ECEN321: Engineering Statistics

Assignment 5

Due: 9:00 a.m., Wednesday 13 May 2020

Bernoulli Distribution

1. (Navidi 4.1.2) A certain brand of dinnerware set comes in three colours: red, white, and blue. Twenty percent of customers order the red set, 45% order the white, and 35% order the blue.

   Let $X = 1$ if a randomly chosen order is for a red set, let $X - 0$ otherwise.

   Let $Y = 1$ if the order is for a white set, let $Y = 0$ otherwise.

   Let $Z = 1$ if the order is for either and red or white set, and let $Z = 0$ otherwise.

   (a) Let p_X denote the success probability for X. Find p_X.

   [1 mark]

   (b) Let p_Y denote the success probability for Y. Find p_Y.

   [1 mark]

   (c) Let p_Z denote the success probability for Z. Find p_Z.

   [1 mark]

   (d) Is it possible for both X and Y to be 1?

   [1 mark]

   (e) Does $p_Z = p_X + p_Y$?

   [1 mark]

   (f) Does $Z = X + Y$? Explain.

   [2 marks]

2. (Navidi 4.1.6) Two dice are rolled. Let $X = 1$ if the dice come up doubles and let $X = 0$ otherwise. Let $Y = 1$ if the sum is 6, and let $Y = 0$ otherwise. Let $Z = 1$ if the dice come up both doubles and with a sum of 6 (that is, double 3), and let $Z = 0$ otherwise.

   (a) Let p_X denote the success probability for X. Find P_X

   [1 mark]

   (b) Let p_Y denote the success probability for Y. Find p_Y

   [1 mark]

   [1 mark]

   (c) Let p_X denote the success probability for Z. Find p_Z

   (d) Are X and Y independent?

   [1 mark]

   (e) Does $p_Z = p_X p_Y$?

   [1 mark]

   (f) Does $Z = XY$? (Show why it is, or why it is not)

   [1 mark]

Binomial Distribution

3. (Navidi 4.2.4) At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

   (a) Find the probability that all 10 of the flights were on time.

   [1 mark]

   (b) Find the probability that exactly eight of the flights were on time.

   [1 mark]

   (c) Find the probability that eight or more of the flights were on time.

   [2 marks]

4. (Navidi 4.2.10) A quality engineer takes a random sample of 100 steel rods from a day's production, and finds that 92 of them meet specifications.

   (a) Estimate the proportion of that day's production that meets specifications, and find the uncertainty in the estimate.

   [2 marks]

   (b) Estimate the number of rods that must be sampled to reduce the uncertainty to 1%.

   [2 marks]

Poisson Distribution

5. (Navidi 4.3.4) Geologists may estimate the time since the most recent cooling of a mineral by counting the number of uranium fission tracks on the surface of the mineral. A certain mineral specimen is of such an age that there should be an average of 6 tracks per cm^2 of surface area. Assume the number of tracks in an area follows a Poisson distribution. Let X represent the number of tracks counted in $1\,\text{cm}^2$ of surface area. Find

   (a) $P(X = 7)$

   [1 mark]

   (b) $P(X \geq 3)$

   [2 marks]

   (c) $P(2 < X < 7)$

   [2 marks]

   (d) μ_x

   [1 mark]

   (e) σ_x

   [1 mark]

6. (Navidi 4.3.10) A chemist wishes to estimate the concentration of particles in a certain suspension. She withdraws $3\,\text{mL}$ of the suspension and counts 48 particles. Estimate the concentration in particles per mL and find the uncertainty in the estimate.

   [2 marks]