Victoria University of Wellington
School of Mathematics and Statistics

MATH 244 — Modelling with Differential Equations — 2019

Assignment 1

Due: 23:59 Wednesday 20 March using the online submission system

Direction fields, first order ODEs. Boyce & DiPrima, Ch 1; Ch 2 section 2.1

1. Consider the list of DEs in Tutorial Q(1), and explain your reasoning in the following subquestions:

   (a) Which DE from that list produced the direction field in Fig (1a)?

   (b) Which DE from that list produced the direction field in Fig (1b)?

2. For the DE $y' + 2y = t$,

   (a) Draw a direction field any way you want (by hand, or use Maple),

   (b) Use the direction field to describe how solutions behave for large positive t,

   (c) Find the general solution by using an integrating factor,

   (d) Use the general solution to describe how solutions behave as $t \to +\infty$.

3. Use a method like the one used in class to solve the ODE $v' = g - \frac{\gamma}{m}\,v$, to solve the IVP

$$y' = 3y - 3, \quad y(0) = 3 \ ,$$

by following this process:

   (a) You could begin, by showing that the ODE can be put in the form

$$\frac{d}{dt}\left(\ln|y-1|\right) = 3$$

   (b) Then make an argument that leads to the general solution

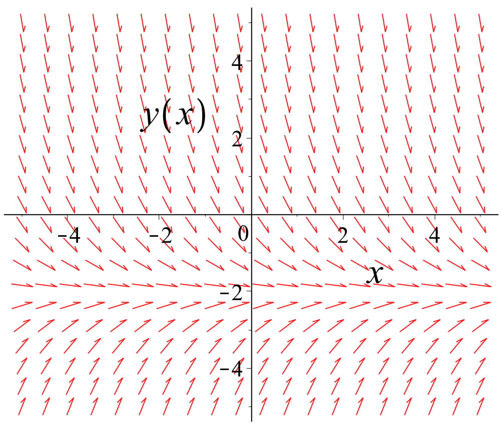
$$y = 1 + Ce^{3t} \ .$$

   (c) Make an argument that C is arbitrary, that is, that C can be any real number.

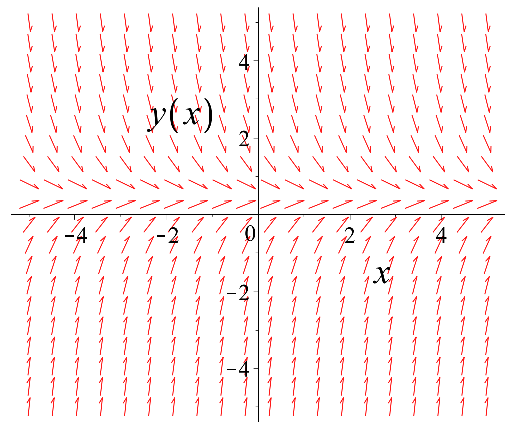
   (d) Verify that this is a solution, by substituting it into the original ODE.

   (e) Then use the initial condition to show that the solution to the IVP is

$$y = 1 + 2e^{3t} \ .$$



(a) Direction Field for unknown DE



(b) Direction Field for unknown DE

Figure 1: Direction Fields for assignment

Tutorial Exercises: Thursday 14–Tuesday 19 March

1. Consider the following list of DEs, and explain your reasoning in the following subquestions:

   (a) $y' = 2y - 1$

   (b) $y' = 2 + y$

   (c) $y' = y - 2$

   (d) $y' = y(y + 3)$

   (e) $y' = y(3 - y)$

   (f) $y' = 1 + 2y$

   (g) $y' = -2 - y$

   (h) $y' = y(y - 3)$

   (i) $y' = 1 - 2y$

   (j) $y' = 2 - y$

   (a) Which DE produced the direction field in Fig (2a)?

   (b) Which DE produced the direction field in Fig (2b)?

2. For the DE $y' + y = t + 2$,

   (a) Draw a direction field any way you want (by hand, or use Maple),

   (b) Use the direction field to describe how solutions behave for large positive t,

   (c) Find the general solution by using an integrating factor,

   (d) Use the general solution to describe how solutions behave as $t \to +\infty$.

3. Use a method like the one used in class to solve the ODE $v' = g - \frac{\gamma}{m}\, v$, to solve the IVP

   $$y' = 2y - 2, \quad y(0) = 2\ ,$$

   by following this process:

   (a) Show that the ODE can be put in the form

   $$\frac{d}{dt}\left(\ln|y - 1|\right) = 2$$

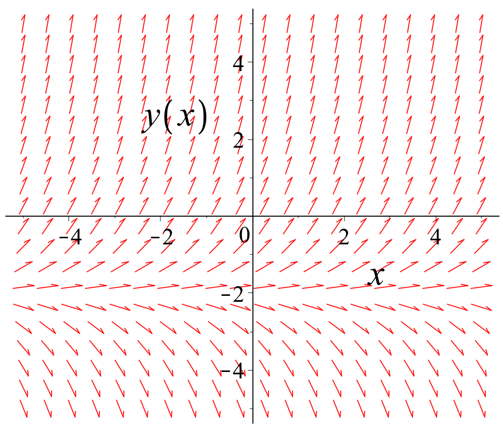
   (b) Make an argument that leads to the general solution

   $$y = 1 + Ce^{2t}\ .$$

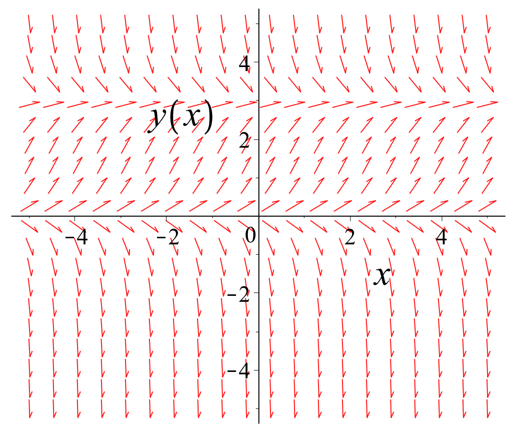
   (c) Verify that this is a solution, by substituting it into the original ODE.

   (d) Then use the initial condition to show that the solution to the IVP is

   $$y = 1 + e^{2t}\ .$$



(a) Direction Field for unknown DE



(b) Direction Field for unknown DE

Figure 2: Direction Fields for tutorial