ENGR 222 **Practice Questions 1** **Topic: Curves**

1 Consider the function $y = 2x^{3/2}$.

(a) Write down the parametric equations describing a curve whose graph is that of $y = 2x^{3/2}$.

(b) Using the parametric equations, determine the tangent line at $x = 1$.

(c) What is the line normal to the graph at $x = 1$.

(d) Determine the arc length s as a function of t using the origin as the starting/reference point on the curve.

(e) Determine the arc length parametrisation of the curve.

2 The curved traced out by a point on the edge of a rolling wheel is known as a cycloid. For a wheel with radius a the curve is described parametrically as

$$(x, y) = (a(t - \sin(t)), a(1 - \cos(t))$$

for the following questions suppose $a = 2$.

(a) Determine the location at $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$ and use this to draw a rough sketch of the curve.

(b) Determine the tangent line at $t = \pi/2$

(c) Determine the normal line at $t = \pi/2$

(d) Calculate the arc length over the interval $0 \leq t \leq 2\pi$

(e) *Challenge:* Determine the arc length parametrisation for $0 \leq t \leq 2\pi$, with $t = 0$ as the starting/reference time.

3 Consider a spiral curve described in polar coordinates by $r = e^{\frac{\theta}{2\pi}}$.

(a) Draw a rough sketch of the curve over $\theta = [0, 2\pi]$

(b) Write out a parametric description of the curve in Cartesian coordinates

(c) Determine the arc length parametrisation of the curve (with $\theta = 0$ as the reference point)

(d) What is the total arc length over $[0, 2\pi]$

4 Consider the curve traced out by the vector valued function

$$\mathbf{r}(t) = 6t\mathbf{i} + 3t^2\mathbf{j} + t^3\mathbf{k}$$

(a) Determine the unit tangent vector (for arbitrary t).

(b) Determine the arc length of the curve over $1 \leq t \leq 4$.

(c) Determine the principle unit normal vector (for arbitrary t).

(d) Determine the binormal vector (for arbitrary t).

(e) Determine the maximum curvature along the curve.

5 Consider the curve described by the vector valued function

$$\mathbf{r}(t) = 6e^{-t}\mathbf{i} - 2e^{-t}\mathbf{j} + 9e^{-t}\mathbf{k}\,.$$

(a) Determine the unit tangent vector (for arbitrary t).

(b) Determine an equation for the arc length s in terms of t using $t = 0$ as the start/reference.

(c) What can you say about the length of the curve over $t \geq 0$?

(d) Determine an equation for t in terms of s.

(e) Substitute the result of the previous question to obtain the arc length parametrisation of the curve.

6 The parametric equations

$$(x, y) = (A\cos(at), B\sin(bt))$$

for positive integers a, b and $A, B \neq 0$ describe a family of Lissajous curves.

Consider a car is driving around a circuit resembling a Lissajous curve with $a = 3$, $b = 2$ and $A = B = 20$ (with units of x, y in metres and t in minutes).

Note: some of the results to the following questions are somewhat complicated and can't be simplified a great deal. Don't spend too much time manipulating the results, just work through making sure you understand the concepts/processes involved.

(a) Determine the velocity vector of the car (for arbitrary t).

(b) Determine the maximum velocity?

(c) Determine the acceleration vector (for arbitrary t).

(d) Decompose the magnitude of acceleration in the tangential and normal directions.

(e) Determine the curvature of the car (for arbitrary t).