ECEN415 Advanced Control Systems Engineering
Assignment Two - 2021

Due 1^{st} of September

Submit pdf documents describing your solutions via the online submission system. Hand written solutions are fine, but these must be scanned and converted to pdf.

Solutions should use standard mathematical notation (not pseudo-matlab), explain your reasoning solutions and include computer generated figures where appropriate. If you do use Matlab routines to derive your answers then you should explain the critical Matlab calls, including input arguments and the pertinent outputs. Your written submitted solutions should not include code unless it is explicitly requested. Note that I will not read your code to understand what you are trying to achieve. Your written answers should contain your complete description of your approach and solutions.

Unless otherwise noted, you can use Matlab (or other suitable tool) to complete these problems. You should upload your code, because if things go wrong I will look at code to try to work out what has happened. (That is, I am not marking your code, but I will use it to try to understand what went wrong.)

Section A - Formative Questions

Questions in this section are intended to be reasonably tractable and are intended to teach you the skills required to do the course, including the questions in section B.

This section comprises 50% of the marks for the assignment, and will be marked only crudely. If you make a reasonable attempt at all the questions you will get full marks for the section. Patchy attempts will yield half marks.

1. A state space system is described by the system matrices

$$A = \begin{bmatrix} 12.5314 & -91.36 & 28.7129 & 59.9628 \\ 21.316 & -115.6631 & 37.5584 & 75.0519 \\ 13.967 & -53.5817 & 15.4161 & 33.4391 \\ 20.5222 & -123.3107 & 42.267 & 79.7156 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.7649 & 1.7649 \\ 2.8318 & 2.8318 \\ 2.2353 & 2.2353 \\ 2.7294 & 2.7294 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.46166 & -4.4635 & 1.9151 & 3.7274 \\ 1.0853 & -12.82 & 4.5753 & 8.3759 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(These matrices are available on the course website so there is no need to enter them by hand).

(a) If the system begins in a state $\boldsymbol{x}(0) = \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}^\mathsf{T}$, use matlab's `lsim` command to plot the response of the system in the time interval $t \in [0, 10]$ s. You should plot the evolution of the system state ($\boldsymbol{x}$) and its output ($\boldsymbol{y}$) as a functions of time.

Hint: BY default, Matlab will plot the system response for you when you use lsim, but make sure that you can extract the required signal from lsim so that you can do other things with them (including plotting them yourself). We will need this later

(b) Replicate the results from above using the matrix exponential to evolve the state, rather than `lsim`.

(c) Use `lsim` to plot the response of the system to the input signal $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^\mathsf{T}$ with

$$u_1 = u(t-1)$$
$$u_2 = 2u(t-5)$$

for u the (Heaviside) unit step function.

(d) Use `c2d` to discretize the system with an appropriate sampling frequency and use `lsim` to plot the response of the discrete time system.

(e) Vary the sampling period and describe the effect on the output.

(f) Find the time response for the discrete time system of there is no input and initial conditions of $\boldsymbol{x} = \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}^\mathsf{T}$. Determine this response using powers of $\boldsymbol{A}$ directly (ie, do not use `lsim`).

2. The phase-shift oscillator circuit shown in figure 1 can be described by the coupled differential equations

$$\dot{v}_{\mathrm{a}} = -\frac{2v_{\mathrm{a}}}{RC} + \frac{v_{\mathrm{b}}}{RC} - \frac{R_2}{R_1}\frac{v_{\mathrm{c}}}{RC}$$
$$\dot{v}_{\mathrm{b}} = \frac{v_{\mathrm{a}}}{RC} - \frac{2v_{\mathrm{b}}}{RC} + \frac{v_{\mathrm{c}}}{RC}$$
$$\dot{v}_{\mathrm{c}} = \frac{v_{\mathrm{b}}}{RC} - \frac{v_{\mathrm{c}}}{RC} - \frac{v_{\mathrm{c}}}{R_1 C}$$

where v_{a},v_{b} and v_{c} are the voltages after the three successive RC stages.

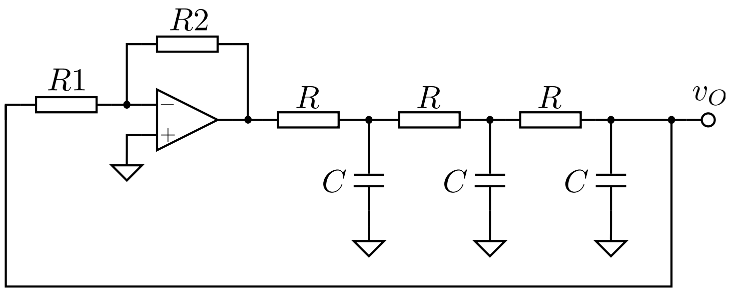

Figure 1: An unbuffered phase-shift oscillator

(a) Write a state space representation for the system.

(b) The ratio $\frac{R2}{R1} \approx 30$ matters if the oscillator is to maintain sustained oscillations. Find the value for this ratio that correctly induces oscillation, making sure that you explain your process.

Hint: You could solve this problem analytically, but don't!

Hint: You should choose some convenient values for R and C, given that the nominal oscillation angular frequency is $\omega_0 = \frac{1}{\sqrt{6}RC}$.

(c) Determine the oscillation frequency of the circuit using the state space model.

(d) Simulate the operation of the oscillator to show that it works as you might expect.

Hint: Think about the initial conditions for the simulation.