both an electrical and a mechanical subsystem. It converts electrical energy into mechanical energy in its motor application. It can be broken down into sub-blocks representing the the *Electrical Subsystem* that then drives the *Mechanical Subsystem* via *Torque Coupling* to produce rotational output (of the shaft). This also produces a back emf, in the form of negative feedback into the system. ✓

Pendulum

The pendulum modelled, is what is known as a Compound Pendulum [1]. Meaning its motion is subject not only to the effects of gravity around its centre of mass (as the arm has mass), but also its inertia in relation to its angular acceleration and the damping (via pivot friction) to its angular velocity. *EXCELLENT DESCRIPTION.*

3 Methodology and Results

In this section you will explain how to solve the problem, that is, how you performed the project. At this early stage you need to be both clear about what you did and why you did it. You will also explain how you evaluated your solution once you have built it. The method of evaluation will be specific to the tasks.

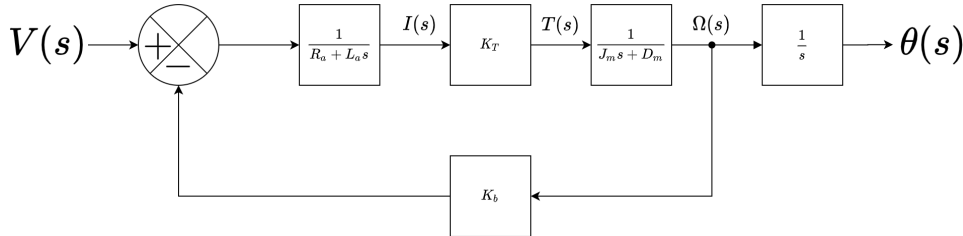## 3.1 DC Motor and Propeller

Figure 1: Block Diagram of DC Motor+Propeller System

As outlined in Gideon Gouws DC motor documentation [2], the DC+propeller's rotational velocity ($\Omega(s)$) system can be modelled by the above block diagram in figure 1. This was then combined and reduced to form a transfer function (representing the sub-system) of inputted voltage to outputted angular velocity.

$$
\begin{aligned}
\frac{\Omega(s)}{V(s)} &= \frac{\frac{K_t}{(R_a+L_a s)(D_m+J_m s)}}{1 + \frac{K_t K_b}{(R_a+L_a s)(D_m+J_m s)}} \\
&= \frac{K_t}{(R_a + L_a s)(D_m + J_m s) + K_t K_b} \\
&= \frac{K_t}{L_a J_m s^2 + (R_a J_m + L_a D_m)s + R_a D_m + K_t K_b} \\
&= \frac{\frac{K_t}{L_a J_m}}{s^2 + \frac{R_a J_m + L_a D_m}{L_a J_a} s + \frac{R_a D_m + K_t K_b}{L_a J_m}}
\end{aligned}
$$

The values of the these physical constants, were measured (multiple times) from the physical system in a previous year then averaged to get the final, utilised values. Using the constants and the derived transfer function, the sub-system was modelled in MATLAB . A multi-step input analysis at V = [1, 2, 3, 4, 5, 6], the results of which are shown in figure 2. This shows the settling time of the system as well as the respective steady state values for each input. The script also computed the steady state gain, $SSG = 80.631551 \frac{rad}{Vs}$ and the Time constant, $\tau = \sqrt{\tau_1 \tau_2} = \sqrt{\frac{1}{\omega_1 \zeta_1} \frac{1}{\omega_2 \zeta_2}} = 0.196093$
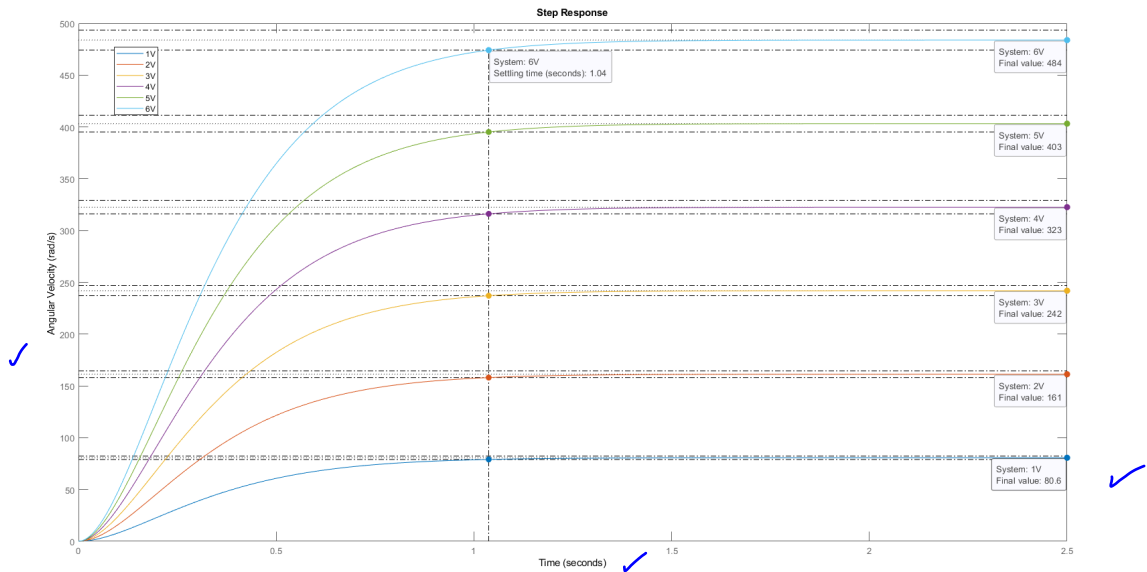
Figure 2: Step response *of WHAT?*

3.2 Pendulum

The pendulum sub-block represents the conversion of input torque (via the thrust provided from the propeller) For the driven pendulum, the torque applied to the pendulum must balance with the existing torques, ie from gravity, damping and the arms moment of inertia. Before the DE is derived, I needed to approximate the torque applied via gravity:

AVOID FIRST PERSON

$$\tau_{gravity} = mgd{\cdot}sin(\theta) = mgd{\cdot}\theta$$

This approximation works for small angles, but quickly diverges in accuracy for $\theta \geq \pi/8 \rightarrow \pi/4$

LABEL

LABEL
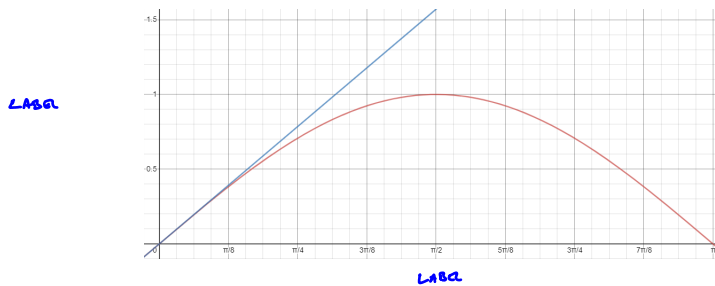
3.2.1 Differential Equation and Transfer Function

The input torque is the thrust force generated by the propeller, F, along the length of the arm, r. This balanced by the 3 components of torque on the arm. Its moment of inertia (J_p) by the rotational acceleration, the linear damping c by the rotational velocity and the force of gravity at the centre of mass (mgd) by the rotation. This forms the below differential equation and following transfer function.

$$\tau(t) = J_p\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + mgd\theta$$

$$T(s) = J_ps^2\Theta(s) + cs\Theta(s) + mgd\Theta(s)$$

$$\frac{\Theta(s)}{T(s)} = \frac{1}{J_ps^2 + cs + mgd}$$

$$= \frac{\frac{1}{J_p}}{s^2 + \frac{c}{J_p}s + \frac{mgd}{J_p}}$$

3

3.2.2 Fitting Undriven Pendulum Curve

All constants but c and J_P could be easily measured. So data measured from the disturbance of a the undriven pendulum and fit the decay exponential, Ae^{-Bt} to get A and B.
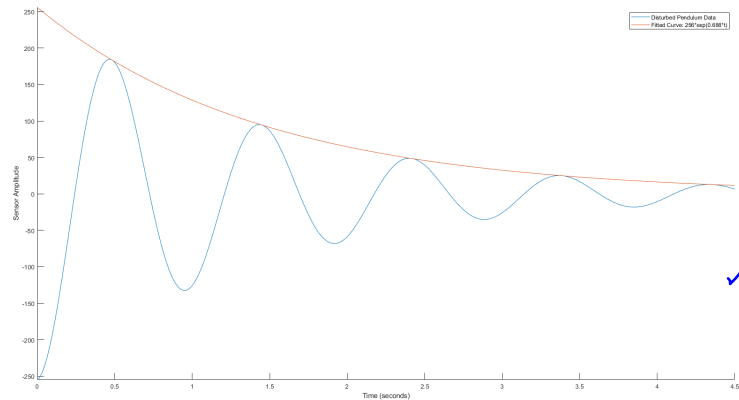

Figure 3: CAPTION

Figure 3 show the curve envelope fit to the oscillation peaks, Setting A to max sensor output then tuning B. This gave values of A = 256, and B = 0.688. By inspection, the period T of the signal can be read. Using it to get $\omega = \frac{2\pi}{1.43 - 0.47}$ UNITS.

3.2.3 Calculating Damping and Inertia

Given that $\omega = \sqrt{\frac{mgd}{J_p} - B^2}$, the moment of inertia and damping coefficient can be calculated:

$$J_p = \frac{mgd}{\omega^2 + B^2 = 0.0053} \qquad c = 2J_pB = 0.0073$$

So now with all the constants know, the step response of this system is plotted below with the peak response and settling time marked. Though this is for a step input of 0.1, to better represent the actual values that it would be driven at.
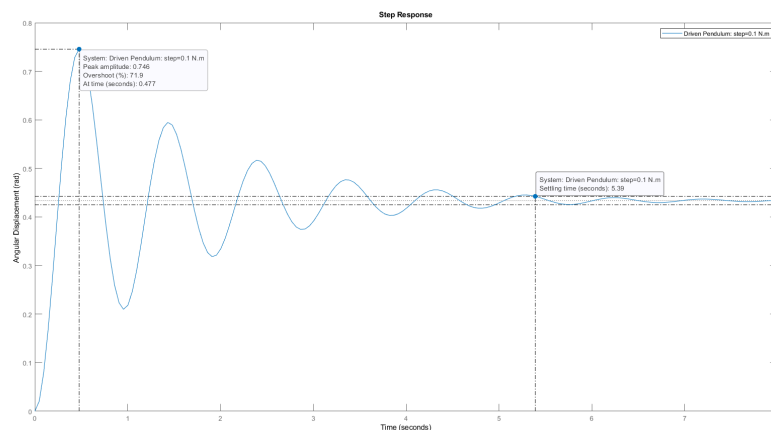

Figure 4: CAPTION.
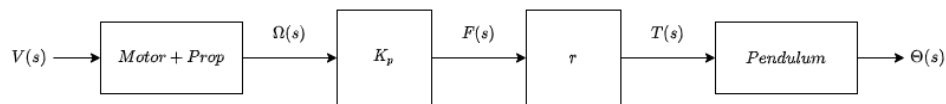
4

Full Open Loop TF

Figure 5: Block Diagram of DC Motor+Propeller System

As shown in figure 5, the combined system needs to first transform rotational velocity into thrust (force), the block to do so we found via a linear approximation of the measured response $\rightarrow K_p$, which is applied over the distance r to provide an input torque to the pendulum. Using the previously derived transfer functions the full system block is shown below.

$$\frac{\Theta(s)}{V(s)} = \frac{\frac{K_t}{L_a J_m}}{s^2 + \frac{R_a J_m + L_a D_m}{L_a J_m} s + \frac{R_a D_m + K_t K_b}{L_a J_m}} \cdot K_p \cdot r \cdot \frac{\frac{1}{J_p}}{s^2 + \frac{c}{J_p} s + \frac{mgd}{J_p}}$$

$$= \frac{\frac{K_t K_p r}{J_p L_a J_m}}{(s^2 + \frac{c}{J_p} s + \frac{mgd}{J_p})(s^2 + \frac{R_a J_m + L_a D_m}{L_a J_m} s + \frac{R_a D_m + K_t K_b}{L_a J_m})}$$
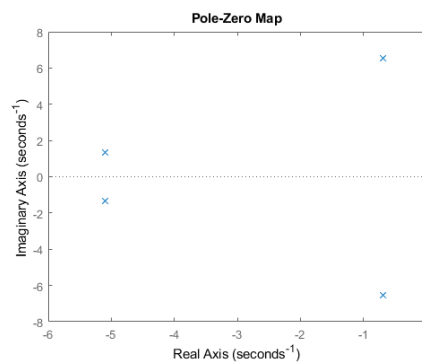

Figure 6:

From the poles shown in figure 6, the response can be approximately described as being quite oscillatory due to complex poles and slow acting due to pole not being very negative. And the simulated results are shown in figure 7, With the peak response, settling time and per input steady state values marked.
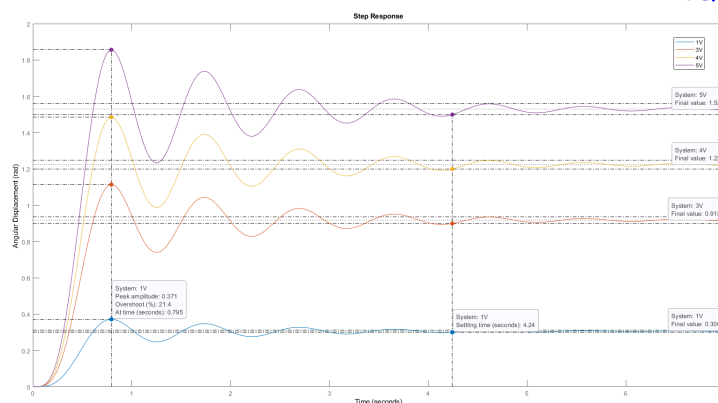

Figure 7:

5

4 Discussion

The sum results of this report is a model of the open loop system relating the input voltage to a DC motor to the resultant angular displacement of the armature. This will be vital when adding a control system to drive the DC motor. Though it should noted that the model is flawed in some specific places.

Firstly, as previously mentioned the torque applied via gravity on the pendulum was approximated to small angles. This usable from $\theta \leq 45°$ but will severely alter the actual response for greater angles.

The relation between the angular velocity of the propeller and its produced thrust was also simpler that it should be physically. And thus the real world system will diverge from this model for greater speeds of the propeller. *By how much?*

Also due to the rough derivation of J_p and c of the system, this model could have errors.

These limitation therefore must be taken into account, possibly in the restriction of operating parameter or necessitating great measurement precision.

5 Conclusions

In conclusion this is workable open loop representation of the system, suitable for use in designing a control system around, but has limitations and approximations that may introduce divergences from reality.

References

[1] R. Fitzpatrick, "The compound pendulum." [Online]. Available: http://farside.ph.utexas.edu/teaching/301/lectures/node141.html

[2] G. Gouws, "Understanding small, permanent magnet brushed dc motors." [Online]. Available: https://ecs.wgtn.ac.nz/foswiki/pub/Courses/ECEN315_2020T1/Assignments/DCmotor_transfer_function.pdf

6 Full MatLab Code

```matlab
%% lab 2 Motor + Prop Sub-System
clear;
clc;

Ra = 6.3;
La = 0.797;
Kb = 0.0043;
Kt = Kb;
Dm = 0.00000553;
Jm = 0.00000241;

numerator = Kt/(Jm*La);
denom_a = 1;
denom_b = (Jm*Ra + Dm*La)/(Jm*La);
denom_c = (Ra*Dm + Kb*Kt) / (Jm*La);

motor_prop = tf(numerator, [denom_a denom_b denom_c])

figure;
hold on
for V = [1 2 3 4 5 6]
step(V*motor_prop)
ylabel("Angular Velocity (rad/s)")
ssg = (numerator/denom_c);

[Wn, Z] = damp(motor_prop); %To get T1 and T2
T1 = 1/(Wn(1)*Z(1));
T2 = 1/(Wn(2)*Z(2));
T = sqrt(T1*T2);

fprintf("V = %d:\nT=%f SSG=%f\n\n",V,T,ssg);

end

%% import Data
opts = delimitedTextImportOptions("NumVariables", 2);

% Specify range and delimiter
opts.DataLines = [1, Inf];
opts.Delimiter = "\t";

% Specify column names and types
opts.VariableNames = ["t", "x"];
opts.VariableTypes = ["double", "double"];

% Specify file level properties
opts.ExtraColumnsRule = "ignore";
opts.EmptyLineRule = "read";

% Import the data
Data = readtable("Data.csv", opts);
CSV_t = Data.t;
CSV_x = Data.x;
clear opts

%% Fit decay curve
figure;
hold on
plot(CSV_t,CSV_x)
A = 256; %max sensor value
B = 0.688;
y = A*exp(-B*CSV_t);
```

```matlab
plot(CSV_t,y)
ylim([-255 256])
ylabel("Sensor Amplitude")
xlabel("Time (seconds)")
hold off


%% Pendulum Sub-System
m = 0.168;
g = 9.8;
d = 0.14;

w = (2*pi)/(1.43-0.47);
Jp = (m*g*d)/(w^2 + B^2);
c = 2*Jp*B;

numerator = 1/Jp;
denominator = [1 c/Jp (m*g*d)/Jp];

pendulum = tf(numerator,denominator)
figure;
step(0.1*pendulum)
ylabel("Angular Displacement (rad)")
%% Combined Open loop system
Kp = 0.0053;
r = 0.165;

combined_sys = motor_prop .* Kp .* r .* pendulum
figure;
pzmap(combined_sys)
figure;
hold on
for V = [1 3 4 5]
step(V*combined_sys)
end
ylabel("Angular Displacement (rad)")
hold off;
```

7 Lab Safety

Hazard	Cause	Probability	Severity	Mitigation
Lacerations from propeller	Touching propeller while in operation	Low	High	Keep human appendages a safe distance away from objects moving at a high speed.
Burning skin	Motor overheats because it is running for too long and someone's skin touches it. Soldering Iron	Moderate	Low	Do not run motors and equipment longer than necessary, and/or for longer than they are capable of running for. If it has been running for too long, turn it off and do not touch until it has cooled down.
Respiratory Problems	Inhaling soldering smoke	High	Moderate	Use fan
Eye-injury/Shrapnel	Plastic shards/other pieces thrown by spinning motor	Moderate	High	Wear eye protection, safety goggles, whenever in range of operating motor
Cutting skin	Standing on bare feet/bare skin coming into contact with sharp objects.	Low	Moderate	Wearing closed shoes at all times in lab
Broken bones and head injury	Tripping on equipment and falling over in a damaging way.	Low/moderate	High	Be aware of surroundings and avoid objects that could cause a person to become physically unstable.
Electric shock	Touching/Messing with the high current mains connection and Incorrect wiring of circuits and malfunctioning electrical devices.	Moderate	High	Double check wiring, keep higher power circuitry isolated, be informed/engaged with what is going on around you
Explosion injuries	Equipment overheats/malfunctions and explodes near humans.	Low	High	If a major malfunction of equipment is noticed, call campus health/111 and evacuate the building immediately.
Dehydration/General Nausea/Fatigue	Not leaving lab, not taking breaks, and/or not having a water bottle	High	Low	Consume adequate amounts of water and food, take breaks and leave the lab environment when needed.
Back Injury	Lifting heavy equipment	Moderate	High	When lifting heavy equipment, do this with sensible lifting processes (e.g. bend knees, 2nd person etc). Or roll it if it has wheels.