ECEN321 : Engineering Statistics
Assignment 6 Submission

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Normal Distribution

1. (Navidi 4.5.4 $X\ N(2,9)$. $\mu = 2,\ \sigma = \sqrt{9} = 3$
   Using the probability table for the normal distr.

   (a) $z = \frac{x-\mu}{\sigma} = \frac{2-2}{3} = 0$
   $P(X \geq 2) = P(Z > 0) = 1 - P(Z) = 1 - 0.5 = 0.5$

   (b) $z = \frac{1-2}{3} = -1/3 \approx -0.33$
   $z = \frac{7-2}{3} = 5/3 \approx 1.67$
   $P(1 \leq X < 7) = P(Z < 1.67) - P(Z < -0.33) = 0.5818$

   (c) $z = \frac{-2.5-2}{3} \approx -1.5$
   $z = \frac{-1-2}{3} \approx -1$
   $P(-2.5 \leq X < 1) = P(Z < -1) - P(Z < -1.5) = 0.0919$

   (d) $z = \frac{-3}{3} = -1$
   $z = \frac{3}{3} = 1$
   $P(-3 \leq X - 2 < 3) = P(Z < 1) - P(Z < -1) = 0.6826$

2. (Navidi 4.5.22) $M = 0.5X + Y\ \mu_X = 0.45, \sigma_X = 0.05,\ \mu_Y = 0.25, \sigma_Y = 0.025$

   (a) $\mu_M = a\mu_X + b\mu_Y = 0.5(0.45) + 0.25 = 0.475$
   $\sigma_M = \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2} = \sqrt{0.5^2(0.05^2) + 0.025^2} = 0.035355$

   (b) $z = \frac{0.5-0.475}{0.035355} = 0.707114$
   $P(M > 0.5) = P(Z > 0.707114) = 1 - P(Z < 0.707114) = 0.2389$

Exponential Distribution

3. (Navidi 4.7.10)

   (a) $\mu = \frac{1}{\lambda}, \lambda = \frac{1}{\mu} \approx 0.33$

   (b) For 5 metres: $5\frac{1}{3} = \frac{5}{3}$
   $P(X = 2) = \frac{(5/3)^2 e^{-5/3}}{2!} \approx 0.26$

Estimation

4. (Navidi 4.9.2)

   Answer: (b) how close repeated values of the estimator are to each other

5. (Navidi 4.9.4)

   (a)
$$\sigma_k^2 = ((n-1)s^2)/k,\ \mu_{aX} = a\mu_X,\ \mu_{s^2} = \sigma^2$$
$$\begin{aligned}\text{Bias} &= \mu_{\sigma_K^2} - \sigma^2 \\ &= \mu_{((n-1)s^2)/k} - \sigma^2 \\ &= ((n-1)/k)\mu_{s^2} - \sigma^2 \\ &= ((n-1)/k)\sigma^2 - \sigma^2 \\ &= \frac{n-1-k}{k}\sigma^2\end{aligned}$$

   (b)
$$\sigma_k^2 = ((n-1)s^2)/k,\ \sigma_{aX}^2 = a^2\sigma_X^2,\ sigma_{s^2}^2 = 2\sigma^4/(n-1)$$
$$\begin{aligned}\text{Variance} &= \sigma_{\sigma_k^2}^2 \\ &= \frac{2(n-1)\sigma^4}{k^2}\end{aligned}$$

   (c)
$$\begin{aligned}MSA_\theta &= Bias^2 + Variance \\ &= (\frac{n-1-k}{k}\sigma^2)^2 + \frac{2(n-1)\sigma^4}{k^2} \\ &= \frac{(n-1-k)^2}{k^2}\sigma^4 + \frac{2(n-1)\sigma^4}{k^2} \\ &= \frac{(n^2 - 2kn + 2k + k^2)\sigma^4}{k^2}\end{aligned}$$