Assignment 5 due: 23:59 pm Wednesday 15 May, online

1. Use the definition to find the Laplace transform of the function:

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & x > 2 \end{cases}.$$

2. (a) Evaluate directly from the definition the Laplace transform of $\cos ax$. You should get $\dfrac{s}{s^2 + a^2}$.

   (b) Use the Laplace transform of $\sin ax$ and the derivative formula to verify the answer in (a).

   (c) Use a trigonometric identity to find $\mathcal{L}\{\cos^2 x\}$.

3. Find the inverse Laplace transforms:

   (a) $\mathcal{L}^{-1}\left\{\left(\dfrac{2}{s} - \dfrac{1}{s^3}\right)^2\right\}$ (b) $\mathcal{L}^{-1}\left\{\dfrac{s+1}{s^2+2}\right\}$ (c) $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+2)(s^2+4)}\right\}$.

4. Use the Laplace transform to solve the initial value problem $y' + 6y = e^{4x}$ subject to $y(0) = 2$, for $y(x)$.

5. Use the First Translation Theorem to find $\mathcal{L}\{x^3 e^{2x}\}$ and hence solve the initial value problem:

$$y'' - 4y' + 4y = x^3 e^{2x}, \quad y(0) = y'(0) = 0.$$

6. Use the Laplace transform and the First Translation Theorem to solve the following initial value problem:

$$y'' - y' = e^{-x} \cos x, \quad y(0) = y'(0) = 0.$$

Tutorial exercises for Thursday 9 May to Tuesday 14 May

1. Find (from the definition) the Laplace transform of the function:

$$f(x) = \begin{cases} 0 & x < a \\ c & a \le x \le b \\ 0 & x > b \end{cases}$$

   where $0 < a < b$ and $c > 0$.

2. Show that $\mathcal{L}\{e^{ax}\}(s) = \dfrac{1}{s-a}$. (Hint: Simplify the integrand before integrating.) For what values of s is the Laplace transform defined?

3. Find the following inverse Laplace transforms:

   (a) $\mathcal{L}^{-1}\left\{\dfrac{(s+1)^3}{s^4}\right\}$ (b) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3+5s}\right\}$.

4. Use the Laplace transform to solve the initial value problem $y' - y = 2\cos 5x$, $y(0) = 0$ for $y(x)$.

5. Use the First Translation Theorem to find $\mathcal{L}\{e^{-2x}\cos 4x\}$ and $\mathcal{L}^{-1}\{\dfrac{2s+5}{s^2+6s+34}\}$.

6. Use the Laplace transform and the First Translation Theorem to solve the initial value problem $y' + 6y = xe^{-x}$ subject to $y(0) = 2$.
