MATH244 Test

During lecture, 19 April 2018

Surname:
First Name:
Student Number:

Please use the spaces provided in this test booklet next to the questions, to give your answers. Attempt all FOUR questions on pages 2–9. You have 45 minutes. You may use page 10 for rough working, and the reverse sides of all pages. Please show all working. Silent calculators may be used, provided you have cleared any alpha-numeric memory. Please indicate where you have used a calculator. A table of formulae is provided with this Test.

Page totals, for marking use only

Page	**Mark**	**max**
p.2		10
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p.4		2
p.5		3
p.6		6
p.7		4
p.8		6
p.9		4
Total		**40**

1. [10]

Consider the differential equation

$$xy' + 2y = \sin x \ .$$

(a) [2]

Without solving, state the largest interval on which a unique solution with initial condition $y(\pi/2) = 0$ exists.

(b) [7]

Find the general solution to the above differential equation, in explicit form.

(c) [1]

If the initial condition is $y(\pi/2) = 0$, solve the resulting initial value problem.

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2. [10]

(a) [5]

Find a value for b such that the following differential equation is exact, and solve it using that value of b:

$$xy^2 + bx^2y + (x + y)\, x^2\, y' = 0\ .$$

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(b) [2]

Consider the first-order differential equation

$$M(x,y) + N(x,y)\,\frac{dy}{dx} = 0\ .$$

Show that if

$$\frac{N_x - M_y}{M} = Q$$

and Q is a function of y only, then the DE has an integrating factor

$$\mu = \exp\left(\int Q(y)\,dy\right)$$

/ 2

(c) [3]

Find an integrating factor that makes the following equation exact, then solve it:

$$1 + \left(\frac{x}{y} - \sin y\right)\frac{dy}{dx} = 0 \, .$$

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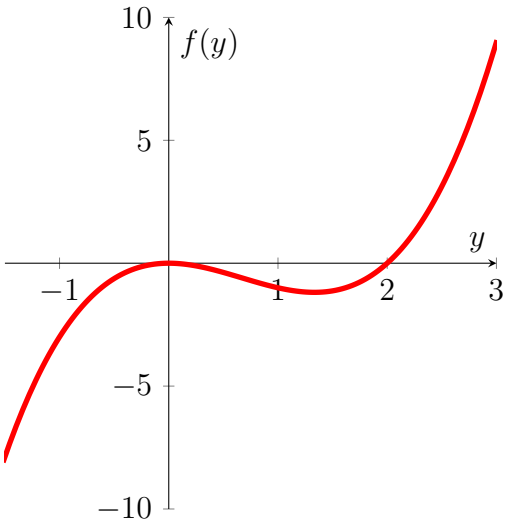
3. [10]

Consider the first-order autonomous differential equation

$$\frac{dy}{dt} = f(y) \ ,$$

where $f(y)$ has just two roots, zero and two, and is as sketched in the figure below:



(a) [2]

State the equilibrium points.

(b) [4]

Draw a phase line, showing equilibrium points and directions that solutions move as t increases.

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(c) [2]

State the stability of each equilibrium point.

(d) [2]

Consider the initial value problem which consists of our autonomous differential equation $dy/dt = f(y)$ with $f(y)$ as sketched at the beginning of Question 3, together with $y(0) = 1$. Describe in words what happens to the solution as $t \to +\infty$.

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4. [10]

(a) A graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming the interest is compounded continuously, and the graduate makes continuous payments at a constant annual rate k, it may be useful to recall that the amount of money $S(t)$ owed satisfies the separable differential equation

$$S'(t) = rS - k \ ,$$

where r is the interest rate and payments are made at a rate k.

i. [5]

Determine the required rate k to have the loan paid off in three years.

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ii. [1]

Calculate the total amount of interest paid.

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(b) [2]

Determine an interval in which a solution of the IVP is sure to exist, without actually solving the IVP:

$$(4 - t^2)y' + 2ty = 3t^2 \ , \quad y(-1) = \pi \ .$$

(c) [2]

Can $y = (\sin t)^2$ be a solution on an interval containing $t = 0$ for the DE

$$y'' + p(t)y' + q(t)y = 0$$

if the coefficient functions are continuous everywhere? Explain your answer.

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****End of Questions****

Use this page and the other side for rough working if needed.