Question 1. System Properties [15 marks]

A linear system H has the input-output pairs shown in Figure 1 below.

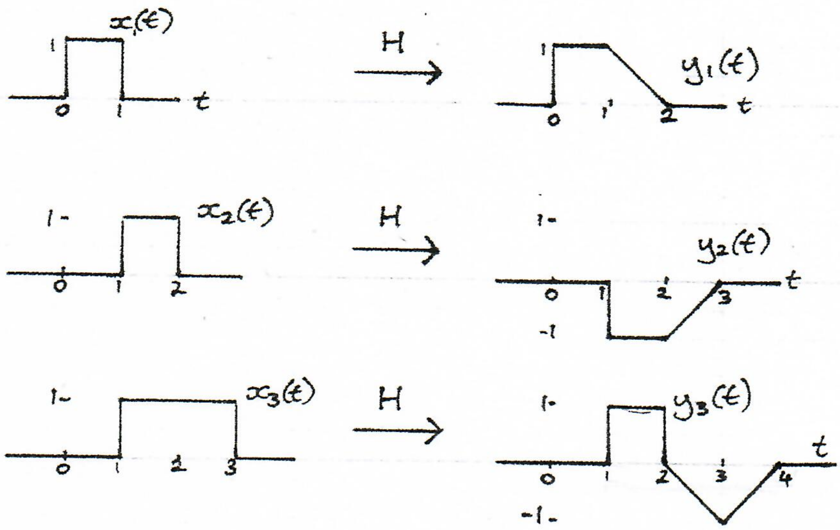


Figure 1: Input/Output signal pairs for system H.

(a) Determine the following. *You must justify your answer to obtain credit.*

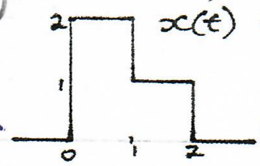
(i) [3 marks] Is the system time invariant? NO $y_2(t+1) \neq y_1(t)$

(ii) [3 marks] Is the system causal? Yes ∵ output is non-zero after input is non-zero.

(iii) [3 marks] Is the system memoryless? NO : output is not a scaled version of the input.

(b) [6 marks] Find and sketch the output, $y(t)$, to the input signal $x(t)$ depicted in Figure 2. *You must explain your approach.*

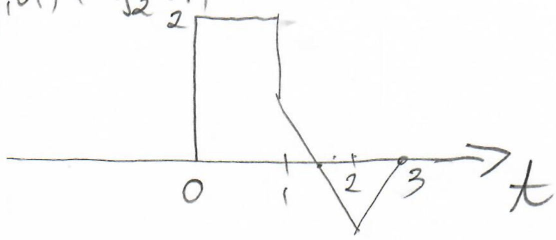
Invoke linearity →



$= 2x_1(t) + x_2(t)$

Figure 2: Input signal for Question 1b.

$y(t) = 2y_1(t) + y_2(t)$



Question 2. Impulse Response of LTI Systems [15 marks]

(a) [3 marks] What is meant by an *impulse response* of an LTI system?

(b) [3 marks] Consider a system where the relationship between an input $x(t)$ and output $y(t)$ is governed by

$$y(t) = x(t-1) - 3x(t+2).$$

Solve for and sketch the impulse response, $h(t)$, of this system.

(c) [9 marks] Consider another system where the input signal

$$x(t) = u(t+0.5) - u(t-0.5)$$

is convolved with the impulse response

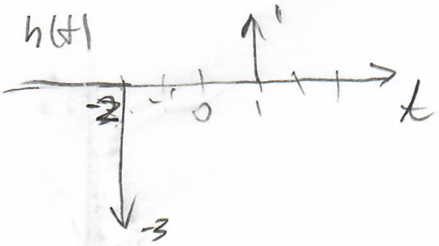
$$h(t) = e^{j\omega_0 t}$$

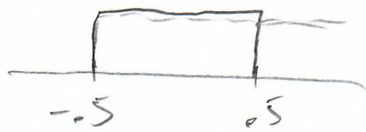
Solve for the output, $y(t)$, of this system. Simplify the final answer as much as possible.

a) Impulse response is the output of a system when the input is an impulse. It is a way to describe the behaviour of an LTI system.

b) By definition $x(t) = \delta(t)$

$$h(t) = \delta(t-1) - 3\delta(t+2)$$





c)
$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} [u(\tau+0.5) - u(\tau-0.5)]\, e^{j\omega_0 (t-\tau)}\, d\tau$$
$$(e^{j\omega_0 t}\, e^{-j\omega_0 \tau})$$
$$= e^{j\omega_0 t} \int_{-0.5}^{0.5} e^{-j\omega_0 \tau}\, d\tau$$
$$= \frac{2}{\omega_0} \sin\left(\frac{\omega_0}{2}\right) e^{j\omega_0 t}$$

2. Continuous Time Fourier Series **(15 marks)**

Consider the periodic signal $x(t)$ shown in fig. 1.

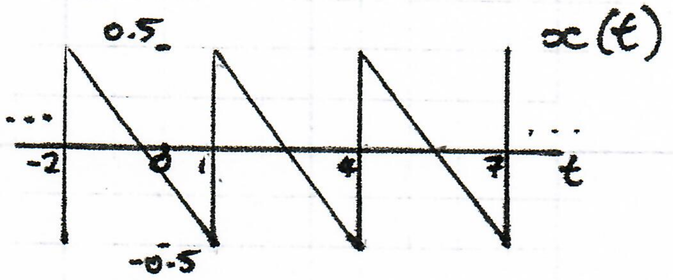


Figure 1: $x(t)$.

(a) By studying fig. 1 determine a_0, the zeroth coefficient in the Fourier series of $x(t)$. Explain your reasoning.

a_0 is the avg. value of $x(t)$ over one period

$\therefore a_0 = 0$

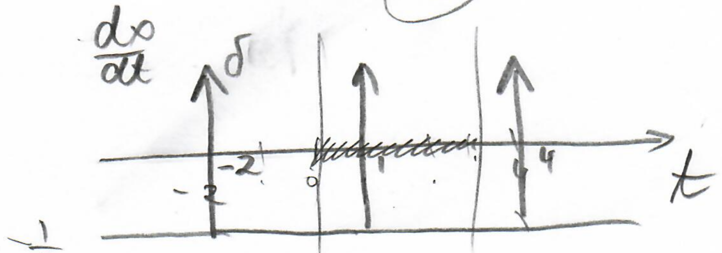
(2 marks)

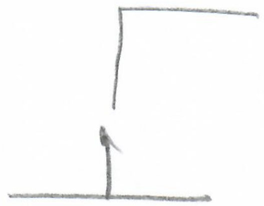
(b) Sketch $\frac{dx}{dt}$, i.e. the first derivative of $x(t)$.

(3 marks)

(c) Find the Fourier series of $\frac{dx}{dt}$.

(6 marks)





c) $a_k = \frac{1}{T}\int_T \frac{dx}{dt} e^{-jk\omega_0 t}\, dt$ $\qquad \omega_0 = \frac{2\pi}{3}$

$= \frac{1}{3}\int_0^3 \left(-\frac{1}{3} + \delta(t-1)\right) e^{-jk\omega_0 t}\, dt$

$= \frac{1}{3}\int_0^3 \delta(t-1) e^{-jk\omega_0 t}\, dt - \frac{1}{9}\int_0^3 e^{-jk\omega_0 t}\, dt$

$= \frac{1}{3} e^{-jk\omega_0}$

$k \neq 0$

$a_0 = 0$

$\frac{dx(t)}{dt} = 0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{1}{3} e^{-jk\omega_0} e^{jk\omega_0 t}$ $\qquad \sum_{-\infty}^{-1} \qquad \sum_{1}^{\infty}$

d) find FS of $x(t)$

from FS property integration

let b_k be the FS coeff of $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

$$\underline{\underline{b_k = \frac{a_k}{jk\omega_0}}}$$

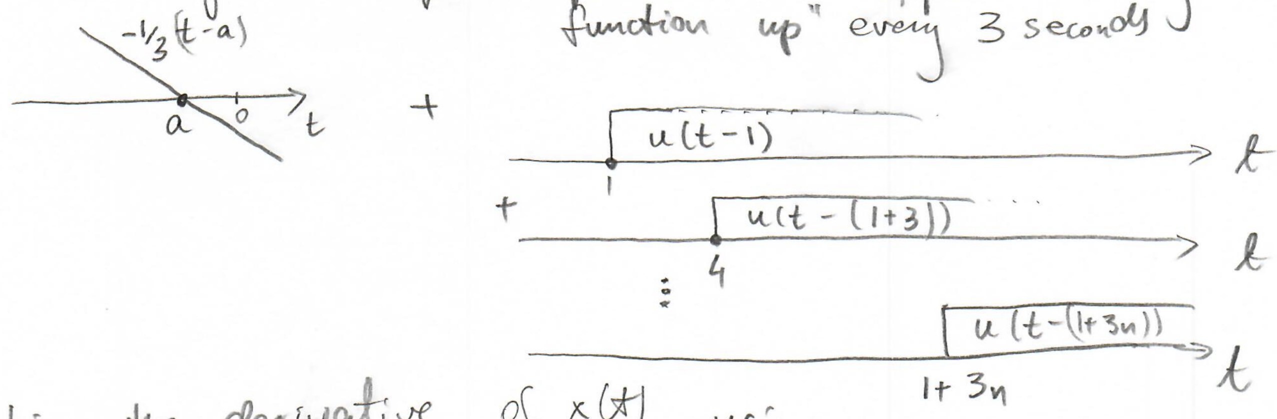
Addendum to part b)

$x(t)$ can be expressed

$$x(t) = -\frac{1}{3}(t-a) + \sum_{n=\infty}^{a} u(t-(1+3n))$$

for some a to give the x-intercept

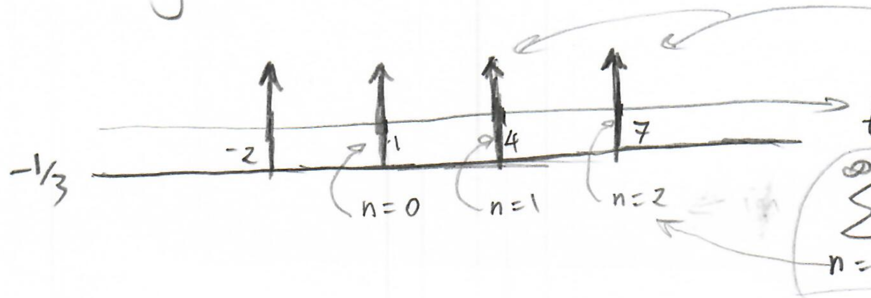
ie a straight line plus periodic unit steps to "bring the function up" every 3 seconds



taking the derivative of $x(t)$ using the above expression gives

$$\frac{dx}{dt} = -\frac{1}{3} + \sum_{n=\infty}^{\infty} \delta(t-(1+3n))$$

plotting this gives us the answer we obtained by 'eye-balling'



$$\sum_{n=\infty}^{\infty} \delta(t-(1+3n))$$