$p = 3x^2 \quad g = x^2$

$\mu = e^{\int 3x^2} = e^{x^3}$

$\frac{d}{dx}\left(e^{x^3} y\right) = e^{x^3} x^2$

$e^{x^3} y = \int e^{x^3} x^2 \quad : u = x^3 \therefore \frac{1}{3} du = x^2 dx$

$= \frac{1}{3} \int e^{u} du = \frac{1}{3} e^{u} = \frac{1}{3} e^{x^3} + C$

$e^{x^3} y = \frac{1}{3} e^{x^3} + C$

$y = \frac{1}{3} + C / e^{x^3}$

$I = \{-\infty, \infty\}$

---

b) $x \frac{dy}{dx} - y = x^2 \sin x \qquad p = \frac{-1}{x} \quad g = x \sin x$

$\hookrightarrow \frac{dy}{dx} - \frac{y}{x} = x \sin x$

$= y' - \frac{1}{x} y = x \sin x$

$\cancel{\mu = e^{\int x \sin x} = e^{-x \cos}}$

$\mu = e^{\int -\frac{1}{x}} = e^{-\ln|x|} = \left(e^{\ln|x|}\right)^{-1}$

$= x^{-1} = \frac{1}{x} = \mu.$

$\frac{d}{dx}\left(\frac{1}{x} y\right) = \frac{1}{x} \cdot x \cdot \sin x$

$\hookrightarrow \frac{1}{x} y = \int \sin x$

$y \frac{1}{x} = -\cos x + C$

$y = -x \cos x + xC$

$I = (-\infty ; \infty)$

---

c) $\cos x \cdot \frac{dy}{dx} + (\sin x) y = 1 \qquad p = \tan x \quad g = \sec x$

$\Rightarrow y' + \frac{\sin x}{\cos x} \cdot y = \frac{1}{\cos x}$

$\mu = e^{\int \tan x} = e^{-\ln|\cos x|} = \left(e^{\ln|\cos x|}\right)^{-1}$

$= y' + \tan(x) \cdot y = \sec(x)$

$= \frac{1}{\cos x} = \sec x$

$\frac{d}{dx}\left(\frac{1}{\cos x} \cdot y\right) = \frac{1}{\cos^2 x}, \quad \frac{1}{\cos x} \cdot y = \int \frac{1}{\cos^2 x}$

$\frac{1}{\cos x} \cdot y = \tan x$

$y = \cos x \cdot \tan x$

$I = [2\pi n, \frac{\pi}{2} + 2\pi n)$

d) $\frac{dr}{d\theta} + r\sec\theta = \cos\theta$ $\quad p = \sec\theta \quad g = \cos\theta$

$r' + \sec\theta\, r = \cos\theta$

$\mu = e^{\int \sec\theta}$

$\frac{d}{d\theta}\left((\tan\theta + \sec\theta) r\right) = (\tan x + \sec\theta)\cos\theta$ $\quad = \exp(\ln|\tan x + \sec x|)$

$\mu = \tan x + \sec x$

$(\tan\theta + \sec\theta) r = \int (\tan\theta + \sec\theta)\cos\theta$

$(\tan\theta + \sec\theta) r = -\cos x + x + c$, $r = \cancel{-\cos\theta + \theta + c} / \tan\theta + \sec\theta$

$r = -\cos\theta + \theta + c \,/\, \tan\theta + \sec\theta$

$I = [2\pi n, \frac{\pi}{2} + 2\pi n)$.

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2. a) $(1+t^4)\frac{dy}{dt} = \frac{t^3}{y}$, $-\frac{t^3}{y} + (1+t^4)\frac{dy}{dt}$

$= y \cdot \frac{dy}{dt} = \frac{t^3}{1+t^4}$,

$\int y\, dy = \int \frac{t^3}{1+t^4}\, dt \quad = \frac{y^2}{2} = \frac{1}{4}\ln|1+t^4| + c$

$y^2 - \frac{1}{2}\ln|1+t^4| = c$

---

b) $e^{-y}\sin t - y'\cos^2 t = 0$

$= \frac{y'}{e^{-y}} = \frac{\sin t}{\cos^2 t}$, $\cancel{\int \frac{y'}{e^{-y}}\, dy = \int \frac{\sin t}{\cos^2 t}\, dt}$

$= \frac{dy}{dt} \cdot \frac{1}{e^{-y}} = \frac{\sin t}{\cos^2 t}$, $\int \underbrace{\frac{1}{e^{-y}}}_{e^y}\, dy = \int \frac{\sin t}{\cos^2 t}\, dt$

$= e^y = \sec t + c$

$e^y - \sec t = c$

3) $y' = \frac{y}{2t}$, $\frac{1}{y} \cdot \frac{dy}{dt} = \frac{1}{2t}$

$\int \frac{1}{y} dy = \int \frac{1}{2t} dt$

$= \ln|y| = \frac{1}{2} \ln|t| + c$

$|y| = |t|^{1/2} * e^{c}$, $y(-1) = 2$

$|2| = |-1|^{1/2} * e^{c}$, $2 = 1 \cdot e^{c}$ $\therefore e^{c} = 2$

so $y = 2x^{1/2}$, this is defined for $x > 0$.