

Assignment 8

due: 23:59 pm on Wednesday 5 June 2019, online

1. For each of the following systems $\mathbf{x}' = A\mathbf{x}$, find a fundamental set of solutions (and prove that it actually is one). Write down the general solution for the system.

(a) $\mathbf{x}' = \begin{pmatrix} 10 & -5 \\ 8 & -12 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{pmatrix} 1 & -8 \\ 1 & -3 \end{pmatrix} \mathbf{x}$.

2. Let $A = \begin{pmatrix} 10 & -5 \\ 8 & -12 \end{pmatrix}$. You are given that a fundamental matrix for the system $\mathbf{x}' = A\mathbf{x}$ is

$$F(t) = \begin{pmatrix} e^{2t} & 2e^{6t} \\ e^{2t} & e^{6t} \end{pmatrix}.$$

Find a particular solution of $\mathbf{x}' = \begin{pmatrix} 10 & -5 \\ 8 & -12 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$, and check your answer is correct.

1. For each of the following, find a fundamental set of solutions (and prove that it actually is one).

(a) $\mathbf{x}' = \begin{pmatrix} 10 & -8 \\ 4 & -2 \end{pmatrix} \mathbf{x}$ and find the unique solution satisfying $\mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

(b) $\mathbf{x}' = \begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{pmatrix} 4 & -5 \\ 5 & -4 \end{pmatrix} \mathbf{x}$ (d) $\mathbf{x}' = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix} \mathbf{x}$.
