

Ecen 315 Chas 4 Damod Eun

Root locus

$$1. G(s) = \frac{s^2 - 3s + 2}{s^2 + 7s + 12}$$

$$Z : 2, 1$$

$$P : -3, -4$$

$P - Z = \emptyset \leftarrow$ no infinches, no conjugate pairs

Breakaway:

$$\text{#) } K = -\frac{s^2 + 7s + 12}{s^2 - 3s + 2} \quad \frac{dK}{ds} = \frac{(2s+7)(s^2 - 3s + 2) - (s^2 + 7s + 12)(2s-3)}{(s^2 - 3s + 2)^2}$$

$$\Rightarrow (2s^3 - 6s^2 + 17s + 11s^2 - 21s + 14) - (2s^3 + 16s^2 + 24s - 7s^2 - 21s - 36)$$

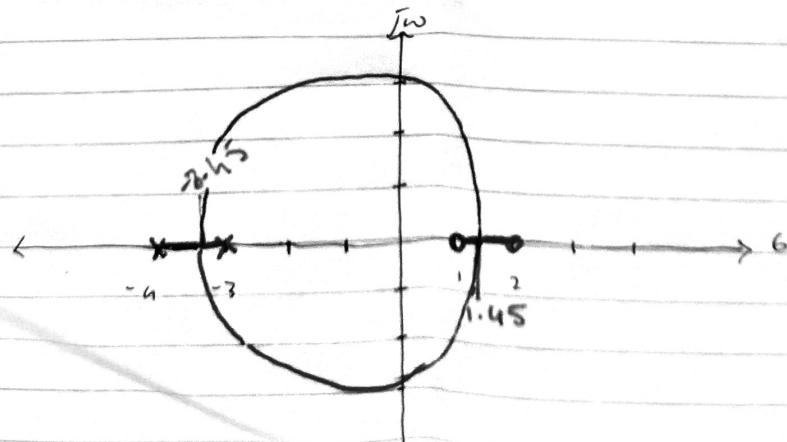
$$= -10s^2 - 20s + 50 \Rightarrow 10s^2 + 20s - 50 = 0 \\ s = (1.45, -3.45)$$

$$\text{#) } \frac{1}{s-2} + \frac{1}{s-1} = \frac{1}{s+3} + \frac{1}{s+4}$$

$$\frac{1}{s-2} + \frac{1}{s-1} - \frac{1}{s+3} - \frac{1}{s+4} = 0$$

$$(s-1)(s+3)(s+4) + (s-2)(s+3)(s+4) - (s-1)(s-2)(s+4) - (s-1)(s-2)(s+3) \\ = 10s^2 + 20s - 50 = 0, s = (1.45, -3.45)$$

Plot:



$$2. \quad G(s) = \frac{s+3}{0.9s^3 + 3s^2 + 5s + 2}$$

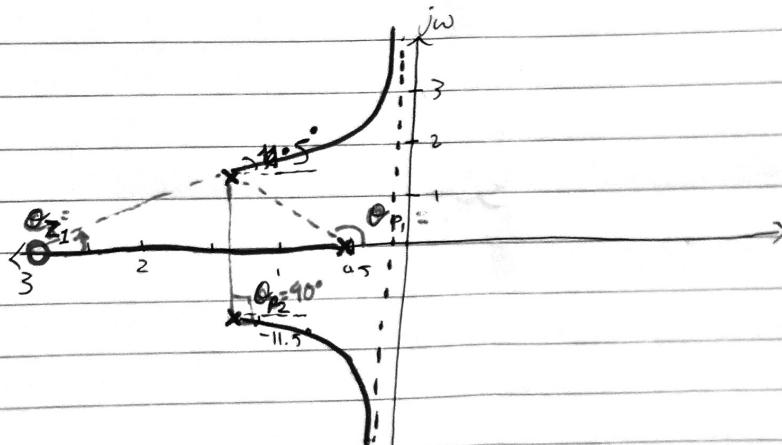
$$Z: -3$$

$$P: -0.55, -1.39 \pm 1.44, R-Z=2$$

Asymptotes.

$$\theta_a = \underbrace{(-0.55 + -1.39 + -1.39)}_2 - (-3) = -0.165$$

$$\theta_a = \frac{(2k+1)\pi}{2}, k=0 \Rightarrow \frac{\pi}{2} = 90^\circ$$



Angles of departure.

$$\theta_D = 180 + \sum \theta_z - \sum \theta_p \Rightarrow 180 + (\theta_{z_1}) - (\theta_{p_1} + \theta_{p_2})$$

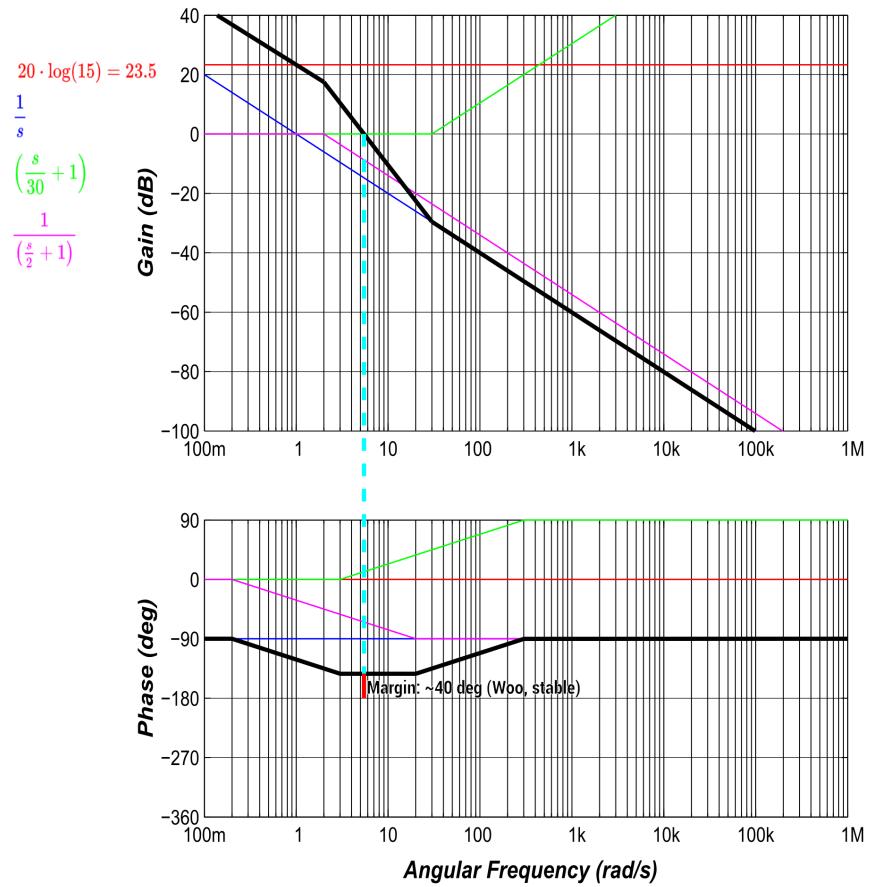
$$\theta_{z_1} = \arg(-1.39 + 1.44j) = 2.16 \angle 41.81^\circ$$

$$\theta_{p_1} = \arg(-1.39 + 1.44j - -0.55) = 1.66 \angle 120.25^\circ$$

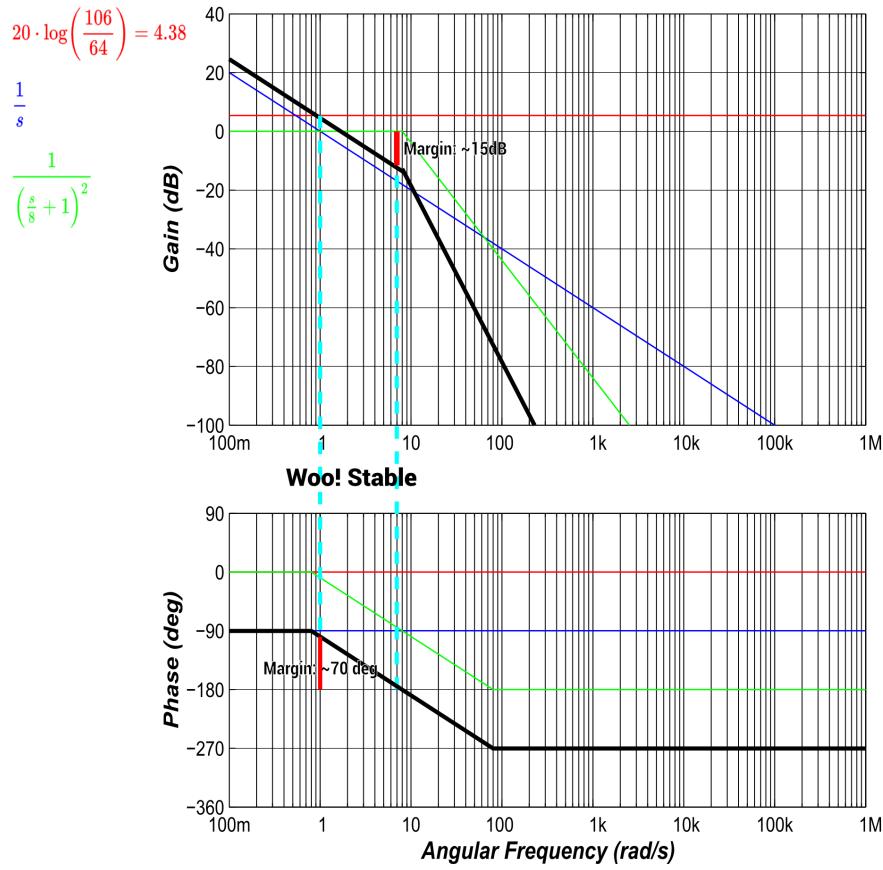
$$\theta_{p_2} = 90^\circ$$

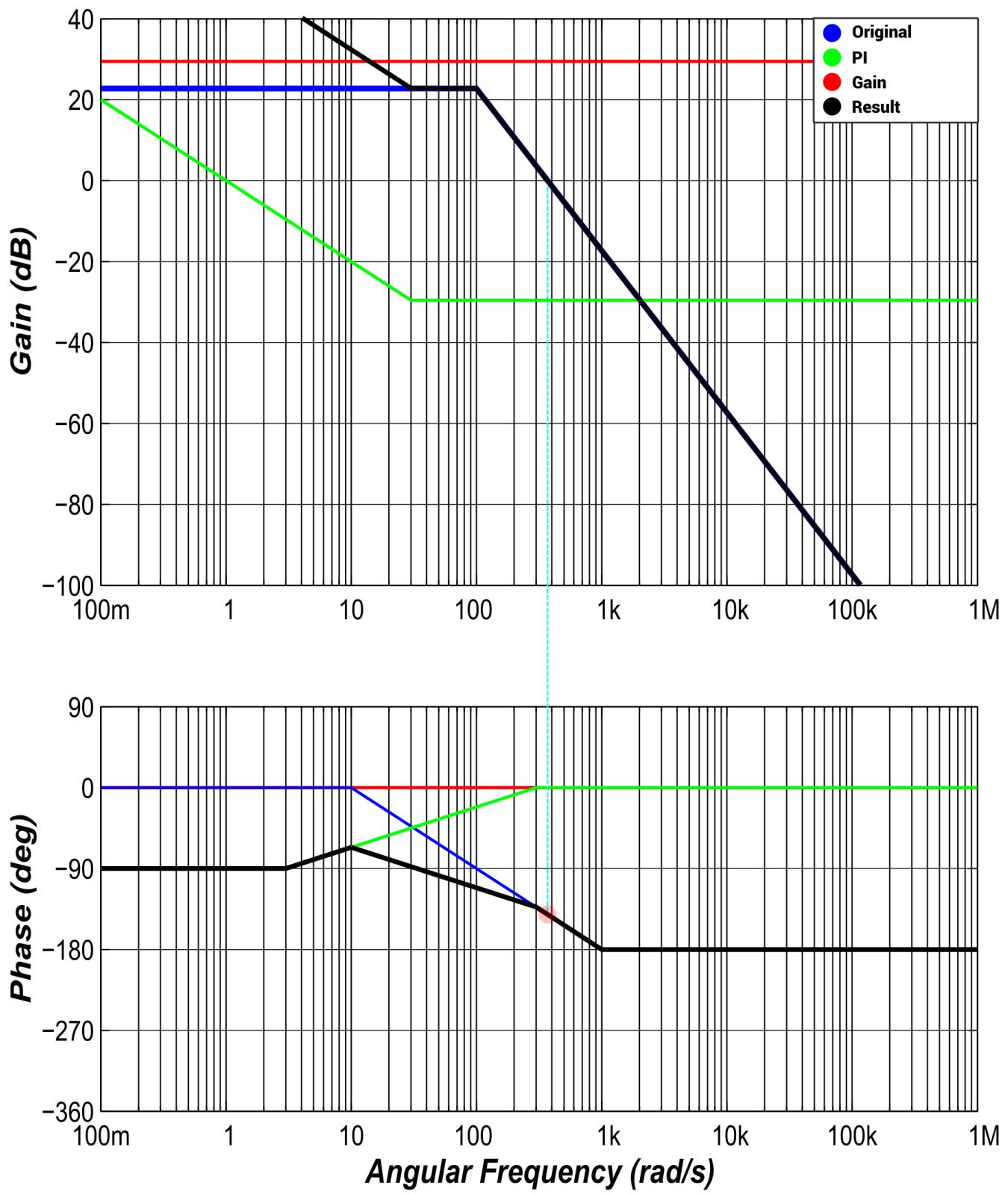
$$\theta_D = 180 + (41.81) - (90 + 120.25) = \pm 11.55^\circ$$

$$G(s) = \frac{s+30}{s(s+2)} = \frac{30\left(\frac{s}{30} + 1\right)}{2s\left(\frac{s}{2} + 1\right)} = 15 \cdot \frac{1}{s} \cdot \left(\frac{s}{30} + 1\right) \cdot \frac{1}{\left(\frac{s}{2} + 1\right)}$$



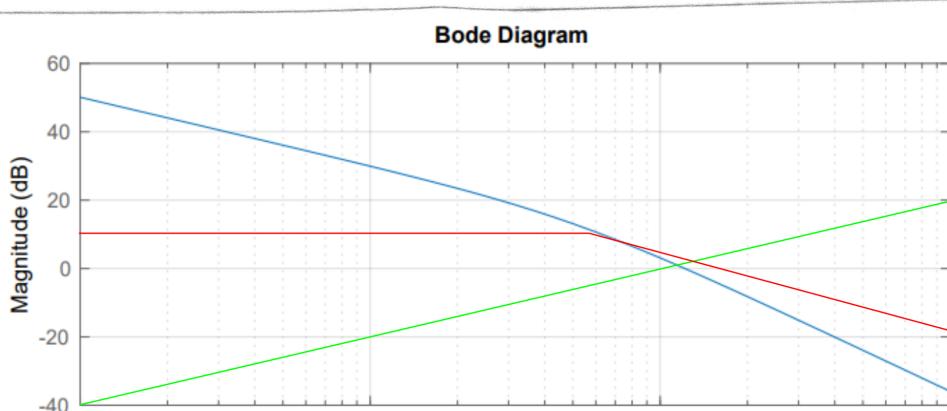
$$G(s) = \frac{106}{s^3 + 16s^2 + 64s} = \frac{106}{64} \cdot \frac{1}{s\left(\frac{s}{8} + 1\right)^2} = \frac{106}{64} \cdot \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{8} + 1\right)^2}$$





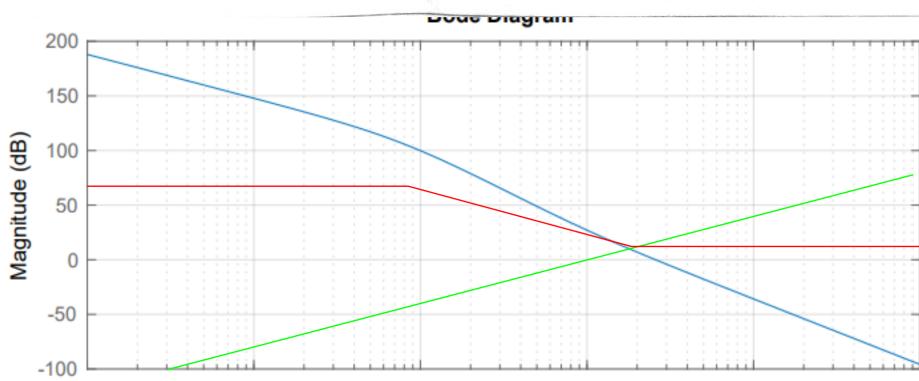
I. \Rightarrow Type 1.

- Step : Zero error
- Parabolic : Infinite error
- Ramp : $\frac{1}{K_V}$, $10 \text{ dB} \rightarrow 3.16 \Rightarrow 0.31\%$



II. Type 2.

- Step & Ramp : Zero error
- Parabolic : $\frac{1}{K_a}$, $70 \text{ dB} \rightarrow 3.16 \times 10^{-4}$



III. Type 0.

- Ramp & Parabolic : Infinite

• Step: $\frac{1}{1+K_p}$, read $14 \text{ dB} \rightarrow K_p = 10^{\frac{14}{20}} = 5.01$

$$\frac{1}{1+5} = \frac{1}{6} = \text{steady state error.}$$