ENGR 222
Assignment 5 Submission

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2. Suppose $T : \mathbb{R}^3 \mapsto \mathbb{R}^4$ is a linear transformation, and

$$T(2,0,-1) = (1,0,2,1)$$
$$T(0,1,1) = (-4,3,1,0)$$
$$T(-3,1,2) = (0,1,-2,0)$$

Find the matrix A so that $T\mathbf{v} = A\mathbf{v}$ for all $\mathbf{v} \in \mathbb{R}^3$

$$A\begin{bmatrix}2\\0\\-1\end{bmatrix} = \begin{bmatrix}1\\0\\2\\1\end{bmatrix},\ A\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}-4\\3\\1\\0\end{bmatrix},\ A\begin{bmatrix}-3\\1\\2\end{bmatrix} = \begin{bmatrix}0\\1\\-2\\0\end{bmatrix}$$

$$A\begin{bmatrix}2&0&-3\\0&1&1\\-1&1&2\end{bmatrix} = \begin{bmatrix}1&-4&0\\0&3&1\\2&1&-2\\1&0&0\end{bmatrix}$$

$$A = \begin{bmatrix}1&-4&0\\0&3&1\\2&1&-2\\1&0&0\end{bmatrix}\begin{bmatrix}2&0&-3\\0&1&1\\-1&1&2\end{bmatrix}^{-1}$$

$$A = \begin{bmatrix}-5&7&-11\\2&-1&4\\1&1&0\\-1&3&-3\end{bmatrix}$$

3. Find the matrix for the projection $\mathbb{R}^3 \mapsto \mathbb{R}^2$ onto yz-plane (i.e., $(x,y,z) \mapsto (y,z)$)

$$x : \begin{bmatrix}1\\0\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\0\end{bmatrix},\ y : \begin{bmatrix}0\\1\\0\end{bmatrix} \mapsto \begin{bmatrix}1\\0\end{bmatrix},\ z : \begin{bmatrix}0\\0\\1\end{bmatrix} \mapsto \begin{bmatrix}0\\1\end{bmatrix}$$

$$A = \begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}$$

4. Find the eigenvalues and eigenvectors for each eigenvalue of the following:

$$\begin{bmatrix}-2&1&-1\\19&-5&4\\43&-13&12\end{bmatrix}$$

```python
from sympy import Matrix
M = Matrix([[-2,1,-1],[19,-5,4],[43,-13,12]])
a = M.eigenvals()
v = M.eigenvects()
```

$$\text{Eigenvalues} : \lambda_1 = -1, \quad \lambda_2 = 2, \qquad \lambda_3 = 4$$

$$\text{Eigenvectors} : \mathbf{v}_1 = \begin{pmatrix}0\\1\\1\end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix}-1\\-1\\3\end{pmatrix},\ \mathbf{v}_3 = \begin{pmatrix}-1\\1\\7\end{pmatrix}$$