Math 244 ~~Ass~~ Ass 6 Daniel Eize 300147549

1) $y'' - 2y' + y = e^{2x}$, $y(0) = 1$, $y'(0) = 0$

1. $s^2 Y - s y(0) - y'(0) = s^2 Y - s$
2. $sY - y(0) = sY - 1$
3. $= Y$
4. $\mathcal{L}(e^{2x}) = \frac{1}{s-2}$

$$\Rightarrow s^2 Y - s - 2sY + 2 + Y = \frac{1}{s-2}$$

$$Y(s^2 - 2s + 1) = \frac{1}{s-2} + s - 2$$

$$Y = \frac{1}{(s-2)(s-1)^2} + \frac{s}{(s-1)^2} - \frac{2}{(s-1)^2}$$

$$\frac{1}{(s-2)(s-1)^2} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$1 = A(s-1)^2 + B(s-2)(s-1) + C(s-2)$$

$$1 = As^2 - 2As + A + Bs^2 - Bs - 2Bs + 2B + Cs - 2C$$

s^2: $0 = A + B \Rightarrow A = -B$

$s^2: \cancel{0 = A + B} \Rightarrow \cancel{A = -B}$

$s^1: \cancel{0 = -2A - 3B + C}$

$s^0: \cancel{1 = A + 2B - 2C}$

1. contd) C by cover up : $s = 1$, $1 = C(-1)$, $C = -1$

A by cover up : $s = 2$, $1 = A(1)^2$, $A = 1$

B : $1 = 1(s-1)^2 + B(s-2)(s-1) - (s-2)$

$1 = s^2 - 2s + 1 + Bs^2 - 3Bs + 2B - s + 2$

$B = -1$

$$y = \frac{1}{s-2} - \frac{1}{s-1} - \frac{1}{(s-1)^2} + \frac{s}{(s-1)^2} - \frac{2}{(s-1)^2}$$

$$y = e^{2x} - e^{x} - xe^{x} + \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right) - 2xe^{x}$$

$$y = e^{2x} \cancel{- e^{x}} \cancel{- xe^{x}} \cancel{+ e^{x}} \cancel{+ xe^{x}} - 2xe^{x}$$

$$y = e^{2x} - 2xe^{x}$$

check? $y' = 2e^{2x} - 2e^{x} - 2xe^{x}$

$y'' = 4e^{2x} - 4e^{x} - 2xe^{x}$ ✓

LHS $\Rightarrow \cancel{4e^{2x}} - \cancel{4e^{x}} - \cancel{2xe^{x}} - \cancel{4e^{2x}} + \cancel{4e^{x}} + \cancel{4xe^{x}} + e^{2x} - \cancel{2xe^{x}}$

it's correct

2. a) $\mathcal{L}\{x\,u(x-2)\}$

$= \mathcal{L}\{(x-2)+2\ u(x-2)\}$

$= e^{-2x}\left(\mathcal{L}(x)(s) + \mathcal{L}\,2\,(s)\right)$

$= e^{-2x}\left(\frac{1}{s^2} + \frac{2}{s}\right)$

2. b) $\mathcal{L}^{-1}\left(\frac{e^{-\pi s}}{s^2+1}\right) \Rightarrow \mathcal{L}^{-1}\left(e^{-\pi s}\cdot\frac{1}{s^2+1}\right)$

2nd T: $= \sin(x-\pi)\,u(x-\pi)$

2. c) $\mathcal{L}^{-1}\left(\frac{e^{-\pi s}}{s^2+2s+2}\right) = \mathcal{L}^{-1}\left(e^{-\pi s}\cdot\frac{1}{s^2+2s+2}\right)$

$= \mathcal{L}^{-1}\left(e^{-\pi s}\cdot\frac{1}{(s+1)^2+1}\right)$

1st + 2nd T: $e^{-x}\sin(x-\pi)\,u(x-\pi)$

3. $f(x) = 1u(x) - 2u(x-1)$

2nd F: $e^{0s}\mathcal{L}(1)(s) - e^{-1s}\mathcal{L}(2)(s)$

$\mathcal{L}$ RHS $= \frac{1}{s} - \frac{2e^{-s}}{s}$

$\mathcal{L}(y'+y) = sY + Y$

$\therefore Y(s+1) = \frac{1}{s} - 2e^{-s}\frac{1}{s}$

$Y = \frac{1}{s(s+1)} - 2e^{-s}\frac{1}{s(s+1)}$, $\frac{1}{s(s+1)} = \frac{a}{s} + \frac{b}{s+1}$

compare: $a = 1$, $b = -1$

$\therefore Y = \frac{1}{s} - \frac{1}{s+1} - \frac{2e^{-s}}{s} + \frac{2e^{-s}}{s+1}$

$y = 1 - e^{-x} - 2u(x-1) + 2(e^{-x}u(x-1))$

$$y = \begin{cases} 1-e^{-x} & : x<1 \\ e^{-x}-1 & : x \geq 1 \end{cases}$$

4b) $\mathcal{L}^{-1}\left(\frac{1}{s^2(s-1)}\right) \Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s} \cdot \frac{1}{s(s-1)}\right)$

partial f: $\frac{1}{s(s-1)} = \frac{a}{s} + \frac{b}{s-1}$, $1 = as - a + bs$

ie, $-a = 1$, $a = -1$

$a = -b$, $b = 1$

so $\Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}\left(\mathcal{L}^{-1}\left(\frac{1}{s-1} - \frac{1}{s}\right)\right)\right)$

$$= \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}\left(e^x - 1\right)\right) = \int_0^x e^\tau - 1 \, d\tau$$

$$= \left[e^x - x\right] - \left[e^0 - 0\right]$$

$$= e^x - x - 1$$

Math 2414 Ass 6

1 2 3 4

1) $y'' - 2y' + y =$

1. $s^2Y - sy(0) - y'$

2. $sY - y(0)$

3.

4. $\mathcal{L}(e^{2x})$ $\frac{1}{s-2}$

$\Rightarrow s^2Y - s - 2sY + \quad = \frac{1}{s-2}$

$Y(s^2 - 2s + 1)$ $\frac{1}{s-2} + s$

$Y = \frac{1}{(s-2)(s-1)^2}$ $\quad 1)^2 - \frac{2}{(s-1)^2}$

$\frac{1}{(s-2)(s-1)^2} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$