ECEN220 - Assignment 1: Solutions

1.21

a) $x(t)$ has been shifted by 1 unit to the right in time. The final graph is in Figure 1

b) $x(2-t)$ can be rewritten as $x(-(t-2))$. This means that $x(t)$ is inverted around $x=0$ and is then shifted to the right by 2 units in time. The final graph is in Figure 1.

c) $x(2t-1)$ can be rewritten as $x(2(t-\frac{1}{2}))$. This means that $x(t)$ has been compressed by a factor of $\frac{1}{2}$ and then shifted to the right by $\frac{1}{2}$ units in time. The final graph is in Figure 1.

d) $x(4-\frac{t}{2})$ can be rewritten as $x((-1)\frac{1}{2}(t-8))$. This means that $x(t)$ has been inverted around $x=0$. Then scaled by a factor of 2. Then shifted to the right by 8 units in time. The final graph is in Figure 1.

e) Recall that the step function is $u(t)=1, \forall t \geq 0$. So

$$[x(t)+x(-t)]u(t) = \begin{cases} x(t)+x(-t) & t \geq 0 \\ 0 \quad t < 0 \end{cases}$$

The final graph is in Figure 1.

f) Recall that $\delta(t)=1$ at $t=0$. This would mean that

$$x(t)\delta(t+3/2) = x(-3/2) \text{ at } t = -\frac{3}{2}$$
$$-x(t)\delta(t-3/2) = -x(3/2) \text{ at } t = \frac{3}{2}$$

From the graph, it can be said that $x(-3/2) = -0.5$ and $-x(3/2) = -0.5$. The graph $x(t)[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})]$ has two impulses at $t=-\frac{3}{2},\frac{3}{2}$, both with a magnitude of -0.5 which is shown in Figure 1.

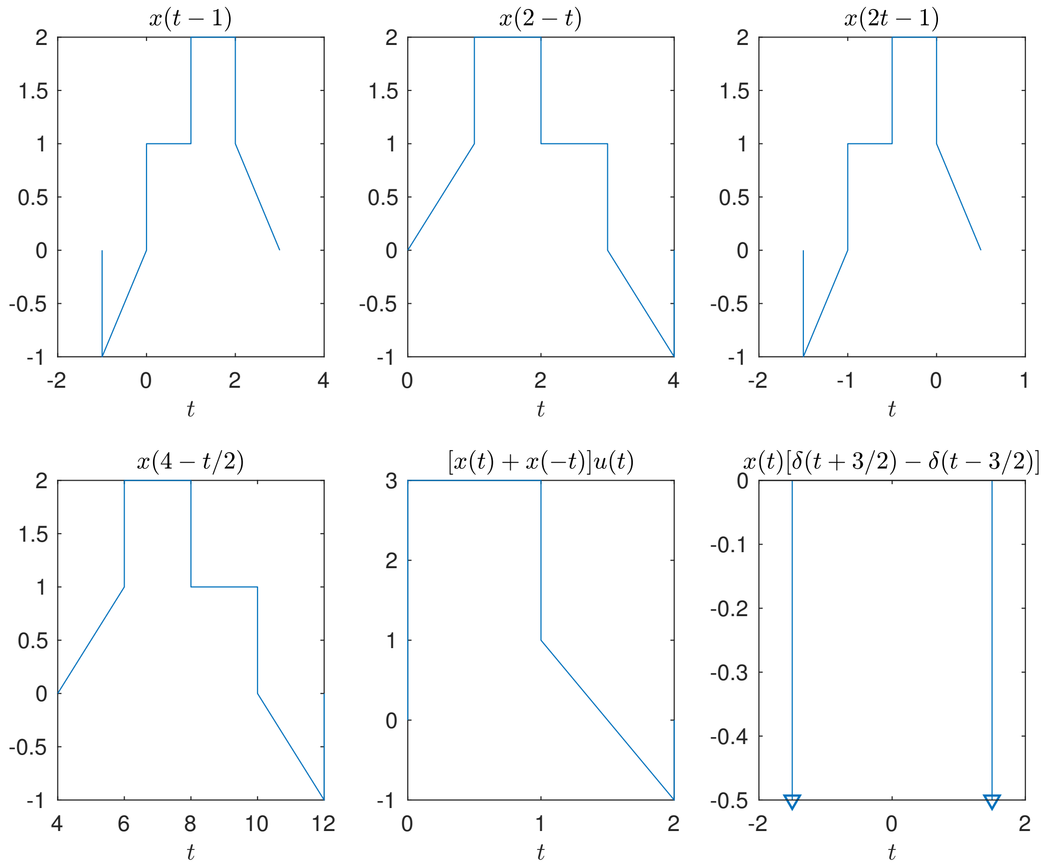

Figure 1: Figures for 1.21

1.22

a) $x[n]$ has been shifted to the right by 4 units in n. The final graph is in Figure 2.

b) $x[3-n]$ can be rewritten as $x[-(n-3)]$. This means that $x[n]$ has been inverted around $n=0$ and then shifted to the right by 3 units in n. The final graph is in Figure 2.

c) Recall that by treating the current DT domain as n', the new mapping to n can be performed by $n = \frac{1}{3}n'$. Since $n \in \mathbb{Z}$, then for any n that doesnt result in an integer due to n', $x[n] = 0$. The final graph is in Figure 2.

d) By treating the current DT domain as n', the new mapping to n can be performed by $n = \frac{n'-1}{3}$. Since $n \in \mathbb{Z}$, then for any n that doesnt result in an integer due to n', $x[n] = 0$. The final graph is in Figure 2.

e) Since the non-zero magnitudes are between $-4 \leq n \leq 3$, then $u[3-n]$ is always positive. Therefore $x[n]u[3-n] = x[n]$. The final graph is in Figure 2.

f) Recall that $\delta[n] = 1$ at $n = 0$. So $x[n-2]\delta[n-2] = x[0]$ at $n = 2$. The final graph is in Figure 2.

g) The expression can be simplified as follows

$$\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n] = \frac{x[n]}{2}(1 + (-1)^n)$$
$$= \begin{cases} x[n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

The final graph is in Figure 2. **Remark:** $(-1)^n$ is even since $(-1)^n = \cos(n\pi)$.

h) Notice that $x[(n-1)^2] = 0$ when $n < 0$ and $n > 2$. Due to the power of 2, any value of n in $n < 0$ and $n > 2$ will result in $x[n] = 0$. The final graph is in Figure 2.

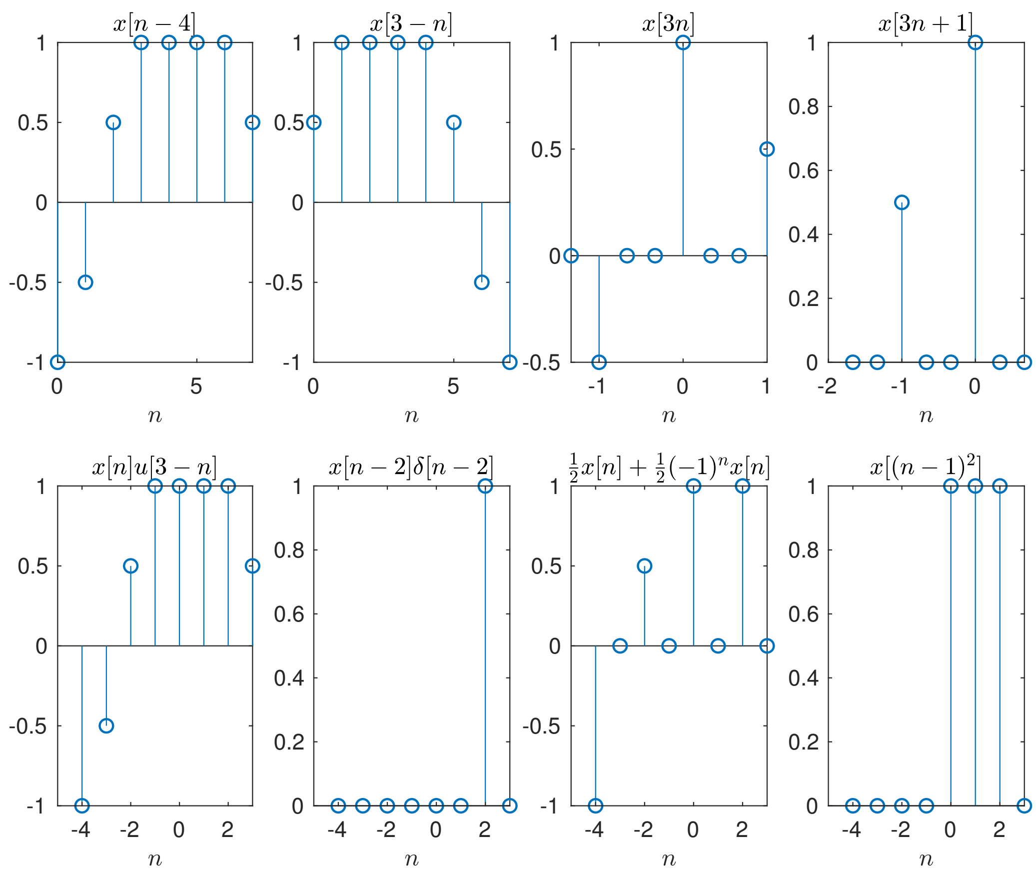

Figure 2: Figures for 1.22

P.T.O

1.25

a)

$$\begin{aligned} 3\cos(4t+\frac{\pi}{3}) &= 3\cos(4(t+T)+\frac{\pi}{3}) \\ &= 3\cos(4t+4T+\frac{\pi}{3}) \end{aligned}$$

Let $T = \frac{\pi}{2}$, then

$$\begin{aligned} 3\cos(4t+\frac{\pi}{3}) &= 3\cos(4t+4(\frac{\pi}{2})+\frac{\pi}{3}) \\ &= 3\cos(4t+2\pi+\frac{\pi}{3}) \\ &= 3\cos(4t+\frac{\pi}{3}) \end{aligned}$$

This function is therefore periodic with a fundamental period of $T_0 = \frac{\pi}{2}$.

b)

$$\begin{aligned} x(t) &= e^{j(\pi t-1)} \\ &= e^{-j}e^{j\pi t} \\ &= e^{-j}(\cos(\pi t)+j\sin(\pi t)) \end{aligned}$$

Check for periodicity:

$$\begin{aligned} e^{-j}(\cos(\pi t)+j\sin(\pi t)) &= e^{-j}(\cos(\pi(t+T))+j\sin(\pi(t+T))) \\ &= e^{-j}(\cos(\pi t+\pi T)+j\sin(\pi t+\pi T)) \end{aligned}$$

Let $T = 2$, then

$$\begin{aligned} e^{-j}(\cos(\pi t)+j\sin(\pi t)) &= e^{-j}(\cos(\pi t+2\pi)+j\sin(\pi t+2\pi)) \\ &= e^{-j}(\cos(\pi t)+j\sin(\pi t)) \end{aligned}$$

Therefore the function is periodic with a fundamental period of $T_0 = 2$.

c)

$$\begin{aligned} x(t) &= \Big[\cos\Big(2t-\frac{\pi}{3}\Big)\Big]^2 \\ &= \Big(\cos\Big(2t-\frac{\pi}{3}\Big)\Big)\Big(\cos\Big(2t-\frac{\pi}{3}\Big)\Big) \\ &= \frac{1}{2}\Big(\cos\Big(2\Big(2t-\frac{\pi}{3}\Big)\Big)+\cos(0)\Big) \quad \textit{Trig. Identity} \\ &= \frac{1}{2}\Big(\cos\Big(4t-\frac{2\pi}{3}\Big)+1\Big) \end{aligned}$$

Check for periodicity:

$$\begin{aligned} \frac{1}{2}\Big(\cos\Big(4t-\frac{2\pi}{3}\Big)+1\Big) &= \frac{1}{2}\Big(\cos\Big(4(t+T)-\frac{2\pi}{3}\Big)+1\Big) \\ &= \frac{1}{2}\Big(\cos\Big(4t+4T-\frac{2\pi}{3}\Big)+1\Big) \end{aligned}$$

Let $T = \frac{\pi}{2}$, then

$$\begin{aligned}\frac{1}{2}\Big(\cos\Big(4t - \frac{2\pi}{3}\Big) + 1\Big) &= \frac{1}{2}\Big(\cos\Big(4t + 2\pi - \frac{2\pi}{3}\Big) + 1\Big)\\ &= \frac{1}{2}\Big(\cos\Big(4t - \frac{2\pi}{3}\Big) + 1\Big)\end{aligned}$$

Therefore the function is periodic with a fundamental period of $T_0 = \frac{\pi}{2}$.

d)

$$\begin{aligned}x(t) &= \mathrm{Ev}\{\cos(4\pi t)u(t)\}\\ &= \frac{1}{2}(\cos(4\pi t)u(t) + \cos(-4\pi t)u(-t))\\ &= \frac{1}{2}(\cos(4\pi t)u(t) + \cos(4\pi t)u(-t)) \quad \textit{cosine is even}\\ &= \frac{\cos(4\pi t)}{2}(u(t) + u(-t))\\ &= \frac{\cos(4\pi t)}{2}\end{aligned}$$

Check for periodicity:

$$\begin{aligned}\frac{\cos(4\pi t)}{2} &= \frac{\cos(4\pi(t+T))}{2}\\ &= \frac{\cos(4\pi t + 4\pi T)}{2}\end{aligned}$$

Let $T = \frac{1}{2}$, then

$$\begin{aligned}\frac{\cos(4\pi t)}{2} &= \frac{\cos(4\pi t + 2\pi)}{2}\\ &= \frac{\cos(4\pi t)}{2}\end{aligned}$$

Therefore this function is periodic with a fundamental period of $T_0 = \frac{1}{2}$.

e)

$$\begin{aligned}x(t) &= \mathrm{Ev}\{\sin(4\pi t)u(t)\}\\ &= \frac{1}{2}(\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t))\\ &= \frac{1}{2}(\sin(4\pi t)u(t) - \sin(4\pi t)u(-t)) \quad \textit{sine is odd}\\ &= \frac{\sin(4\pi t)}{2}(u(t) - u(-t))\\ &= \begin{cases}\frac{\sin(4\pi t)}{2} & t \geq 0\\ -\frac{\sin(4\pi t)}{2} & t < 0\end{cases}\end{aligned}$$

Due to the difference of functions in different areas of the time domain, this function is not periodic.

f)

$$x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} u(2t-n)$$

Due to the step function, this function is not periodic as there will be a section whereby the function is zero and a section where it is non-zero.

1.26

a)

$$x[n] = \sin\left(\sin\left(\frac{6\pi n}{7} + 1\right)\right)$$

Check for periodicity:

$$\begin{aligned}\sin\left(\sin\left(\frac{6\pi n}{7} + 1\right)\right) &= \sin\left(\sin\left(\frac{6\pi(n+N)}{7} + 1\right)\right)\\ &= \sin\left(\sin\left(\frac{6\pi n}{7} + \frac{6\pi N}{7} + 1\right)\right)\end{aligned}$$

Let $N = 7$, then

$$\begin{aligned}\sin\left(\sin\left(\frac{6\pi n}{7} + 1\right)\right) &= \sin\left(\sin\left(\frac{6\pi n}{7} + 3(2\pi) + 1\right)\right)\\ &= \sin\left(\sin\left(\frac{6\pi n}{7} + 1\right)\right)\end{aligned}$$

Therefore this function is periodic with a fundamental period of $N_0 = 7$.

b)

$$x[n] = \cos\left(\frac{n}{8} + \pi\right)$$

Check for periodicity:

$$\begin{aligned}\cos\left(\frac{n}{8} + \pi\right) &\stackrel{?}{=} \cos\left(\frac{n+N}{8} + \pi\right)\\ &\neq \cos\left(\frac{n}{8} + \frac{N}{8} + \pi\right)\end{aligned}$$

Since this is a discrete function, there is no possible value of N that satisfies this equality. Therefore this function is not periodic.

c)

$$x[n] = \cos\left(\frac{\pi}{8} n^2\right)$$

Check for periodicity:

$$\begin{aligned}\cos\left(\frac{\pi}{8} n^2\right) &= \cos\left(\frac{\pi}{8}(n+N)^2\right)\\ &= \cos\left(\frac{\pi}{8}(n^2 + 2nN + N^2)\right)\\ &= \cos\left(\frac{\pi}{8} n^2 + \frac{\pi}{8} 2nN + \frac{\pi}{8} N^2\right)\end{aligned}$$

Let $N = 8$, then

$$\begin{aligned}\cos\left(\frac{\pi}{8}n^2\right) &= \cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}2n8 + \frac{\pi}{8}(8)^2\right)\\ &= \cos\left(\frac{\pi}{8}n^2 + n2\pi + 4(2\pi)\right)\\ &= \cos\left(\frac{\pi}{8}n^2\right)\end{aligned}$$

Therefore this function is periodic with a fundamental period of $N_0 = 8$.

d)

$$\begin{aligned}x[n] &= \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)\\ &= \frac{1}{2}\Big(\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}n\right) + \cos\left(\frac{\pi}{2}n - \frac{\pi}{4}n\right)\Big)\\ &= \frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n\right) + \cos\left(\frac{\pi}{4}n\right)\Big)\end{aligned}$$

Check for periodicity:

$$\begin{aligned}\frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n\right) + \cos\left(\frac{\pi}{4}n\right)\Big) &= \frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}(n+N)\right) + \cos\left(\frac{\pi}{4}(n+N)\right)\Big)\\ &= \frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n + \frac{3\pi}{2}N\right) + \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right)\Big)\end{aligned}$$

Let $N = 8$, then

$$\begin{aligned}\frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n\right) + \cos\left(\frac{\pi}{4}n\right)\Big) &= \frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n + 6(2\pi)\right) + \cos\left(\frac{\pi}{4}n + 2\pi\right)\Big)\\ &= \frac{1}{2}\Big(\cos\left(\frac{3\pi}{2}n\right) + \cos\left(\frac{\pi}{4}n\right)\Big)\end{aligned}$$

Therefore this function is periodic with a fundamental period of $N_0 = 8$.

e)

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$

Check for periodicity:

$$\begin{aligned}2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right) &= 2\cos\left(\frac{\pi}{4}(n+N)\right) + \sin\left(\frac{\pi}{8}(n+N)\right) - 2\cos\left(\frac{\pi}{2}(n+N) + \frac{\pi}{6}\right)\\ &= 2\cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right) + \sin\left(\frac{\pi}{8}n + \frac{\pi}{8}N\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N + \frac{\pi}{6}\right)\end{aligned}$$

Let $N = 16$, then

$$\begin{aligned}2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right) &= 2\cos\left(\frac{\pi}{4}n + 2(2\pi)\right) + \sin\left(\frac{\pi}{8}n + 2\pi\right) - 2\cos\left(\frac{\pi}{2}n + 2\pi + \frac{\pi}{6}\right)\\ &= 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)\end{aligned}$$

Therefore this function if periodic with a fundamental period of $N_0 = 16$.

1.31

a) Notice that the signal $x_2(t)$ can be constructed using signal $x_1(t)$ as follows

$$x_2(t) = x_1(t) + -x_1(t-2)$$

. Through linearity, since y_1 is the result of passing x_1 through some system, then

$$y_2(t) = y_1(t) - y_1(t-2)$$

. The graph $y_2(t)$ is in Figure 3.

b) Similar to 1.31a), the signal $x_3(t)$ can be constructed as follows

$$x_3(t) = x_1(t) + x_1(t-1)$$

. And using linearity, the signal $y_3(t)$ is

$$y_3(t) = y_1(t) + y_1(t+1)$$

. The graph $y_3(t)$ is in Figure 3

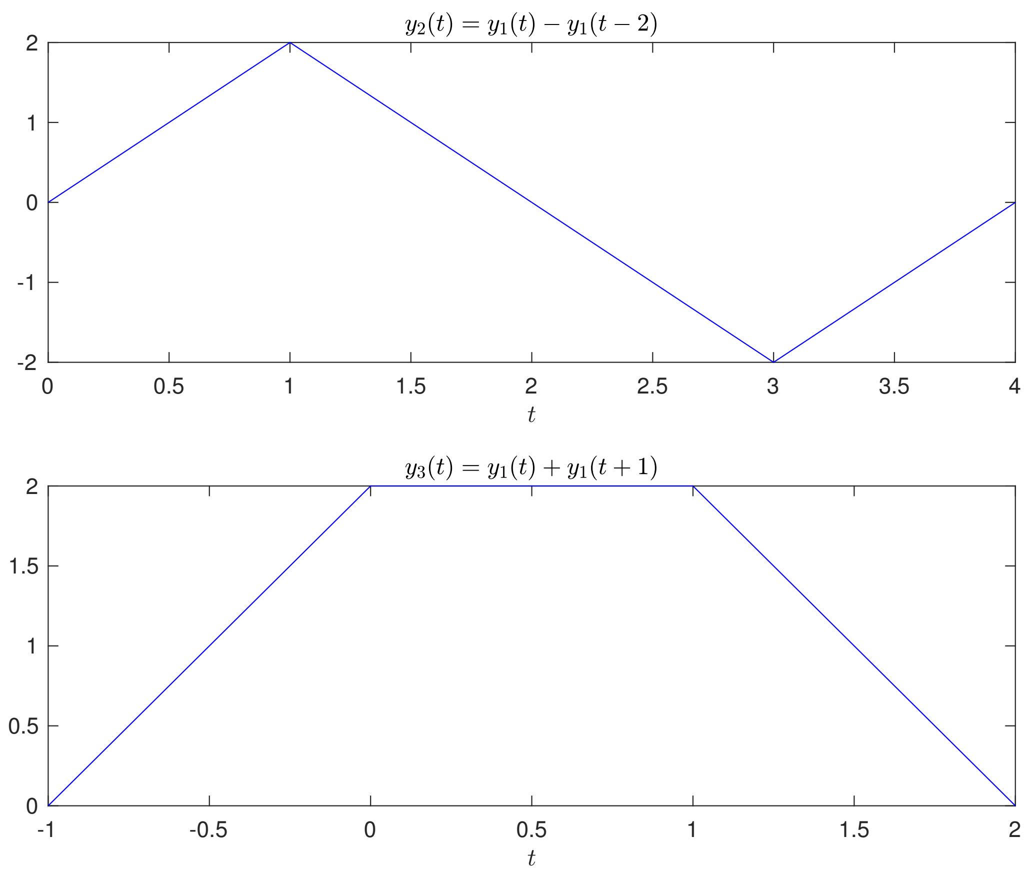

Figure 3: Figures for 1.31

1.37

a) Let $z = t + \tau$, then $\tau = z - t$ and $d\tau = dz$. So,

$$\begin{aligned}\phi_{xy}(t) &= \int\limits_{-\infty}^{\infty} x(z)y(-t+z)dz \\ &= \int\limits_{-\infty}^{\infty} y(-t+z)x(z)dz \\ &= \phi_{yx}(-t)\end{aligned}$$

b) From 1.37a), it can be said that since $\phi_{xy}(t) = \phi_{xy}(-t)$, then $\phi_{xy}(t)$ is an even function. Let $y = x$, then $\phi_{xx}(t)$ is also an even function with no odd part. That is, $\text{odd}\{\phi_{xx}(t)\} = 0$.

c) Knowing that $y(t) = x(t+T)$, the original problem becomes

$$\phi_{xy}(t) = \int\limits_{-\infty}^{\infty} x(t+\tau)x(\tau+T)\ d\tau.$$

Let $T = z + \tau$, then $\tau = z - T$ and $d\tau = dz$. Then,

$$\begin{aligned}\phi_{xy}(t) &= \int\limits_{-\infty}^{\infty} x(t+z-T)x(z-T+T)\ dz \\ &= \int\limits_{-\infty}^{\infty} x((t-T)+z)x(z)\ dz \\ &= \phi_{xx}(t-T)\end{aligned}$$

Secondly, let $x = y$, then

$$\phi_{yy}(t) = \int\limits_{-\infty}^{\infty} y(t+\tau)y(\tau)\ d\tau.$$

Knowing that $y(t) = x(t+T)$, then

$$\begin{aligned}\phi_{yy}(t) &= \int\limits_{-\infty}^{\infty} x(t+\tau+T)x(\tau+T)\ d\tau \\ &= \int\limits_{-\infty}^{\infty} x(t+(\tau+T))x(\tau+T)\ d\tau \\ &= \phi_{xx}(t)\end{aligned}$$