VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

CLASS TEST 2 – 2020

TRIMESTER 1

ECEN 321

ENGINEERING STATISTICS

Time Allowed: THIRTY TWO HOURS

OPEN BOOK

Permitted materials: All.

Instructions: Attempt ALL Questions.
NOT all questions have the same marks value.
There are 50 marks in total.

1. Distributions **(6 marks)**

Derive an expression for the mean and variance of a continuous random variable X which is uniformly distributed on the interval $[a, b]$.

(Note: There are *no* marks for the answers, only for the working).

(a) Mean:

(2 marks)

(b) Variance:

(4 marks)

2. Uncertainties **(5 marks)**

We wish to measure a change between two independent measurements, expressed as a proportion of the initial value i.e., $y = (x_2 - x_1)/x_1$.

(a) If $x_1 = 1.11 \pm 0.01$ and $x_2 = 0.99 \pm 0.01$, what is the value of y? **(1 mark)**

(b) What is the uncertainty in y? **(4 marks)**

3. Exponential Distribution **(7 marks)**

(a) Derive the formula for the mean of an exponential distribution having parameter λ.

(There are *no* marks for the answers, only for the working.) **(4 marks)**

(b) The distance between flaws on a long cable is exponentially distributed with mean 10 m. Find the probability that the distance between two flaws is greater than 15 m. **(3 marks)**

4. Estimation **(8 marks)**

(a) List two desirable properties of estimators. **(2 marks)**

(b) Give mathematical definitions of the two quantities that govern whether or not an estimator has these two properties. **(4 marks)**

(c) Explain why these two properties are desirable. **(2 marks)**

5. Maximum Likelihood **(7 marks)**

Suppose you have flipped a biased coin N times, and observed k heads. Derive the maximum likelihood estimate of the probability p of the coin producing heads.

(Note: if you know that both heads and tails are possible, then there are better Bayesian ways of approaching this problem, but to keep this simple, just stick to the maximum likelihood estimate).

6. Confidence Intervals **(6 marks)**

A simple random sample of 15 small cars were subjected to a head-on collision test, and 11 of them were "written off" (i.e., the cost of repairs was greater than the value of the car). Another sample of 12 large cars was subjected to the same test, and 4 of them were written off. Find a 96% confidence interval for the difference in proportions of the small cars and large cars that were written off.

(Even though use of the central limit theorem in this question is tenuous to say the least – use it!)

7. Hypothesis Tests **(7 marks)**

An instrument is properly calibrated if the mean measurement error is zero. In a sample of 49 measurements the sample mean of the error is 20 μV, with a standard deviation of 36 μV. We wish to form a test of whether or not the instrument is properly calibrated.

(a) State carefully an appropriate null hypothesis and the corresponding alternate hypothesis.

(2 marks)

(b) Find the P value for the test. **(3 marks)**

(c) Is it plausible that the instrument is calibrated, or are you convinced that it is out of calibration? Explain your reasoning. **(2 marks)**

8. Traffic **(4 marks)**

A car insurance company is setting up a 0800 number and call centre for new customers. They estimate that during the busy hour the (Poisson) call arrival rate is 3 calls per minute. The mean call duration (assumed exponentially distributed) is 7 minutes. The company assumes that customers who encounter congestion on calling will immediately call their competitor, and don't want this to happen for any more than 5% of callers.

(a) What is the busy hour traffic load? (provide the value and the units)

(2 marks)

(b) How many operators does the call centre require to meet the availability target? **(2 marks)**

Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
4.0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Student t distribution

	$\Pr[T \leq t]$				
r	0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

Chi-square distribution

	$\Pr[X \leq x]$					
r	0.01	0.025	0.05	0.95	0.975	0.99
1	0.000	0.001	0.004	3.841	5.024	6.635
2	0.020	0.051	0.103	5.991	7.378	9.210
3	0.115	0.216	0.352	7.815	9.348	11.345
4	0.297	0.484	0.711	9.488	11.143	13.277
5	0.554	0.831	1.145	11.070	12.833	15.086
6	0.872	1.237	1.635	12.592	14.449	16.812
7	1.239	1.690	2.167	14.067	16.013	18.475
8	1.646	2.180	2.733	15.507	17.535	20.090
9	2.088	2.700	3.325	16.919	19.023	21.666
10	2.558	3.247	3.940	18.307	20.483	23.209
11	3.053	3.816	4.575	19.675	21.920	24.725
12	3.571	4.404	5.226	21.026	23.337	26.217
13	4.107	5.009	5.892	22.362	24.736	27.688
14	4.660	5.629	6.571	23.685	26.119	29.141
15	5.229	6.262	7.261	24.996	27.488	30.578
16	5.812	6.908	7.962	26.296	28.845	32.000
17	6.408	7.564	8.672	27.587	30.191	33.409
18	7.015	8.231	9.390	28.869	31.526	34.805
19	7.633	8.907	10.117	30.144	32.852	36.191
20	8.260	9.591	10.851	31.410	34.170	37.566
21	8.897	10.283	11.591	32.671	35.479	38.932
22	9.542	10.982	12.338	33.924	36.781	40.289
23	10.196	11.689	13.091	35.172	38.076	41.638
24	10.856	12.401	13.848	36.415	39.364	42.980
25	11.524	13.120	14.611	37.652	40.646	44.314
26	12.198	13.844	15.379	38.885	41.923	45.642
27	12.879	14.573	16.151	40.113	43.195	46.963
28	13.565	15.308	16.928	41.337	44.461	48.278
29	14.256	16.047	17.708	42.557	45.722	49.588
30	14.953	16.791	18.493	43.773	46.979	50.892

Erlang B Traffic Table

	Pr[B]											
N	0.0001	0.0005	0.001	0.005	0.01	0.02	0.05	0.1	0.15	0.2	0.30	0.40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109	6.514	8.191
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230	7.856	9.800
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369	9.213	11.42
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86	13.33	16.31
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04	14.72	17.95
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22	16.11	19.60
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41	17.50	21.24
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61	18.90	22.89
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85	27.33	32.81
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06	28.74	34.46
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28	30.15	36.12
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50	31.56	37.78
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72	32.97	39.44
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94	34.39	41.10
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16	35.80	42.76
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39	37.21	44.41
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61	38.63	46.07
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84	40.05	47.74
40	21.37	23.41	24.44	27.38	29.01	31.00	34.60	38.79	42.48	46.15	54.24	64.35
50	28.87	31.29	32.51	35.98	37.90	40.26	44.53	49.56	54.03	58.51	68.46	80.99
60	36.62	39.40	40.80	44.76	46.95	49.64	54.57	60.40	65.63	70.90	82.70	97.63
70	44.58	47.68	49.24	53.66	56.11	59.13	64.67	71.29	77.26	83.32	96.95	114.3
80	52.69	56.10	57.81	62.67	65.36	68.69	74.82	82.20	88.91	95.75	111.2	130.9
90	60.92	64.63	66.48	71.76	74.68	78.31	85.01	93.15	100.6	108.2	125.5	147.6
100	69.27	73.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

Erlang C Traffic Table

	Pr[W]											
N	0.0001	0.0005	0.001	0.005	0.01	0.02	0.05	0.1	0.15	0.2	0.30	0.40
1	.0001	.0005	.0010	.0050	.0100	.0200	.0500	.1000	.1500	.2000	.3000	.4000
2	.0142	.0319	.0452	.1025	.1465	.2103	.3422	.5000	.6278	.7403	.9390	1.117
3	.0860	.1490	.1894	.3339	.4291	.5545	.7876	1.040	1.231	1.393	1.667	1.903
4	.2310	.3533	.4257	.6641	.8100	.9939	1.319	1.653	1.899	2.102	2.440	2.725
5	.4428	.6289	.7342	1.065	1.259	1.497	1.905	2.313	2.607	2.847	3.241	3.569
6	.7110	.9616	1.099	1.519	1.758	2.047	2.532	3.007	3.344	3.617	4.062	4.428
7	1.026	1.341	1.510	2.014	2.297	2.633	3.188	3.725	4.103	4.406	4.897	5.298
8	1.382	1.758	1.958	2.543	2.866	3.246	3.869	4.463	4.878	5.210	5.744	6.178
9	1.771	2.208	2.436	3.100	3.460	3.883	4.569	5.218	5.668	6.027	6.600	7.065
10	2.189	2.685	2.942	3.679	4.077	4.540	5.285	5.986	6.469	6.853	7.465	7.959
11	2.634	3.186	3.470	4.279	4.712	5.213	6.015	6.765	7.280	7.688	8.336	8.857
12	3.100	3.708	4.018	4.896	5.363	5.901	6.758	7.554	8.099	8.530	9.212	9.761
13	3.587	4.248	4.584	5.529	6.028	6.602	7.511	8.352	8.926	9.379	10.09	10.67
14	4.092	4.805	5.166	6.175	6.705	7.313	8.273	9.158	9.760	10.23	10.98	11.58
15	4.614	5.377	5.762	6.833	7.394	8.035	9.044	9.970	10.60	11.09	11.87	12.49
16	5.150	5.962	6.371	7.502	8.093	8.766	9.822	10.79	11.44	11.96	12.77	13.41
17	5.699	6.560	6.991	8.182	8.801	9.505	10.61	11.61	12.29	12.83	13.66	14.33
18	6.261	7.169	7.622	8.871	9.518	10.25	11.40	12.44	13.15	13.70	14.56	15.25
19	6.835	7.788	8.263	9.568	10.24	11.01	12.20	13.28	14.01	14.58	15.47	16.18
20	7.419	8.417	8.914	10.27	10.97	11.77	13.00	14.12	14.87	15.45	16.37	17.10
21	8.013	9.055	9.572	10.99	11.71	12.53	13.81	14.96	15.73	16.34	17.28	18.03
22	8.616	9.702	10.24	11.70	12.46	13.30	14.62	15.81	16.60	17.22	18.19	18.96
23	9.228	10.36	10.91	12.43	13.21	14.08	15.43	16.65	17.47	18.11	19.10	19.89
24	9.848	11.02	11.59	13.16	13.96	14.86	16.25	17.51	18.35	19.00	20.02	20.82
25	10.48	11.69	12.28	13.90	14.72	15.65	17.08	18.36	19.22	19.89	20.93	21.76
26	11.11	12.36	12.97	14.64	15.49	16.44	17.91	19.22	20.10	20.79	21.85	22.69
27	11.75	13.04	13.67	15.38	16.26	17.23	18.74	20.08	20.98	21.68	22.77	23.63
28	12.40	13.73	14.38	16.14	17.03	18.03	19.57	20.95	21.87	22.58	23.69	24.57
29	13.05	14.42	15.09	16.89	17.81	18.83	20.41	21.82	22.75	23.48	24.61	25.50
30	13.71	15.12	15.80	17.65	18.59	19.64	21.25	22.68	23.64	24.38	25.54	26.44
40	20.58	22.33	23.17	25.44	26.58	27.84	29.77	31.48	32.61	33.48	34.83	35.89
50	27.80	29.86	30.86	33.49	34.80	36.26	38.47	40.42	41.70	42.69	44.21	45.40
60	35.30	37.63	38.76	41.73	43.20	44.83	47.29	49.46	50.88	51.97	53.65	54.96
70	42.99	45.58	46.83	50.10	51.73	53.52	56.21	58.57	60.12	61.31	63.14	64.56
80	50.84	53.68	55.03	58.60	60.36	62.30	65.21	67.75	69.42	70.70	72.66	74.18
90	58.82	61.88	63.34	67.18	69.07	71.15	74.26	76.98	78.76	80.12	82.21	83.83
100	66.91	70.19	71.75	75.84	77.85	80.06	83.37	86.25	88.13	89.58	91.78	93.49

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