ECEN415 Advanced Control Systems Engineering
Assignment One - 2021

Due 25^{th} of July

Submit pdf documents describing your solutions via the online submission system. Hand written solutions are fine, but these must be scanned and converted to pdf.

Solutions should use standard mathematical notation (not pseudo-matlab), explain your reasoning solutions and include computer generated figures where appropriate. If you do use Matlab routines to derive your answers then you should explain the critical Matlab calls, including input arguments and the pertinent outputs. Your written submitted solutions should not include code unless it is explicitly requested. Note that I will not read your code to understand what you are trying to achieve. Your written answers should contain your complete description of your approach and solutions.

Unless otherwise noted, you can use Matlab (or other suitable tool) to complete these problems. You should upload your code, because if things go wrong I will look at code to try to work out what has happened. (That is, I am not marking your code, but I will use it to try to understand what went wrong.)

Section A - Formative Questions

Questions in this section are intended to be reasonably tractable and are intended to teach you the skills required to do the course, including the questions in section B.

This section comprises 50% of the marks for the assignment, and will be marked only crudely. If you make a reasonable attempt at all the questions you will get full marks for the section. Patchy attempts will yield half marks.

1. Sketch Nyquist plot (or use Matlab) and use the diagram to assess the stability of the following transfer functions. Consider whether each system is stable as written, and whether it could be driven unstable by increasing or decreasing the gain sufficiently.

   (a) $G_1(s) = \dfrac{20(s^2+s+0.5)}{s(s+1)(s+10)}$

   (b) $G_2(s) = \dfrac{20(s^2+s+0.5)}{s(s-1)(s+10)}$

   (c) $G_3(s) = \dfrac{s^2+3}{(s+1)^2}$

(d) $G_4(s) = \dfrac{3(s+1)}{s(s-10)}$.

Hint: (The last of these is tricky because of what happens in the region where $G(s)$ approaches infinity (at $s = 0$). It could circle at complex infinity in the negative or (mostly) positive parts of the $G(s)$ plane. Consider what is happening as we pass the pole at $s = 0$ on the Nyquist contour.)

2. A system has a nominal transfer function $G(s) = \dfrac{4}{s+2}$, and a delay of 0.2 s. We can define a model such as this in matlab using the pair of commands

```
>> G = zpk([],-2,4);
>> G.IODelay = 0.2;
```

You will find that some, but not all, Matlab commands will work with systems including an IODelay in this way.

(a) Plot the Nyquist diagram for $G(s)$ and use it to discuss the stability of $G(s)$ if we were to enclose it in a feedback loop and increase the proportional gain. Is the system closed loop stable as written? If so, then how much gain could we add before it became unstable? If not then by how much must we reduce the gain to achieve stability?

(b) Plot the step response of the closed loop system and its proportionally controlled version. Show one stable and one unstable condition. Does the system behave as predicted by the Nyquist diagram?

(c) How much additional time delay could be added to the original $G(s)$ before it becomes unstable?

3. Consider the low pass filter $G(s) = \dfrac{1000}{s+1000}$ which has a corner frequency at 1000 rads^{-1} and unity dc gain.

Find the equivalent z-domain transfer functions using each of the conversion methods discussed in the notes, and compare the time and frequency domain performances. That is, compare the impulse/step and bode plots as appropriate.

Section B - Summative Questions

Questions in this section are intended to be more open ended. This section is also worth 50%, but will be marked with higher resolution than the formative section.

1. Consider the transfer function of a pure delay $D(s) = \mathrm{e}^{st_\mathrm{d}}$, where t_d is the length of the delay.

   (a) [2 marks] At what angular frequency does the delay cause a phase shift of -90°?

   (b) [4 marks] Consider a system having a unity gain bandwidth of 1000 rads^{-1}that is able to suffer only a 15°degradation before it no longer meets its stability requirements. How much pure delay could be added to the system before it becomes unstable?

   (c) [4 marks] Repeat the above analysis using first and second order Padé approximants to model the delay.

   Hint: You should be able to solve the first part analytically, but Matlab is likely to be useful for the latter parts. Though it is not strictly necessary, you may find pade useful.

2. [10 marks] A system has transfer function $G(s) = \dfrac{15}{(s+1)(s+2)}$ and is to be used in a sampled data system *without* an antialiasing filter.

   Use a root locus diagram to discuss the stability of the system with and without the effect of the sampling.

   If you were to use a sampling rate that is ten times the unity gain frequency of the system, how much additional gain could be added to the system before it became unstable?

   Hint: You can use the first order Padé approximant in this problem.

3. [20 marks]

   You are given a continuous time control system design that has a closed loop bandwidth (ie unity gain bandwidth) of 10 kHz and are told that the system can only tolerate an additional 10° of phase delay at the unity gain frequency.

   The system is subject to interference, so you must make sure that any aliased signals are attenuated by at least 40 dB.

   Your task is to design the sampling system to be used in implementing the continuous time controller in a microcontroller. You are told that you should try to sample at least ten times faster than the unity gain frequency, and that the board has space for at most a second order antialiasing filter. Because of the nature of the particular plant, you do not need a reconstruction filter.

   Choose a sampling frequency and a corner frequency for the anti-aliasing filter in the system. You should aim to achieve the lowest sampling frequency you can achieve.

   Hint: For simplicity use a Butterworth anti-aliasing filter (which is normally a good choice of filter topology), or even cascaded first order filters. A second order Butterworth low pass filter with corner frequency ω_c has transfer function

$$G_\mathrm{b}(s) = \frac{\omega_\mathrm{c}^2}{s^2 + \sqrt{2}\omega_\mathrm{c} s + \omega_\mathrm{c}^2}$$