School of Mathematics and Statistics
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ENGR 222 **Assignment 2** **Due: Thursday 18 March 11:59pm**

1 The following questions are concerned with the function

$$f(x,y) = -2x^3 + 3x^2y + 2y^3 - 9y + 5$$

(a) Determine the first order partial derivative of $f(x,y)$.

(b) Determine the second order partial derivatives of $f(x,y)$.

(c) Find all of the critical points of $f(x,y)$.
Note: you don't need to classify them.

(d) Classify the critical point $(0, \sqrt{3/2})$.

2 Quick questions

(a) Determine the directional derivative of $f(x,y,z) = e^x \cos(y)(1-z)^2$ at the origin in the direction $\mathbf{u} = (0.36, 0.48, 0.8)$ (which is a unit vector).

(b) Determine the local linear approximation of $f(x,y,z) = (1+x)(1-y^2)(1-z)^2$ at the point $(1,2,3)$.

(c) Determine the 2nd degree Taylor polynomial of $f(x,y) = e^{-x^2-y^2}$ at the point $(1,1)$.

(d) Consider examining the value of $f(x,y) = x^3+y^3-4x-2y+1$ along the curve $(x(t), y(t)) = (t^3-2t, t^2)$. Determine the gradient of f along this curve when $t=1$.

(e) Consider the surface described by $z = x^2 + xy - y^4$. Determine the equation describing the tangent plane to the surface at the point $(x,y) = (2,1)$.

3 Double integrals

(a) Determine the integral of $f(x,y) = e^{-x}\cos(y)$ over the rectangular region $R = \{(x,y) : x \in [0,2]\,,\ y \in [-\pi/2, \pi/2]\}$.

(b) Determine the integral of $f(x,y) = \sin(x+y)$ over the triangular region for which $x \geq 0$, $y \geq 0$ and $x + y \leq \pi$.

(c) Determine the area of the region $R = \{(x,y) : e^{y/3} \leq x \leq 10 + \sin(y)\,,\ y \in [0,5]\}$ using a double integral.

(d) Determine the average of $f(x,y) = 3y-2x$ over the region $R = \{(x,y) : 0 \leq y \leq 4-x^2\,,\ x \in [-2,2]\}$.

(e) Determine the surface area of the surface described by $z = \sqrt{9-x^2}$ over the region $R = \{(x,y) : 0 \leq y \leq x\,,\ x \in [0,3]\}\}$.

4 Lab question:

This question will involve some Python coding, please include relevant code as part of your submission (copy and paste the code and any plots, or use a screenshot if needed). If you're not sure what needs to be included please ask. You should not need to include more than 100 lines of code, and ideally only around half of that.

(a) In this question you will investigate the numerical approximation of a derivative.

(i) Consider estimating $\frac{df}{dx}$ for the function $f(x) = e^{\cos(\pi x^2)}$ at $x = 1/\sqrt{2}$.
It may be useful for you to know that $\frac{df}{dx} = -\sqrt{2}\pi$ (you don't need to show this).
Estimate the derivative via

$$\frac{df}{dx} \approx \frac{f(x+h) - f(x)}{h}$$

for several different h to produce a plot of the error of the approximation vs h (similar to the Lab02 notebook).
From this plot, determine roughly the value of h which gives the best estimate of the derivative.

(ii) Repeat the previous question but instead using the numerical formula

$$\frac{df}{dx} \approx \frac{f(x+h) - f(x-h)}{2h}.$$

(iii) Estimate the second derivative of $f(x)$ at $x = 1/\sqrt{2}$ via the formula

$$\frac{d^2 f}{dx^2} \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

As before, produce a plot of the error vs h (noting the exact answer is $2\pi(\pi - 1)$, again you aren't expected to derive this), and comment on what the best h appears to be and the magnitude of the error for this particular h.

(b) In this question you will investigate the numerical approximation of an integral.

(i) Consider the integral

$$\int_0^{10} x e^{-\sqrt{x}}\, dx\,.$$

Estimate its value using the trapezoidal rule by breaking the interval into $n = 10$ sub-intervals having equal width.
Repeat for $n = 20, 40, 80, 160$ and based on the results estimate the true value of the integral rounded to 3 decimal places.

(ii) Repeat the previous part question but using Simpson's rule instead.
How many correct decimal digits are you comfortable estimating based on these results?

(iii) Use the quad function from scipy.integrate to estimate the value of the integrals

$$\int_0^{10} x e^{-\sqrt{x}}\, dx\,, \qquad \int_0^{100} x e^{-\sqrt{x}}\, dx\,, \qquad \int_0^{1000} x e^{-\sqrt{x}}\, dx\,.$$

What value does the result appear to converge to?
Lastly use quad to directly estimate the integral

$$\int_0^{\infty} x e^{-\sqrt{x}}\, dx\,.$$