4.

$$aug = \begin{pmatrix} 1 & -1 & 2 & 4 & \bigg| & 2 \\ 3 & -3 & 1 & 2 & \bigg| & 1 \\ 2 & -1 & 1 & 0 & \bigg| & -1 \\ 2 & -6 & 1 & 10 & \bigg| & 9 \end{pmatrix}$$

Solved using sympy:

```
from sympy import symbols,linsolve, Matrix
x,y,z,w = symbols("x,y,z,w")

aug = Matrix([[1,-1,2,4,2],[3,-3,1,2,1],[2,-1,1,0,-1],[2,-6,1,10,9]])
linsolve(aug,x,y,z,w)
```

$$(x, y, z, w) = (2w - 2, 2w - 2, 1 - 2w, w)$$

5.

$$aug = \begin{pmatrix} 1 & 2 & 2 & 3 & \bigg| & 0 \\ -4 & -8 & -8 & -9 & \bigg| & 0 \\ 2 & 4 & 1 & 0 & \bigg| & 3 \\ 1 & 2 & -2 & -4 & \bigg| & 4 \end{pmatrix}$$

Solved using sympy:

```
from sympy import symbols,linsolve, Matrix
x,y,z,w = symbols("x,y,z,w")

aug = Matrix([[1,2,2,3,0],[-4,-8,-8,-9,0],[2,4,1,0,3],[1,2,-2,-4,4]])
linsolve(aug,x,y,z,w)
```

$$(x, y, z, w) = (2 - 2y, y, -1, 0)$$

7. $aCH_4 + bO_2 \rightarrow cCO_2 + dH_2O$

$$C : a = c \rightarrow 1a + 0b - 1c + 0d = 0$$
$$H : 4a = 2d \rightarrow 4a + 0b + 0c - 2d = 0$$
$$O : 2b = 2c + d \rightarrow 0a + 2b - 2c + 1d = 0$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(a, b, c, d) = (1, 2, 1, 2)$$

8.

$$x : (1, 0) \rightarrow (-1, 0.3)$$
$$y : (0, 1) \rightarrow (0.2, 0.5)$$
$$A = \begin{pmatrix} -1 & 0.2 \\ 0.3 & 0.5 \end{pmatrix}$$

10. Steps:

- Counter clockwise rotation of 20°
- reflection about x-axis
- Clockwise rotation of 20°

$$cos(20) = 0.93969, \; sin(20) = 0.34202$$

$$\begin{pmatrix} cos(20) & -sin(20) \\ sin(20) & cos(20) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} cos(20) & sin(20) \\ -sin(20) & cos(20) \end{pmatrix}$$
$$= \begin{pmatrix} 0.93969 & -0.34202 \\ 0.34202 & 0.93969 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0.93969 & 0.34202 \\ -0.34202 & 0.93969 \end{pmatrix}$$
$$= \begin{pmatrix} 0.7660 & 0.6428 \\ 0.6428 & -0.7660 \end{pmatrix}$$