ECEN220 - Assignment 3 Solutions

2.28

(a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$

- Causal since $h[n] = 0$ for $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-\frac{1}{5}} = \frac{5}{4} < \infty$$

- $h[n]$ is stable.

(b) $h[n] = (0.8)^n u[n] = \left(\frac{4}{5}\right)^n u[n+2]$

- Not causal since $h[n] \neq 0$ for $n < -2$

$$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n = \frac{1}{1-\frac{4}{5}} = 5 < \infty$$

- $h[n]$ is stable.

(c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

- Not causal as $h[n] \neq 0$ for $n<0$

$$\sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} 2^n = \infty$$

- $h[n]$ is not stable.

(d) $h[n] = 5^n u[3-n]$

- Not causal since $h[n] \neq 0$ for $n<0$

$$\sum_{n=-\infty}^{3} 5^n = \sum_{n=-3}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-\frac{1}{5}} + \left(\frac{1}{5}\right)^{-3} + \left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{5}\right)^{1}$$

$$= \frac{625}{4} < \infty$$

- $h[n]$ is stable

(e) $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$

- Causal since $u[n-1]=0$ for $n<1$. So $h[n]=0$ for $n<0$.

- The second term in the summation has $(1.01)^n$ which is unbounded as $n \to \infty$. So $h[n]$ is not stable.

(f) $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[1-n]$
$= \left(-\frac{1}{2}\right)^n u[n] + \left(\frac{101}{100}\right)^n u[1-n]$

- Not causal since $u[1-n]=1$ for $n<1$. So $h[n] \neq 0$ for $n<0$.

$$\sum_{n=-\infty}^{\infty} \left| \left(-\frac{1}{2}\right)^n u[n] + \left(\frac{101}{100}\right)^n u[1-n] \right|$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{101}{100}\right)^n + \sum_{n=0}^{1} \left| \left(-\frac{1}{2}\right)^n + \left(\frac{101}{100}\right)^n \right| + \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{100}{101}\right)^n - 1 + |1+1| + \left| -\frac{1}{2} + \frac{101}{100} \right| + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - 1 - \frac{1}{2}$$

$$= \frac{1}{1-\frac{100}{101}} - 1 + 2 + \frac{51}{100} + \frac{1}{1-\frac{1}{2}} - 1 - \frac{1}{2}$$

$$= \frac{10301}{100} = 103.01 < \infty$$

- $h[n]$ is stable.

(g) $h[n] = n\left(\frac{1}{3}\right)^n u[n-1]$

- Causal because $u[n-1]=0$ for $n<1$. So $h[n]=0$ for $n<0$

$$\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} n\left(\frac{1}{3}\right)^n = \frac{1/3}{\left(1-\frac{1}{3}\right)^2} = \frac{3}{4} < \infty$$

- $h[n]$ is stable.

2.29

(a) $h(t) = e^{-4t} u(t-2)$

- Causal since $h(t) = 0$ for $t < 0$.

$$\int_{2}^{\infty} e^{-4t}\,dt = \left[-\frac{1}{4}e^{-4t}\right]_2^{\infty} = \frac{1}{4}e^{-8} < \infty$$

- $h(t)$ is stable.

(b) $h(t) = e^{-6t} u(3-t)$

- Not causal since $u(3-t) = 1$ for $t < 3$. So $h(t) \neq 0$ for $t < 0$.

$$\int_{-\infty}^{3} e^{-6t}\,dt = \left[-\frac{1}{6}e^{-6t}\right]_{-\infty}^{3} = -\frac{1}{6}e^{-18} + \frac{1}{6}e^{\infty} = \infty$$

- $h(t)$ is not stable

(c) $h(t) = e^{-2t} u(t+50)$

- Not causal since $u(t+50) = 1$ for $t > -50$. So $h(t) \neq 0$ for $t < 0$.

$$\int_{-50}^{\infty} e^{-2t}\,dt = \left[-\frac{1}{2}e^{-2t}\right]_{-50}^{\infty} = \frac{1}{2}e^{100} < \infty$$

- $h(t)$ is stable

(d) $h(t) = e^{2t}u(-1-t)$

- Not causal since $u(-1-t)=1$ for $t<-1$.

$$\int_{-\infty}^{-1} e^{2t}\,dt = \left[\frac{1}{2}e^{2t}\right]_{-\infty}^{-1} = \frac{1}{2}e^{-2} < \infty$$

- $h(t)$ is stable.

(e) $h(t) = e^{-6|t|}$

- Not causal since $|t|$ would mean that $h(t) \neq 0$ for $t<0$.

$$\int_{-\infty}^{\infty} e^{-6|t|}\,dt = 2\int_0^{\infty} e^{-6t}\,dt = \frac{1}{3}\left[-e^{-6t}\right]_0^{\infty} = \frac{1}{3} < \infty$$

- $h(t)$ is stable

(f) $h(t) = te^{-t}u(t)$

- Causal since $h(t)=0$ for $t<0$.

$$\int_0^{\infty} te^{-t}\,dt = \left[-te^{-t} - e^{-t}\right]_0^{\infty}$$

$$= 1 < \infty$$

	D	I
+	t	e^{-t}
−	1	$-e^{-t}$
+	0	e^{-t}

- $h(t)$ is stable.

(g) $h(t) = \left(2e^{-t} - e^{\frac{t-100}{100}}\right)u(t)$

- Causal since $h(t)=0$ for $t<0$

$$\int_0^{\infty} 2e^{-t} - e^{\frac{t}{100}-1}\,dt$$

- Not stable since $e^{\infty}=\infty$ in the second term of the integral

2.46

$$\frac{dx(t)}{dt} = -6e^{-3t}u(t-1) + 2e^{-3}\delta(t-1)$$
$$= -6e^{-3t}u(t-1) + 2e^{-3}\delta(t-1)$$
$$= -3x(t) + 2e^{-3}\delta(t-1)$$

We are given that

$$x(t) \longrightarrow y(t)$$

which means that

$$-3y(t) + 2e^{-3}\delta(t-1) \longrightarrow -3y(t) + 2e^{-3}h(t-1)$$

By direct comparison to the equation in the problem, it can be said that

$$2e^{-3}h(t-1) = e^{-2t}u(t)$$
$$h(t-1) = \frac{1}{2}e^{-3}e^{-2t}u(t)$$
$$\therefore h(t) = \frac{1}{2}e^{-3}e^{-2(t+1)}u(t+1)$$

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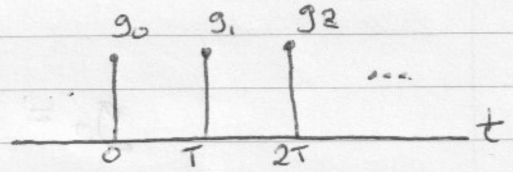
$$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT)$$

$$g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT)$$

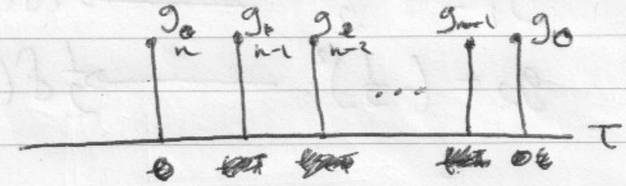
(a) Since $y(t) = x(t) * h(t)$ then $y(t) = y(t) * g(t) * h(t)$.
This would mean that $g(t) * h(t) = \delta(t)$.

The easiest way to compute $g(t) * h(t)$ is by visual inspection.

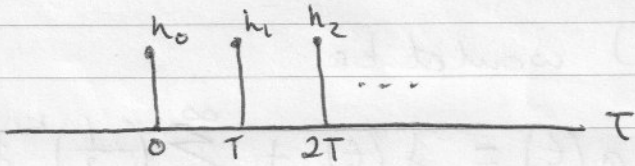
$$g[t] = \sum_{k=0}^{\infty} g_k \delta(t - kT)$$



$g[t-\tau] =$



$h[\tau] =$



As $g(t-\tau)$ passes over $h(\tau)$, the overlapping region is

$$h(t) * g(t) = \sum_{k=0}^{n} h_k g_{n-k} \delta(t - nT) = \begin{cases} 1, & n = 0 \\ 0, & n = 1, 2, 3, \ldots \end{cases}$$

desired $\delta(t)$

So for $n=0$: $h_0 g_0 = 1$

$$\boxed{g_0 = \frac{1}{h_0}}$$

for $n=1$: $h_0 g_1 + h_1 g_0 = 0$

$$g_1 = -\frac{h_1 g_0}{h_0}$$

$$\boxed{g_1 = -\frac{h_1}{h_0^2}}$$

for $n=2$: $h_0 g_2 + h_1 g_1 + h_2 g_0 = 0$

$$g_2 = -\frac{1}{h_0}\left(h_1 g_1 + h_2 g_0\right)$$
$$= -\frac{1}{h_0}\left(-\frac{h_1^2}{h_0^2} + \frac{h_2}{h_0}\right)$$

(b) Substituting $h_i = \begin{cases} 1 & i=0 \\ 1/2 & i=0 \\ 0 & i \geq 2 \end{cases}$, we get

$$g_0 = 1 \longrightarrow \delta(t) \text{ at } k=0$$
$$g_1 = -\frac{1}{2} \longrightarrow -\tfrac{1}{2}\delta(t-T) \text{ at } k=1$$
$$g_2 = \left(-\tfrac{1}{2}\right)^2 \longrightarrow \tfrac{1}{2}\delta(t-2T) \text{ at } k=2$$
$$\vdots$$

So $g(t)$ would be

$$g(t) = \delta(t) + \sum_{k=1}^{\infty}\left(-\tfrac{1}{2}\right)^k \delta(t-kT)$$

(c) In the time domain, a delay "T" is denoted
(i) by $\delta(t-T)$. By examining $y_k(t)$, the expression for $h(t)$ can be easily obtained

$$y_1(t) = x_0(t)\alpha\delta(t-T)$$

$$y_2(t) = x_1(t) + \alpha\delta(t-T)\,y_1(t)$$
$$= x_1(t) + \alpha^2\delta(t-T)\,x_0(t)$$
$$\vdots$$

So the expression $h(t)$ such that $y(t) = x(t) * h(t)$ is

$$h(t) = \sum_{k=0}^{\infty}\alpha^k \delta(t-T)$$

(ii) If $0<\alpha<1$, then $\alpha^k<1$ and the infinite summation converges. However if $\alpha>1$, then the infinite summation diverges.

(iii) Using the same equations in (c) but rearranging for $x_i(t)$,

$$y_2(t) - \alpha\delta(t-T)y_1(t) = x_1(t)$$

It can then be said that

$$g(t) = 1 - \alpha\,\delta(t-T)$$

(d) Let $x_1[n] = \delta[n]$, then.

$$y[n] = x_1[n] * h[n] = h[n]$$

Let $x_2[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-N]$, then.

$$\begin{aligned} y[n] &= \left(\tfrac{1}{2}\delta[n] + \tfrac{1}{2}\delta[n-N]\right) * h[n] \\ &= \tfrac{1}{2}\delta[n]*h[n] + \tfrac{1}{2}\delta[n-N]*h[n] \\ &= \tfrac{1}{2}h[n] + h[n] \\ &= h[n] \end{aligned}$$

Same