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1. a) Zero Slope @ -2

$y > -2 \Rightarrow N^-$

$y < -2 \Rightarrow P^+$

possibly: (b) $y' = 2 + y$

$(y' = 0 = 2 + {}^-2)$

(g) $y' = {}^-2 - y$

$(y' = 0 = {}^-2 - {}^-2)$

	b	g
$y > {}^-2$	P^+	N^-
$y < {}^-2$	N^-	P^+

$\therefore$ $y' = {}^-2 - y$ (g) matching slope field 1a

b) Zero Slope @ 0.5

$y > 0.5 \Rightarrow N^-$

$y < 0.5 \Rightarrow P^+$

possibly: (a) $y' = 2y - 1$

$\emptyset = 2(0.5) - 1$

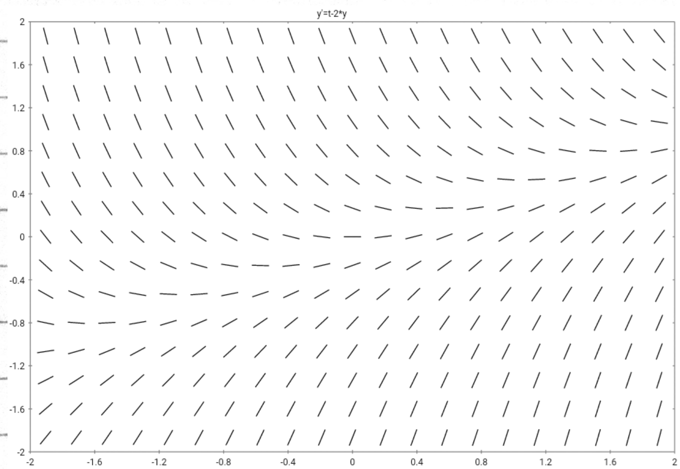
(i) $y' = 1 - 2y$

$\emptyset = 1 - 2(0.5)$

	a	i
$y > 0.5$	P^+	N^-
$y < 0.5$	N^-	P^+

$\therefore$ $y' = 1 - 2y$ (a) matchs fig 1b.

2. DE: $y' + 2y = t$ or $\frac{dy}{dt} + 2y = t$



b) looks to tend towards $y = t/2$ (as $t \to \infty$)

c) $\frac{dy}{dt} + 2y = t \qquad \mu = e^{\int 2\,dt} = e^{2t}$

$$\Rightarrow \frac{d}{dt}\left(e^{2t} y\right) = e^{2t} t \rightarrow e^{2t} y = \int e^{2t} t \, dt \qquad \left[\int uv' = uv - \int u'v \ (u = t\ u' = 1,\ v' = e^{2t},\ v = \tfrac{1}{2}e^{2t})\right]$$

$$= \frac{t}{2} e^{2t} - \int \frac{1}{2} e^{2t} \, dt$$

$$e^{2t} y = \frac{t}{2} e^{2t} - \frac{1}{4} e^{2t} + C$$

$$y = \frac{t}{2} - \frac{1}{4} + C/e^{2t}$$

d) as $t \to \infty$: $\frac{C}{e^{2t}} \to \emptyset$, $-\frac{1}{4}$ becomes less relevant.

$\therefore$ $y = \frac{t}{2}$

3. $y' = 3y - 3$, $y(0) = 3$

a) $y' = 3(y-1)$

$y'/_{y-1} = 3$ or $\frac{1}{y-1}\,dy = 3$

so $\int \frac{1}{y-1}\,dy = \ln|y-1)$

$d/dt(\ln|y-1|) = 3$

b) $\int \frac{d}{dt}(\ln|y-1|) = \int 3$

$= \ln|y-1| = 3t + c$

$\exp(\ln|y-1|) = e^{3t} \cdot e^{c}$ $(e^{c} = C)$

$= |y-1| = Ce^{3t}$

as C can control the sign we can lose the $|x|$

$y - 1 = Ce^{3t}$

$y = 1 + Ce^{3t}$

c) C is reliant on initial conditions (in order to accurately describe the system) and as $y(t)$ is a continuous function C can be anywhere in $\mathbb{R}$.

d) $\frac{d}{dt}(1 + Ce^{3t})$

$= 3Ce^{3t}$

✓

sub: $y' = 3y - 3$

$= 3(1 + Ce^{3t}) - 3$

$y' = \cancel{3} 3Ce^{3t} - \cancel{3}$

e) $y(0) = 3$

$y = 1 + Ce^{3t} = 3$

$= 1 + Ce^{3(0)} = 3$

$= 1 + Ce^{0}$

$= 1 + C = 3$

$C = 2$ $\longrightarrow$ for $y(0) = 3$, $y = 1 + 2e^{3t}$