prove/derive $\quad y(t) = x(t) * h(t) \overset{\mathcal{F}}{\rightleftharpoons} Y(\omega) = X(\omega) H(\omega)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$: convolution

$$Y(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau \right] e^{-j\omega t}\, dt$$: fourier transform

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau) e^{-j\omega t}\, dt \right] d\tau$$

rearrange due to indentical intergration limits and bring out $x(\tau)$ as result.

$$\left[\underbrace{p = t - \tau}_{\text{substitution}}, \quad dt = dp, \quad t = p + \tau \right]$$

$$Y(\omega) = \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(p) e^{-j\omega(p+\tau)}\, dp \right] d\tau$$: note $e^{-j\omega(p+\tau)} = e^{-j\omega p} e^{-j\omega\tau}$ and bring out

$$Y(\omega) = \underbrace{\int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau}\, d\tau}_{\mathcal{F}\{x\}} \times \underbrace{\int_{-\infty}^{\infty} h(p) e^{-j\omega p}\, dp}_{\mathcal{F}\{h\}}$$ by definition

$$\therefore\ Y(\omega) = X(\omega) H(\omega)$$