ENGR 222
Assignment 2 Submission

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1. **Multivariate Function**

$$f(x,y) = -2x^3 + 3x^2y + 2y^3 - 9y + 5$$

(a)
$$f_x = -6x^2 + 6xy$$
$$f_y = 3x^2 + 6y^2 - 9$$

(b)
$$f_{xy} = 6x$$
$$f_{xx} = -12x + 6y$$
$$f_{yy} = 12y$$

(c)
$$f_x = -6x^2 + 6xy = 0$$
$$f_y = 3x^2 + 6y^2 - 9 = 0$$
by inspection $(x = y = 1, -1)$
$$for\ x = 0,$$
$$f_x = 0$$
$$f_y = 6y^2 - 9 = 0$$
$$\therefore\ y = \sqrt{9/6} = \sqrt{\frac{3}{2}}$$
$$for\ y = 0:$$
$$f_x = -6x^2 = 0$$
$$f_y = 3x^2 - 9 = 0$$
no x

critical points $\Rightarrow [(1,1), (-1,-1), (0, \sqrt{\frac{3}{2}})]$

(d) Second Partials test:
$$D = f_{xx}(0, \sqrt{\frac{3}{2}}) \cdot f_{yy}(0, \sqrt{\frac{3}{2}}) - f_{xy}^2(0, \sqrt{\frac{3}{2}})$$
$$f_{xx} = -12x + 6y, f_{yy} = 12y, f_{xy} = 6x$$
$$D = (-12(0) + 6\left(\sqrt{\frac{3}{2}}\right))(12\left(\sqrt{\frac{3}{2}}\right)) - (6(0))^2$$
$$= (0 + 3\sqrt{6})(6\sqrt{6}) - 0$$
$$= 108$$

$D > 0$ and $f_{xx} > 0$ therefore, this critical point is a local minimum.

2. **Quick questions**

(a) $f(x,y,z) = e^x cos(y)(1-z)^2,\ \mathbf{u} = (0.36, 0.48, 0.8)$
$$D_{\mathbf{u}} = f_x u_1 + f_y u_2 + f_z u_3$$

$$f_x = e^x cos(y)(1-z)^2$$
$$f_x(0,0,0) = 1 \times 1 \times 1 = 1$$
$$f_y = -e^x sin(y)(1-z)^2$$
$$f_y(0,0,0) = -1 \times 0 \times 1 = 0$$
$$f_z = 2e^x cos(y)(z-1)$$
$$f_z(0,0,0) = 2 \times 1 \times -1 = -2$$

$$D_{\mathbf{u}} = 1(0.36) + 0(0.48) + -2(0.8) = -1.24$$

(b) $f(x,y,z) = (1+x)(1-y^2)(1-z)^2,\ \mathbf{p} = (1,2,3)$
$$L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0)$$
$$+ f_y(x_0,y_0,z_0)(y-y_0)$$
$$+ f_z(x_0,y_0,z_0)(z-z_0)$$

$$f(\mathbf{p}) = (1+1)(1-2^2)(1-3)^2 = -24$$

$$f_x = (1-y^2)(1-z)^2$$
$$f_x(\mathbf{p}) = (1-2^2)(1-3)^2 = -12$$
$$f_y = (1+x)(-2y)(1-z)^2$$
$$f_y(\mathbf{p}) = (1+1)(-2(2))(1-3)^2 = -32$$
$$f_z = 2(1+x)(1-y^2)(z-1)$$
$$f_z(\mathbf{p}) = 2(1+1)(1-2^2)(3-1) = -24$$

$$L(\mathbf{p}) = -24 + (-12)(x-1) + (-32)(y-2) + (-24)(z-3)$$
$$= 124 - 12x - 32y - 24z$$

(c) $f(x,y) = e^{-x^2-y^2} = e^{-x^2}e^{-y^2},\ \mathbf{p} = (1,1)$
$$L(x,y) = f(\mathbf{p}) + f_x(\mathbf{p})(x-x_0) + f_y(\mathbf{p})(y-y_0)$$
$$p_2(x,y) = L(x,y) + \frac{1}{2}\left[(x-x_0)^2 f_{xx}(\mathbf{p}) + 2(x-x_0)(y-y_0)f_{xy}(\mathbf{p}) + (y-y_0)^2 f_{yy}(\mathbf{p})\right]$$

$$f_x = -2xe^{-x^2}e^{-y^2}$$
$$= -2xe^{-x^2-y^2}$$
$$f_y = -2ye^{-x^2}e^{-y^2}$$
$$= -2ye^{-x^2-y^2}$$

$$f_{xx} = e^{-y^2}(-2(e^{-x^2}) + -2x(-2xe^{-x^2}))$$
$$= (4x^2-2)e^{-x^2-y^2}$$
$$f_{yy} = (4y^2-2)e^{-x^2-y^2}$$
$$f_{xy} = -2xye^{-x^2-y^2}$$

$$L(\mathbf{p}) =$$
$$p_2(\mathbf{p}) =$$

(d)

(e)

3. **Double integrals**

(a)

(b)

(c)

(d)

(e)

4. **Lab question**

(a) i.

ii.

iii.

(b) i.

ii.

iii.