# Control of a Motorised Pendulum

## Lab 2: The transfer function of a DC motor.

### 1. Previous - Identification of sub systems from Lab 1

In the first lab you have tried to model this relatively complex system by breaking it down into smaller sub-systems. There are a number of ways in which this can be done (and not one single absolutely correct answer). One possible model for these sub-systems is shown below:

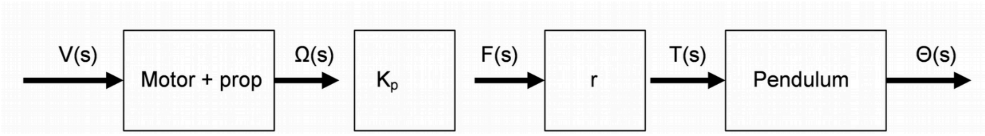



Figure 1: Block diagram of the system model.

1. The electric motor, together with the inertia and damping of the propeller will reach a certain angular velocity after application of a voltage to the motor.
2. Due to this angular velocity of the prop, together with its shape and size, it will produce a certain thrust (force).
3. This force will be converted to a torque by the length of the pendulum arm.
4. The torque on the pendulum will produce an angular displacement of the pendulum.

We can now model each of these subsystems and attempt to measure (estimate) the system characteristics of each.

### 2. Deriving motor transfer function

The block diagram representation of the standard model for an armature controlled, permanent magnet DC motor is given by [1]:

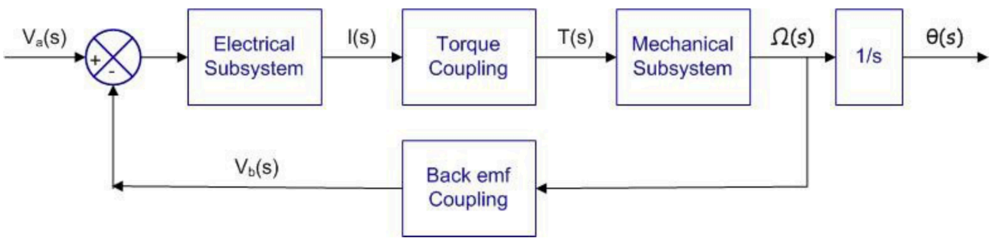
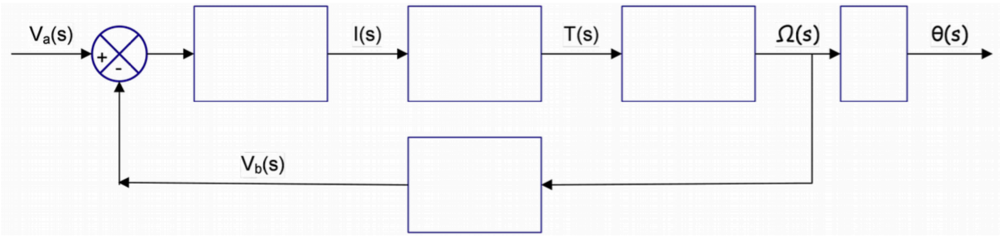



Figure 2: Sub-systems that can be identified in the electric motor.

(a) For each of these blocks now write down the transfer that will describe the input-output characteristics of that block.




(b) By reducing down the block diagram, show that the transfer function, relating angular velocity, $\Omega_m$, to input voltage, $V_a$, of the motor will then be given by:

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{\frac{Kt}{J_m L_a}}{s^2 + \frac{J_m R_a + D_m L_a}{J_m L_a} s + \frac{R_a D_m + K_t K_b}{J_m L_a}}$$

where:

$L_a$ is the armature inductance,
$R_a$ is the armature resistance,
$K_t$ is the torque constant,
$J_m$ is the load inertia,
$D_m$ is the damping coefficient of the load and
$K_b$ is the back emf constant.

### 3. Simulation of the motor response

(a) The table below has been partially completed with estimated values from previous years. Use these variables to express the motor transfer function:

| Motor Parameters | Average Value |
|---|---|
| $R_a$ $(\Omega)$ | 6.3 |
| $L_a$ $(H)$ | 0.797 |
| $K_b$ $(V.s/rad) = K_t$ $(N.m/A)$ | 0.0043 |
| $D_m$ $(N.m.s/rad)$ | 0.00000553 |
| $J_m$ $(kgm^2)$ | 0.00000241 |
| | |
| $K_t/(J_m L_a)$ | |
| | |
| $(J_m R_a + D_m L_a)/J_m L_a$ | |
| | |
| $(R_a D_m + K_b K_t)/(J_m L_a)$ | |

(b) In matlab, simulate the response of this transfer to a unit step in the input voltage. Also do the simulation for steps of 2, 3, 4, 5 and 6V in the voltage. For each case calculate the time constant of the system, the settling time, steady state value of $\omega$ as well as the steady state gain. What is the unit for the steady state gain?

## References

[1] Understanding small brushed DC motors, G.J. Gouws, August 2008