ECEN321: Engineering Statistics
Assignment 3 Submission

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Jointly Distributed Random Variables

1. (Navidi 2.6.4,6)

   (a) $p_X(x) = \sum_y p(x,y)$
   $p_X(0) = 0.48,\ p_X(1) = 0.25,\ p_X(2) = 0.17,\ p_X(3) = 0.1$
   $p_X(x) = 0,\ elsewhere$

   (b) $p_Y(y) = \sum_x p(x,y)$
   $p_Y(0) = 0.34,\ p_Y(1) = 0.27,\ p_Y(2) = 0.22,\ p_Y(3) = 0.17$
   $p_Y(y) = 0,\ elsewhere$

   (c) $p(x,y) = P(X = x, Y = y)$ and if X and Y are independent then $p(x,y) = p_X(x) \cdot p_Y(y)$
   $P(X = 0, Y = 0) = 0.15$
   $p_X(0) \cdot p_Y(0) = 0.48 \times 0.34 = 0.1632$
   $\therefore X, Y$ are not independent

   (d) $\mu_X = 0p_X(0) + 1p_X(1) + 2p_X(2) + 3p_X(3) = 0 + 0.25 + 0.34 + 0.3 = 0.89$
   $\mu_Y = 0p_Y(0) + 1p_Y(1) + 2p_Y(2) + 3p_Y(3) = 0 + 0.27 + 0.44 + 0.51 = 1.22$

   (e) $\sigma_X^2 = 0^2 p_X(0) + 1^2 p_X(1) + 2^2 p_X(2) + 3^2 p_X(3) - \mu_X^2 = 0 + 0.25 + 0.68 + 0.9 - 0.89^2 = 1.0379$
   $\sigma_X = \sqrt{1.0379} = 1.018774$

   $\sigma_Y^2 = 0^2 p_Y(0) + 1^2 p_Y(1) + 2^2 p_Y(2) + 3^2 p_Y(3) - \mu_Y^2 0 + 0.27 + 0.88 + 1.53 - 1.22^2 = 1.1916$
   $\sigma_Y = \sqrt{1.1916} = 1.091604$

   (f) $Cov(X,Y) = \mu_{XY} - \mu_X \cdot \mu_Y$

$$\begin{aligned}
\mu_{XY} = &(0)(0)p_{X,Y}(0,0) + (0)(1)p_{X,Y}(0,1) + (0)(2)p_{X,Y}(0,2) + (0)(3)p_{X,Y}(0,3) \\
&+ (1)(0)p_{X,Y}(1,0) + (1)(1)p_{X,Y}(1,1) + (1)(2)p_{X,Y}(1,2) + (1)(3)p_{X,Y}(1,2) \\
&+ (2)(0)p_{X,Y}(2,0) + (2)(1)p_{X,Y}(2,1) + (2)(2)p_{X,Y}(2,2) + (2)(3)p_{X,Y}(2,2) \\
&+ (3)(0)p_{X,Y}(3,0) + (3)(1)p_{X,Y}(3,1) + (3)(2)p_{X,Y}(3,3) + (3)(3)p_{X,Y}(3,3) \\
\\
&= 0 + 0 + (1)(0.07) + (2)(0.05) + (3)(0.04) + 0 + (2)(0.05) \\
&+ (4)(0.04) + (6)(0.02) + 0 + (3)(0.03) + (6)(0.02) + (9)(0.01) \\
\mu_{XY} = &\ 0.97,\ \mu_X = 0.89,\ \mu_Y = 1.22
\end{aligned}$$

   $Cov(X,Y) = 0.97 - 0.89 \times 1.22 = -0.1158$

(g) $\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{-0.1158}{1.112098} = -0.104128$

(h) $p_{Y|X}(y|1) = \frac{p(1,y)}{p_X(1)}$

$p_{Y|X}(y=0|1) = \frac{0.09}{0.25} = 0.36$ $\qquad$ $p_{Y|X}(y=1|1) = \frac{0.07}{0.25} = 0.28$

$p_{Y|X}(y=2|1) = \frac{0.05}{0.25} = 0.2$ $\qquad$ $p_{Y|X}(y=3|1) = \frac{0.04}{0.25} = 0.16$

(i) $p_{X|Y}(x|2) = \frac{p(x,2)}{p_Y(2)}$

$p_{X|Y}(x=0|2) = \frac{0.11}{0.22} = \frac{1}{2}$ $\qquad$ $p_{X|Y}(x=1|2) = \frac{0.05}{0.22} = \frac{5}{22}$

$p_{X|Y}(x=2|2) = \frac{0.04}{0.22} = \frac{2}{11}$ $\qquad$ $p_{X|Y}(x=3|2) = \frac{0.02}{0.22} = \frac{1}{11}$

(j) $E(Y|X=1) = \sum_y y \cdot p_{Y|X}(y|1)$
$(0)0.36 + (1)0.28 + (2)0.2 + (3)0.16 = 1.16$

(k) $E(X|Y=2) = \sum_x x \cdot p_{X|Y}(x|2)$
$0\frac{1}{2} + 1\frac{5}{22} + 2\frac{2}{11} + 3\frac{1}{11} = \frac{19}{22}$

2. (Navidi 2.6.16)

(a) $P(X > 1 \text{ and } Y > 1) = \displaystyle\int_1^\infty \int_1^\infty xe^{-(x+xy)}\,\mathrm{d}x\mathrm{d}y$

$= \displaystyle\int_1^\infty \left(\frac{-e^{-y-1}y - 2e^{-y-1}}{(-y-1)^2}\right)\mathrm{d}y$

$= \frac{1}{2e^2} \approx 0.06766...$

(b) $f_X(x) = \displaystyle\int_{-\infty}^\infty f_{XY}(x,y)\,\mathrm{d}y$

$= \displaystyle\int_0^\infty xe^{-(x+xy)}\,\mathrm{d}y$

$= -e^{-x-xy}\big|_0^\infty = 0 - (-e^{-x})$

$f_X(x) = e^{-x}$

(c) $f_Y(y) = \displaystyle\int_{-\infty}^\infty f_{XY}(x,y)\,\mathrm{d}x$

$= \displaystyle\int_0^\infty xe^{-(x+xy)}\,\mathrm{d}x$

$= e^{-x-xy}\frac{1}{-1-y}x - e^{-x-xy}\frac{1}{(-1-y)^2}\big|_0^\infty = 0 - (-\frac{1}{(-y-1)^2})$

$f_Y(y) = \frac{1}{(-y-1)^2}$

(d) $f(1,1) = e^(2) = 0.13533..$
$f_X(1) \cdot f_Y(1) = (e^{-1}) \cdot (\frac{1}{(-2)^2}) = \frac{1}{4e} = 0.09196$
$f(x,y) \neq f_X(x) \cdot f_Y(y) \ \therefore NO$