ECEN321: Engineering Statistics
Assignment 2 Submission

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Counting Methods

1. (Navidi 2.2.10)

   $\frac{N!}{K_1!K_2!K_3!} = \frac{15!}{6!5!4!} = 630,630$

Conditional Probability

2. (Navidi 2.3.10)
   $P(A \cap N^c) = 0.2 \;\; P(A^c \cap N) = 0.7 \;\; P(A \cap N) = 0.1$

   (a) $P(A) = P(A \cap N^c) + P(A \cap N) = 0.3$

   (b) $P(N) = P(A^c \cap N) + P(A \cap N) = 0.8$

   (c) $P(N|A) = \frac{P(A \cap N)}{P(A)} = \frac{0.1}{0.3} = 0.33\bar{3}$

   (d) $P(A|N) = \frac{P(A \cap N)}{P(N)} = \frac{0.1}{0.8} = 0.125$

   (e) $P(N^c|A) = \frac{P(A \cap N^c)}{P(A)} = \frac{0.2}{0.3} = 0.667$

   (f) $P(A^c|N) = \frac{P(A^c \cap N)}{P(N)} = \frac{0.7}{0.8} = 0.875$

3. (Navidi 2.3.20)
   $P(G) = 0.7 \;\; P(M) = 0.2 \;\; P(P) = 0.1$
   $P(A|G) = 0.005 \;\; P(A|M) = 0.01 \;\; P(A|P) = 0.025$

   (a) $P(G \cap A) = P(A|G) * P(G) = 0.005 * 0.7 = 0.0035$

   (b)

   $$\begin{aligned} P(A) &= P(G \cap A) + P(M \cap A) + P(P \cap A) \\ &= P(A|G) * P(G) + P(A|M) * P(M) + P(A|P) * P(P) \\ &= 0.005 * 0.7 + 0.01 * 0.2 + 0.025 * 0.1 \\ P(A) &= 0.008 \end{aligned}$$

   (c) $P(G|A) = \frac{P(G \cap A)}{P(A)} = \frac{0.0035}{0.008} = 0.4375$

4. (Navidi 2.3.32)
$P(f) = 0.0002 \;\; P(P^c|f) = 0.995 \;\; P(P|f^c) = 0.99$

   (a)
$$\begin{aligned}
P(f|P^c) &= \frac{P(P^c|f)P(f)}{P(P^c|f)P(f) + P(P^c|f^c)P(f^c)} \\
&= \frac{P(P^c|f)P(f)}{P(P^c|f)P(f) + [1 - P(P|f^c)][1 - P(f)]} \\
&= \frac{(0.995)(0.0002)}{(0.995)(0.0002) + [1 - 0.99][1 - 0.0002]} \\
P(f|P^c) &= 0.01952
\end{aligned}$$

   (b) i. If a bottle fails, it has a probability of having a flaw of 0.01952

   (c)
$$\begin{aligned}
P(f^c|P) &= \frac{P(P|f^c)P(f^c)}{P(P|f^c)P(f^c) + P(P|f)P(f)} \\
&= \frac{P(P|f^c)P(f^c)}{P(P|f^c)P(f^c) + [1 - P(P^c|f)]P(f)} \\
&= \frac{(0.99)(0.9998)}{(0.99)(0.9998) + (1 - 0.995)(0.0002)} \\
P(f^c|P) &= 0.999998989698
\end{aligned}$$

   (d) ii. Given that the bottle passed the inspection it has a probability of being flawless of 0.999998989698

   (e) Having a high false positive rate of fault detection (a) is fine, as long as the chance of the inspection passing only a flawless bottle is high (c)

5. (Navidi 2.4.2)

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.1	0.05

   (a) $P(X \leq 2) = \sum_{n\leq 2} P(X = n) = 0.4 + 0.3 + 0.15 = 0.85$
   (b) $P(X > 1) = \sum_{n>1} P(X = n) = 0.15 + 0.1 + 0.05 = 0.3$
   (c) $\mu_X = \sum_x xP(X = x) = 0(0.4) + 1(0.3) + 2(0.15) + 3(0.1) + 4(0.05) = 1.1$
   (d) $\sigma_X^2 = (\sum_x x^2 P(X = x)) - \mu_X^2 = (0^2(0.4) + 1^2(0.3) + 2^2(0.15) + 3^2(0.1) + 4^2(0.05)) - 1.1^2 = 1.39$

6. (Navidi 2.4.14)

   (a) $P(X > 25) = \int_{25}^{30} \frac{x}{250}\,\mathrm{d}x = \left.\frac{x^2}{500}\right|_{25}^{30} = 0.55$
   (b) $\mu_X = \int_{20}^{30} \frac{x^2}{250}\,\mathrm{d}x = \left.\frac{x^3}{750}\right|_{20}^{30} = 25.33\bar{3}$
   (c) $\sigma_X^2 = \int_{20}^{30} \frac{x^3}{250}\,\mathrm{d}x - \mu_X^2 = \left.\frac{x^4}{1000}\right|_{20}^{30} - (25.33\bar{3})^2 = 8.22\bar{2}$
   (d) $\sigma_X = 2.867442$

(e)

$$F(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t$$

$$x < 20, F(x) = \int_{-\infty}^{x} 0\,\mathrm{d}t = 0$$

$$\begin{aligned}
20 \le x < 30, F(x) &= \int_{-\infty}^{20} 0\,\mathrm{d} + \int_{20}^{x} \frac{t}{250}\,\mathrm{d}t \\
&= 0 + \frac{t^2}{500}\bigg|_{20}^{x} \\
F(x) &= \frac{x^2}{500} - \frac{4}{5}
\end{aligned}$$

$$\begin{aligned}
x \ge 30, F(x) &= \int_{-\infty}^{20} 0\,\mathrm{d} + \int_{20}^{30} \frac{t}{250}\,\mathrm{d}t + \int_{30}^{x} 0\,\mathrm{d}t \\
&= 0 + \frac{t^2}{500}\bigg|_{20}^{30} + 0 \\
&= 0 + 1 + 0 \\
F(x) &= 1
\end{aligned}$$

(f) $P(X > 28) = \int_{28}^{30} \frac{x}{250}\,\mathrm{d}x = \frac{x^2}{500}\bigg|_{28}^{30} = 0.232$

Linear Functions of Random Variables

7. (Navidi 2.5.6)

   $\sigma = 7 \times 10^{-15}$

   $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = \frac{7\times10^{-15}}{\sqrt{2}} = 4.9497475 \times 10^{-15}$