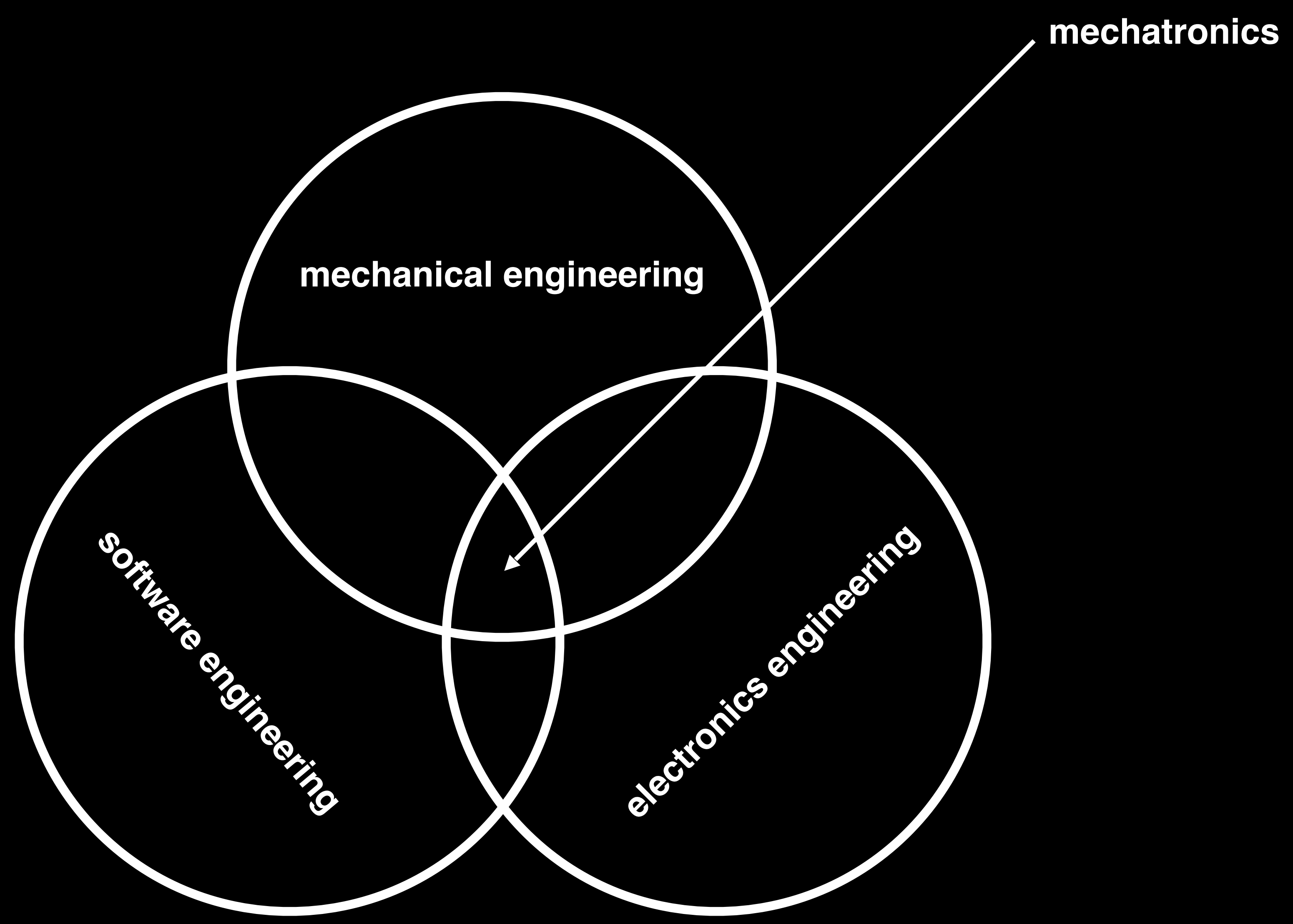
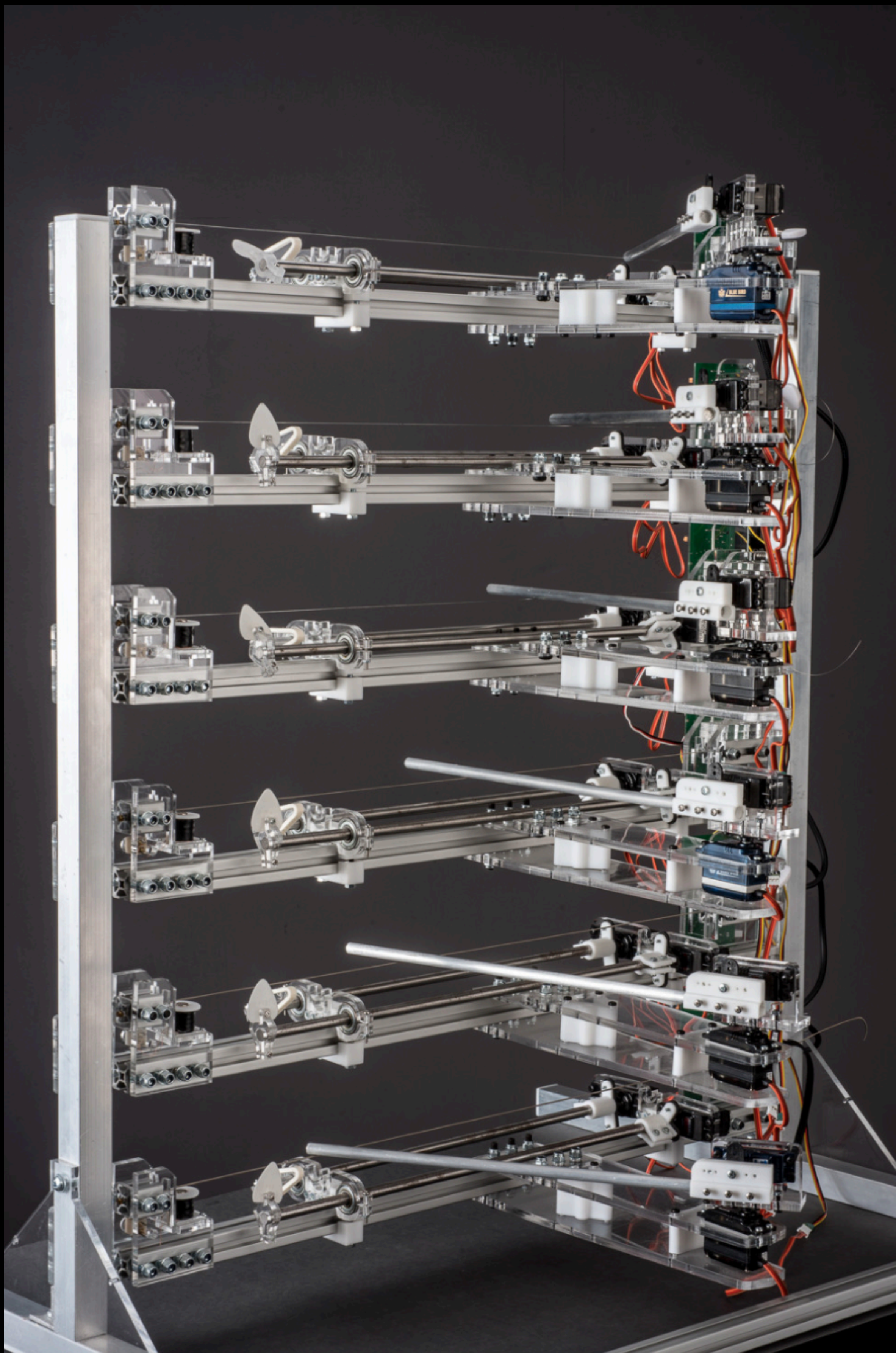
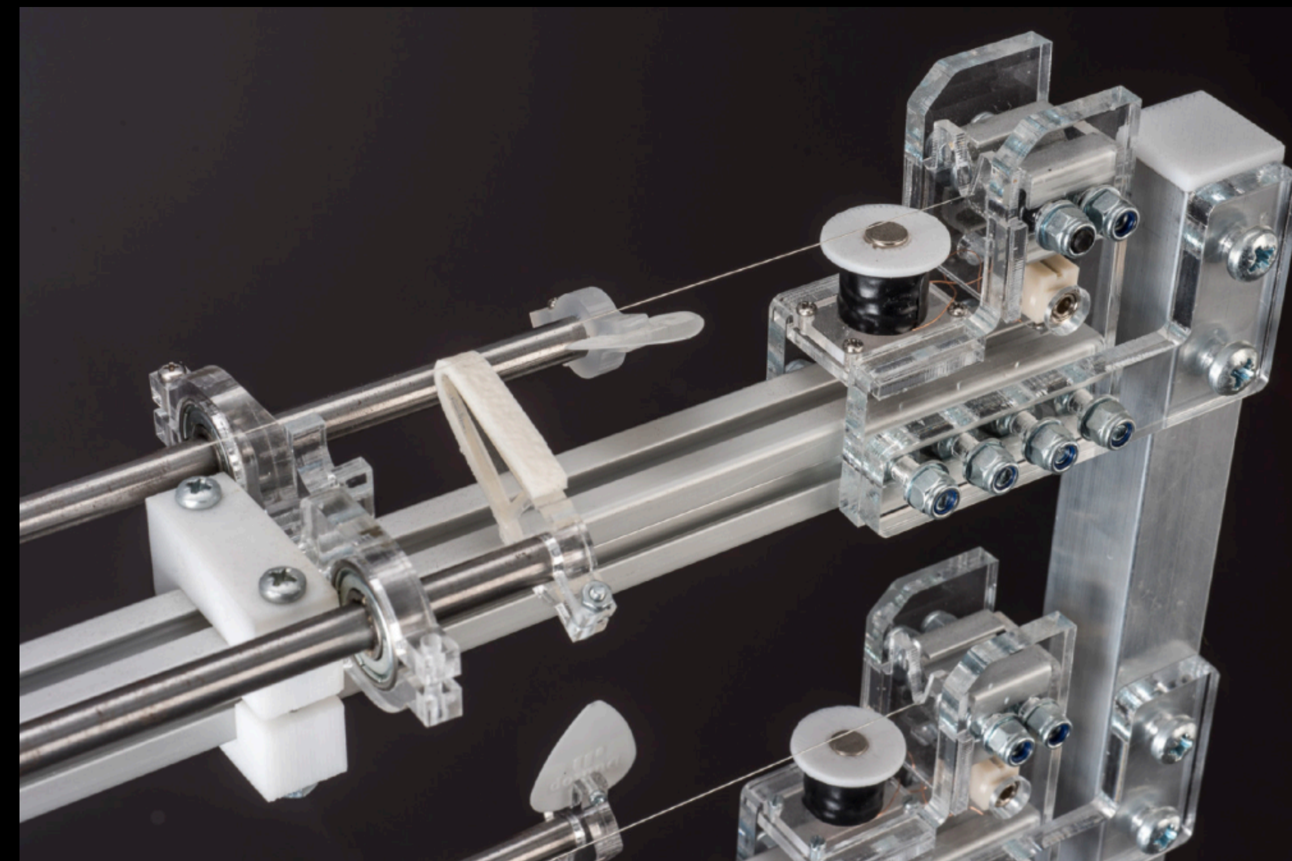
ECEN425

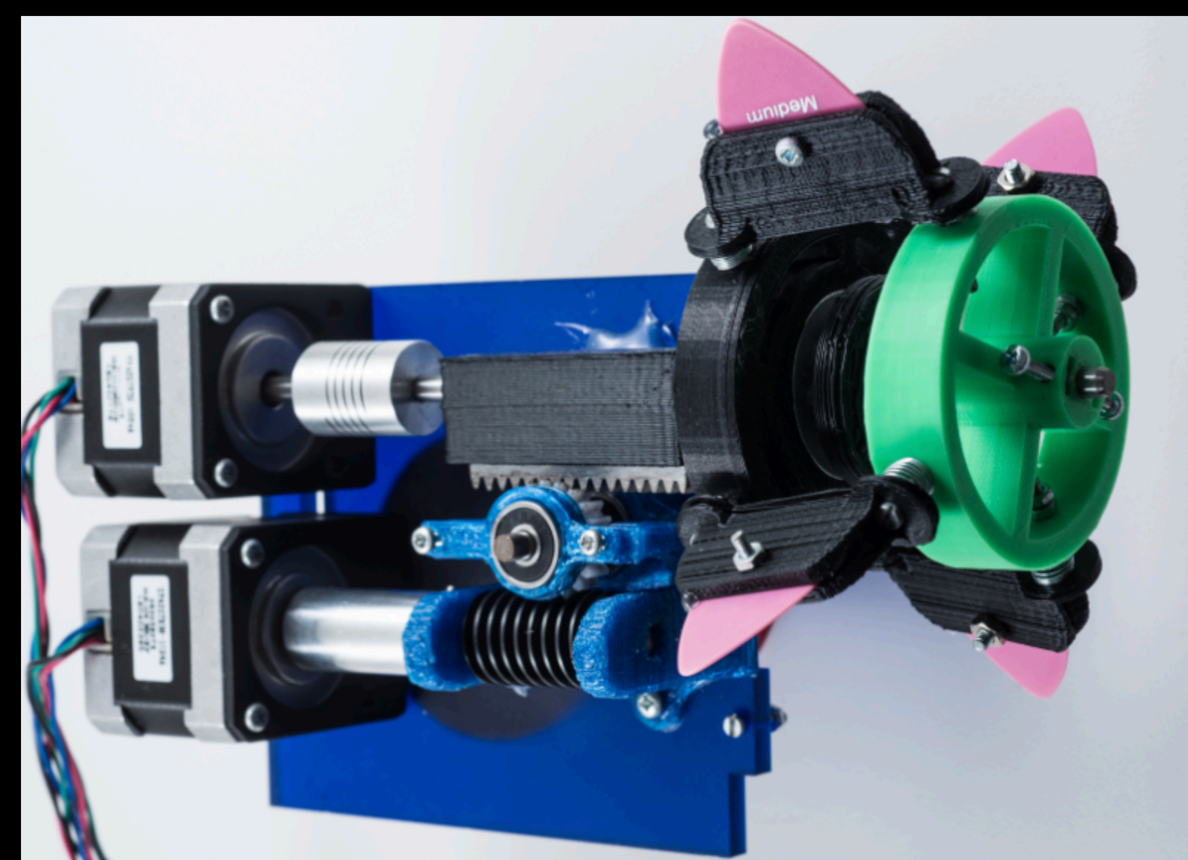


- jim.murphy@vuw.ac.nz
- 92 Fairlie Terrace, Room 202


REMO





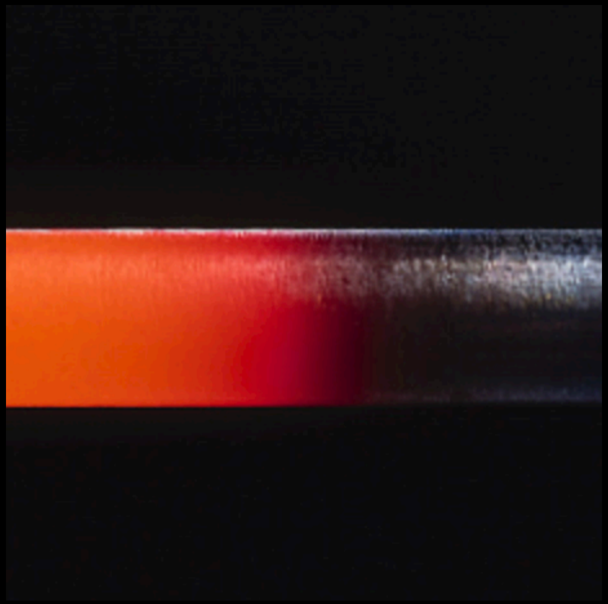

Medium

- jim.murphy@vuw.ac.nz
- 92 Fairlie Terrace Room 202
- No fixed office hours: schedule a meeting via email!
- Lecture notes and assignments will be posted to Blackboard

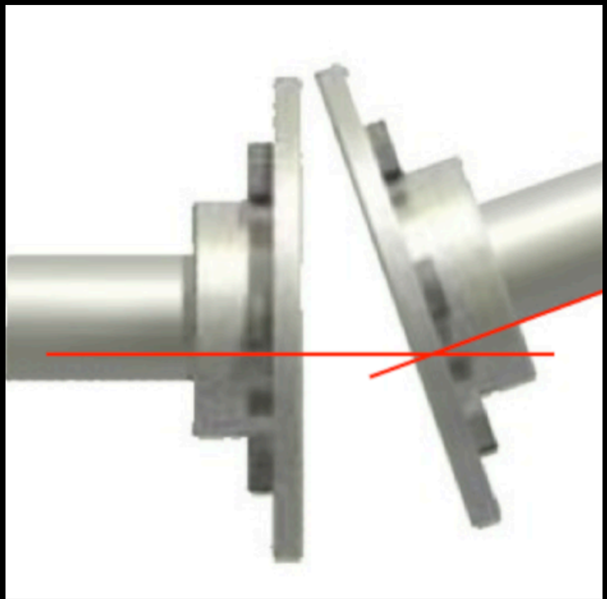
UNCERTAINTIES

CORROSION

WEAR

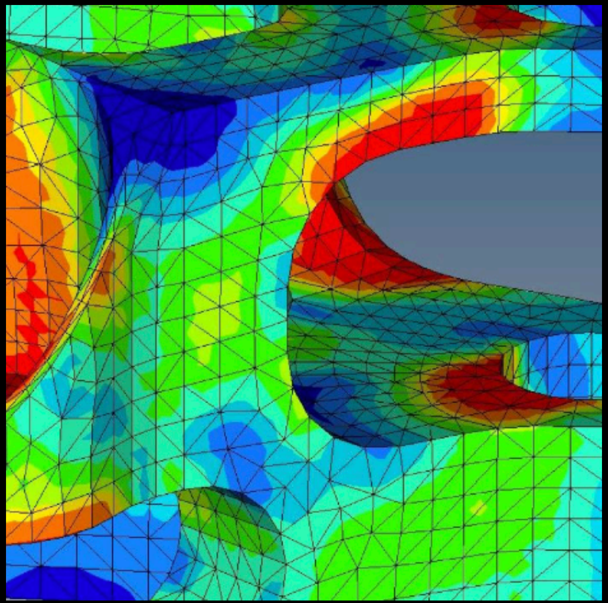


TEMPERATURE



MISALIGNMENT

MANUFACTURING ERRORS



MODEL ERRORS

- One way to deal with these inevitable errors is to use a design factor and safety factor.
- This essentially involves building the system stronger than needed for normal use.
- It can be helpful to be able to estimate the worst case scenario.

DESIGN FACTOR

- Designing with a design factor allows uncertainties to be addressed (so long as *absolute uncertainties* can be estimated)

Design Factor

Point at which the system fails

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}}$$

Parameter to not exceed

Or… given the loss of function parameter and a design factor, we can arrive at a maximum allowable parameter

$$\text{Maximum allowable parameter} = \frac{\text{loss-of-function parameter}}{n_d}$$

DESIGN FACTOR

An electric scooter's maximum load is known to an uncertainty of ±20 percent *(absolute uncertainty 1)*. The load that will cause breakage/failure is known within ±15 percent *(absolute uncertainty 2)*. The load causing failure is nominally 2000 N. Determine the design factor and the maximum allowable load that will offset the absolute uncertainties.

The maximum allowable load must decrease to 1/1.2
(addresses absolute uncertainty 1, ±20 percent)
We might see a loss-of-function load as great as 1/0.85
(addresses absolute uncertainty 2, ±15 percent)

$$n_d = \frac{1/0.85}{1/1.2} = 1.4$$

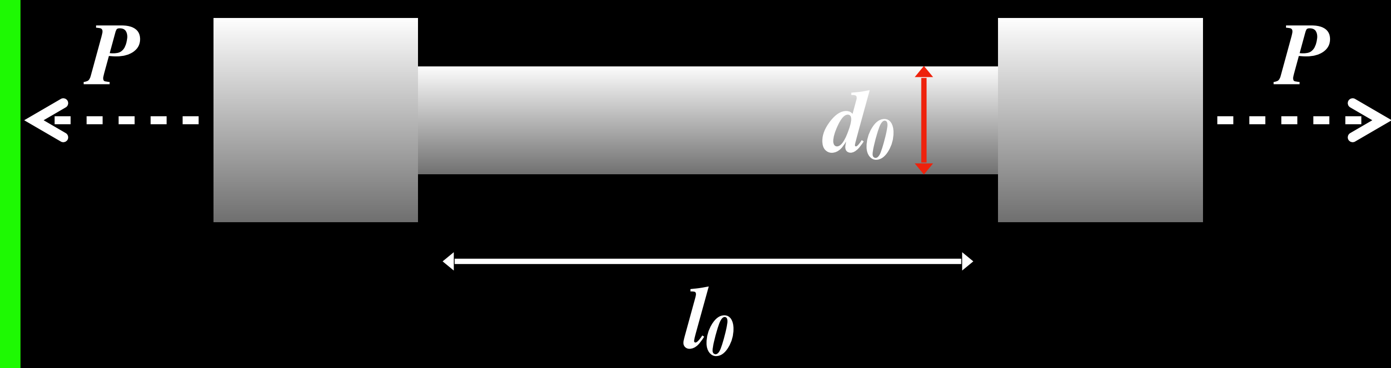
$$\text{Max. allowable load} = \frac{2000}{1.4} = 1428\text{ N}$$

- This is a deterministic approach. Stochastic (statistics-based) approaches that focus on the design's statistical survivability are also commonly used. These require many samples.

MATERIALS

- Deciding upon a material is a crucial step (often the first) in a design process.
  - This affects everything from cost to dimension to manufacturing process.
- To arrive at an understanding of allowable stresses in a component, we need to know how a material behaves.
  - What & how are the points where the material fails when subjected to stresses?

Tensile Test

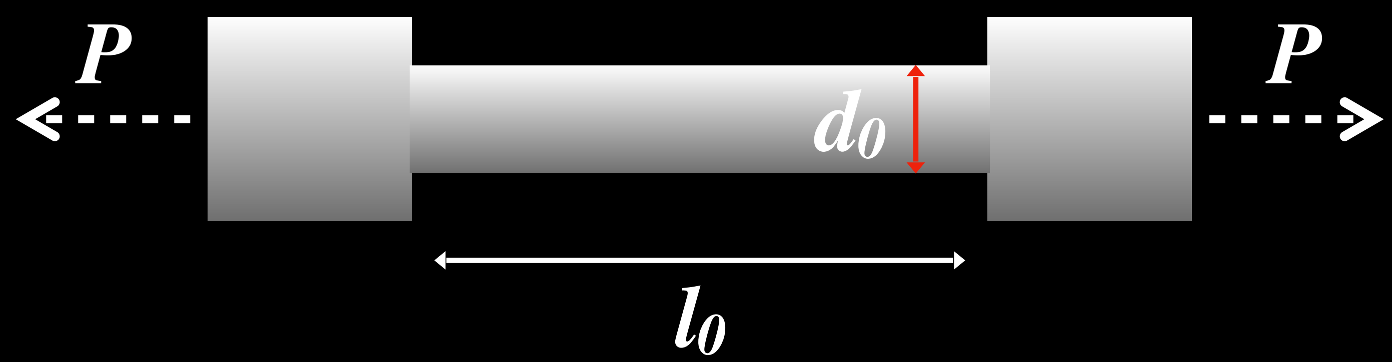


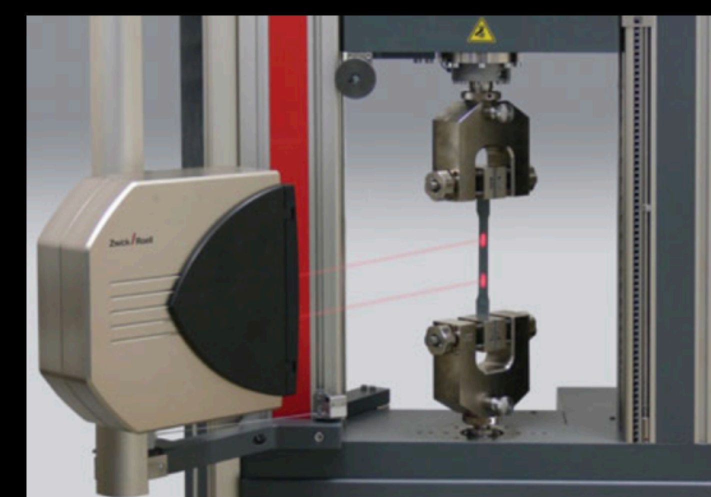
Load P is converted to stress σ by

$$\sigma = \frac{P}{A_0} \quad \text{where} \quad A_0 = \frac{1}{4}\pi d_0^2$$

By measuring how the material responds to this stress, the relationship between input stress σ and strain ϵ can be determined

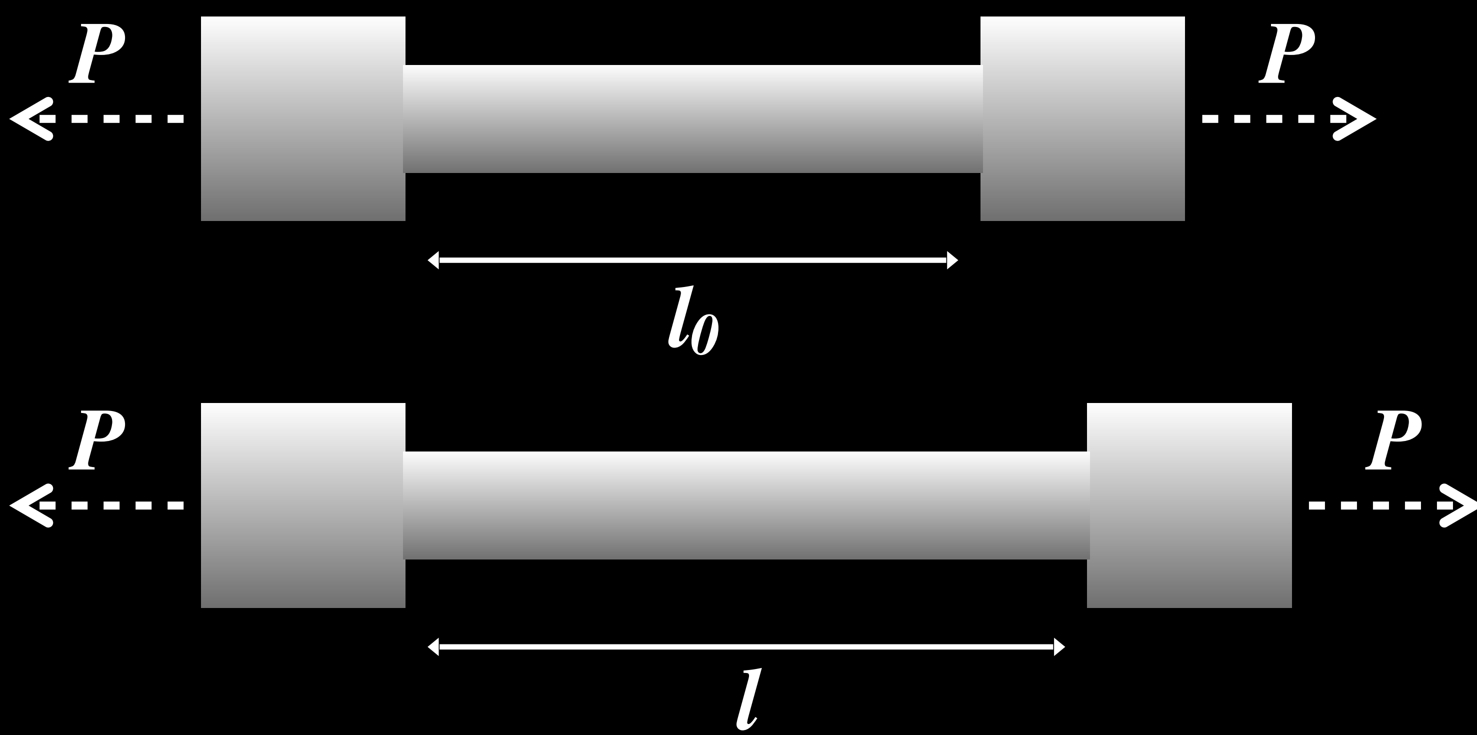
TENSION TEST

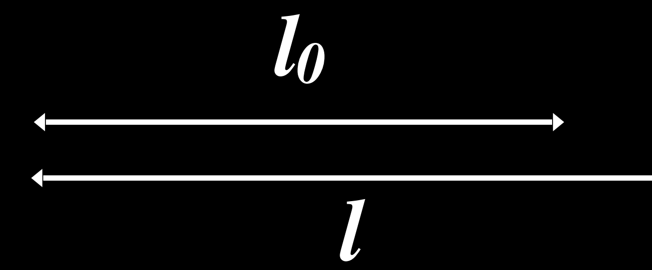




- The relationship between stress and strain can be measured with a tension tester unit.
  - This places the sample between two arms that compress/extend.
  - Change in material length l is measured
    - Measurement is conducted with an extensometer.

STRAIN

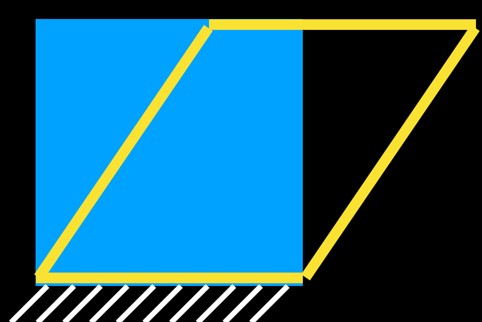




$$\delta = l - l_0$$

Deformation

Normal strain

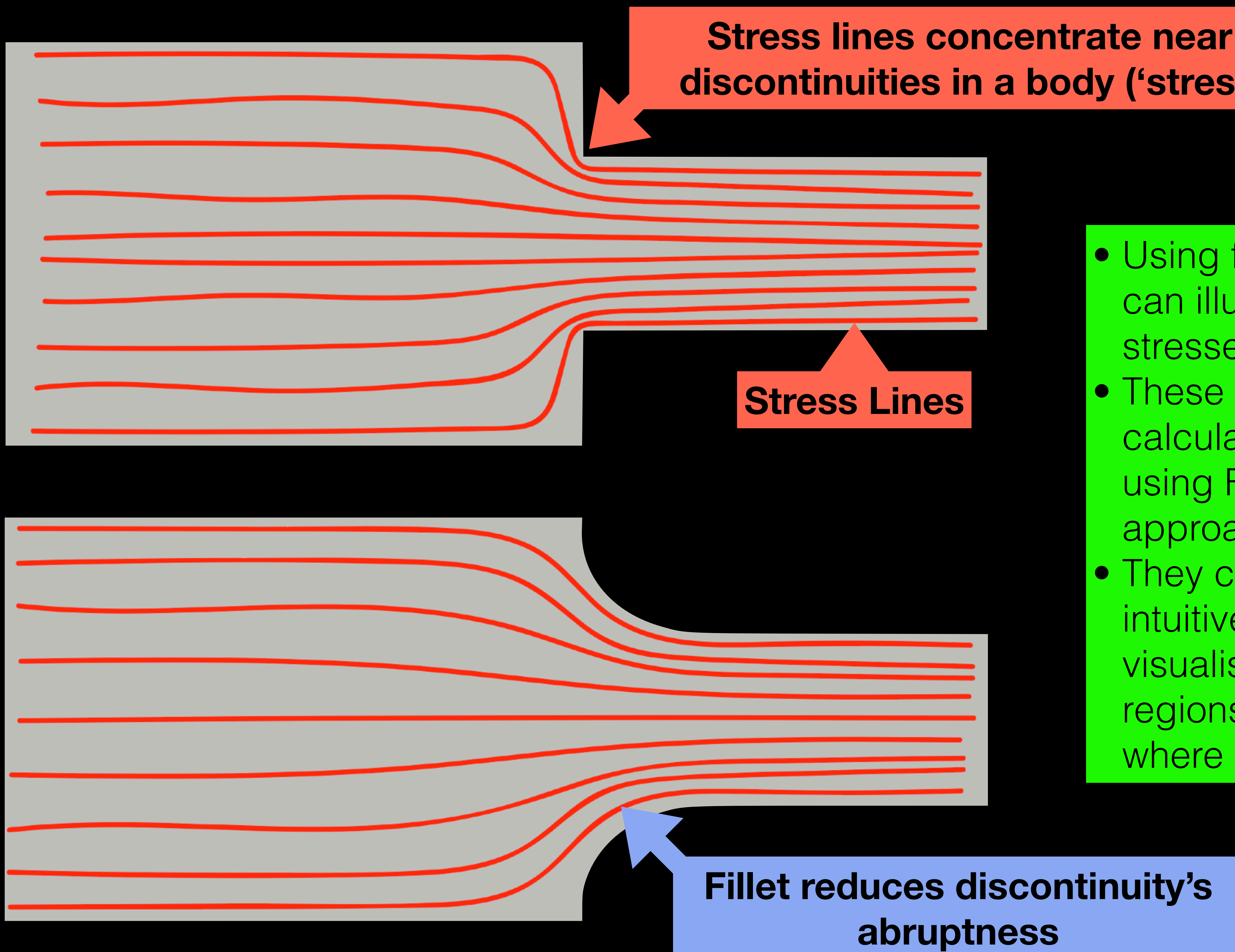


Shear strain

Rather than be described by the actual deformation length (l-l_0), normal strain ϵ is described by the change in unit length of the material

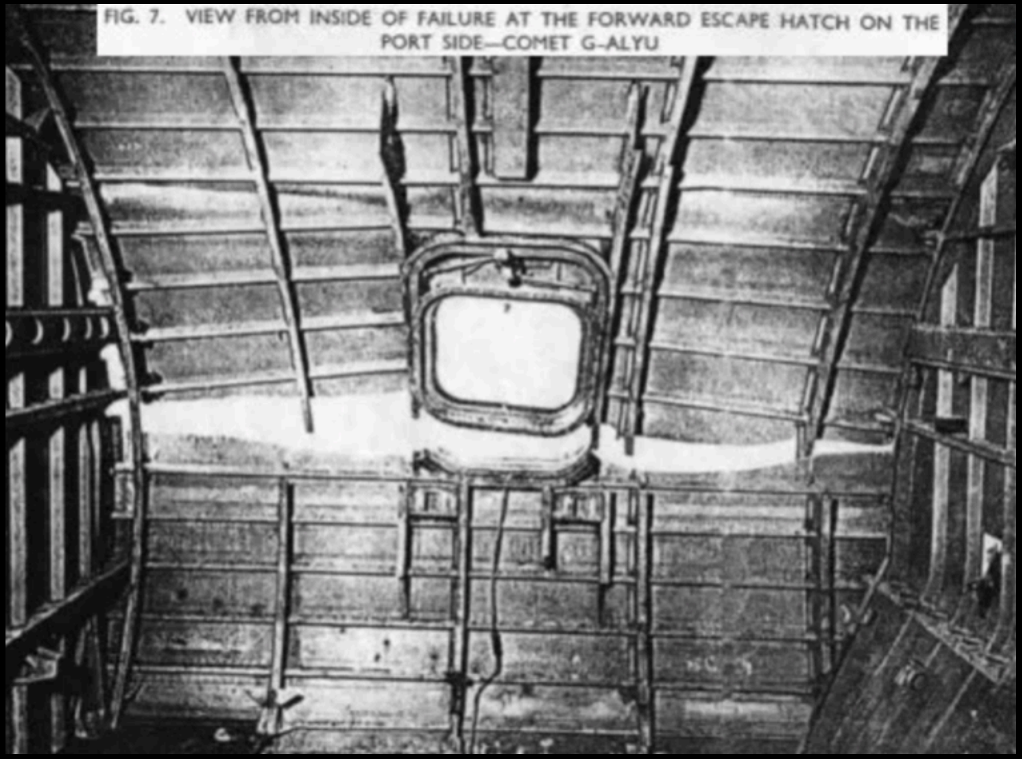
$$\epsilon = \frac{\delta}{l_0}$$

STRESS CONCENTRATION



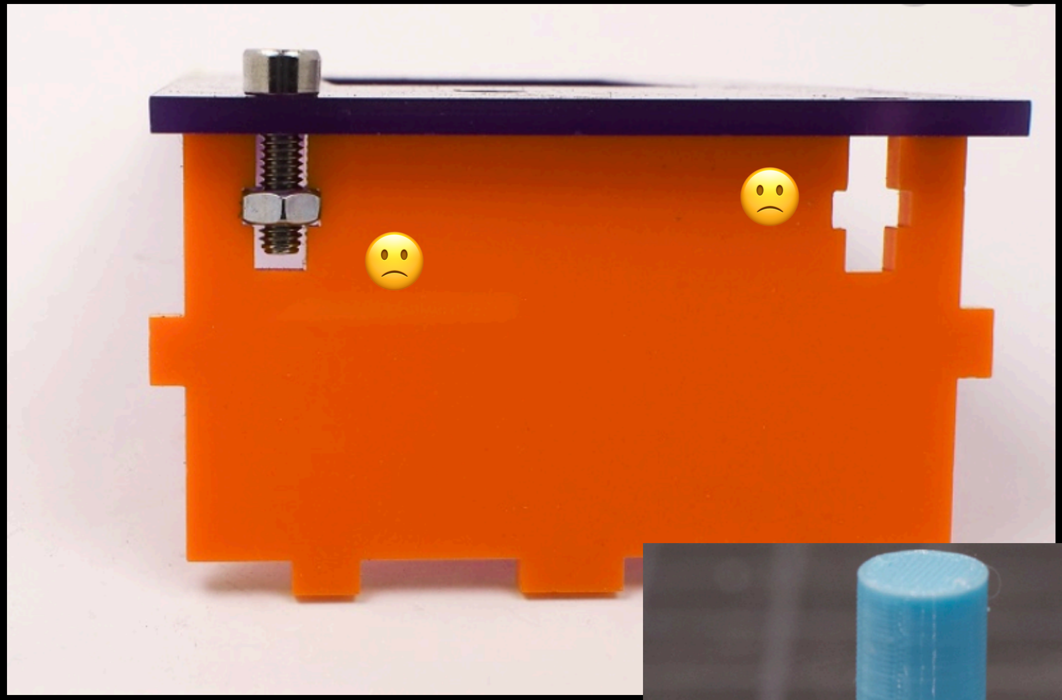
- Using force lines, we can illustrate the stresses in a body.
- These can be calculated (usually using FEA approaches).
- They can provide an intuitive, qualitative visualisation of regions in a body where stresses rise.

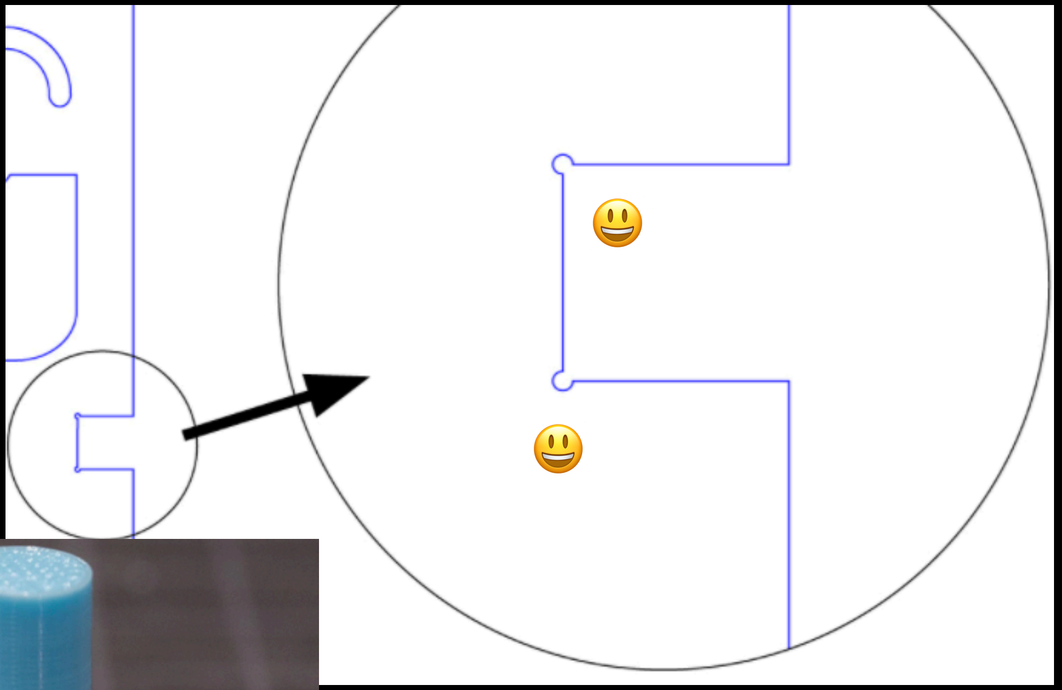
STRESS CONCENTRATION

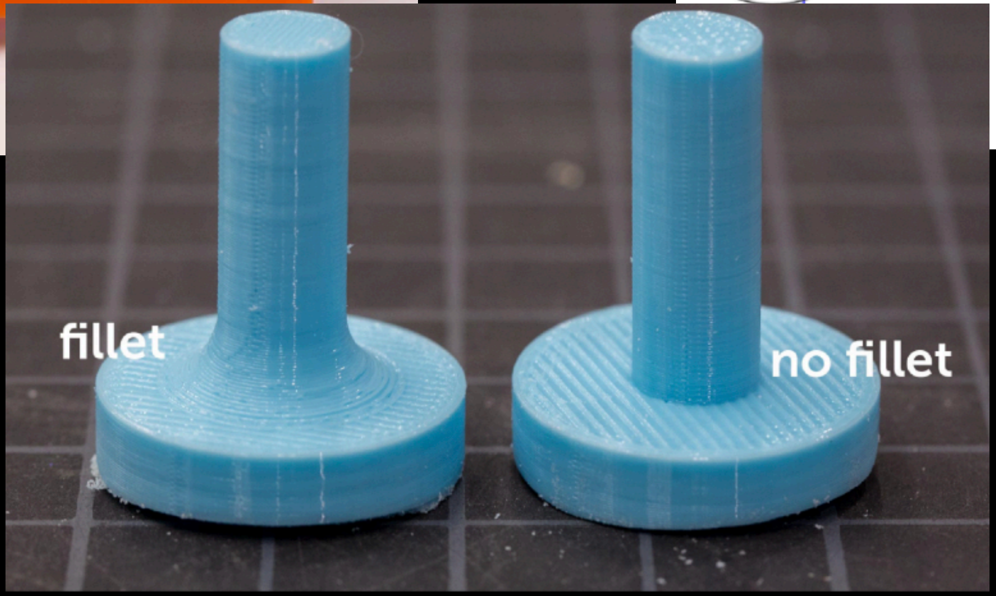


STRESS CONCENTRATION

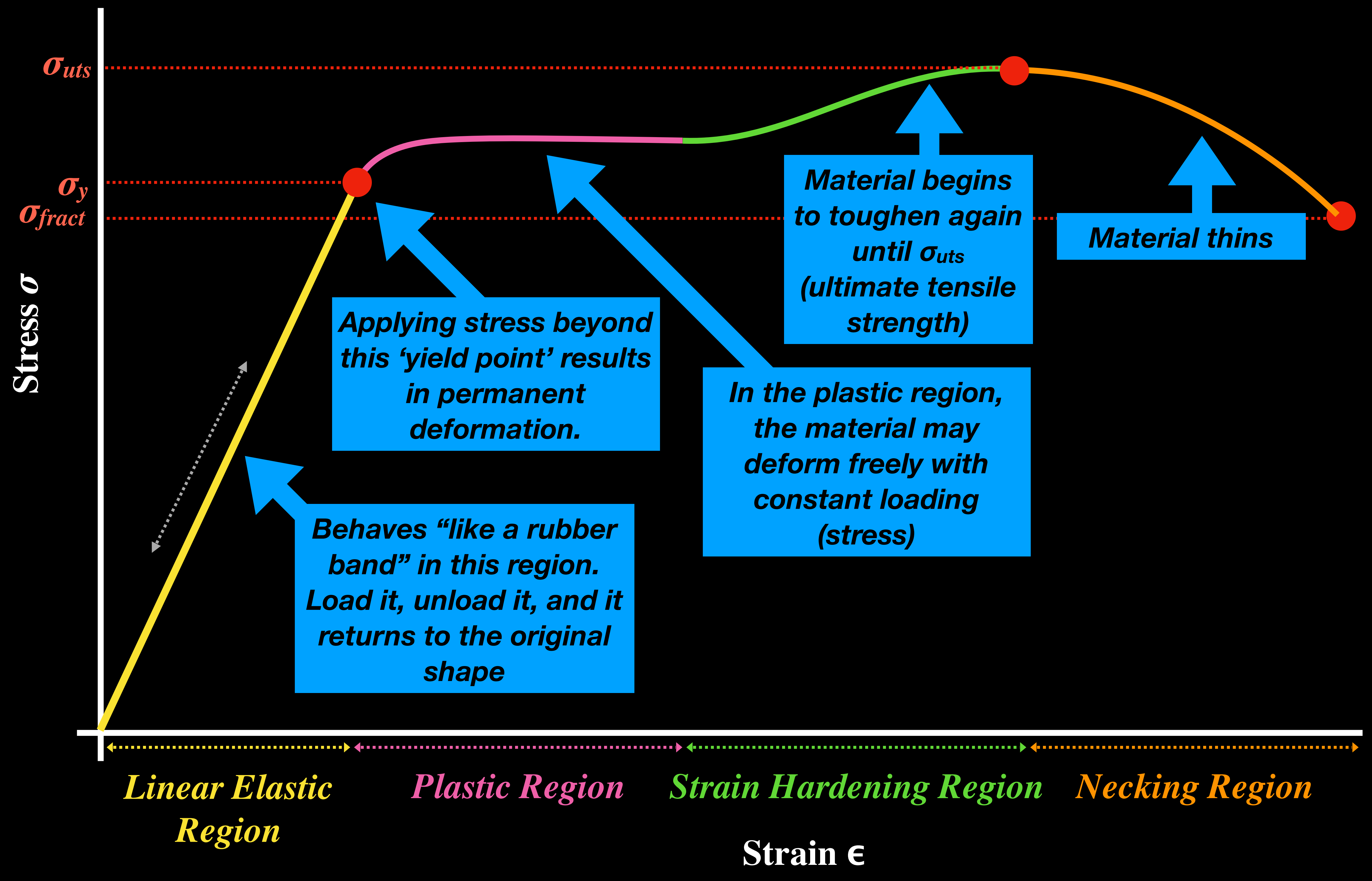
- When laser cutting, reduce stress concentration by avoiding abrupt angular changes.
- Add fillets when 3D printing to more evenly distribute stresses







STRESS-STRAIN DIAGRAM



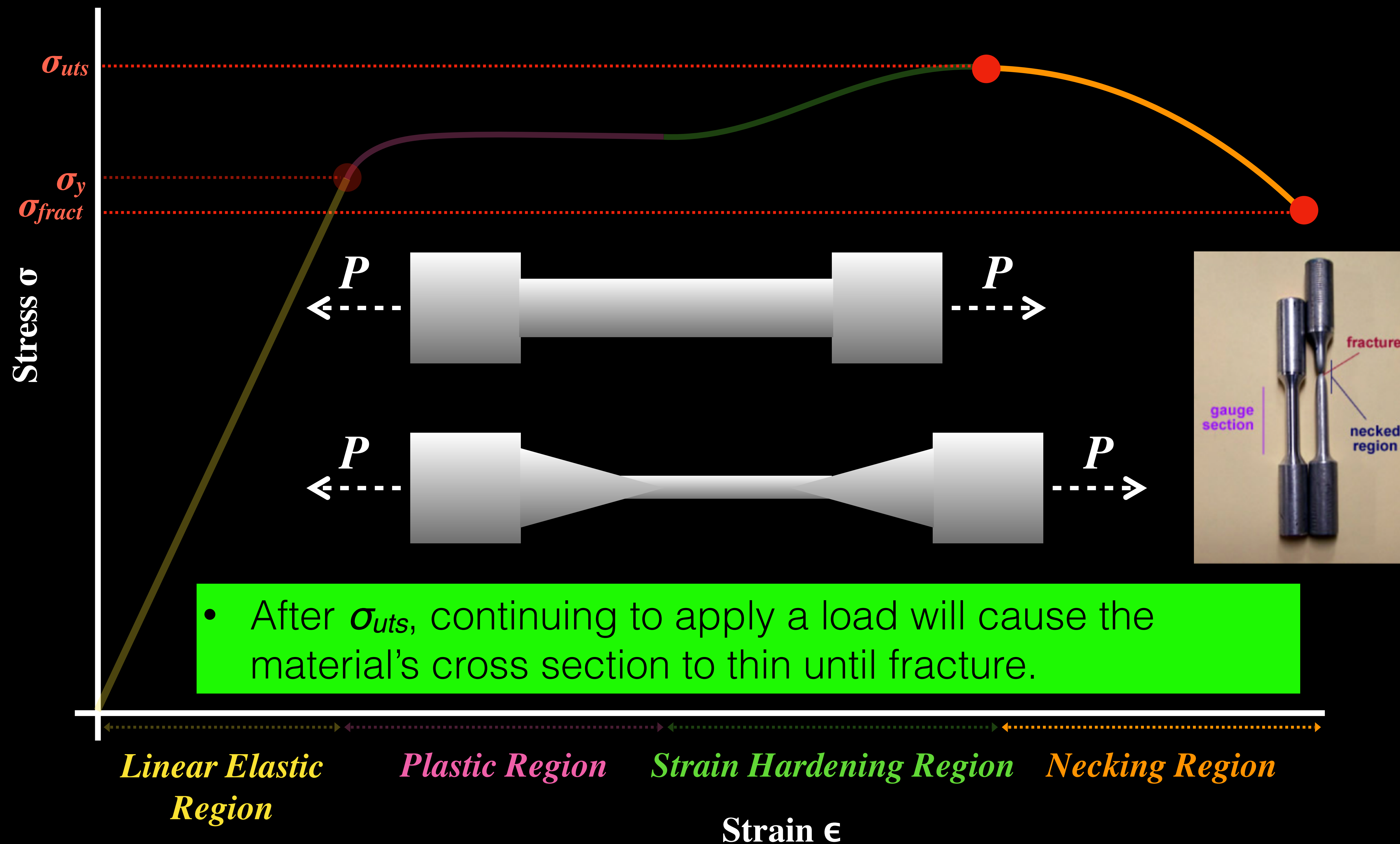
A NOTE ABOUT UNITS

FORCE
Unit: Newtons (N)
1 N = 1 kg · 1 m/s^2
(In engineering, we often use kN)

STRESS (σ, sigma)
Unit: Pascal in SI (PSI in Imperial units)
(Newtons per square metre: 1 N/m^2)
MPa and GPa are common in mech. engineering.

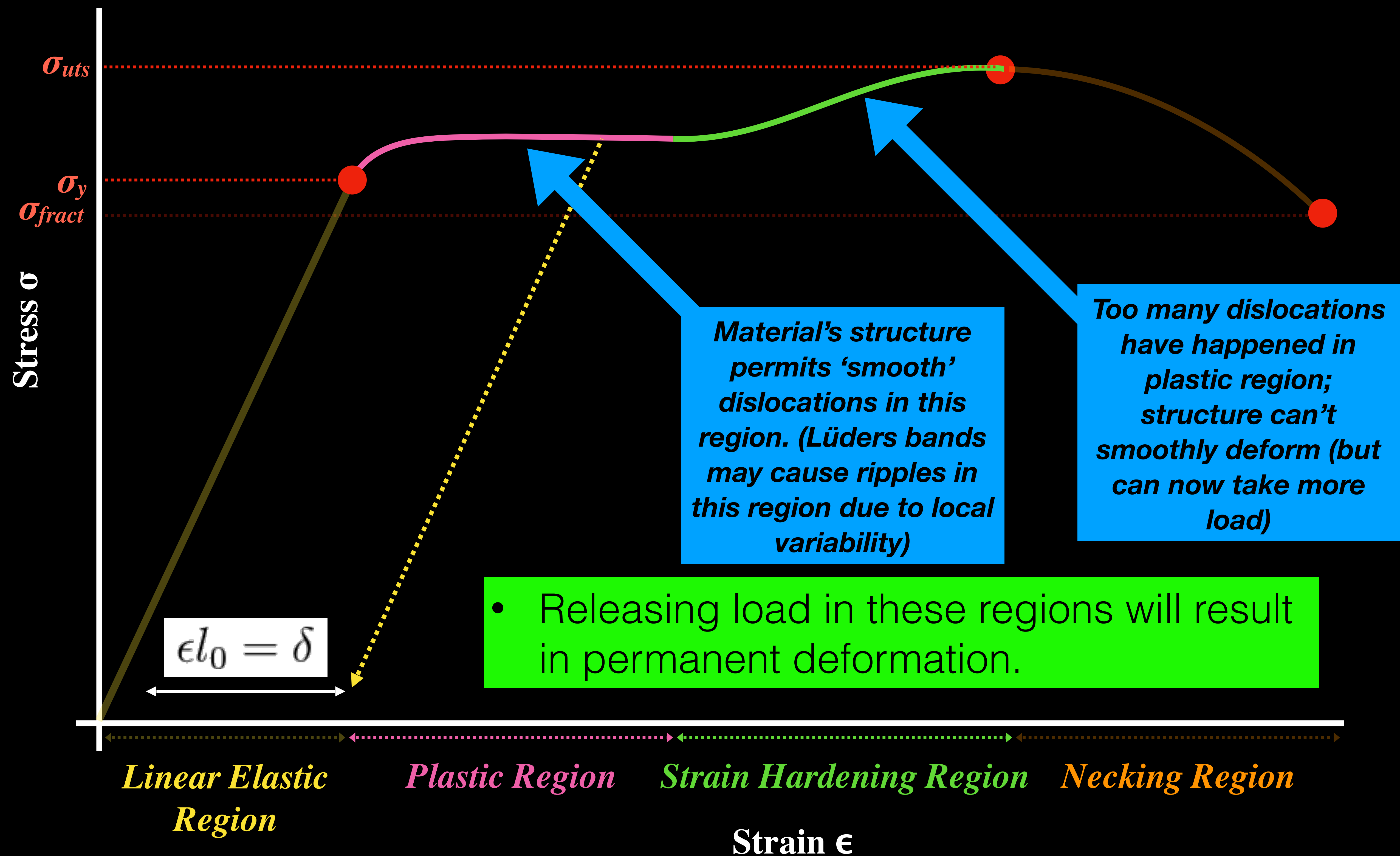
STRAIN (ϵ, epsilon)
Unit: dimensionless (ratio of two lengths), but you often see it as a ratio of lengths (e.g., m/m)

NECKING REGION

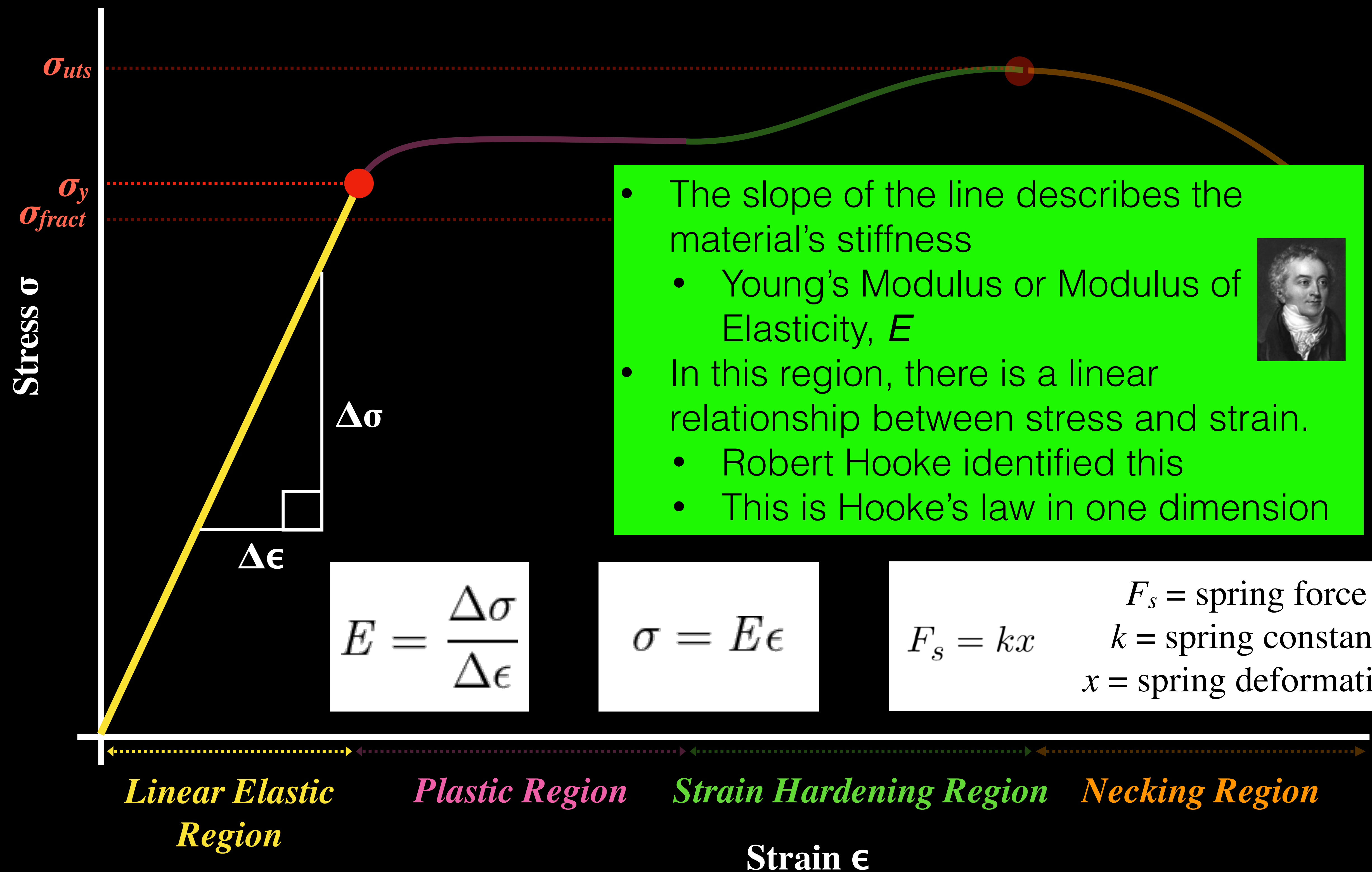


- After σ_{uts}, continuing to apply a load will cause the material's cross section to thin until fracture.

PLASTIC & STRAIN HARDENING REGIONS



LINEAR ELASTIC REGION



- The slope of the line describes the material's stiffness
  - Young's Modulus or Modulus of Elasticity, *E*

- In this region, there is a linear relationship between stress and strain.
  - Robert Hooke identified this
  - This is Hooke's law in one dimension

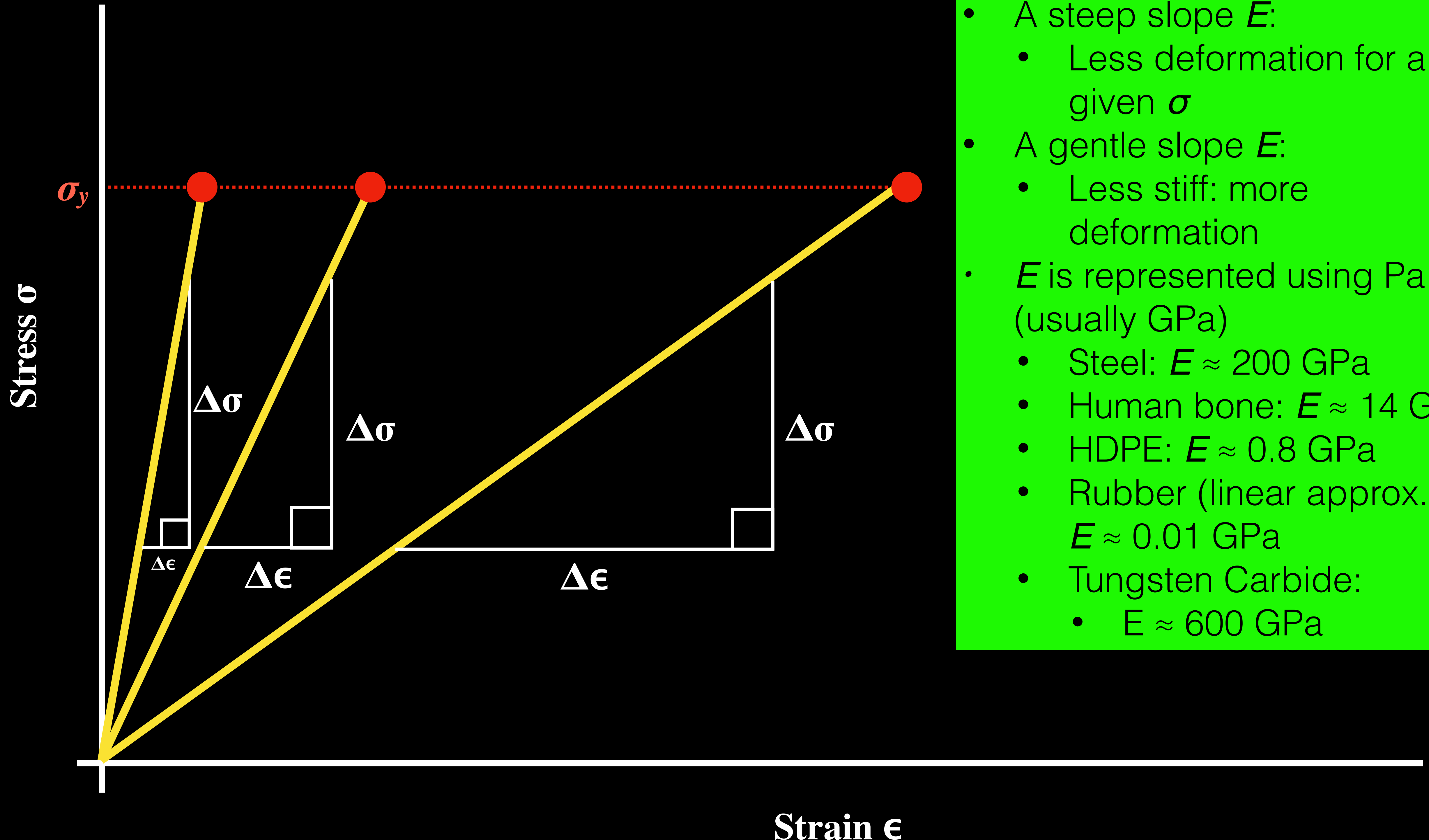
$$E = \frac{\Delta\sigma}{\Delta\epsilon}$$

$$\sigma = E\epsilon$$

$$F_s = kx$$

F_s = spring force
k = spring constant
x = spring deformation

MODULUS OF ELASTICITY

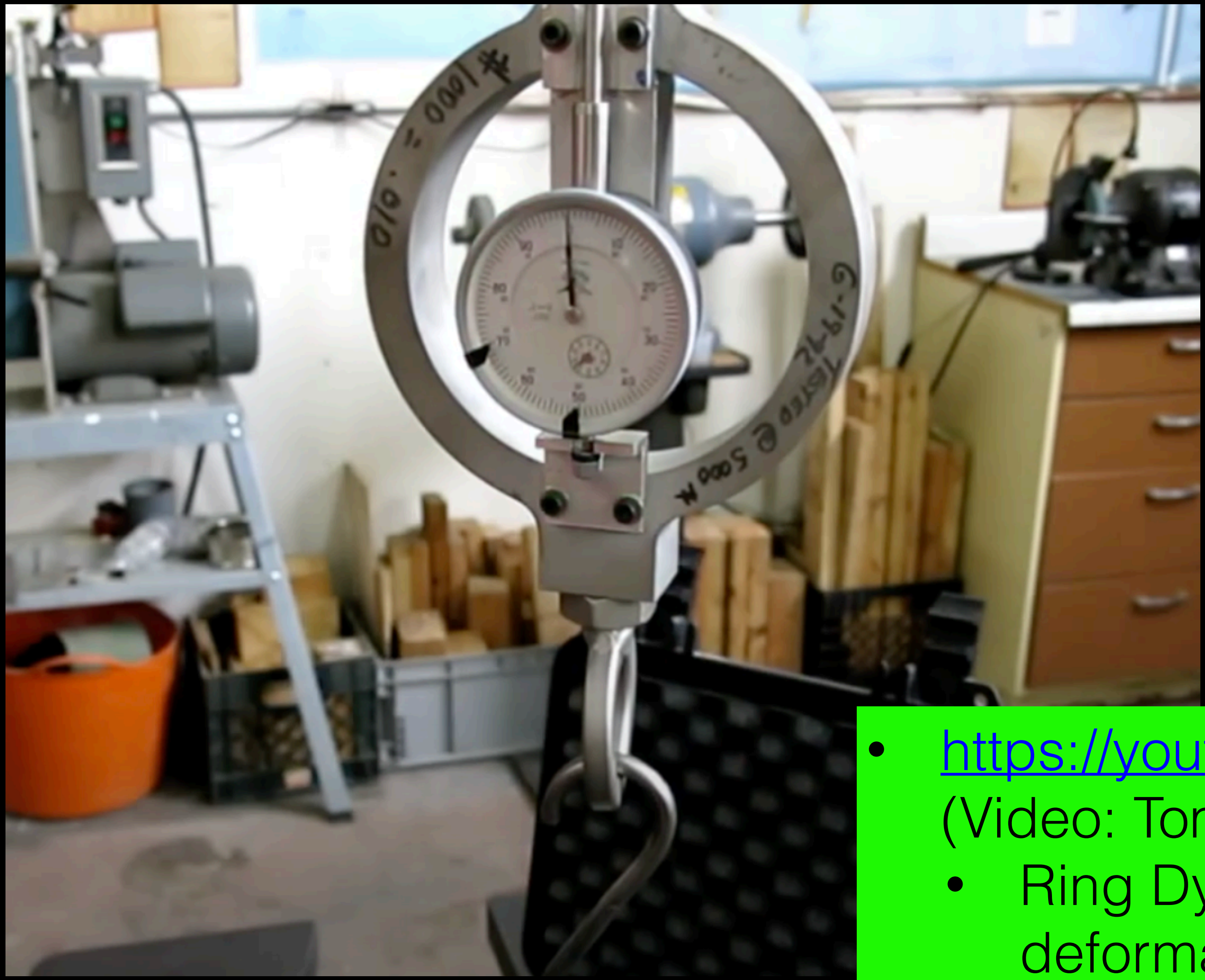


- A steep slope *E*:
  - Less deformation for a given *σ*
- A gentle slope *E*:
  - Less stiff: more deformation
- *E* is represented using Pa (usually GPa)
  - Steel: *E* ≈ 200 GPa
  - Human bone: *E* ≈ 14 GPa
  - HDPE: *E* ≈ 0.8 GPa
  - Rubber (linear approx.): *E* ≈ 0.01 GPa
  - Tungsten Carbide:
    - E ≈ 600 GPa

VIDEO: TENSION TEST

https://youtu.be/D8U4G5kcpcM?t=86

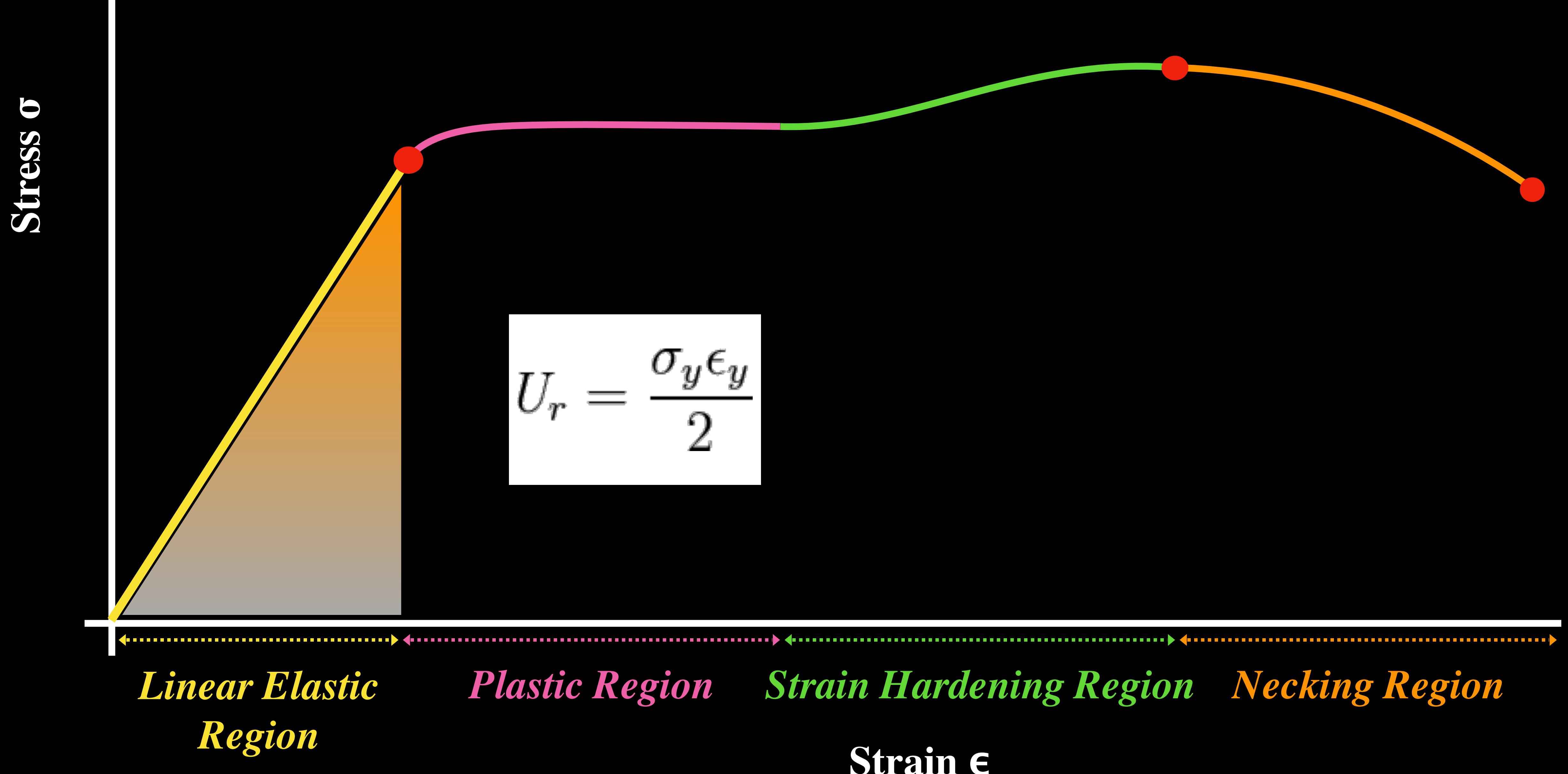
OBSERVING MATERIAL YIELD



- https://youtu.be/kvkFWEuFxck?t=440 (Video: Tom Lipton)
  - Ring Dynamometer - elastic deformation in aluminium ring is measured with a dial indicator.
  - Hook is loaded (stressed), and permanent deformation is induced.

RESILIENCE

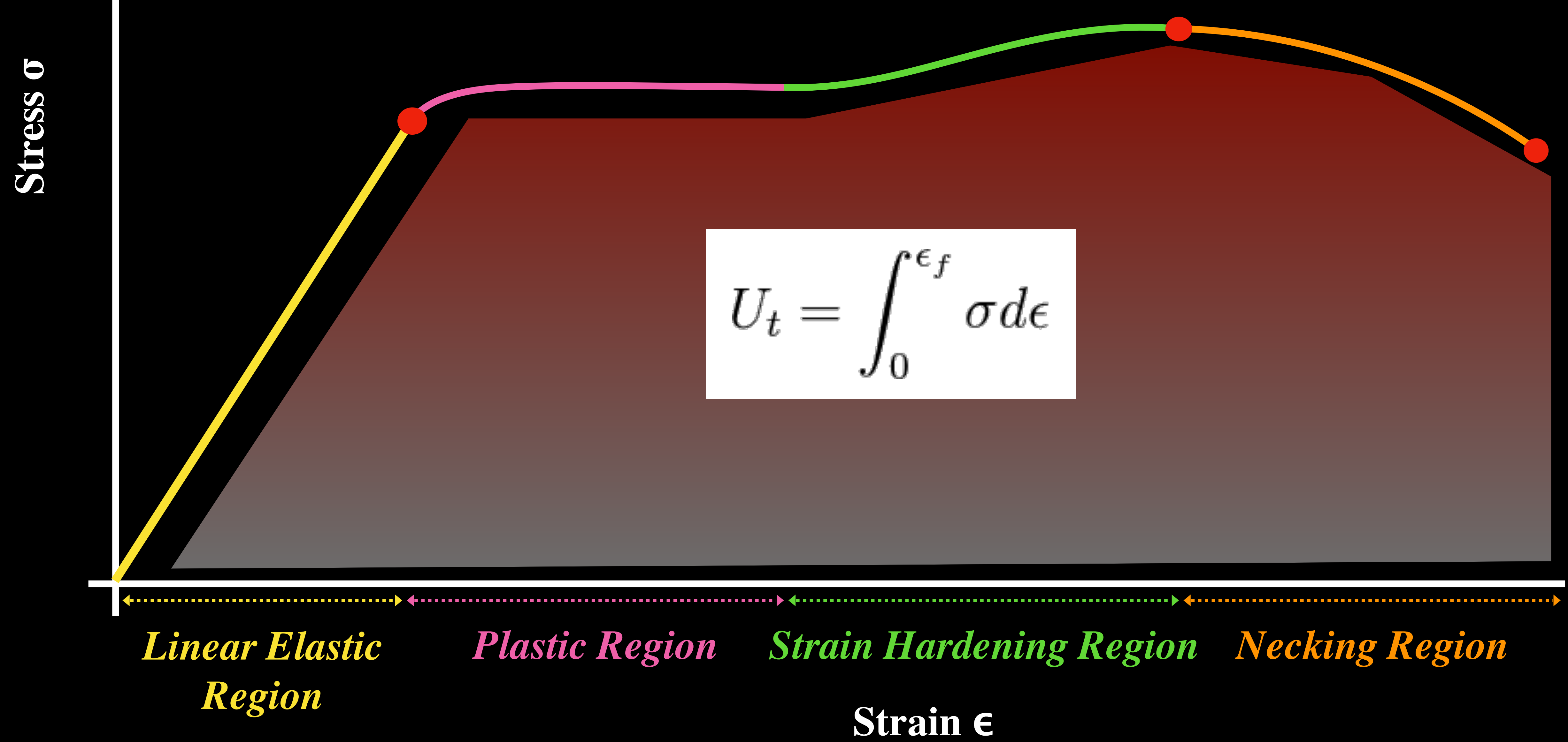
- The amount of energy required to arrive at different points in a material's stress/strain curve can be determined by finding the area under different parts of the curve.
- Area under Linear Elastic Region: amount of energy required to make the material yield.
    - This is called the Modulus of Resilience (***U_r***, sometimes ***u_r***), with units of J*m^{-3}
- Given the same ***σ_y***, a stiffer material will have a lower ***U_r*** than a less stiff material.



$$U_r = \frac{\sigma_y \epsilon_y}{2}$$

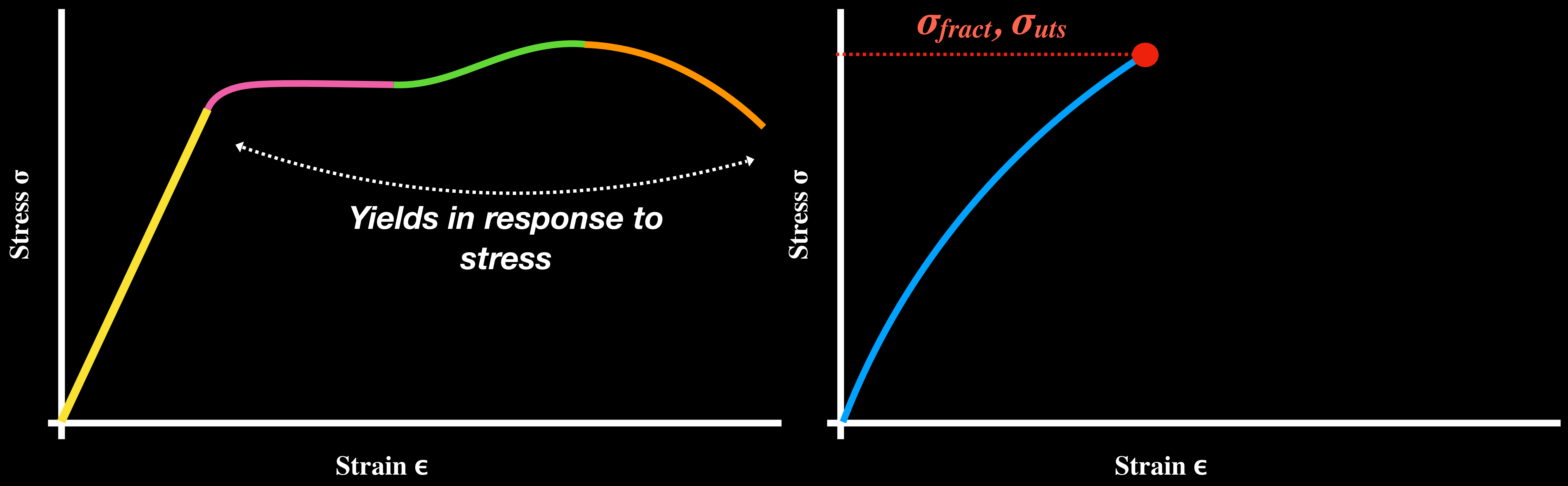
TOUGHNESS

- The amount of energy required to reach the point of fracture can be determined by finding the area under the entire stress-strain diagram.
  - This value is known as the modulus of toughness, U_t
  - Unlike the linear modulus of resilience, this is usually found using numerical integration approximations, graphical approaches, etc.
- For most structural mechatronic engineering situations, we focus more on U_r



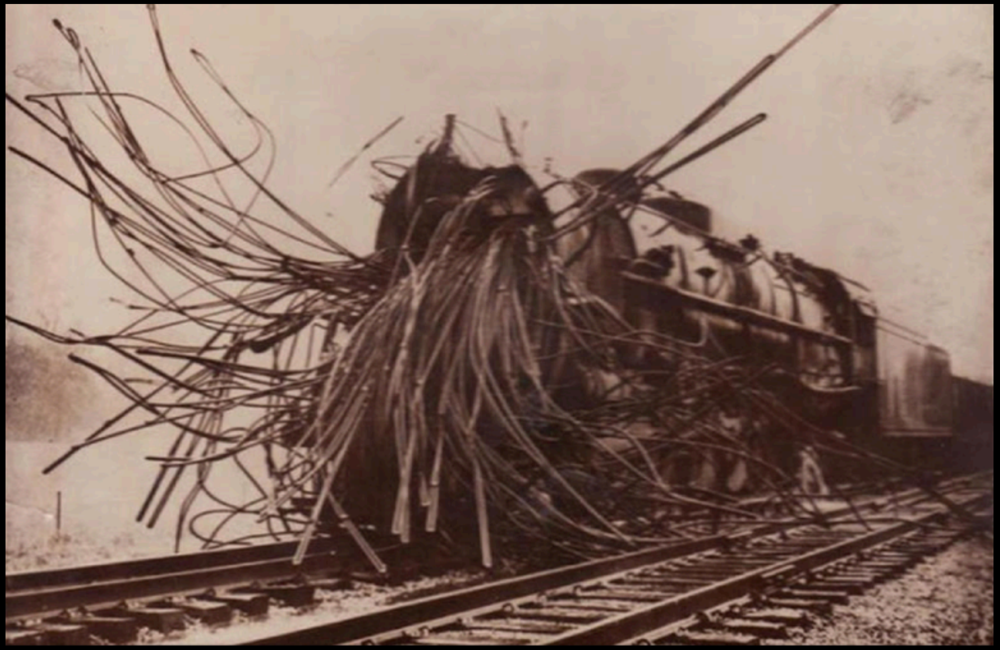
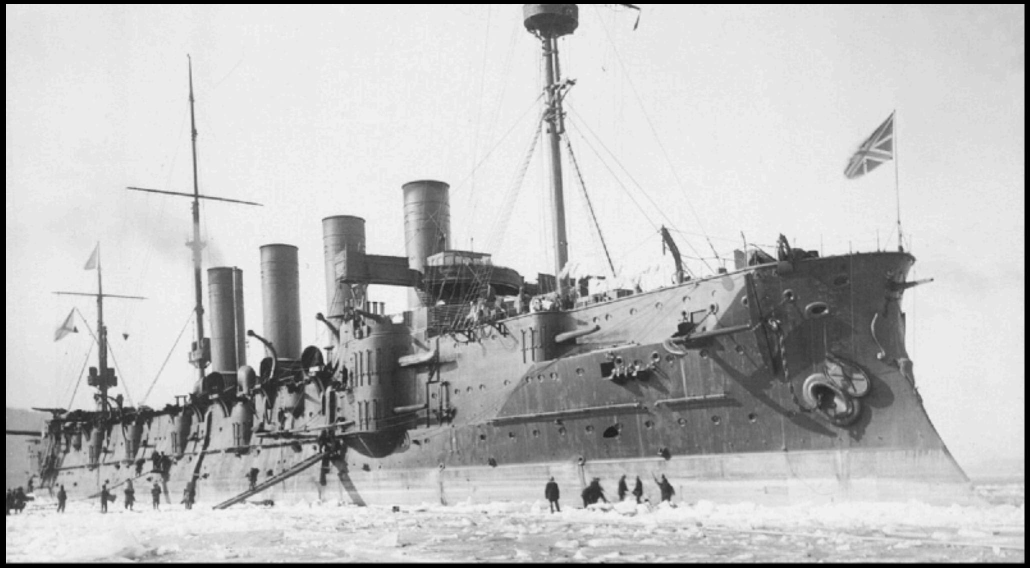
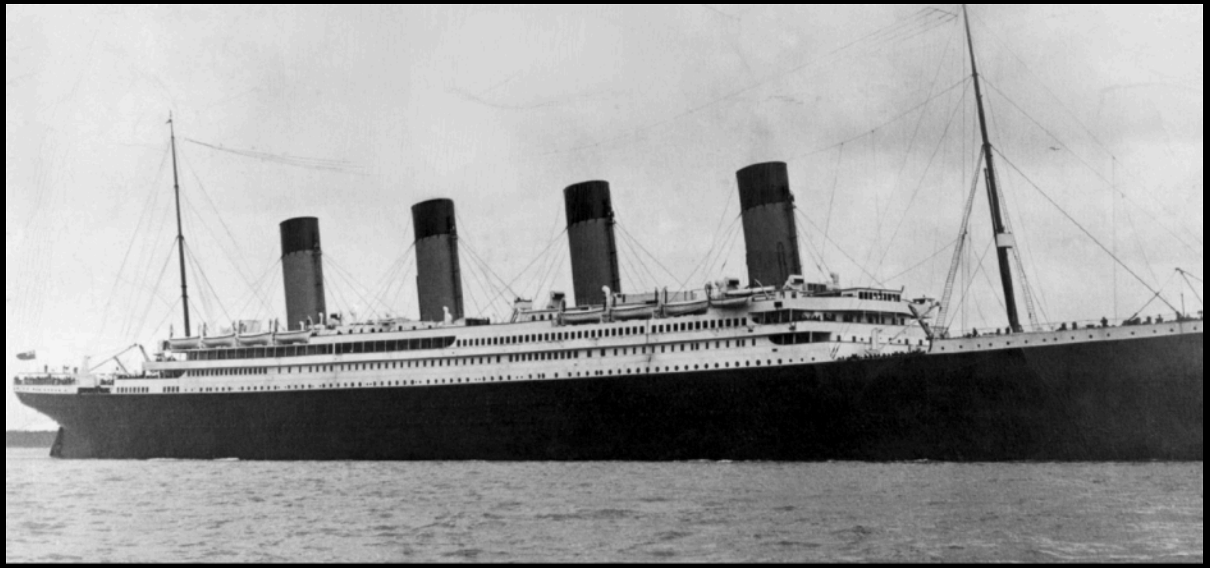
$$U_t = \int_0^{\epsilon_f} \sigma \, d\epsilon$$

DUCTILE/BRITTLE

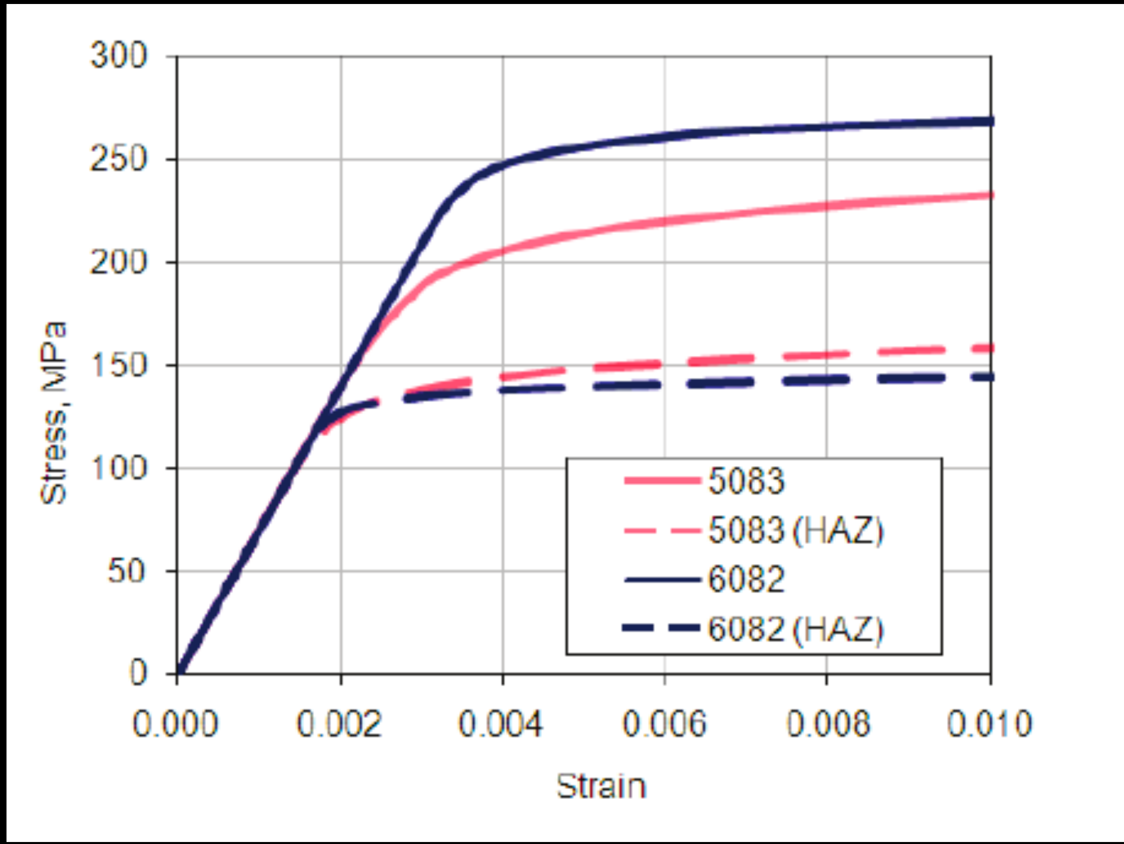


- Ductile materials exhibit yielding in response to stress.
  - Silver (Ag): very ductile
- A non-ductile material has the same point of ultimate tensile strength as its fracture point.
  - Failure occurs in the elastic region.
    - E.g., Glass: flex then break, no plastic deformation

DUCTILE/BRITTLE

EXAMPLE STRESS-STRAIN CURVE



https://www.researchgate.net/figure/Material-stress-strain-curves-for-aluminium-alloys-5083-H116-and-6082-T6-in-the-parent_fig2_233118591

Linear elastic region, affected by "Heat Affected Zones" in different aluminium alloys

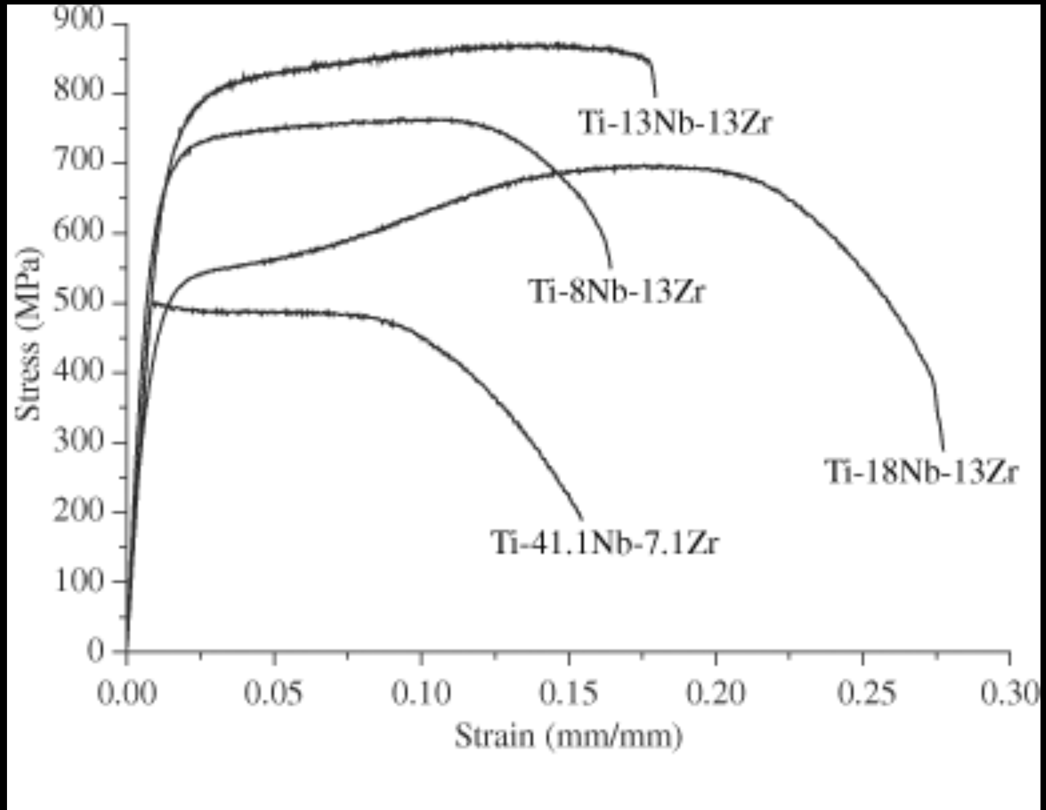


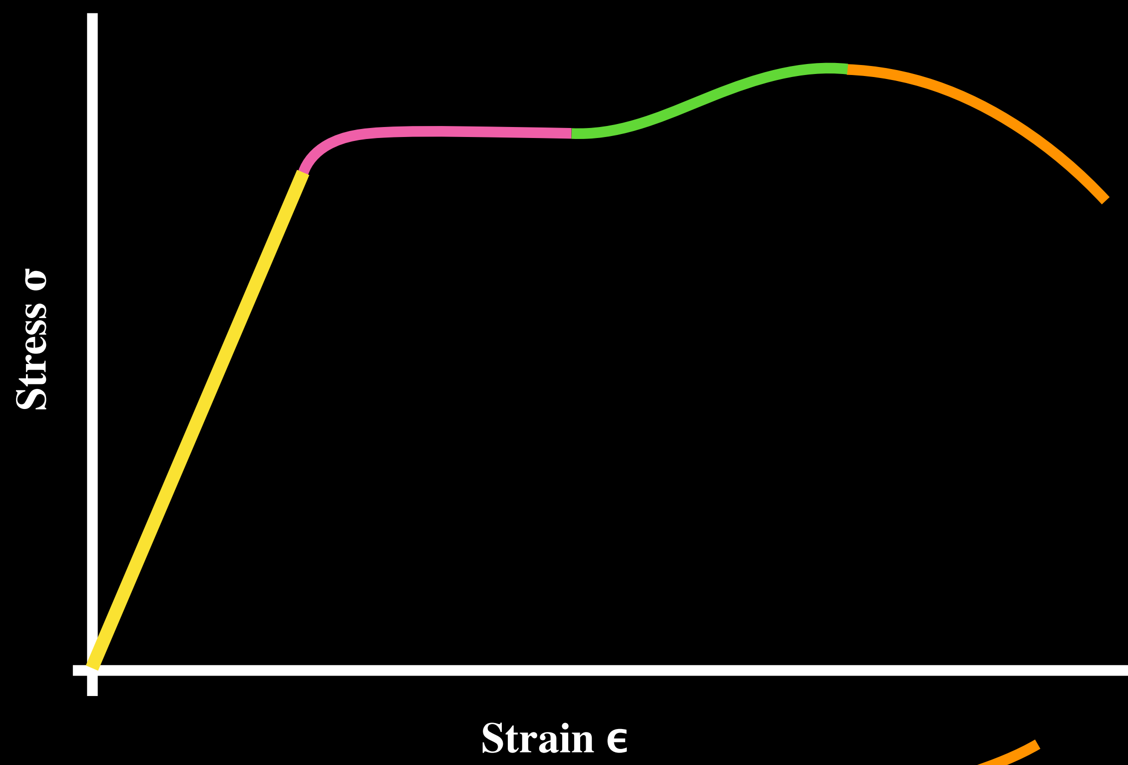
Figure 2. Full stress-strain curves for Ti-Nb-Zr alloys.

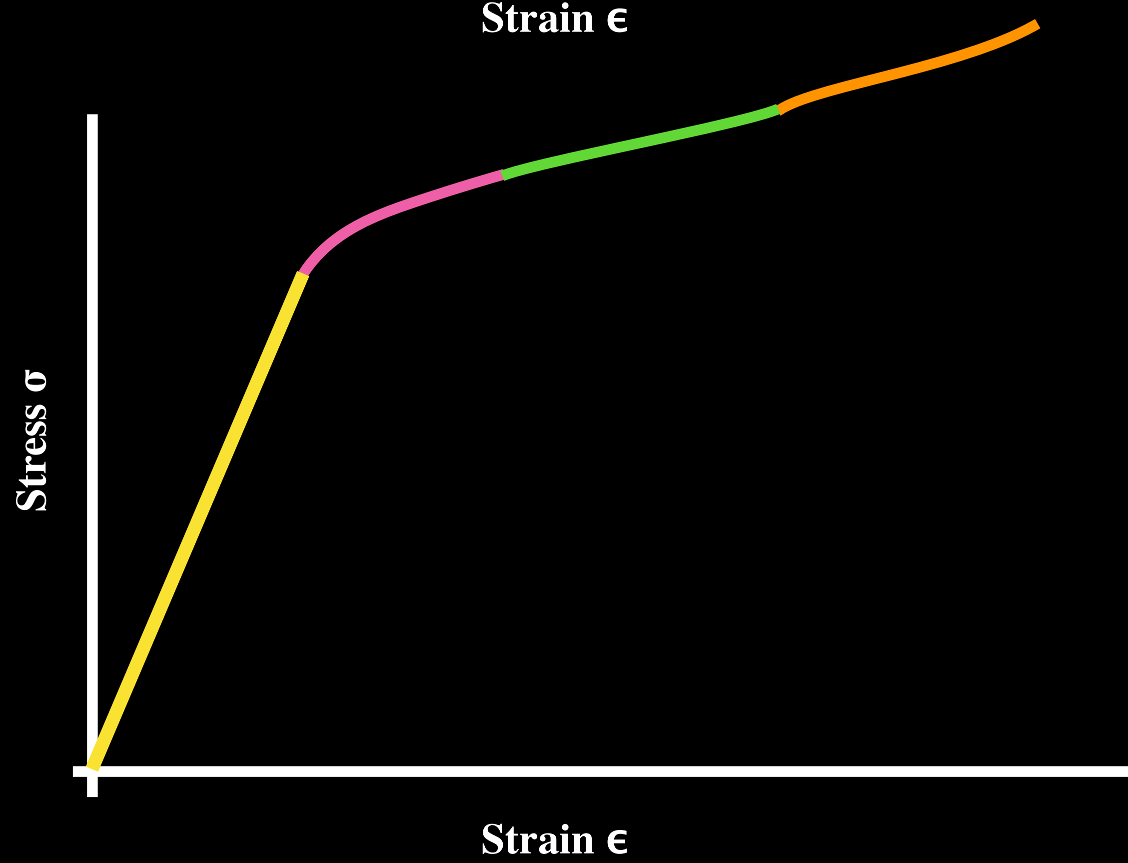
http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1516-14392005000400013

Stress-strain curves for various Titanium alloys. Tradeoff between cost, ultimate tensile strength, machinabilty, etc.

TRUE VS. ENGINEERING STRESS-STRAIN

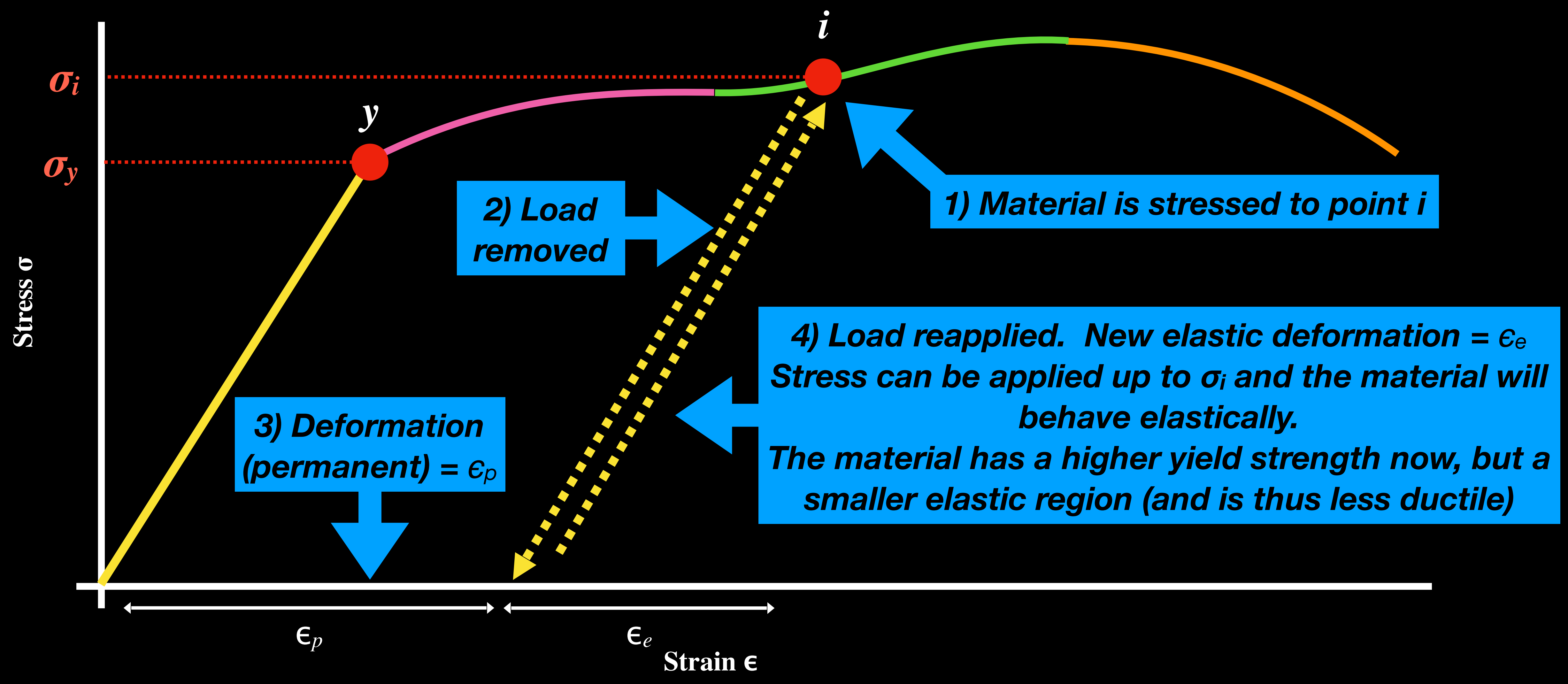
- Up to now, we have been assuming that the diameter of the sample doesn't change.
    - This assumption leads to apparent flat or decreasing slopes in stress-strain diagrams.
- As the sample is stressed, the diameter (and area) decreases.
    - If this decrease in area is taken into account, then the stress-strain diagram would show an upward slope until fracture.
        - Such a diagram is called a "true stress-strain diagram"





COLD WORKING

- There are many ways to work materials with *high temperatures*
  - *High temperatures*: temps. above recrystallisation temperature
  - These approaches tend to greatly affect the material's structure and, thus, its whole stress-strain curve.
- Working a material at normal temperatures ('cold working'):
  - Stress is applied to material into its plastic region.



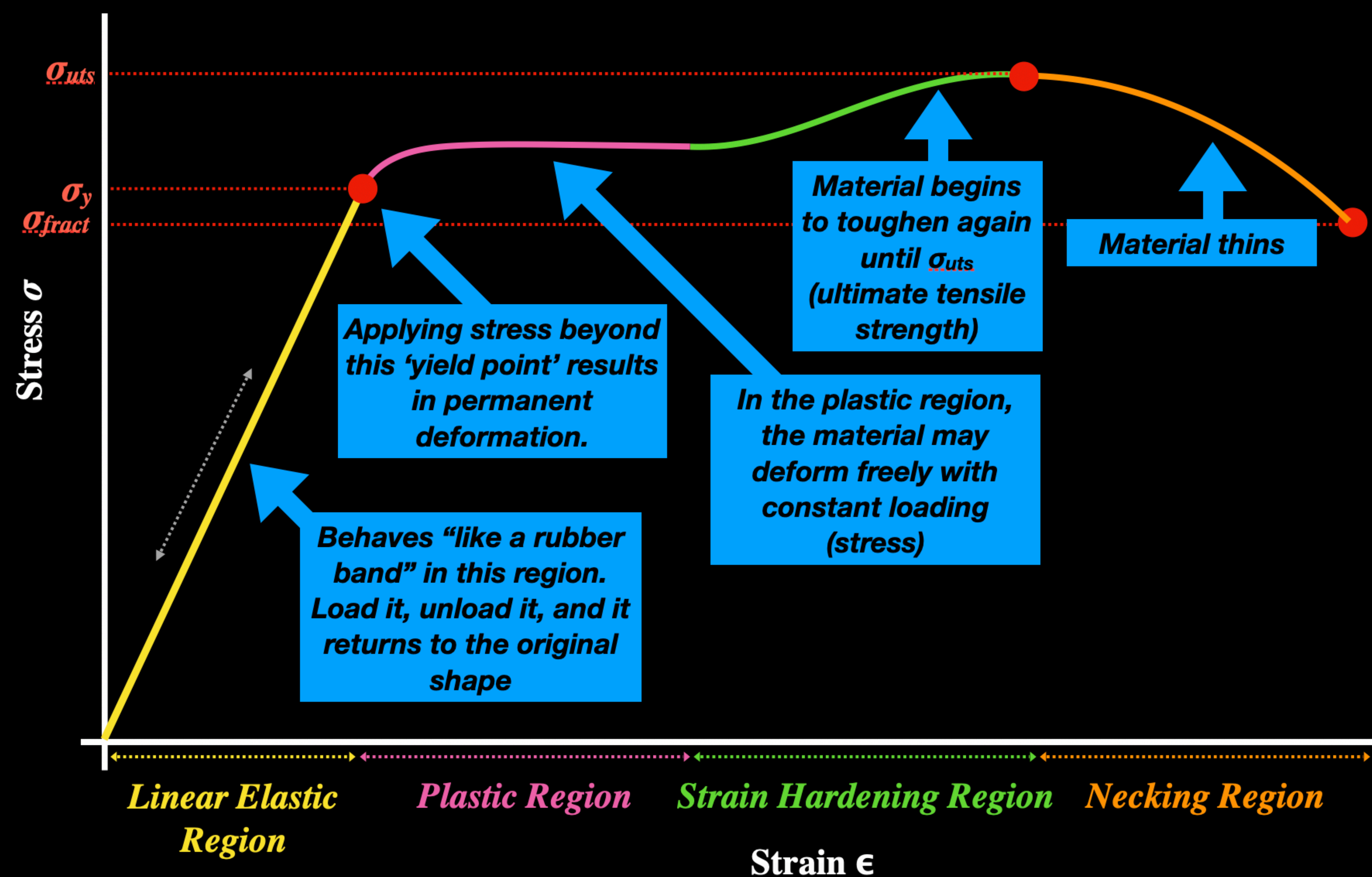
TODAY & TOMORROW

- Today:
  - Materials III: Hardness & Impact Behaviour
- Thursday: Materials IV: Deformation and Shear Stress
  - Gearbox assignment & Mechanical Principles assignment handed out
- (Next week: SolidWorks. SolidWorks videos will be posted to Blackboard. Jim will pop around to your labs for optional help sessions during Tues + Thurs lecture times)

MECHANICAL PRINCIPLES I

- **Assignment due end of week 9**
  - Bring completed assignment to lecture…
    - …or drop it off at my office (slide under the door if necessary)
- Show all work; attach to assignment sheets.
  - If using Excel or similar, include any formulae used
- Covers material through tomorrow's lecture (all 'materials' lectures)
- Assignment will be posted to Blackboard on Wednesday

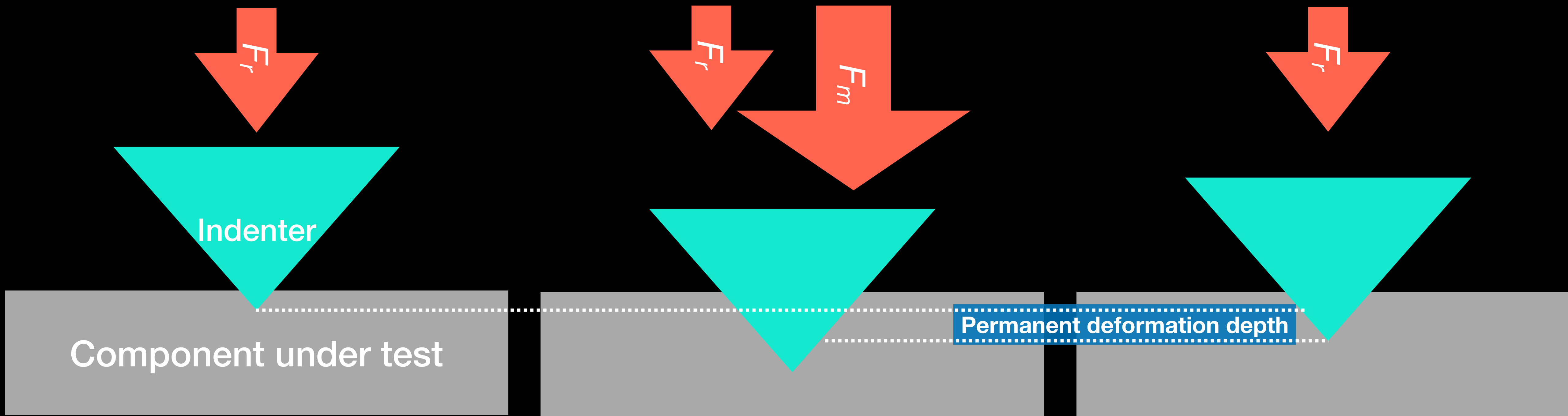
STRESS STRAIN DIAGRAM



HARDNESS

- Hardness: how difficult a material is to penetrate with a pointed tool.
  - It is difficult to conduct tensile tests (or similar):
    - Expensive equipment
    - Time-consuming
    - Destructive
  - We can use hardness tests as a way to more easily arrive at a material's ultimate strength.
- Hardness tests can be conducted at a very small scale (examining only a tiny portion of a material) up through very large-scale tests that use a large pointed tool to deform the material.
  - These large-scale tests are typically used in mechatronics engineering applications.
    - "Macro-hardness tests"
  - Two types dominate hardness testing in engineering:
    - Rockwell & Brinell hardness

ROCKWELL HARDNESS



1) The indenter is brought into contact with the component under test with an "Initial Force" specified according to the indenter material; this provides a precise known starting position.

2) Applied force is increased (with addition of a "Major Force," an amount specified by the indenter material).

3) Major Force is released. The resulting permanent deformation depth is measured. This depth is then input into an equation whose variables depend on the indenter material.

- Advantage: easily tested by a single apparatus that can be completely automated.
  - Very popular for the testing of parts on a production line.
- Disadvantage: tests only a small area (may give an incomplete picture)
  - Also, there are many different Rockwell Hardness scales that depend on the indenter used! Things can get confusing, particularly in assemblies with many different materials.

ROCKWELL HARDNESS

- Rockwell scales: A, B, C, etc.
  - A scale indenter: diamond cone (60 kg load)
  - B scale indenter: ~1.6 mm diameter ball (100 kg load)
  - C scale indenter: diamond cone (150 kg load)
- Key takeaway: Rockwell Hardness tests are relative to other tests on the same scale only.

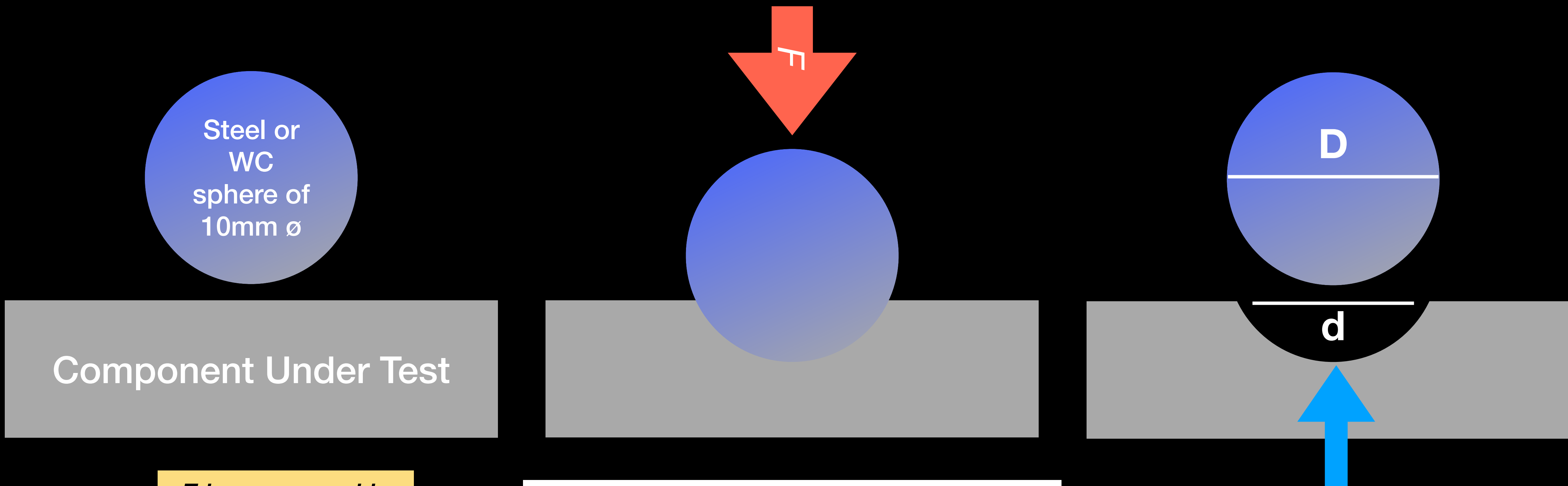
Rockwell Hardness Scales

Scale	Indenter	Minor Load *F0* kgf	Major Load *F1* kgf	Total Load *F* kgf	Value of *E*
A	Diamond cone	10	50	60	100
B	1/16" steel ball	10	90	100	130
C	Diamond cone	10	140	150	100
D	Diamond cone	10	90	100	100
E	1/8" steel ball	10	90	100	130
F	1/16" steel ball	10	50	60	130
G	1/16" steel ball	10	140	150	130
H	1/8" steel ball	10	50	60	130
K	1/8" steel ball	10	140	150	130
L	1/4" steel ball	10	50	60	130
M	1/4" steel ball	10	90	100	130
P	1/4" steel ball	10	140	150	130
R	1/2" steel ball	10	50	60	130
S	1/2" steel ball	10	90	100	130
V	1/2" steel ball	10	140	150	130

BRINELL HARDNESS

- A good approach to use when avg. hardness is desired (due to a much larger indenter than is used in Rockwell hardness testing)
- This test is a bit more difficult/slow to automate (it usually requires manual measurement of the indentation).
  - Like Rockwell, it is largely non-destructive.
- Hardness Hb equals the load applied to the sphere divided by the (spherical!) surface area of the resulting indentation.

BRINELL HARDNESS



F is expressed in kgf (1 kgf is equal to 9.80665 N.)

$$BHN = \frac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

Diameter of spherical depression is measured.

- A good approach to use when avg. hardness is desired (due to a much larger indenter than is used in Rockwell hardness testing)
- This test is a bit more difficult/slow to automate (it usually requires manual measurement of the indentation).
  - Like Rockwell, it is largely non-destructive.
- Hardness Hb equals the load applied to the sphere divided by the (spherical!) surface area of the resulting indentation.

BRINELL HARDNESS

Relationship between Brinell Hardness and ultimate strength of a material is (roughly) linear.

$$S_u = 3.4\, H_b\, \mathrm{MPa}$$

Approximation for steels

$$S_u = 1.58\, H_b - 86\, \mathrm{MPa}$$

Approximation for cast irons*

D. E. Krause, "Gray Iron—A Unique Engineering Material," ASTM Special Publication 455, 1969, pp. 3–29, as reported in Charles F. Walton (ed.), Iron Castings Handbook, Iron Founders Society, Inc., Cleveland, 1971, pp. 204, 205.

MATERIAL HARDNESS

Rockwell B80: B scale (soft metals: aluminium, soft steel, etc.)

43 Products How can

About Carbon Steel, Alloy Steel, Spring Steel, and Cast Iron
More

Tight-Tolerance Ultra-Machinable 12L14 Carbon Steel Rods

- Yield Strength: 65,000 psi
- Hardness: Rockwell B80 (Medium)
- Heat Treatable: Yes
- Max. Hardness After Heat Treatment: Rockwell C65
- Specifications Met: ASTM A108

Ready for turning in a lathe, these rods are precision ground and held to a strict straightness tolerance. The lead additive acts as a lubricant, which allows 12L14 carbon steel to withstand very fast machining. It's used to fabricate a wide variety of machine parts.

CAD For technical drawings and 3-D models, click on a part number.

https://www.mcmaster.com/metals/steel/tight-tolerance-ultra-machinable-12l14-carbon-steel-rods/

MATERIAL HARDNESS

Rockwell C60: C scale (Hard steels)

Hardened Undersized High-Speed M2 Tool Steel Rods

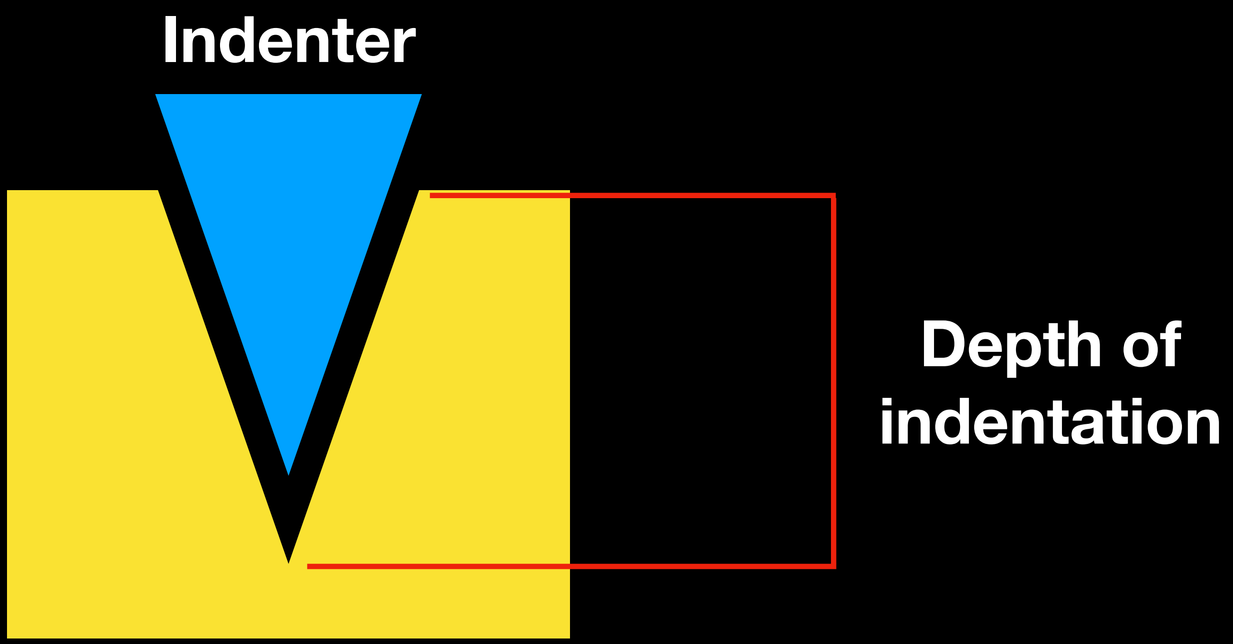
- Yield Strength: Not Rated
- Hardness: Rockwell C60 (Very Hard)

These rods, also known as drill blanks, have an undersized diameter for machining your own jobbers'-length drill bits. They're hardened for increased abrasion and impact resistance. M2 tool steel offers a nice balance of wear resistance and machinability. It has a high molybdenum content, which allows it to maintain sharp cutting edges even at elevated temperatures.

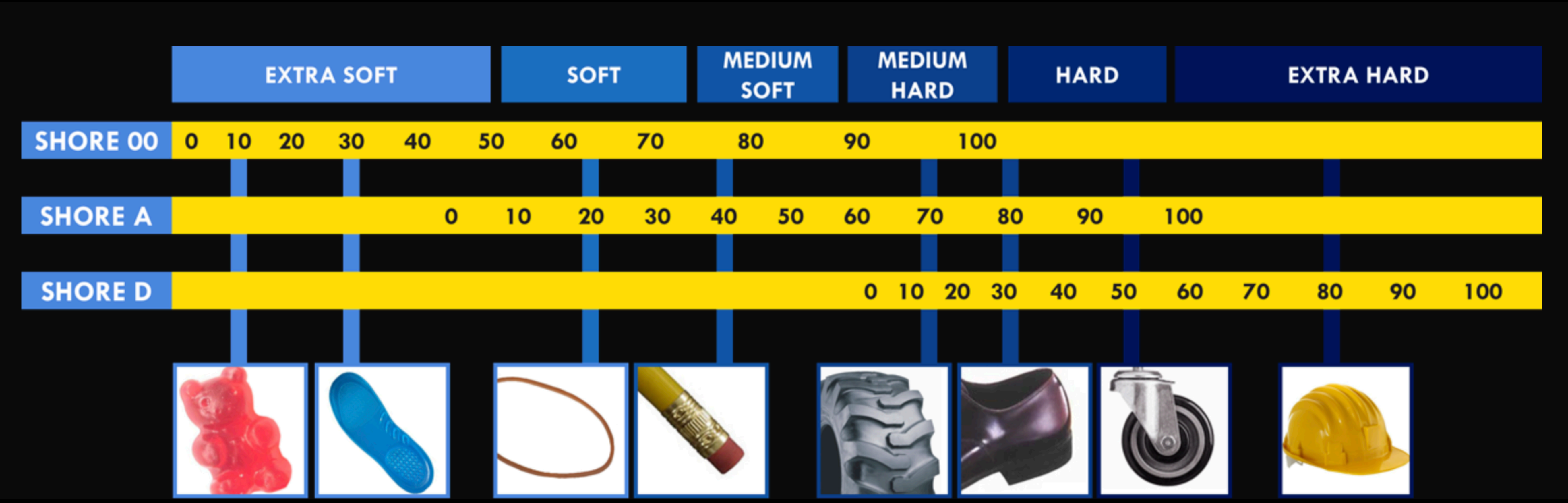
https://www.mcmaster.com/metals/tool-steel/hardened-undersized-high-speed-m2-tool-steel-rods-9/

HARDNESS OF "SOFT" MATERIALS

- For rubbers, polymers, and other 'soft' materials, the Shore Hardness scales are used.
  - There are multiple different shapes of indenters (A, C, D, B, M, E, O, OO, DO, OOO, OOO-S); no need to know all of these… OO, A, and D are by far the most common. Scales are unrelated and overlap.
  - A: Flattened cone-shaped indenter (35 degree), diameter of 1.40 mm, spring force of 8.05 N; OO = 1.20 mm sphere, 1.111 N spring force (for very soft materials).
  - D: Cone shaped indenter (30 degree), 1.40 mm diameter, spring force: 44.45 N
- Tests examine indentation depth after 15 seconds of indenter application.
  - If the indenter presses 2.54 mm into a material, then the durometer reads 0; if the indenter presses 0 mm into a material, the durometer reads 100.



SHORE DUROMETER



SHORE DUROMETER

Durometer: from 30A to 70A (using the Durometer A scale)

251 Products

Multipurpose Neoprene Rubber Sheets and Strips

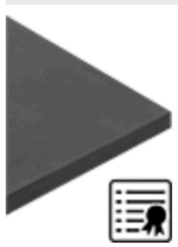
Neoprene is also known as chloroprene. It offers good oil and abrasion resistance. Softer durometer rubber has better conformability; harder durometer rubber is more wear resistant.

Crisscross texture rubber provides a nonslip gripping surface on both sides.

Buffed texture rubber is rough on one side to accept adhesive and smooth on the other.

Rubber **with material certification** comes with a traceable lot number and cure date.

Sheets



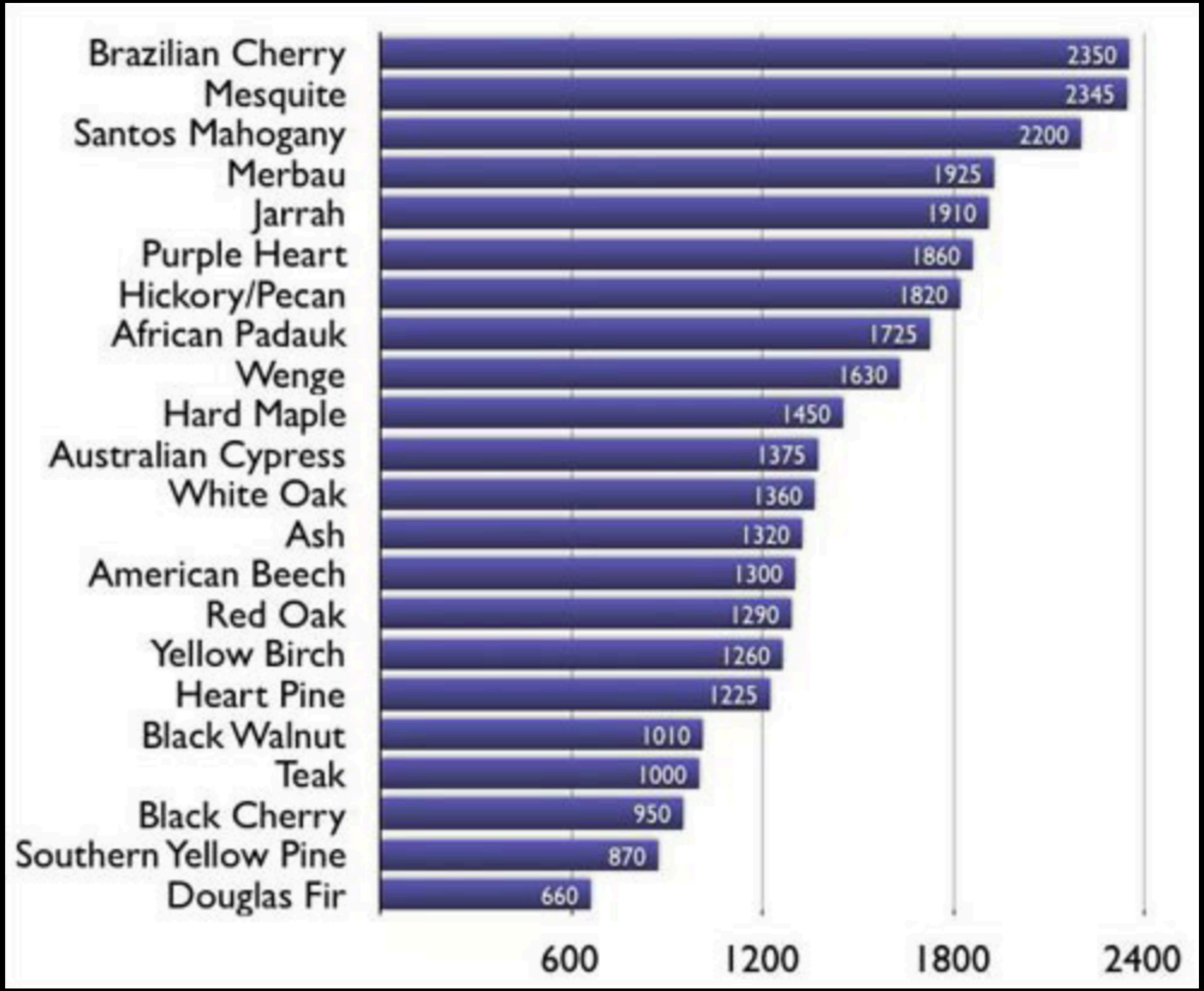
- Color: Black
- Temperature Range: -30° to 200° F
- Tensile Strength: 900 psi
- For Use Outdoors: Yes

			6" × 6"		12" × 12"		12" × 24"	
Thick.	Thick. Tolerance	Choose a Durometer		Each		Each		Each
Material Certification with Traceable Lot Number and Cure Date								
1/64"	-0.010" to +0.010"	50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N11	$5.92	1370N31	$9.28	1370N51	$12.61
1/32"	-0.010" to +0.010"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N12	6.21	1370N32	10.41	1370N52	13.76
1/16"	-0.016" to +0.016"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N13	6.75	1370N33	11.73	1370N53	15.83
3/32"	-0.016" to +0.016"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N14	7.65	1370N34	13.49	1370N54	19.93
1/8"	-0.020" to +0.020"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N15	9.12	1370N35	16.55	1370N55	24.92
3/16"	-0.031" to +0.031"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N16	10.84	1370N36	20.77	1370N56	32.58
1/4"	-0.031" to +0.031"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N17	13.60	1370N37	27.33	1370N57	41.86
3/8"	-0.047" to +0.047"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N18	17.37	1370N38	34.40	1370N58	58.47
1/2"	-0.047" to +0.047"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N19	21.92	1370N39	43.33	1370N59	74.01
3/4"	-0.094" to +0.094"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N21	31.29	1370N41	65.48	1370N61	111.36
1"	-0.100" to +0.100"	30A (Soft), 40A (Medium Soft), 50A (Medium), 60A (Medium Hard), 70A (Hard)	1370N22	39.78	1370N42	86.83	1370N62	148.48

https://www.mcmaster.com/rubber/rubber/multipurpose-neoprene-rubber-sheets-and-strips/

WOOD HARDNESS

- Wood is heterogeneous: different behaviour with grain, tangential to grain, etc.
  - Janka Scale: How much force (kN) is needed to press a steel ball, 11.27 mm in diameter halfway into a piece of wood.
  - Many caveats due to heterogeneity: wood dimensions, wood location on tree, knot content, moisture content, etc. are all controlled.



https://www.builddirect.com/learning-center/wp-content/uploads/2010/03/Janka_Chart.jpg

OTHER HARDNESS TESTS

- Vickers
  - Pyramidal indentation
  - Wide dynamic range
  - Developed in 1921 as easier alternative to Brinell
  - Pyramidal indenter's dimensions are measured
- Mohs Hardness Scale
  - Used by ceramics engineers, geologists, mineralogists
  - A material at a given Mohs number will scratch all materials with lower numbers.
- Knoop Hardness Scale
  - Used for measuring brittle, thin materials
  - Diamond micro-indentations are made

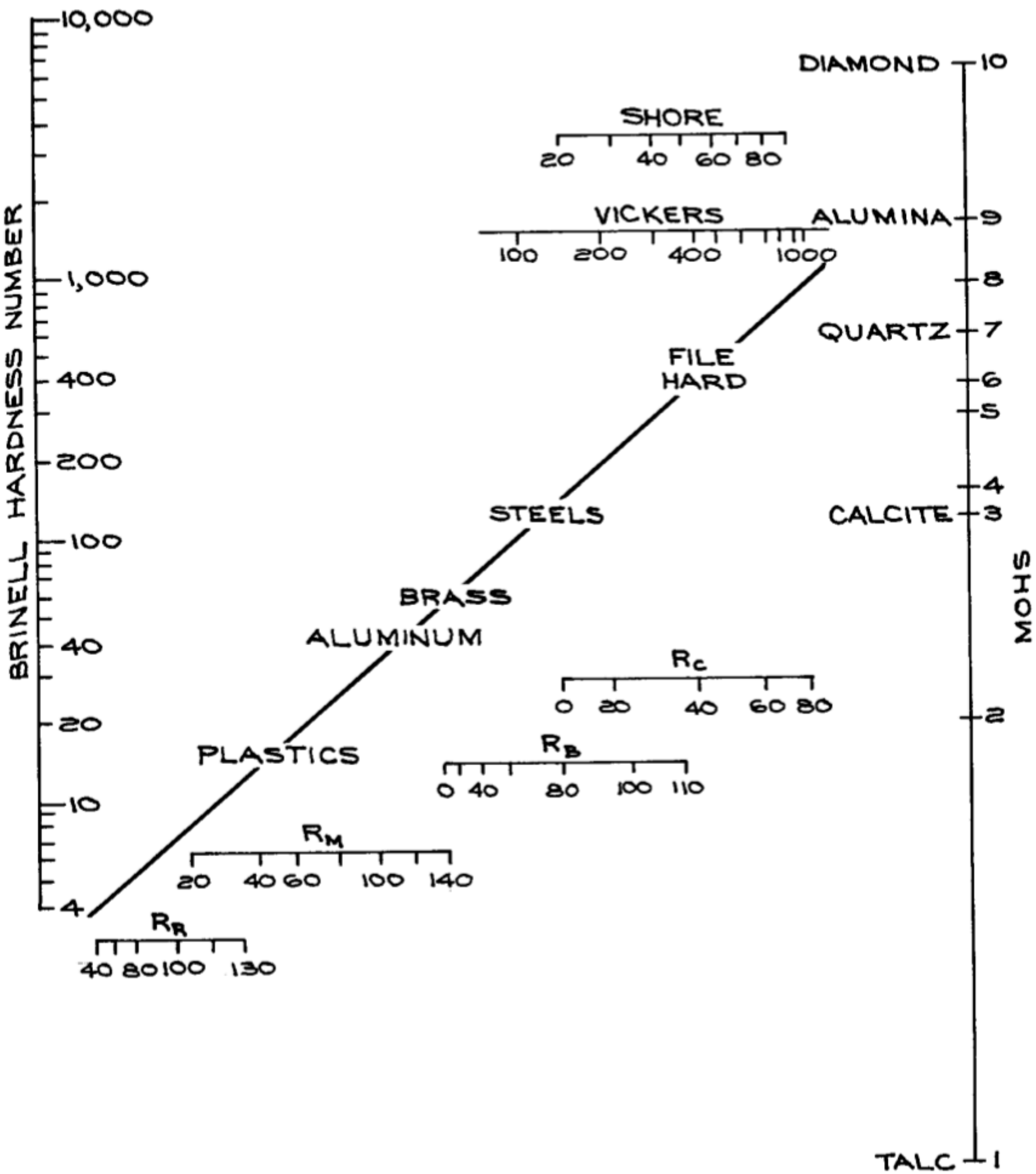
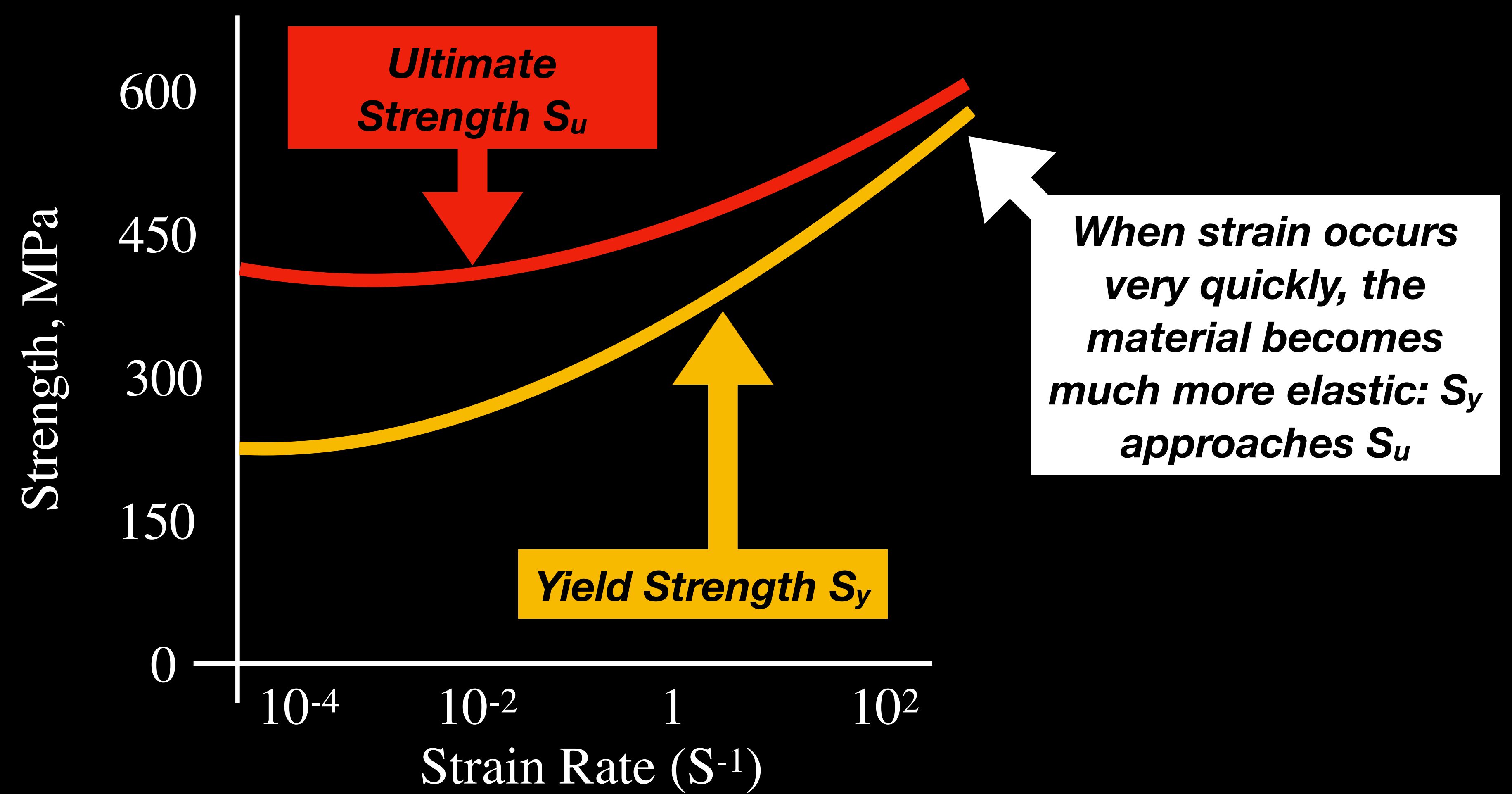


Figure 1.13 Approximate relation of Brinell (BHN), Rockwell (R_R, R_M, R_B, R_C), Vickers (VHN), Shore, and Mohs hardness scales.

"Building Scientific Apparatus" 4th Edition, Moore et al.

IMPACT BEHAVIOUR

- The preceding studies examined material behaviour under static (slowly applied) stresses.
  - If the stress applied at a rate slower than 1/3rd of the fundamental vibrational frequency of the material, the load is said to be static.
  - Faster than this, and this suddenly applied stress is said to be dynamic.
- The stress-strain relationship changes under impact conditions.

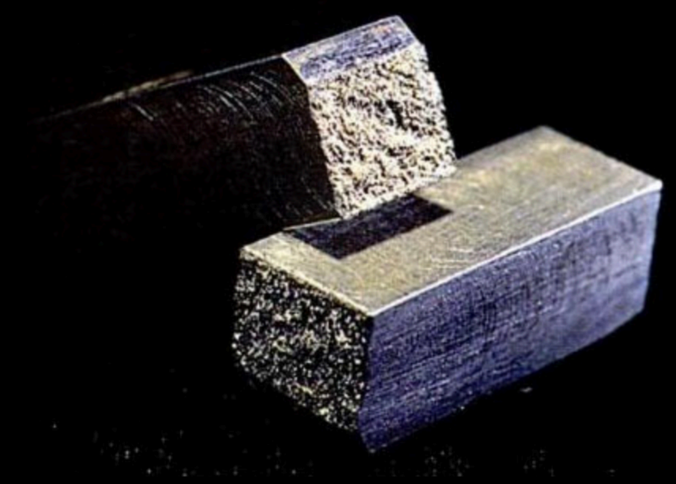


Graph from 2-8, Shipley's Mechanical Engineering Design

IMPACT TESTING

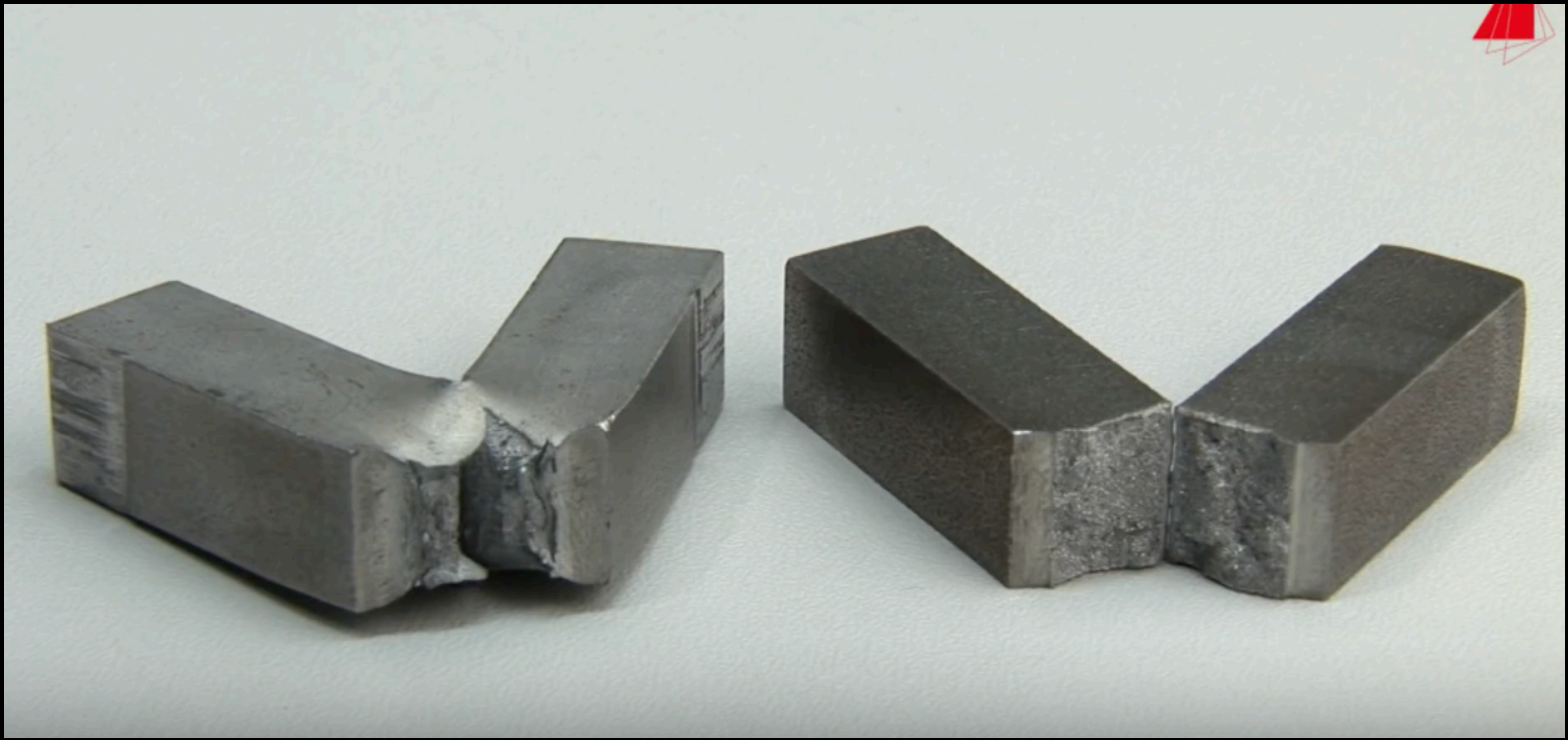
- In the Charpy Test, a swinging pendulum apparatus is usually used to test a material's impact behaviour.
  - A pendulum is raised to a known height and is released, swinging against a material.
  - The material is prepared with a v-shaped groove of specified dimensions.
- The distance that the pendulum bounces back (or follows through) from the material shows how much energy is absorbed the material in an impact.
  - More bounce-back/follow-through = less energy absorbed; more brittle material.
- The material's failure mode can be examined for empirical evidence of ductility/brittleness.
  - *Used to forensically analyse Titanic steel!*
- This test is quite sensitive to temperature and dimension errors (in the v-groove); more modern fracture analysis approaches have come to dominate high-consequence applications (alternatively, use many samples of a Charpy test… expensive, though)
- For more information: https://www.tf.uni-kiel.de/matwis/amat/iss/kap_3/backbone/r3_2_2.html

Brittle: No elastic deformation. Clean granular break.



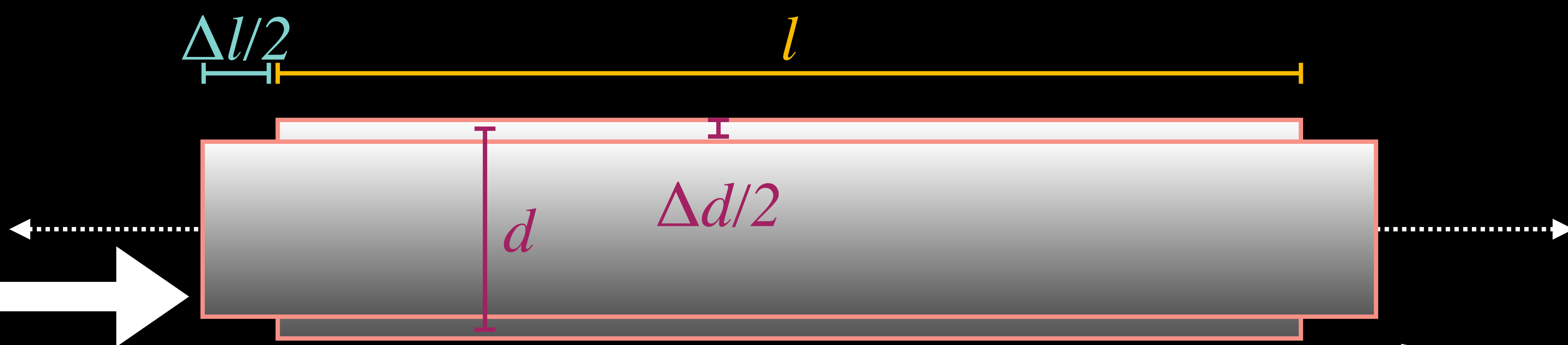
Ductile: evidence of elastic deformation prior to fracture

VIDEO: CHARPY TEST



https://www.youtube.com/watch?v=tpGhqQvftAo

LAT/LONG DEFORMATION



Elongation in loaded (axial) dimension

Contraction in lateral dimension (cross section)

$$\epsilon_{long} = \frac{\Delta l}{l_0}$$

$$\epsilon_{lat} = -\frac{\Delta d}{d_0}$$

Negative value due to the contraction of the material.

- If the initial length l and change in length Δl are known, and the initial diameter d and change in diameter Δd are known…
  - …longitudinal strain ε_{long} and latitudinal strain ε_{lat} may be found.

NEXT WEEK (UPDATED)

- No lecture Monday (Public Holiday)
- Tuesday: meet here for Gearbox Assignment discussion
- Thursday: SolidWorks help session (I'll pop by Co239 and AM219 to help with SolidWorks; SolidWorks videos will be posted beforehand on Blackboard)

POISSON'S RATIO

- Poisson observed isotropic & homogeneous materials
  - He noted that the ratio between ε_{long} and ε_{lat} is a constant.
  - This constant is known as Poisson's Ratio, v (nu)
- A lower value of v means that there is less contraction when the specimen is placed under tension and less expansion when the specimen is compressed.
  - Most materials have Possion's Ratios between 0 and 0.5.
- Cork's low v ($v \approx 0.0$) means that wine bottle corks fit easily without expansion.
- v becomes important when considering fits of materials under tension and compression.
  - e.g., pipes under pressure
  - or precision fits of loaded parts
- (Auxetic: material with negative Poisson's ratio (!!))

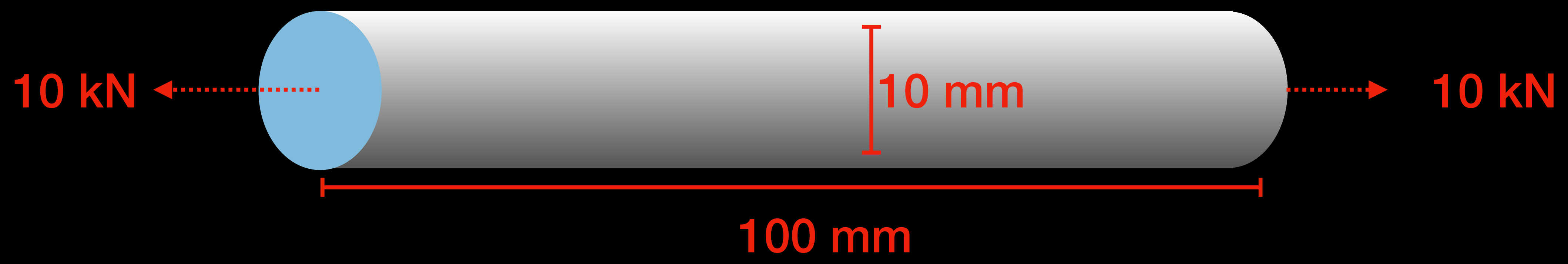
$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

MATERIAL	POISSON'S RATIO (v)
Lead	0.425
Phosphor Bronze	0.349
Carbon steel	0.292
Glass	0.245

POISSON'S RATIO

- A carbon steel rod (circular cross section; *E* of 200 GPa, *v* = 0.292) has a diameter of 10 mm and an initial length of 100 mm. Find the diameter of the rod when an axial load of 10 kN is applied.

1) Make a drawing.



POISSON'S RATIO

- A carbon steel rod (circular cross section; *E* of 200 GPa, *v* = 0.292) has a diameter of 10 mm and an initial length of 100 mm. Find the diameter of the rod when an axial load of 10 kN is applied.

2) Calculate normal stress & strain in the rod.

kN/mm² = GPa

$$\sigma = \frac{P}{A_0} \quad \text{where} \quad A_0 = \frac{1}{4}\pi d_0^2 \quad = \frac{10\ kN}{\frac{\pi}{4}(10\ mm)^2} = 0.1273\ \frac{kN}{mm^2}$$

Hooke's Law: relates stress to strain

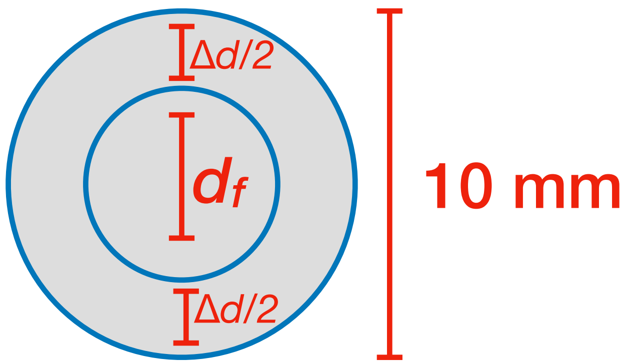
$$\sigma = E\epsilon \qquad \epsilon_{long} = \frac{\sigma}{E} = \frac{0.1273\ GPa}{200\ GPa} = 0.0006365$$

POISSON'S RATIO

- A carbon steel rod (circular cross section; *E* of 200 GPa, *v* = 0.292) has a diameter of 10 mm and an initial length of 100 mm. Find the diameter of the rod when an axial load of 10 kN is applied.

3) Calculate ε_{lat} using Poisson's Ratio

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$



$$\epsilon_{lat} = (-0.292)(0.0006365)$$
$$\epsilon_{lat} = -0.0001859$$

Negative value: it constricted.

POISSON'S RATIO

- A carbon steel rod (circular cross section; *E* of 200 GPa, *v* = 0.292) has a diameter of 10 mm and an initial length of 100 mm. Find the diameter of the rod when an axial load of 10 kN is applied.

4) Calculate the change in diameter

$$\epsilon = \frac{\delta}{l_0} \quad \epsilon_{lat} = \frac{\Delta d}{d_0} \quad \Delta d = (-0.0001859)(10\ mm)$$

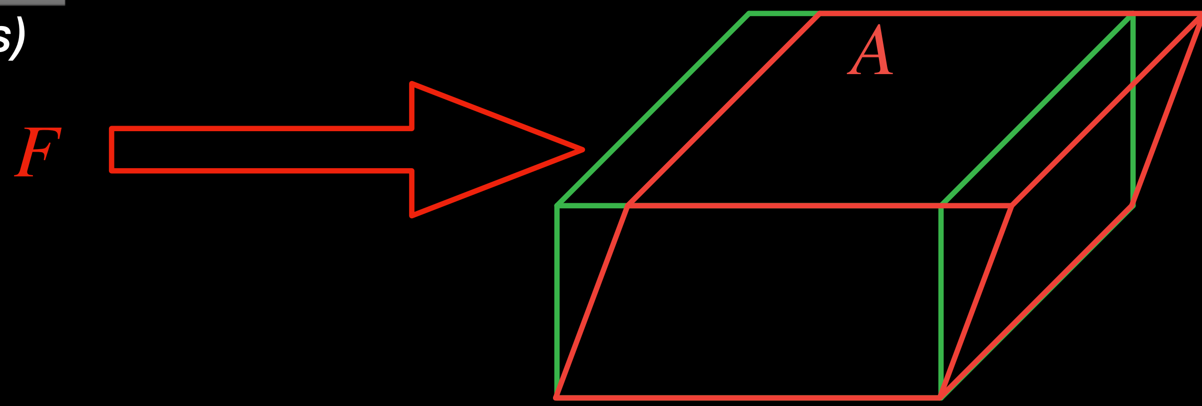
$$\Delta d = -0.001859\ mm$$

5) Calculate the final diameter

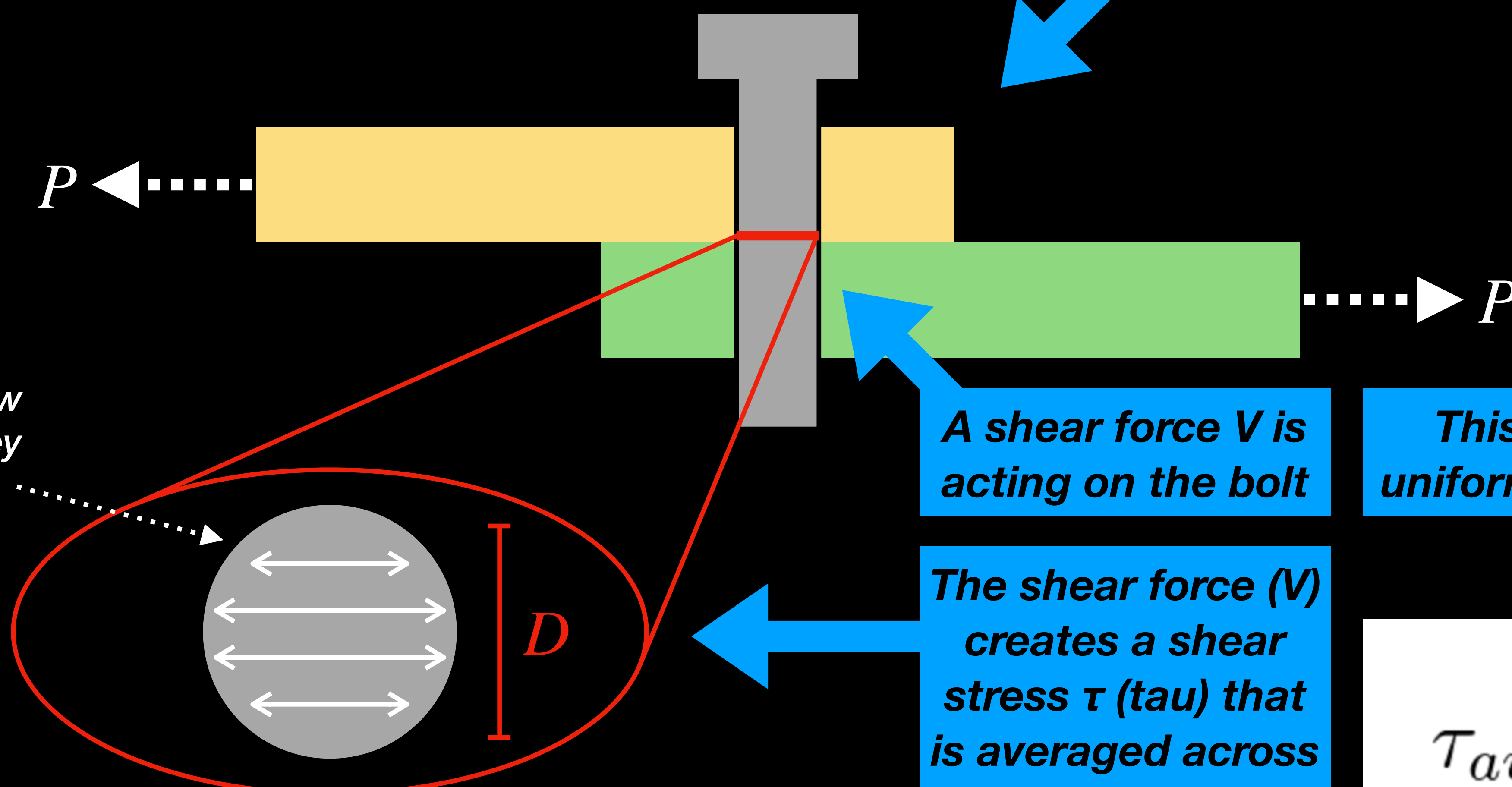
$$\Delta d = d_f - d_0 \quad d_f = \Delta d + d_0 \quad \boxed{d_f = 9.9981\ mm}$$

SHEAR STRESS

(Tensile stress)



A single shear connection: Two plates with force transferred at one plane. The whole load is transferred through the single bolted connection (P/1)



Note how these stress lines are moving perpendicular to how they would were they normal stresses

A shear force V is acting on the bolt

This assumes a uniform shear stress

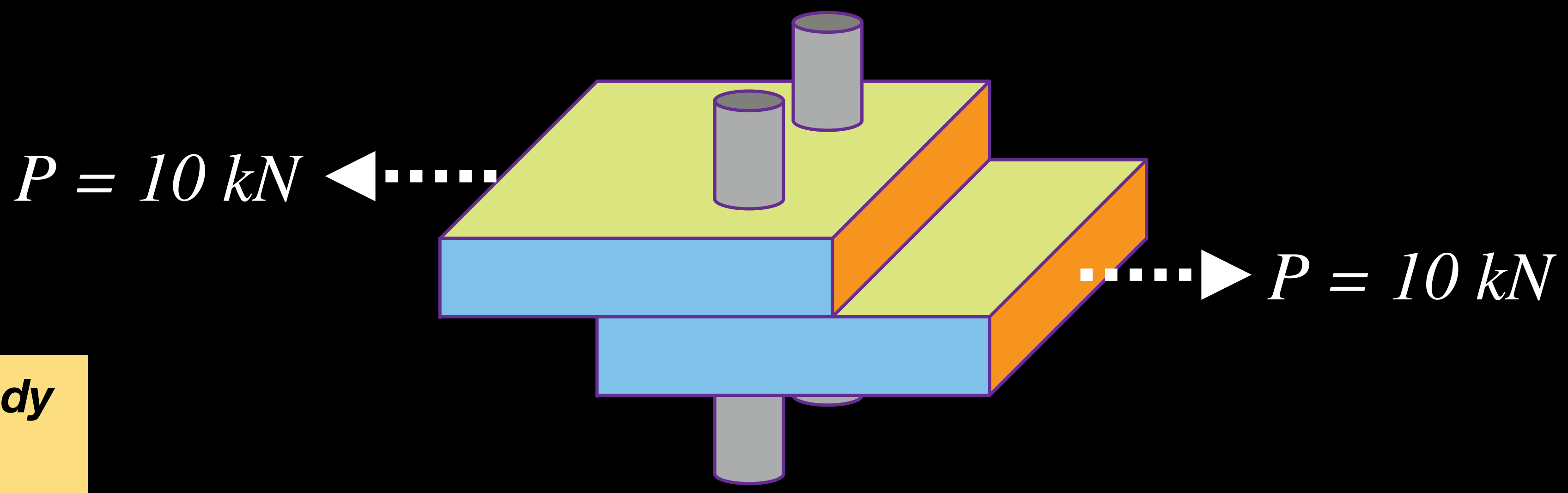
The shear force (V) creates a shear stress τ (tau) that is averaged across the bolt's cross sectional area A

$$\tau_{avg} = \frac{V}{A}$$

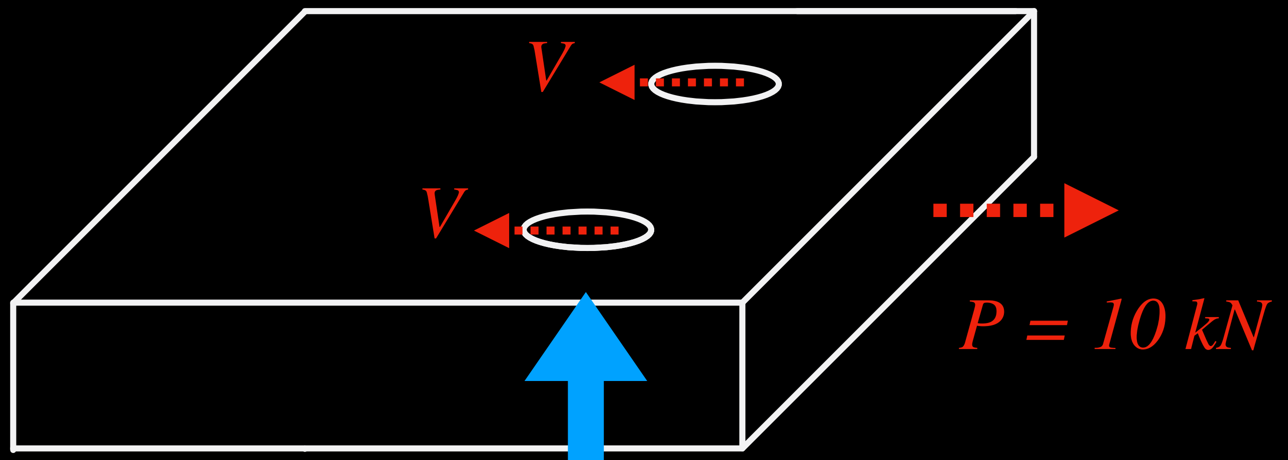
Note: With correct preload, the bolt/rivet brings the materials into contact such that their friction provides much of the strength

EXAMPLE: SHEAR STRESS

- Two pieces of 3 mm thick Perspex are joined with two 3 mm rods. What is the average shear stress in the rods when 10 kN of load (in tension to the two pieces of Perspex) is applied?



1) Draw a free body diagram of the bottom (or top) plate.



Normal load P results in two areas of shear load V

$$V = \frac{P}{2} = 5\ kN$$

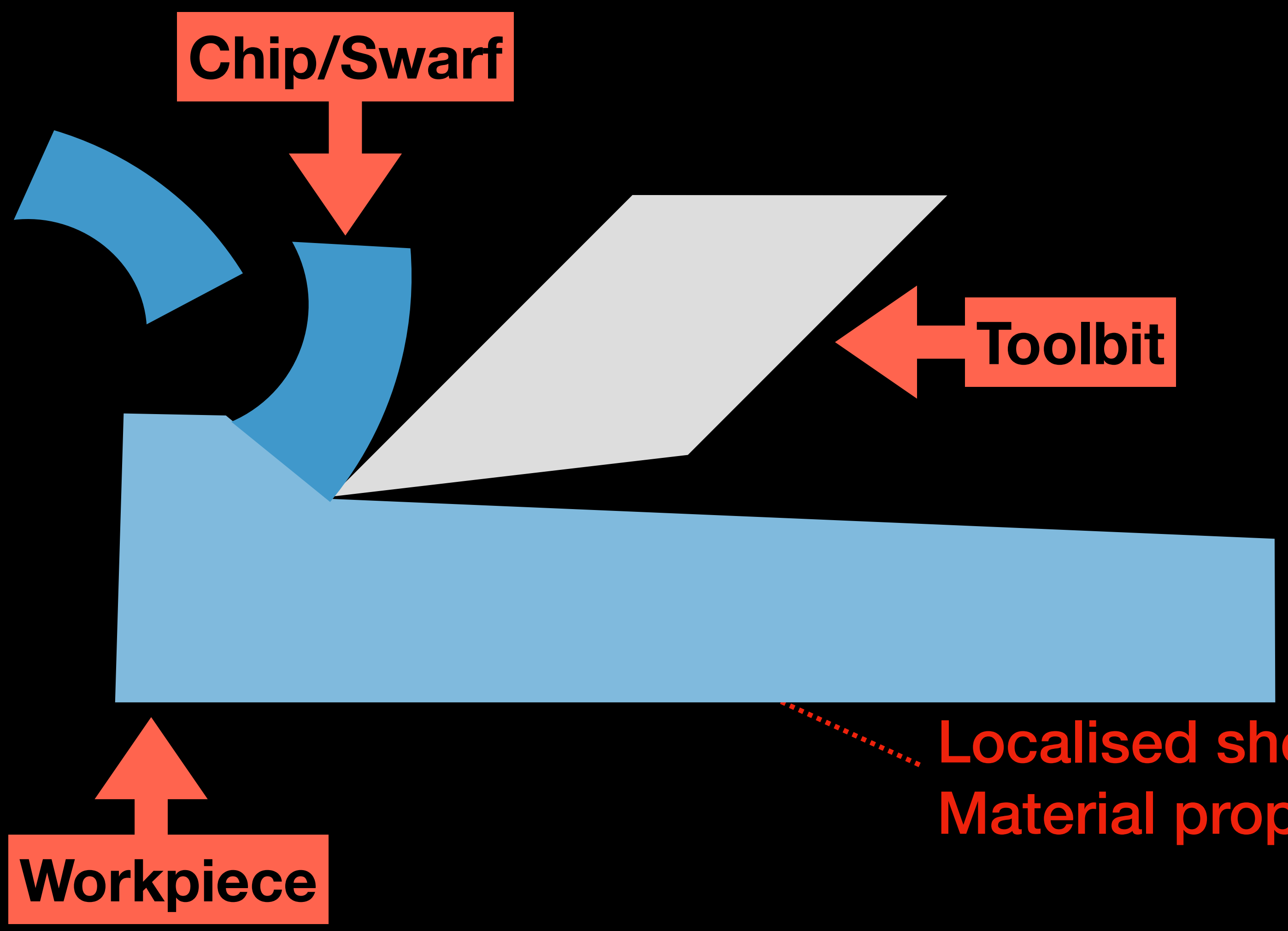
2) Find the average shear stress.

$$\tau_{avg} = \frac{V}{A}$$

$$A = \frac{\pi}{4}(3\ mm)^2 = 7.065\ mm^2$$

$$\tau_{avg} = \frac{5\ kN}{7.065\ mm^2} = 0.7071\ GPa$$

CONVENTIONAL MACHINING



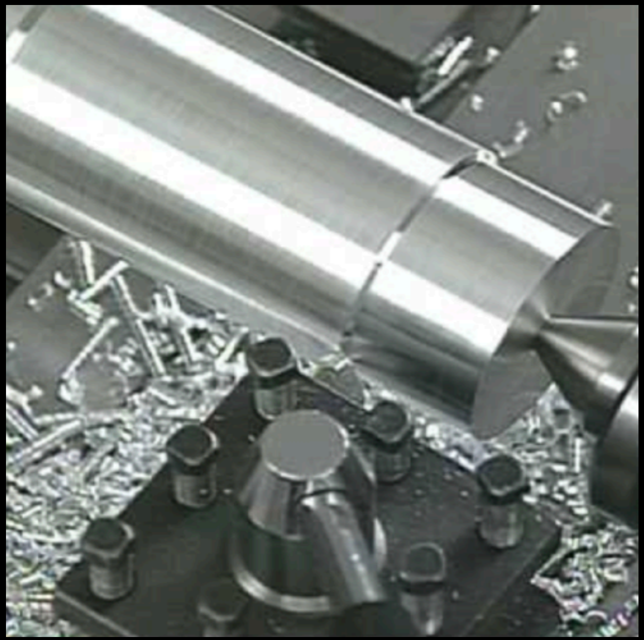
Machining: A process in which unwanted material is removed from a workpiece in the form of chips.

Localised shear stresses fracture the workpiece.
Material properties & cutter shape dictate chip shape.

MACHINING PROCESSES

https://www.youtube.com/watch?v=HflaISnqHOk

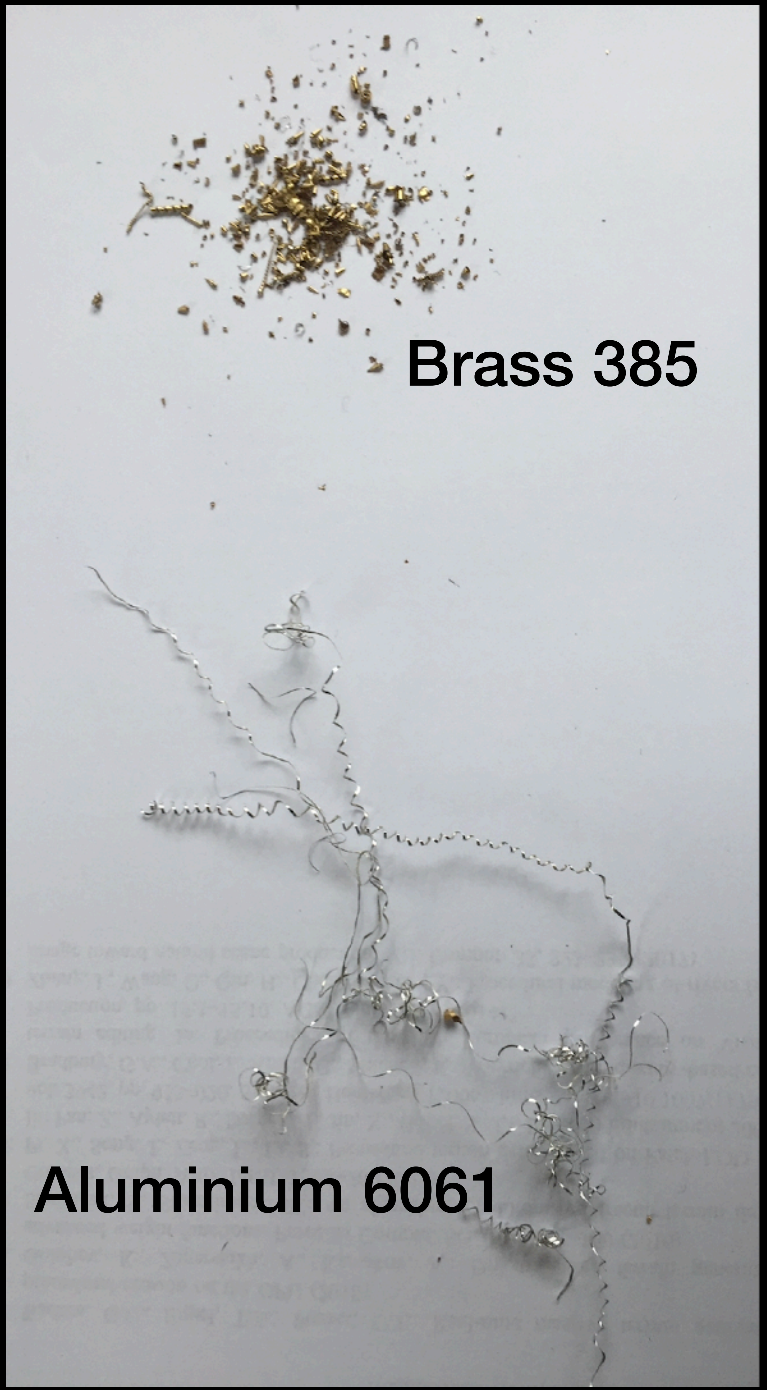
MACHINING TYPE	TOOL BEHAVIOUR	WORKPIECE BEHAVIOUR
Milling	Tool rotates	Workpiece moves along a straight line
Turning	Tool moves along a straight line	Workpiece rotates
Drilling	Tool rotates and moves along a straight line	Workpiece is stationary
Shaping	Tool moves along a straight line; cutting edge is perpendicular to cut surface	Workpiece is stationary (sometimes moves instead of tool)
Broaching	Tool moves along a straight line; cutting edge is parallel to cut surface	Workpiece is stationary (sometimes moves instead of tool)



https://www.youtube.com/watch?v=J63dZsw7Ia4

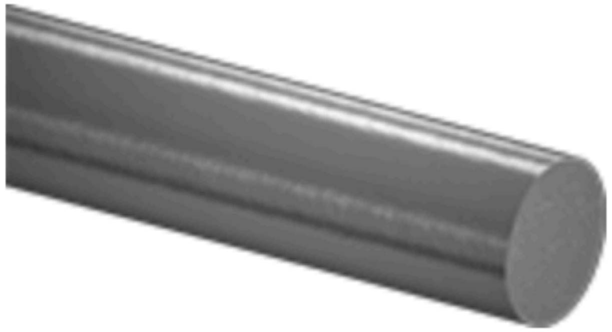
MACHINABILITY

- When it comes to manufacturing a component using machining processes, it's very important to consider how the material responds to this machining.
    - A material with good machinability (free machining): material can be removed at low power, with low tooling wear, good chip behaviour, and a good surface finish.
- Machinability is a hybrid of a few more fundamental material properties: hardness, ductility, strength/toughness, etc.
- Different alloys can have very different machinabilities.
- Practically, machinability is often among the most critical criteria in material choice.



FREE MACHINING BRASS

Easy-to-Machine Architectural 385 Brass Rods



- Yield Strength:
  Inch sizes: 16,000 psi
  Metric sizes: 60,000 psi
- Hardness:
  Inch sizes: Rockwell B40 (Soft)
  Metric sizes: Rockwell B55 (Soft)
- Temper:
  Inch sizes: H02 (1/2 Hard)
  Metric sizes: 1/2 Hard
- Heat Treatable: No
- Specifications Met:
  Inch sizes: ASTM B455
  Metric Sizes: European Standard EN 12164

Often called architectural bronze, 385 brass is easy to machine and has excellent formability when heated. It is typically used for handrails, ornamental trim, and hardware, such as hinges and lock bodies.

BRASS 385 (https://www.mcmaster.com/brass-alloy-385/)

FREE MACHINING AL

Easy-to-Machine 2011 Aluminum Rods

- Yield Strength: 38,000 psi
- Hardness: Brinell 115 (Soft)
- Temper: T3
- Fabrication: Cold Drawn
- Specifications Met: ASTM B211

2011 has the best machinability of all the aluminum alloys. It is the most selected aluminum for screws, tube fittings, hose parts, and other items that require extensive machining.

MACHINING STAINLESS STEEL

- Many advantages of stainless steel:
  - Corrosion-resistant
  - Relatively strong (esp. amongst corrosion-resistant materials!)
  - Many alloys are relatively inexpensive
- Can be difficult to machine:
  - Prone to work-hardening during machining
  - Best machined with modern tungsten carbide tooling

MACHINING THERMOPLASTICS

- Thermoplastics have a very wide range of responses to machining
- Considerations: thermal expansion when heated by machining processes
- Chip/swarf can be difficult to control
- Cutting tool must be very sharp - dull tools melt materials
- PMMA (Perspex): brittle
- UHMWPE: soft, poor chip-breaking, cheap
- Acetal/Delrin/Polyoxymethylene: good machinability - excellent finish, more expensive, one of the best thermoplastics for machining.

CHOOSING MATERIALS

- When choosing metals, a number of properties must be considered:
  - Strength, hardness, ductility…
  - But also:
  - Strength-to-weight ratio (strength/mass)
  - Corrosion resistance
  - Electrical conductivity
  - Machinability
  - Behaviour at high temperatures
  - Cost
- We've looked at many material properties now…
  - … we'll take a quick break next week to look at some CAD/CAM approaches…
  - …and will spend the first lecture in Week 9 looking at selecting materials based upon these properties.

MACHINING VIDEOS!

- Oxtoolco (toolmaking and metrology): https://www.youtube.com/channel/UCZC9LGZLfyjrKT4OZne-JNw
- Applied Science (Instrumentation and exotic materials machining): https://www.youtube.com/channel/UCivA7_KLKWo43tFcCkFvydw
- ROBRENZ (toolmaking): https://www.youtube.com/channel/UCn4U3aEr6L2nLe1m_3as6JQ
- Dan Gelbart (general fabrication): https://www.youtube.com/channel/UCYA1VjSKXgNVh03wjw_HSRA
- This Old Tony (fun, general machining & CNC): https://www.youtube.com/channel/UC5NO8MgTQKHAWXp6z8Xl7yQ
- Clickspring (general machining, lots of non-ferrous work): https://www.youtube.com/channel/UCworsKCR-Sx6R6-BnIjS2MA

TYPES OF GEARS

SPUR GEARS: Teeth parallel to the axis of rotation. Simplest type (and the type we'll mostly be studying)

HELICAL GEARS: Teeth inclined to axis of rotation: gradual engagement of teeth means they're quieter. Thrust loads develop.

BEVEL GEARS: Teeth on a conical surface. For transmitting power between perpendicular/ intersecting shafts

WORM GEARS: A screw-like worm's direction of rotation dictates the direction of rotation of a pinion / worm-wheel. Locking

Spur Gears

- Spur gears are used to transmit motion between parallel shafts
- They are usually cylindrical (flat); the teeth project radially.

Spur Gears

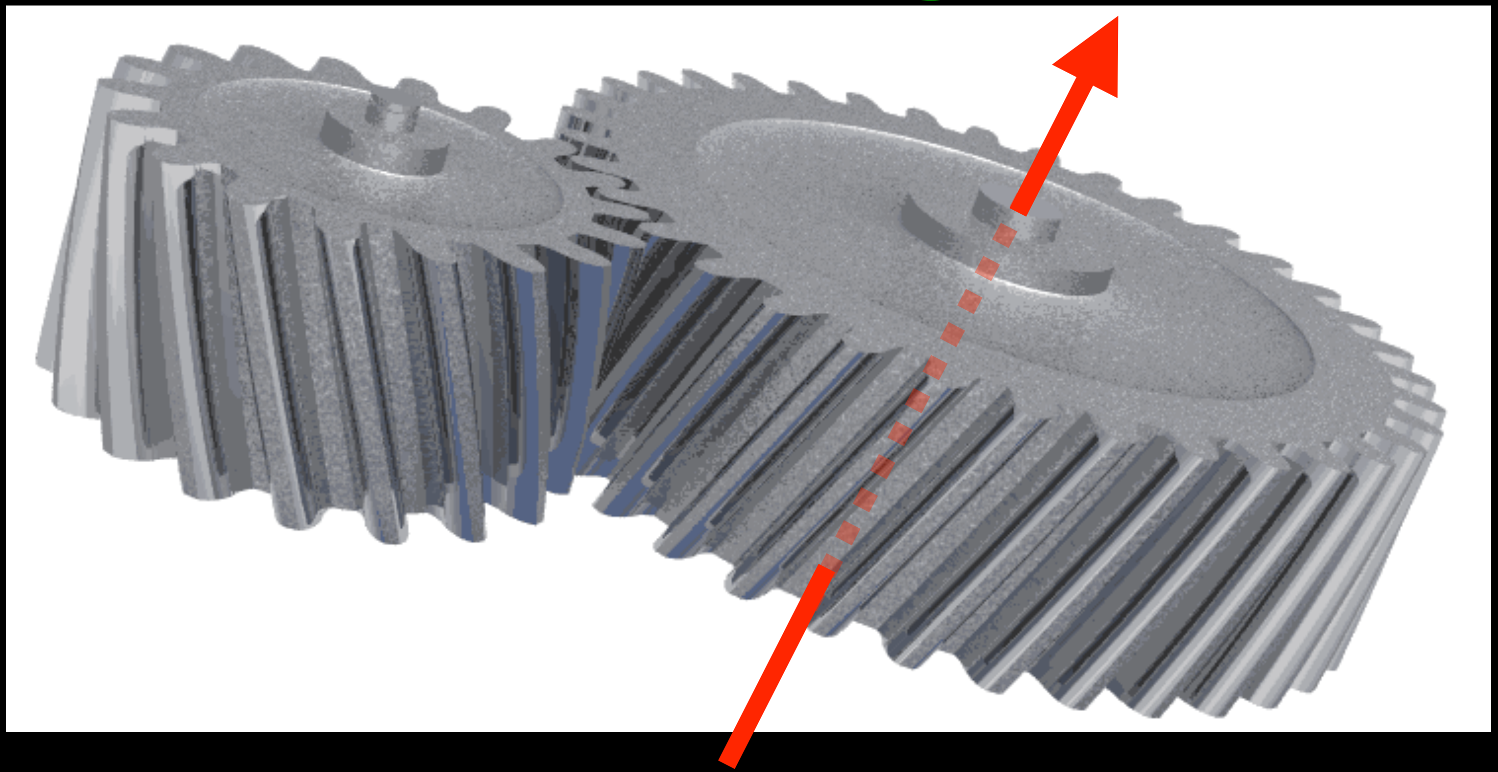
Spur gear with keyway and grub screw.

- Spur gears are prone to noise.
- If they aren't manufactured with high precision, gear meshing can be rough.

Helical gears

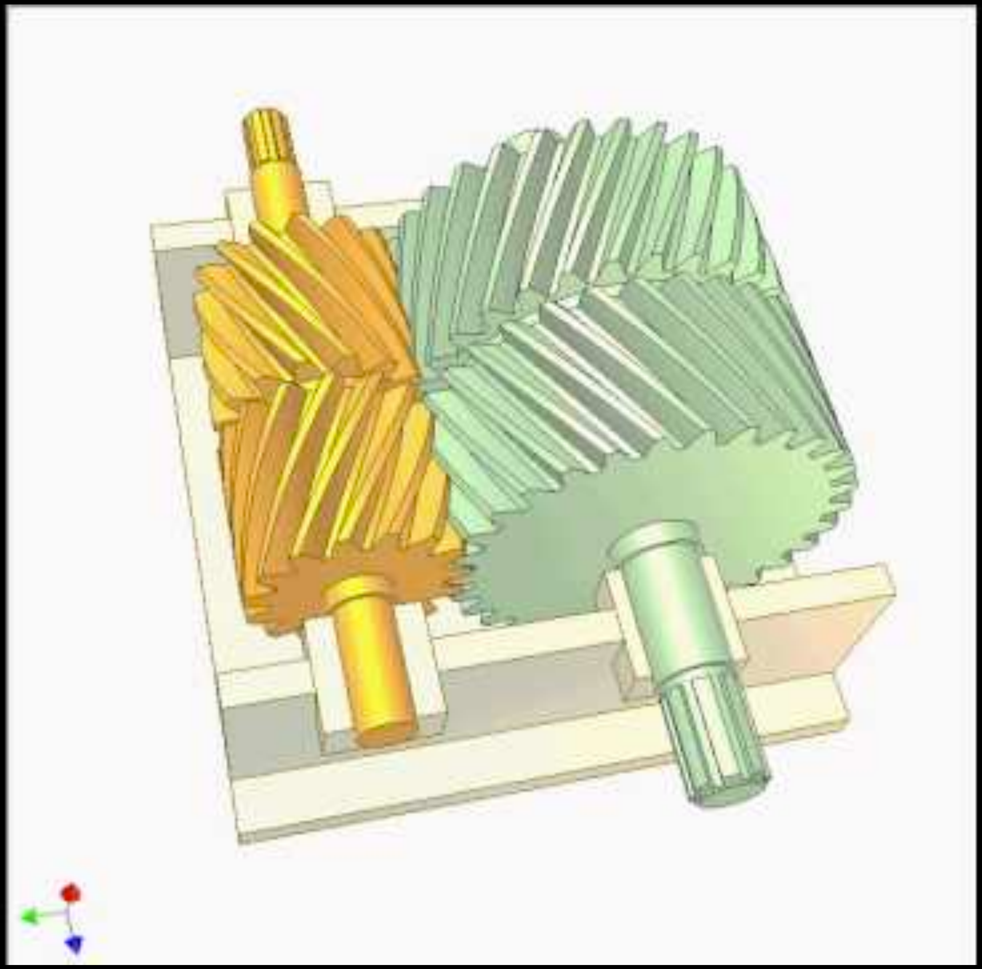
- Helical gears are an improvement on spur gears.
- The teeth are cut at an angle, allowing for a more gradual (and smoother) meshing between gears.
  - This allows for greater speed and reduced noise.
  - Also good for high torque applications: more tooth area in contact.

Helical gears



- Somewhat more difficult to manufacture than spur gears.
- When turning, tooth angle results in a thrust force along the axis of the gear.
    - Must be compensated for with an adequate thrust bearing.

Double helical gears



- Also known as herringbone gears.
- The teeth are 'V' shaped (a reflected helical gear).
    - These angles create opposing thrust forces, canceling each other out.

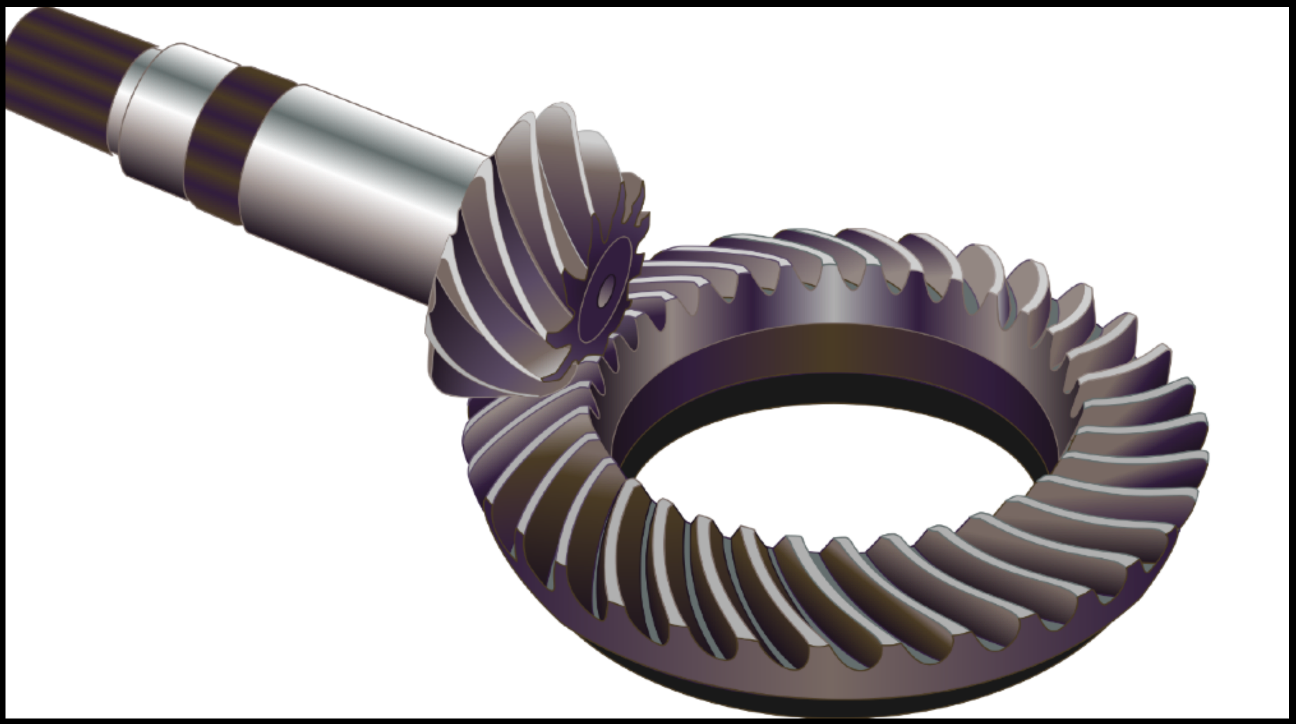
High speed & right angles with Helical gears

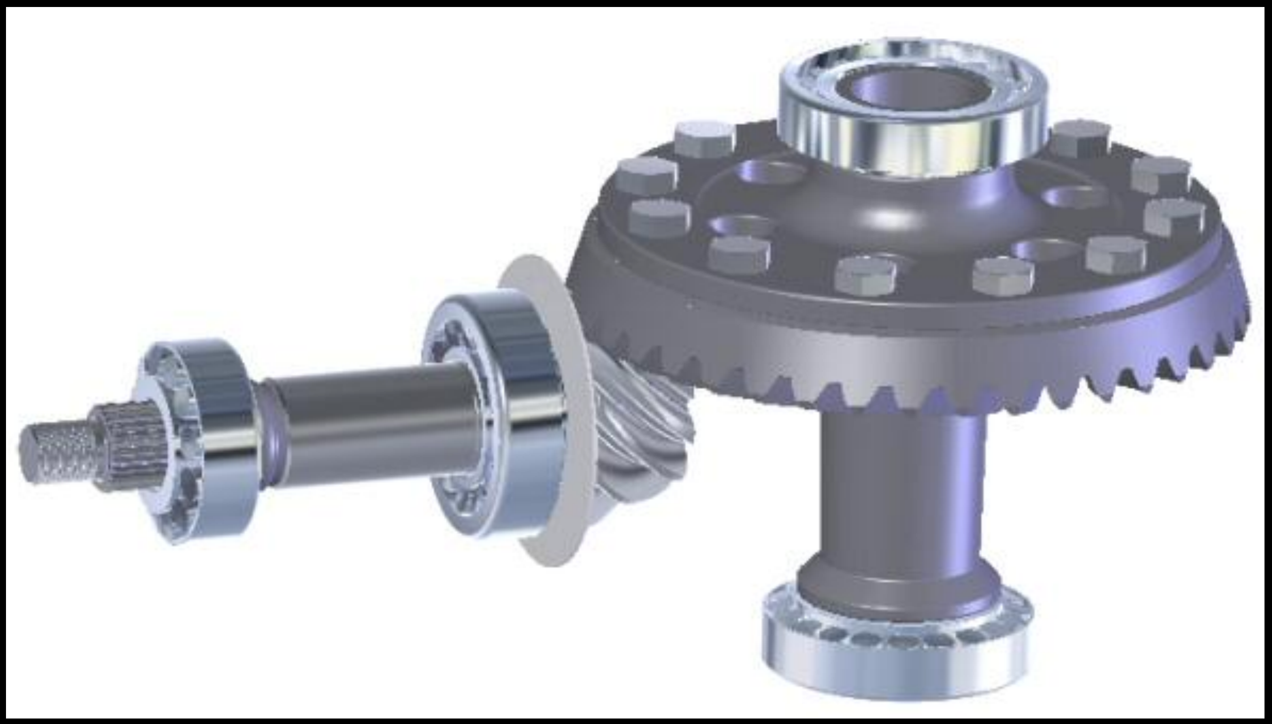
- With correct angles of gear teeth, helical gears can be used to adjust the rotation angle by 90 degrees.
- Helical gears: popular in high speed applications.
  - High speed: pitch line velocity > 1500 m/min; rotational speed of pinion > 3600 rpm

Bevel gears

- Bevel gears allow for changing the rotation direction between shafts.
  - 90 degrees is common, though other angles are available.

Helical/Spiral Bevel gears



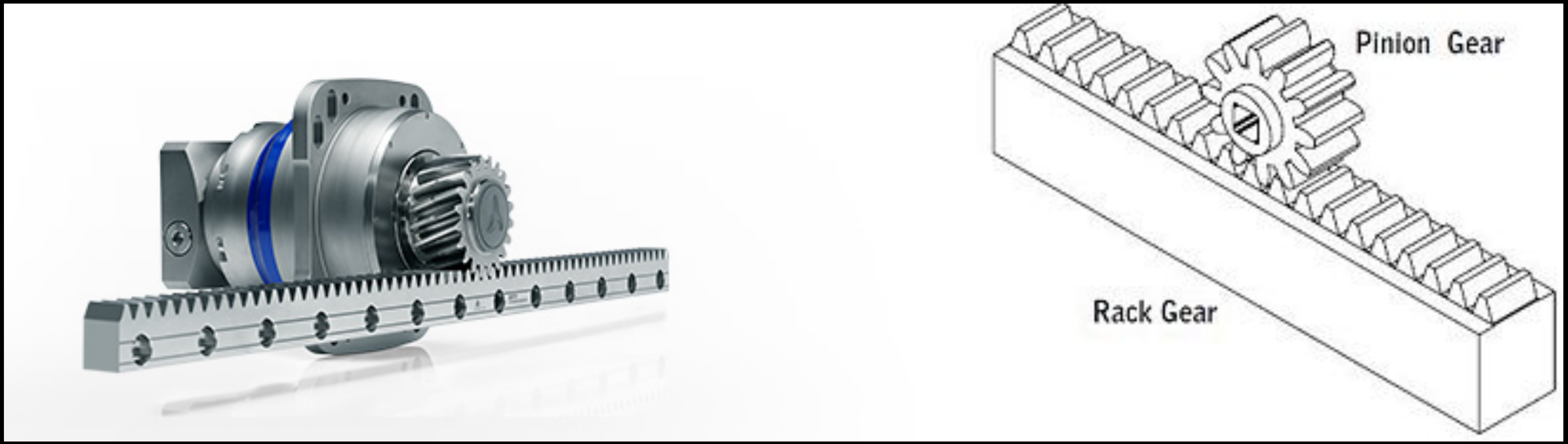


- For high speed applications, spiral (helical) bevel gears are used.
- Display "handedness" (Right hand = clockwise inclination of tooth from axis through tooth midpoint)
  - High-torque, high-speed applications (e.g., turboshafts on helicopters)

Worm gears

- Worm gears also allow for right angle power transmission between shafts.
  - The worm is a screw-like device with helical threads.
- Very compact size allows for high gear ratios.
- Worm gear can turn the spur gear.
  - Spur gear can't (easily) turn the worm gear.
  - This allows spur shaft to be locked into place unless worm is being driven.
- Advantages: Low noise, high power transmission
- Disadvantages: Increased friction; requires higher starting torque.

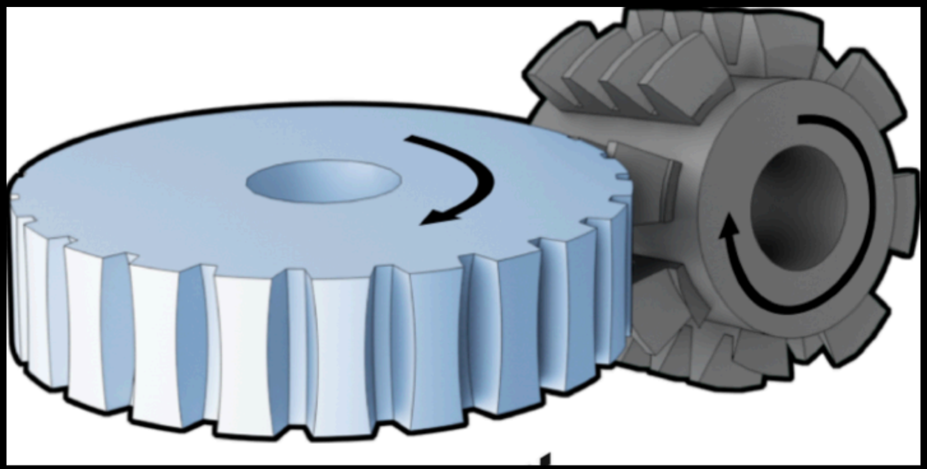
Rack and Pinion Gears



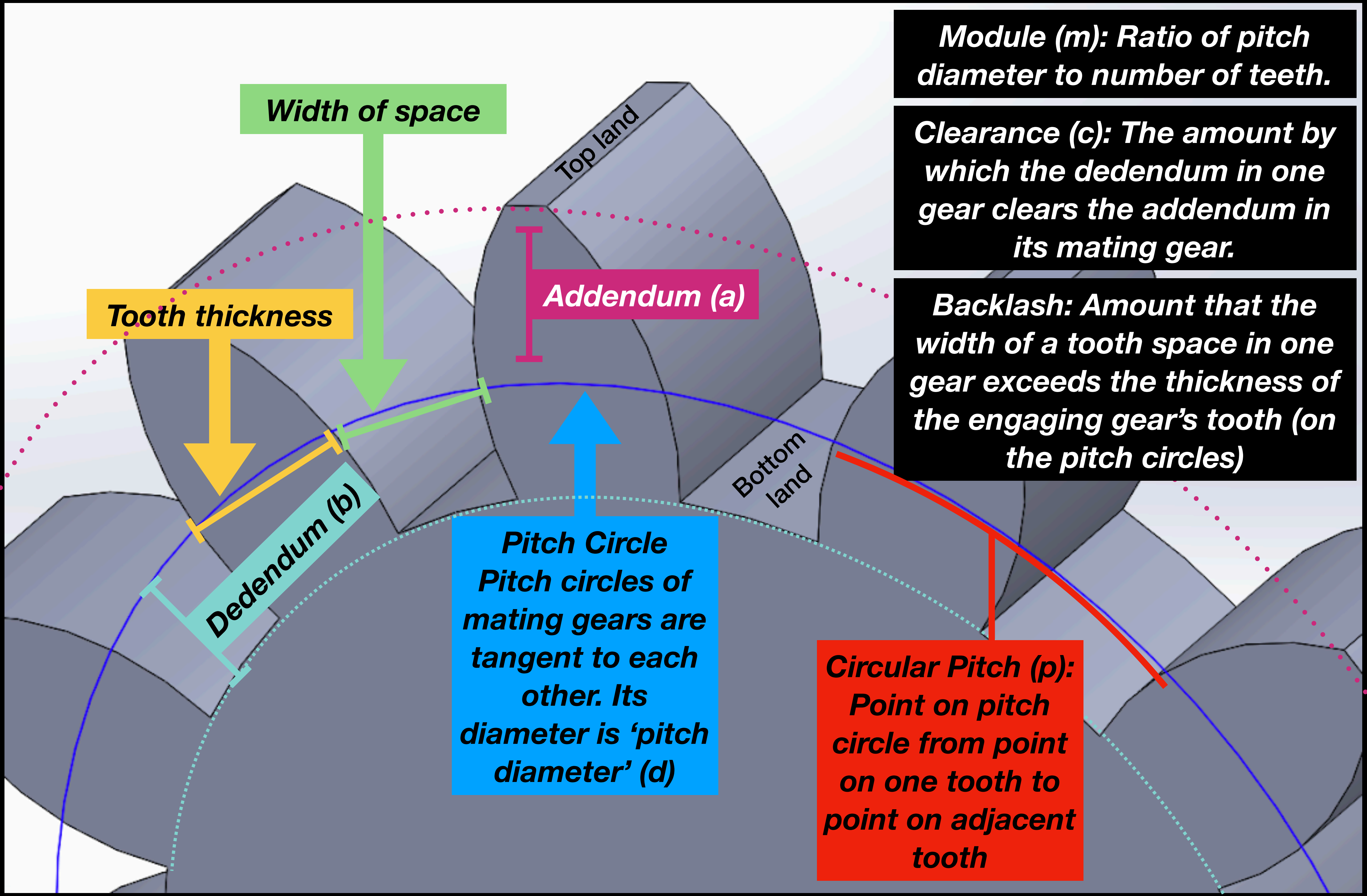
- One way of converting rotational motion into linear motion
- The circular pinion engages the teeth on the flat bar - the rack.
  - The rack is an ‘unrolled’ spur gear.
- Found in the steering mechanism of many cars.
- Must have some form of mechanical limit built in, to prevent rack from slipping past pinion.

Gear Manufacture

- Hobbing:
  - Fast to manufacture (multiple teeth formed at once). Very accurate profiles.
  - Can not produce ‘exotic’ gear tooth shapes
- Milling:
  - Dedicated cutter is used, teeth are cut one at a time.
  - Cheap tooling, relatively slow.
- Forming:
  - Metal is ‘pressed’ (moved away through plastic deformation)
    - Cheap but inaccurate - might require post-processing machining steps (e.g., turning to diameter)



GEAR NOMENCLATURE



TYPES OF GEARS

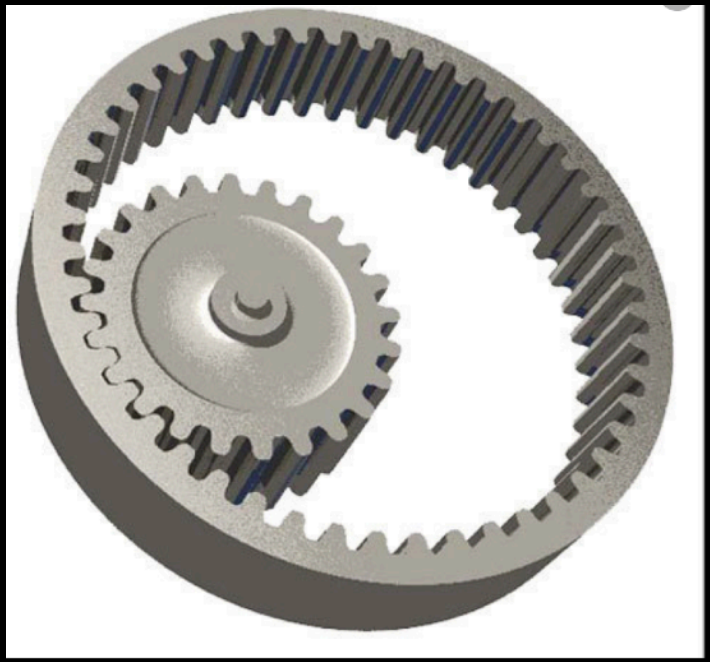
SPUR GEARS: Teeth parallel to the axis of rotation. Simplest type (and the type we'll mostly be studying)

HELICAL GEARS: Teeth inclined to axis of rotation: gradual engagement of teeth means they're quieter. Thrust loads develop.

BEVEL GEARS: Teeth on a conical surface. For transmitting power between perpendicular/ intersecting shafts

WORM GEARS: A screw-like worm's direction of rotation dictates the direction of rotation of a pinion / worm-wheel. Locking

Spur Gears



- Spur gears are used to transmit motion between parallel shafts
- They are usually cylindrical (flat); the teeth project radially.

Spur Gears

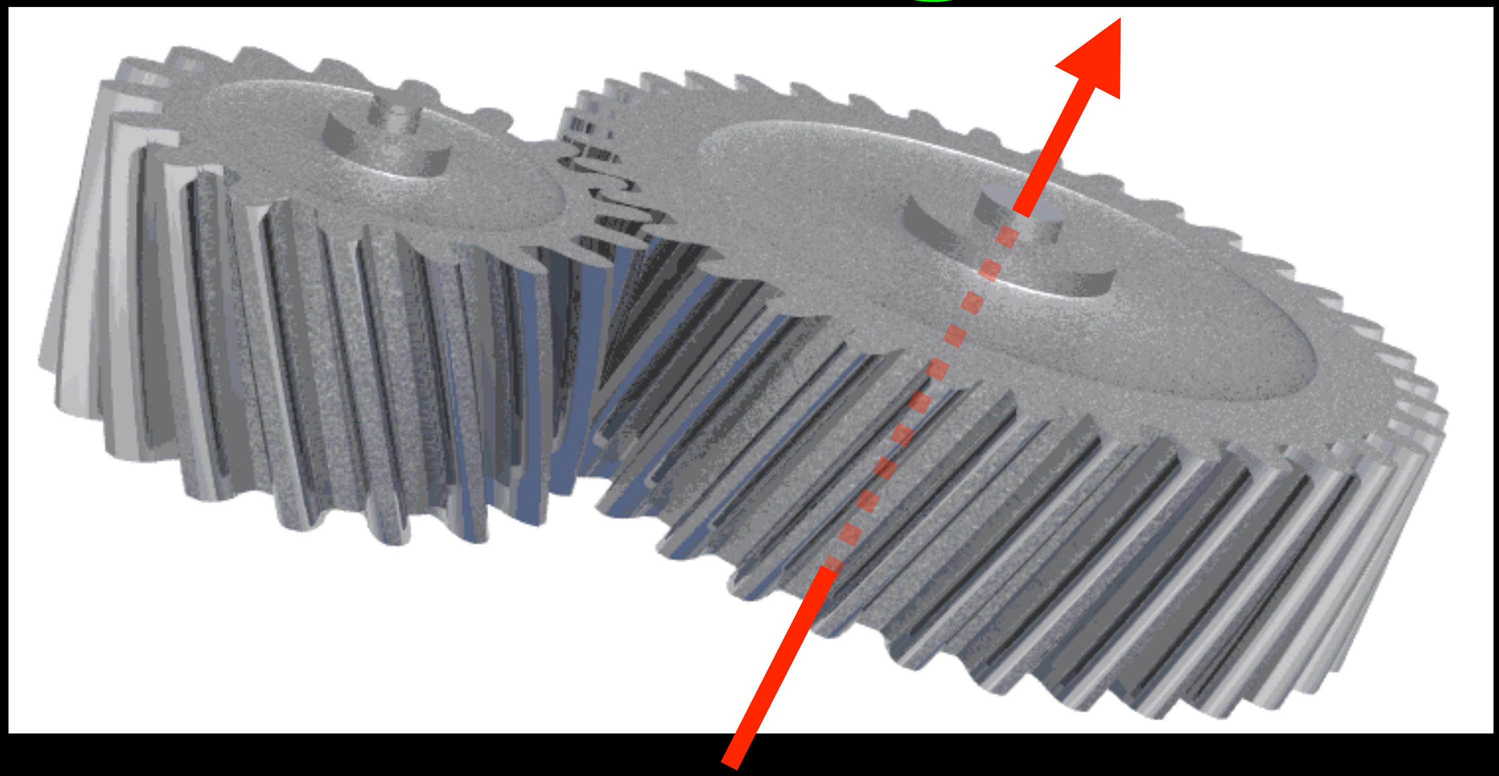
Spur gear with keyway and grub screw.

- Spur gears are prone to noise.
- If they aren't manufactured with high precision, gear meshing can be rough.

Helical gears

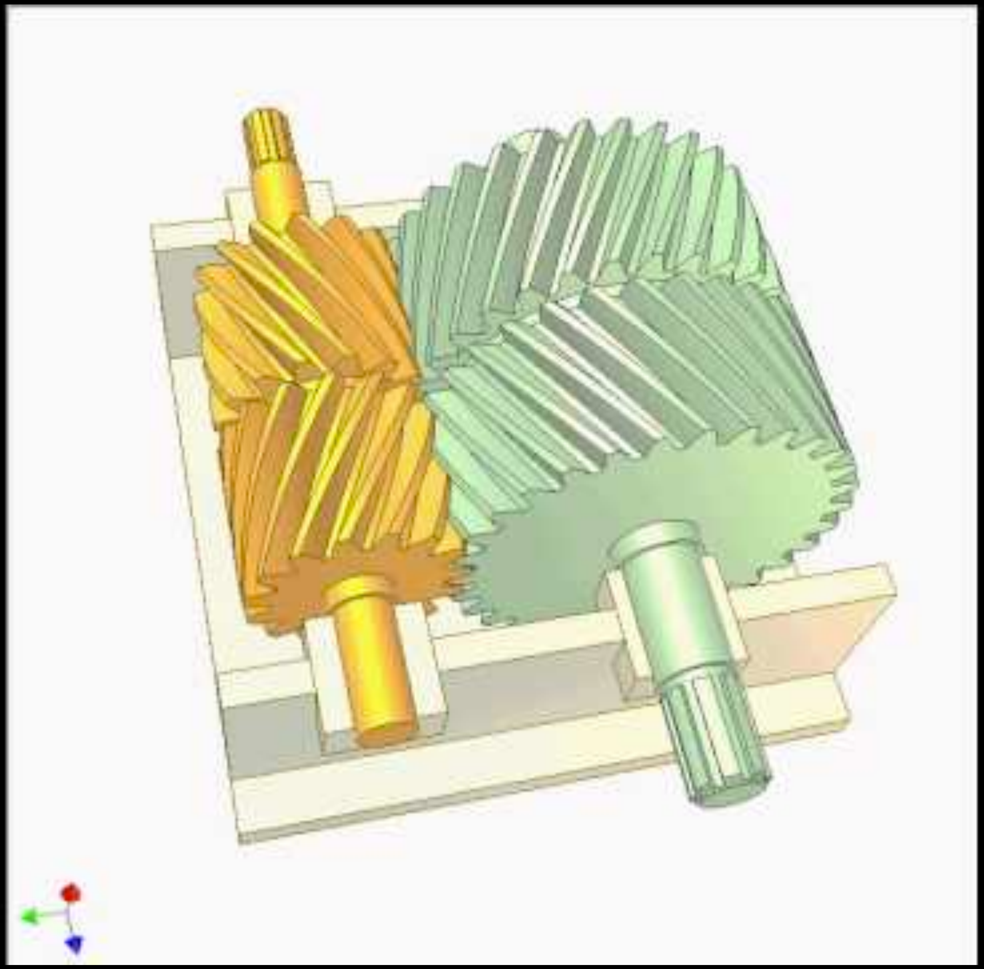
- Helical gears are an improvement on spur gears.
- The teeth are cut at an angle, allowing for a more gradual (and smoother) meshing between gears.
  - This allows for greater speed and reduced noise.
  - Also good for high torque applications: more tooth area in contact.

Helical gears



- Somewhat more difficult to manufacture than spur gears.
- When turning, tooth angle results in a thrust force along the axis of the gear.
    - Must be compensated for with an adequate thrust bearing.

Double helical gears



- Also known as herringbone gears.
- The teeth are ‘V’ shaped (a reflected helical gear).
  - These angles create opposing thrust forces, canceling each other out.

High speed & right angles with Helical gears

- With correct angles of gear teeth, helical gears can be used to adjust the rotation angle by 90 degrees.
- Helical gears: popular in high speed applications.
  - High speed: pitch line velocity > 1500 m/min; rotational speed of pinion > 3600 rpm

Bevel gears

- Bevel gears allow for changing the rotation direction between shafts.
    - 90 degrees is common, though other angles are available.

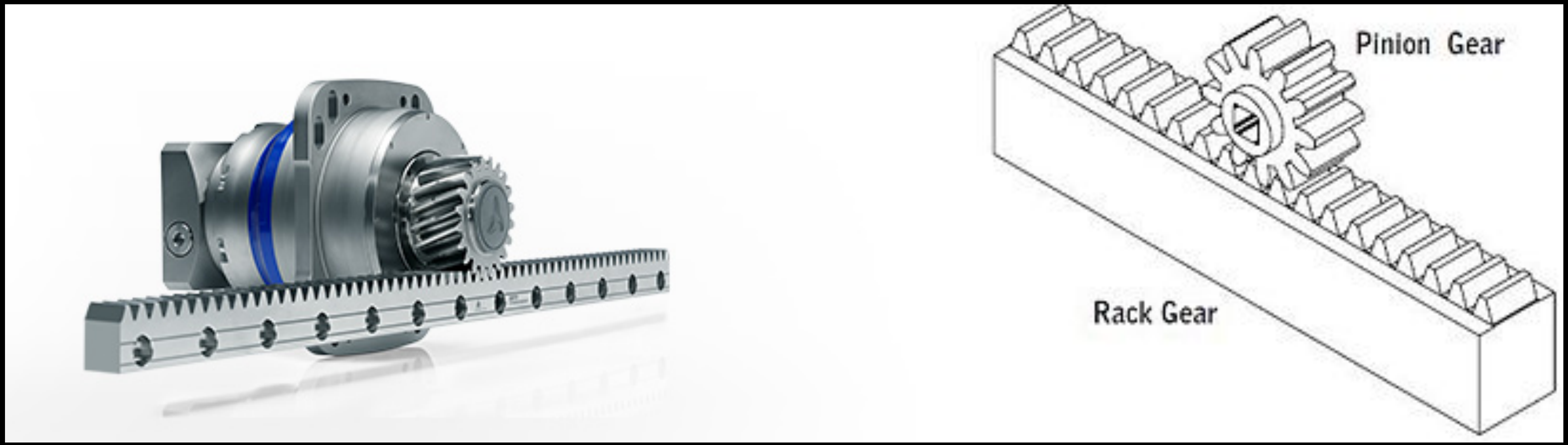
Helical/Spiral Bevel gears

- For high speed applications, spiral (helical) bevel gears are used.
- Display “handedness” (Right hand = clockwise inclination of tooth from axis through tooth midpoint)
  - High-torque, high-speed applications (e.g., turboshafts on helicopters)

Worm gears

- Worm gears also allow for right angle power transmission between shafts.
  - The worm is a screw-like device with helical threads.
- Very compact size allows for high gear ratios.
- Worm gear can turn the spur gear.
  - Spur gear can’t (easily) turn the worm gear.
  - This allows spur shaft to be locked into place unless worm is being driven.
- Advantages: Low noise, high power transmission
- Disadvantages: Increased friction; requires higher starting torque.

Rack and Pinion Gears



- One way of converting rotational motion into linear motion
- The circular pinion engages the teeth on the flat bar - the rack.
  - The rack is an ‘unrolled’ spur gear.
- Found in the steering mechanism of many cars.
- Must have some form of mechanical limit built in, to prevent rack from slipping past pinion.

Gear Manufacture

- Hobbing:
  - Fast to manufacture (multiple teeth formed at once). Very accurate profiles.
  - Can not produce ‘exotic’ gear tooth shapes
- Milling:
  - Dedicated cutter is used, teeth are cut one at a time.
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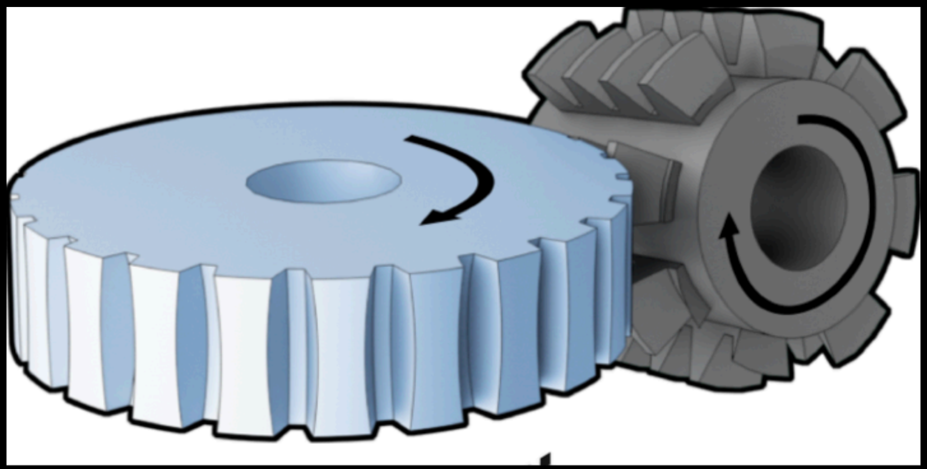
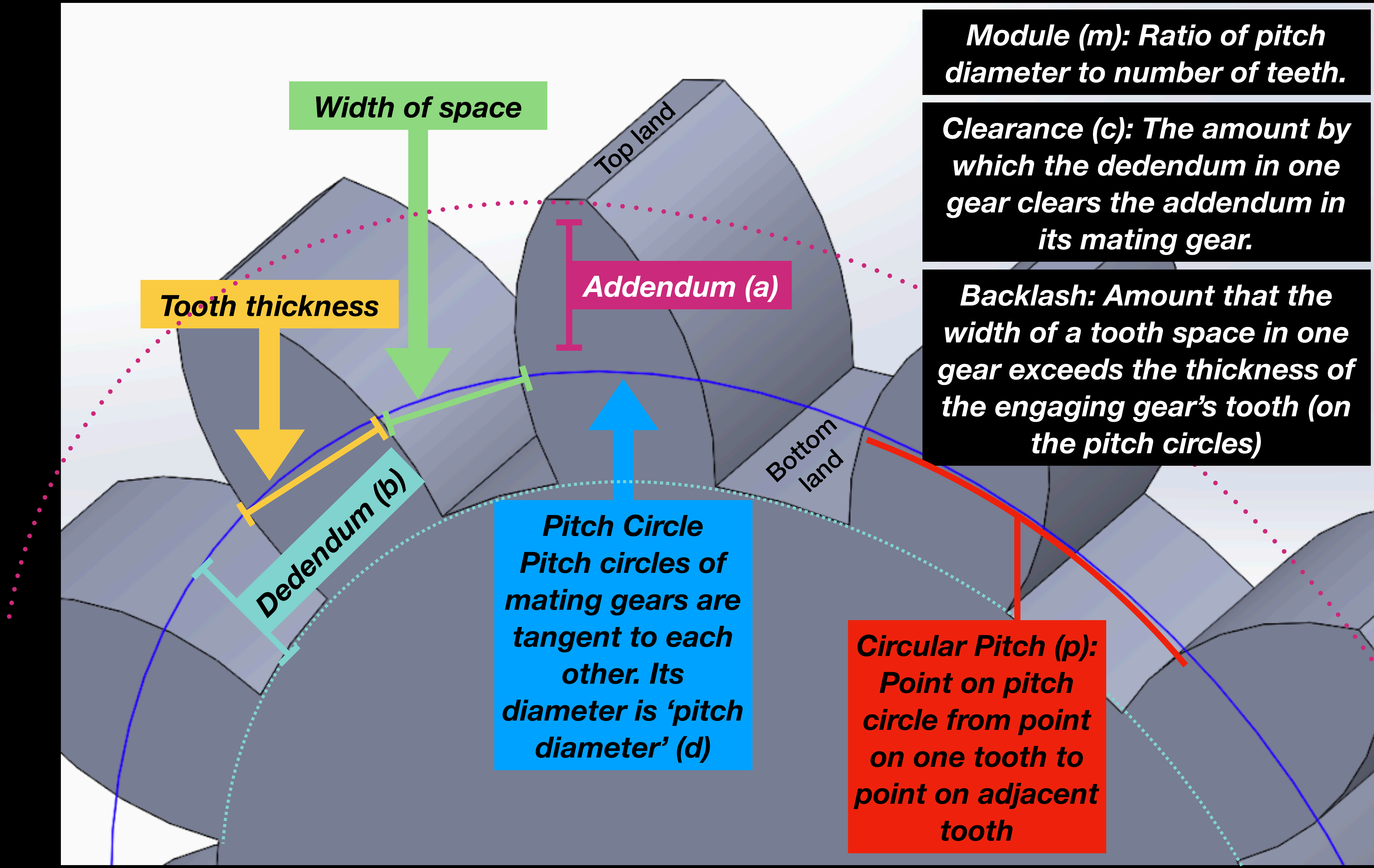


GEAR NOMENCLATURE



Module (m): Ratio of pitch diameter to number of teeth.

Clearance (c): The amount by which the dedendum in one gear clears the addendum in its mating gear.

Backlash: Amount that the width of a tooth space in one gear exceeds the thickness of the engaging gear's tooth (on the pitch circles)

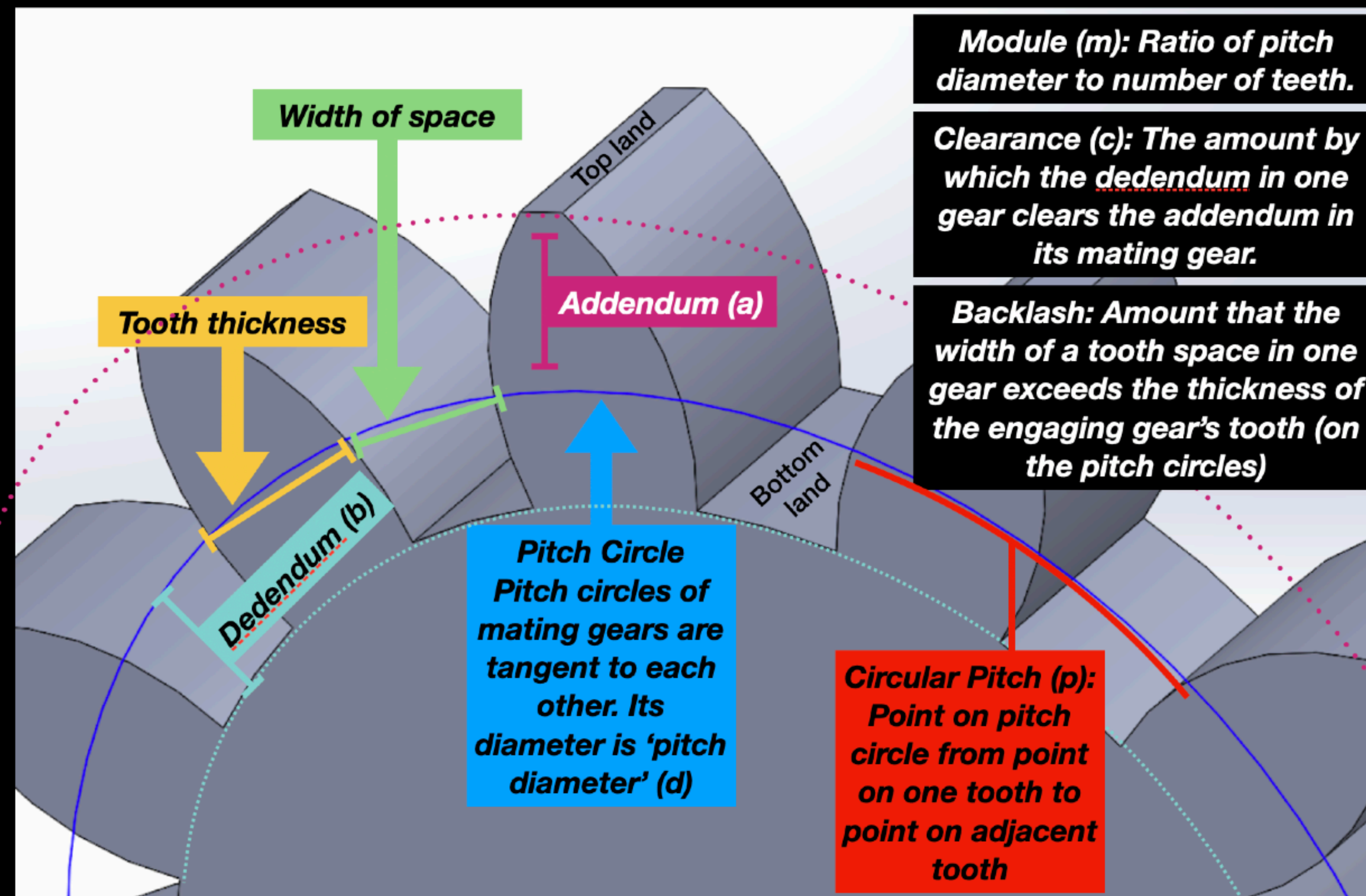
BASIC GEAR RELATIONS

$$m = \frac{d}{N}$$

A bigger module: larger teeth.

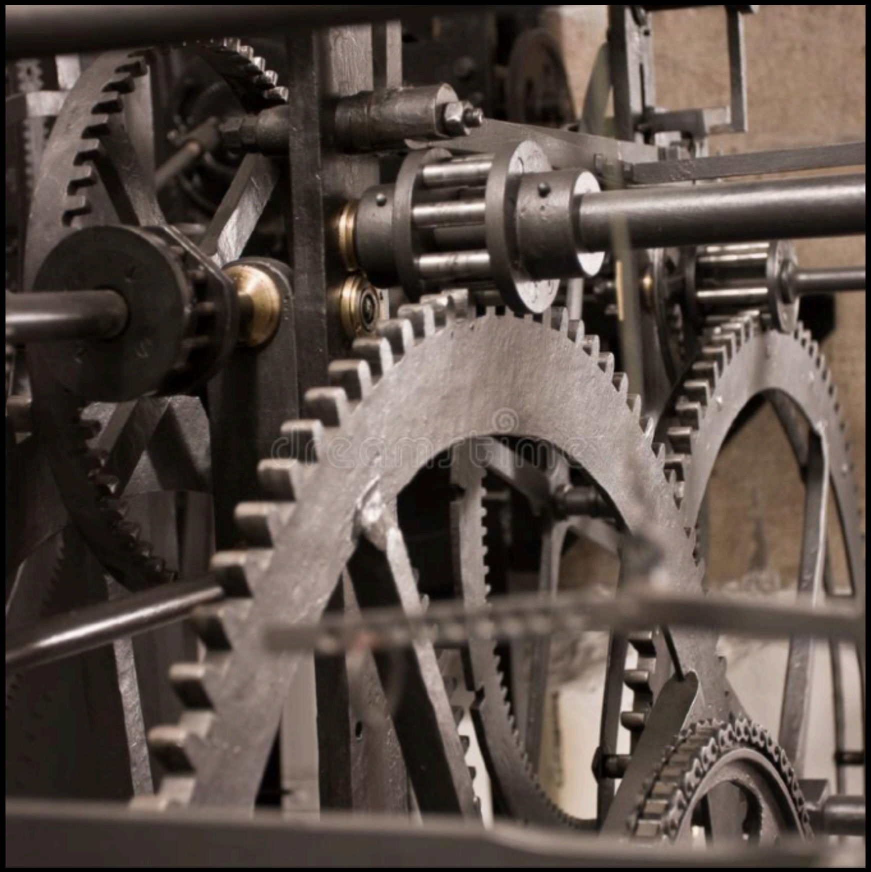
$$p = \frac{\pi d}{N} = \pi m$$

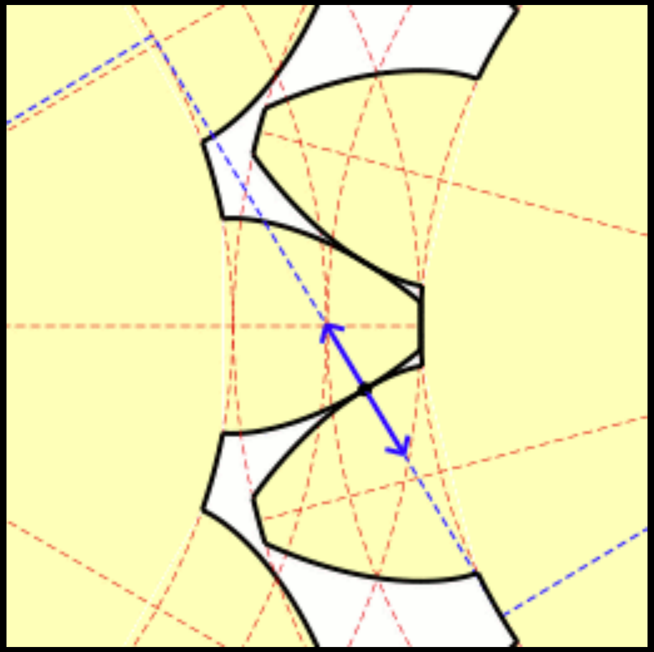
N = number of teeth



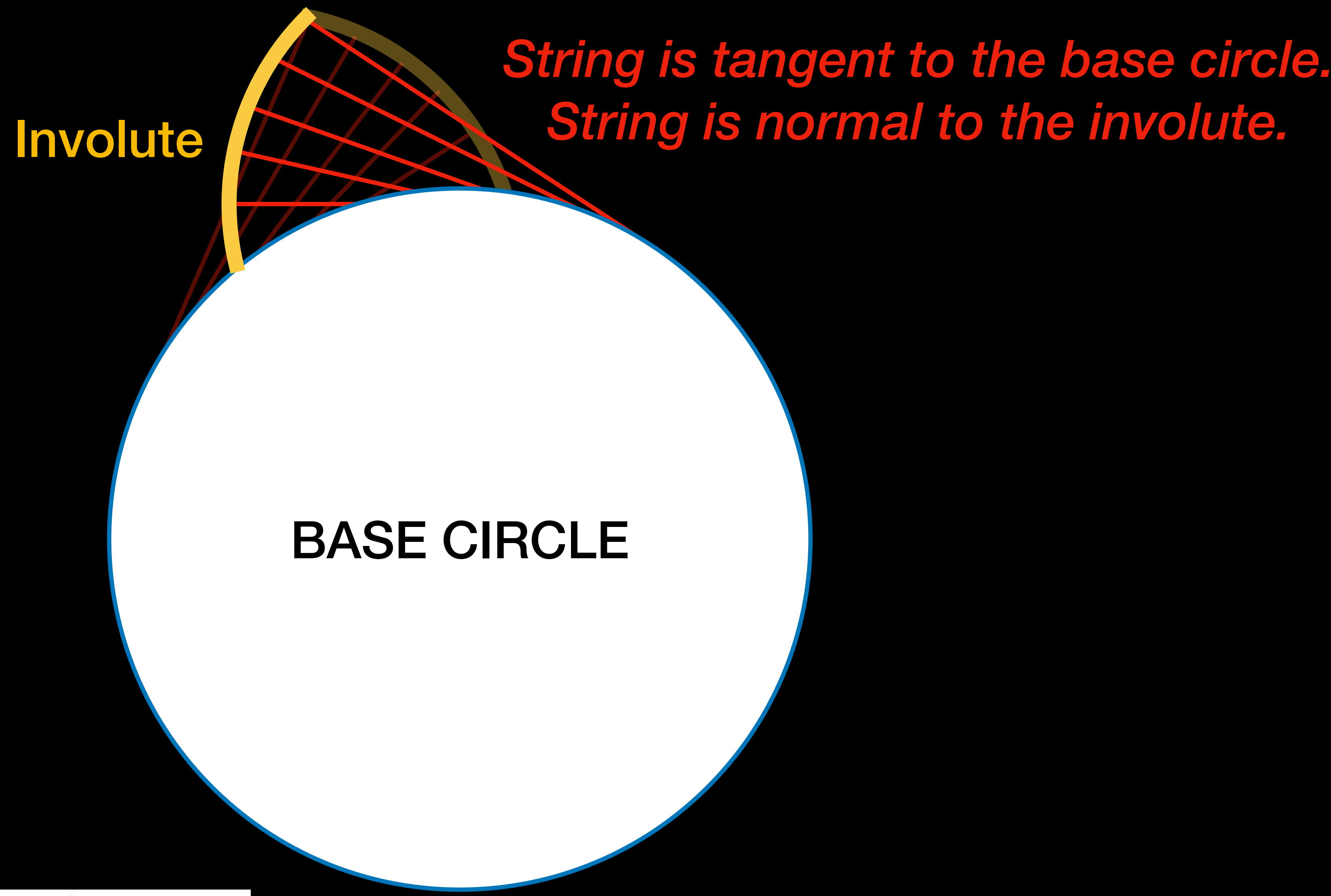
INVOLUTE PROFILE

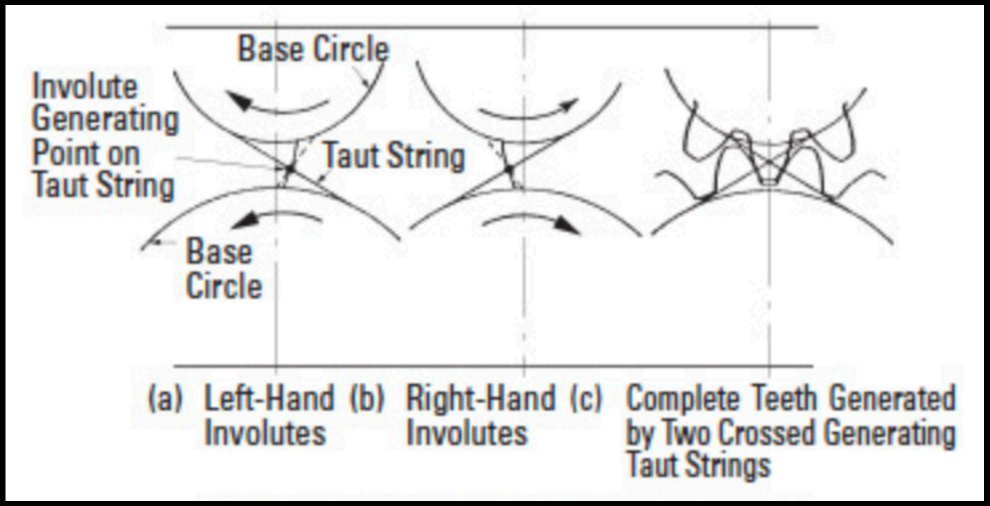
- While many gear profiles can be designed that mesh, they often don't allow "conjugate action"
  - Conjugate action: Constant angular velocity ratio during meshing.
  - This is essential in high-speed, high-load applications.
- The most common gear tooth profile that allows conjugate action is the involute profile.
  - A guide to drawing involute profiles (using Unwin's Construction approximation) is on Blackboard
  - *(SolidWorks toolbox gears: approximate involute profiles)*
- Involute: smooth meshing, resistant to errors in gear centre-distance spacing



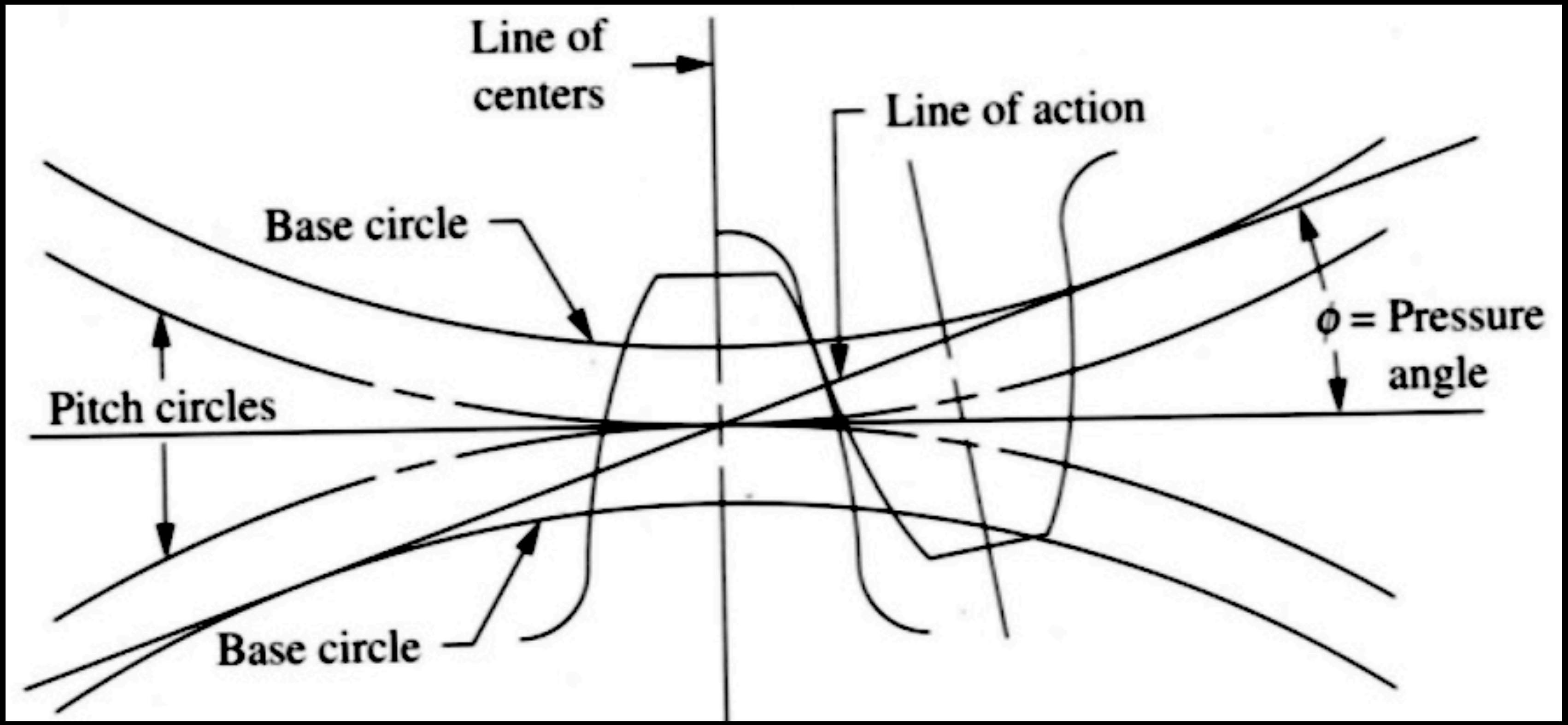


INVOLUTE PROFILE





INVOLUTE NOMENCLATURE



LINE OF CENTRES: The line connecting two meshing gears' centres.

BASE CIRCLE: Circle from which the involute profile is formed (e.g., by unwinding a taut string from it)

POINT OF CONTACT: In an involute gear, a single (sliding) point of contact exists between both gears.

LINE OF ACTION: A line tangent to both base circles that follows the point of contact

PRESSURE ANGLE: The angle formed between a line normal to the line connecting gear centres and the line of action

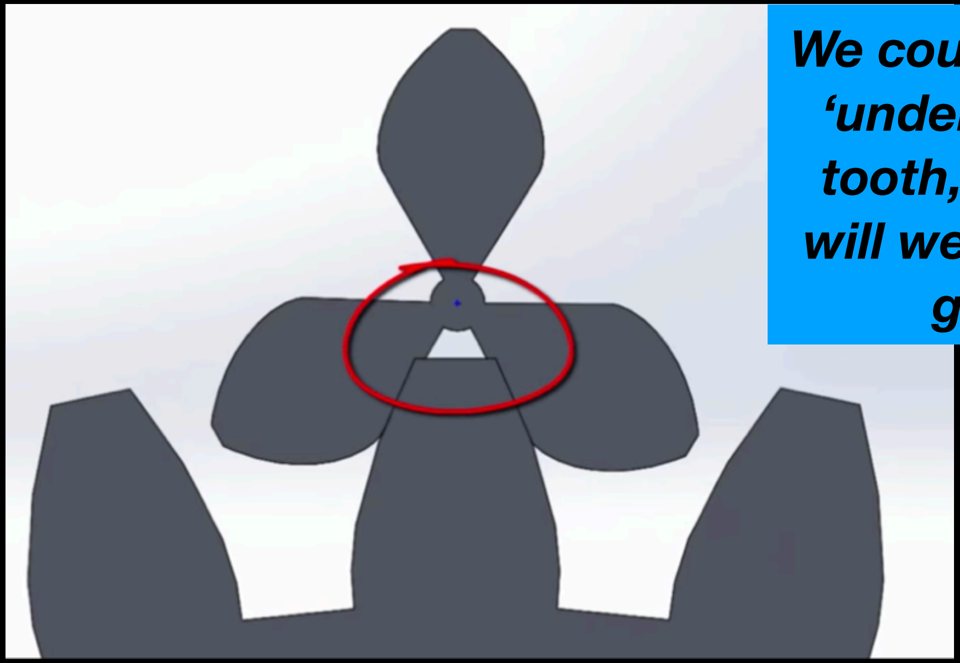
GEAR STANDARDS

- Module is the ratio of pitch circle to number of teeth.
  - In order to mesh, two gears must have the same module.
- Involute gears with different pressure angles won't correctly mesh.
- Since modules and pressure angles must be shared between mating gears , there are some commonly used standards, called 'tooth systems.'
  - Standard modules for spur gears: 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
  - A pressure angle of 20° is typical for modern spur gears. Less commonly, pressure angles of 22.5° and 25° are used.
  - Historically, a 14.5° pressure angle was common; now considered obsolete.

$$m = \frac{d}{N}$$

INTERFERENCE

- With involute gears, if the gear's base circle is too small, a mating gear's teeth may intrude with it.
  - Both gears' base circles need to be large enough to allow the mating gears' teeth to move with conjugate action.
  - The practical upshot is that, for a given gear size, there is a minimum number of teeth below which there will be interference.
  - It can be important to know this minimum number, in order to design the most compact gearboxes possible.



We could further 'undercut' the tooth, but this will weaken the gear.

https://www.youtube.com/watch?v=s1V9-RFuf0I

For a spur gear with a one-to-one ratio (equal teeth on both gears), the smallest number of teeth w/out interference N_p:

$$N_p = \frac{2}{3sin^2\phi}(1 + \sqrt{1 + 3sin^2\phi})$$

Where ϕ is the pressure angle *(Often 20º)*

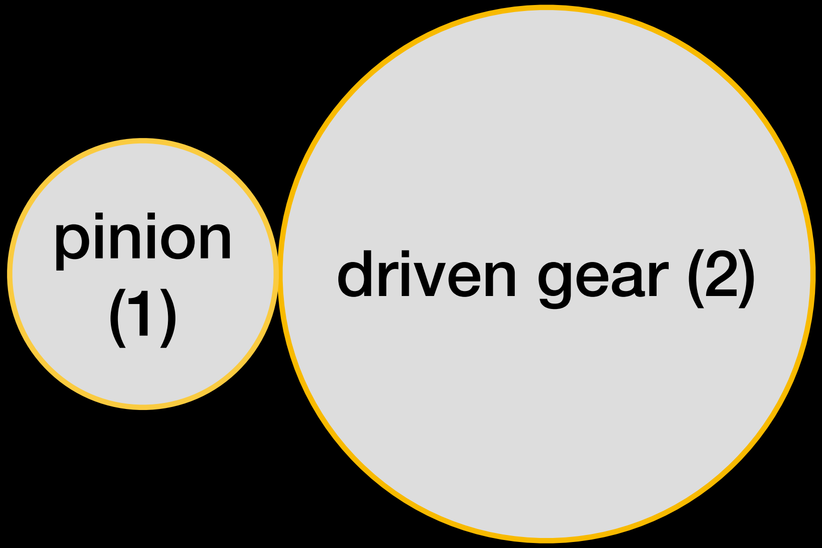
If a gear has more teeth than its pinion, the smallest number of teeth w/out interference N_p:

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

Where ϕ is the pressure angle and m is the the ratio of gear teeth over pinion teeth (N_G/N_P)

GEAR TRAINS I

Given a pinion (1) driving a gear (2): the speed of the driven gear (*n_2*, in rpm) can be determined if: the speed of the pinion (*n_1*) and the numbers of teeth (N1 and N2) or the pitch diameters (d1 and d2) are known.

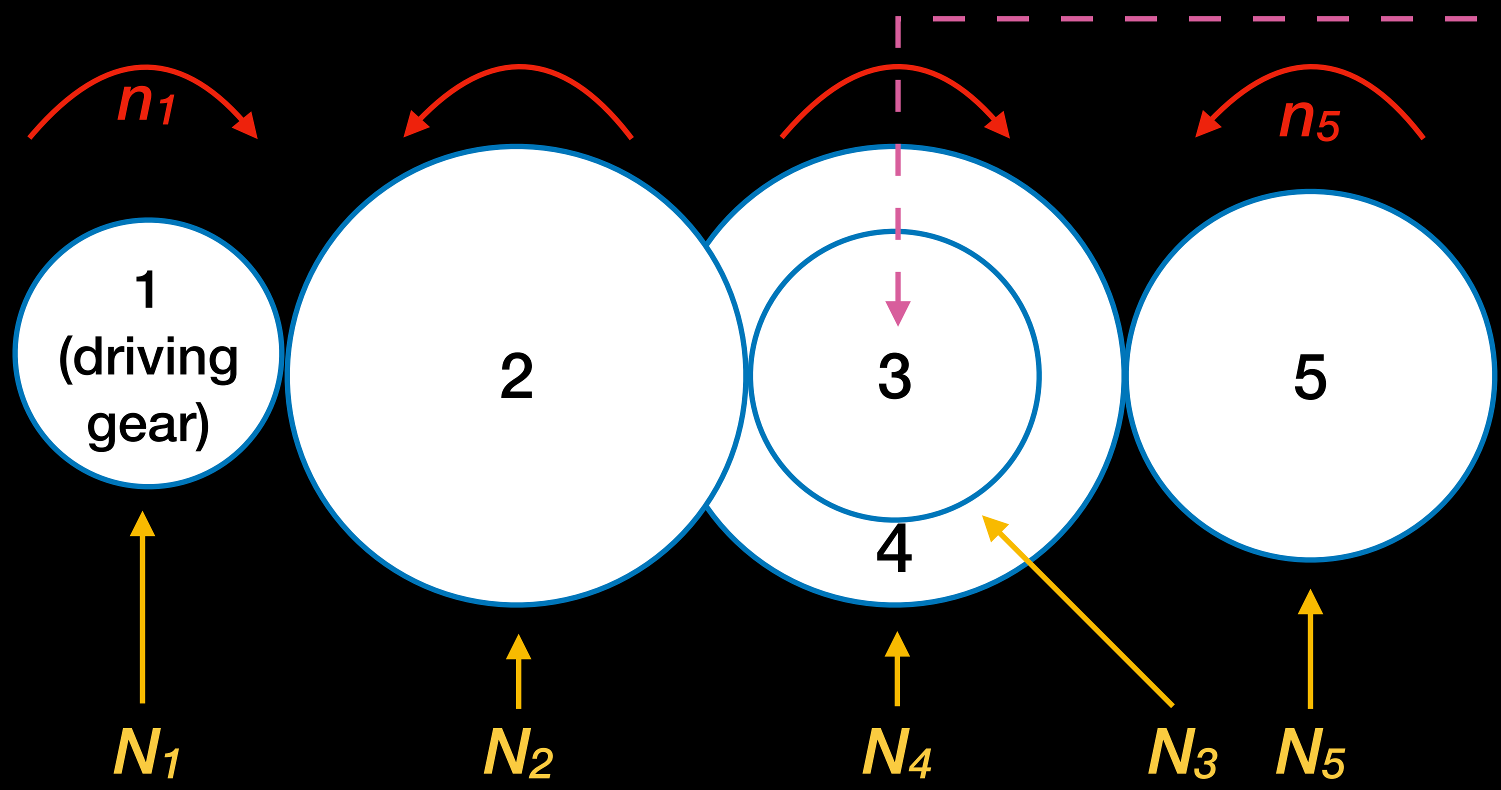


$$n_2 = \left| \frac{N_1}{N_2} n_1 \right| = \left| \frac{D_1}{D_2} n_1 \right|$$

This equation works for any gear type: spur, helical, bevel, etc.

Absolute value signs allow for any rotation direction to be used.

GEAR TRAINS II



Without this 'double reduction gear' (both fixed to axle), the interim gear 2 & 4 will act only as idlers.

If we know the speed of the driving gear (n_1), then what is the speed of gear 5 (n_5)?

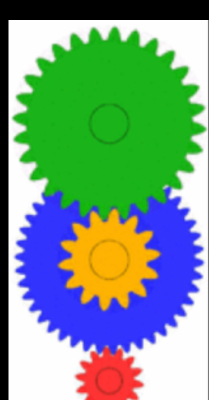
Driver gears

Driven gears

$$\frac{N_1}{N_2} \quad \frac{N_2}{N_3} \quad \frac{N_4}{N_5}$$

$$n_5 = -\frac{N_1}{N_2}\frac{N_2}{N_3}\frac{N_4}{N_5}n_1$$

Negative: direction of rotation is opposite to input



Double reduction gear

TRAIN VALUE

Driver gears

Driven gears

$$\frac{N_1}{N_2} \qquad \frac{N_2}{N_3} \qquad \frac{N_4}{N_5}$$

$$n_5 = -\frac{N_1}{N_2}\frac{N_2}{N_3}\frac{N_4}{N_5}n_1$$

We can abstract the ratio of the products of driving tooth numbers (or pitch diameters) to driven tooth numbers (or pitch diameters) as *e*, the train value

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

Pitch diameters may be substituted for tooth numbers.

e **is positive if the last gear rotates in the same direction/sense as the first; it's negative (as in the example above) if it rotates in the opposite sense**

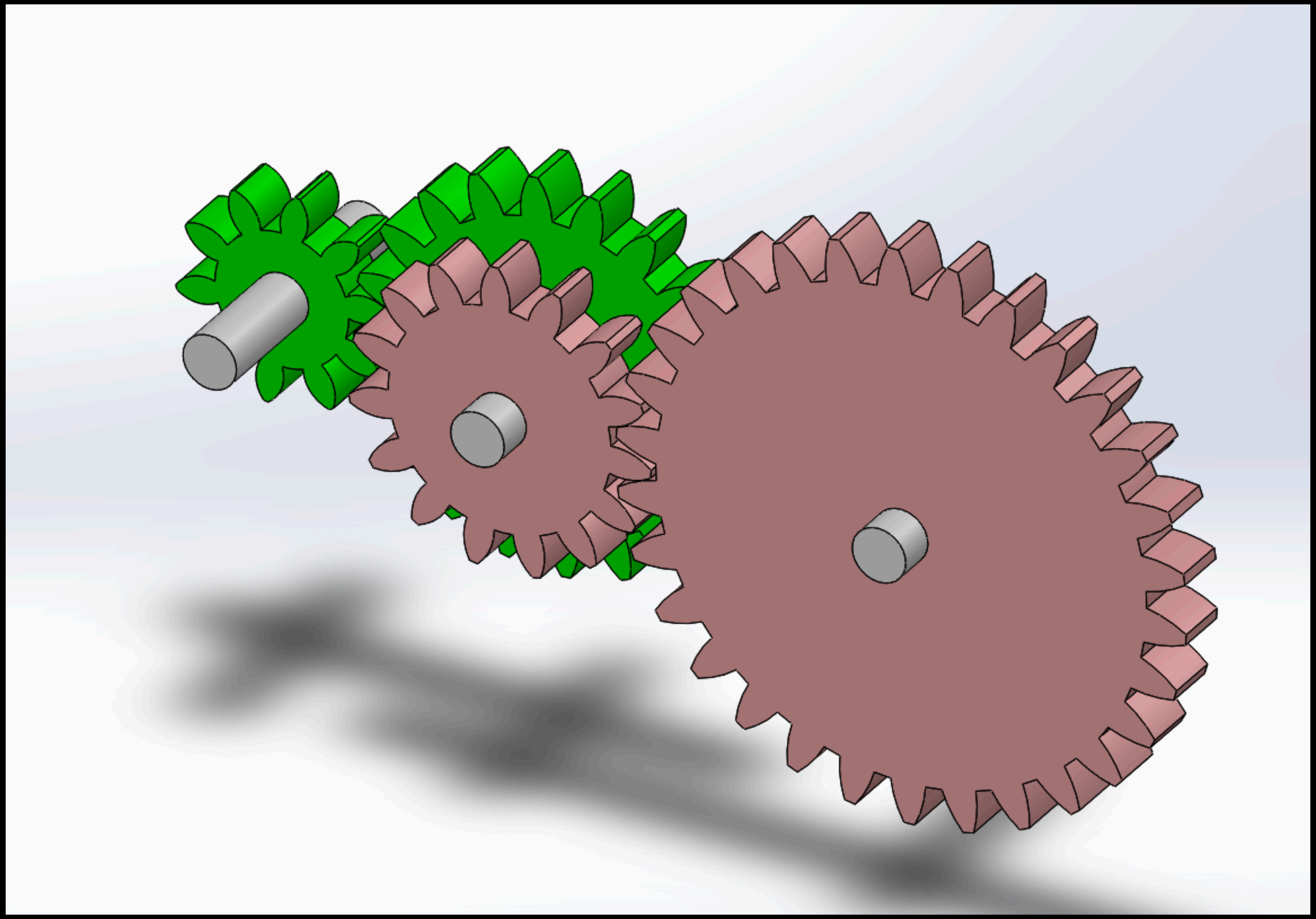
Speed of last gear in the train

$$n_L = e n_F$$

Speed of first gear in the train

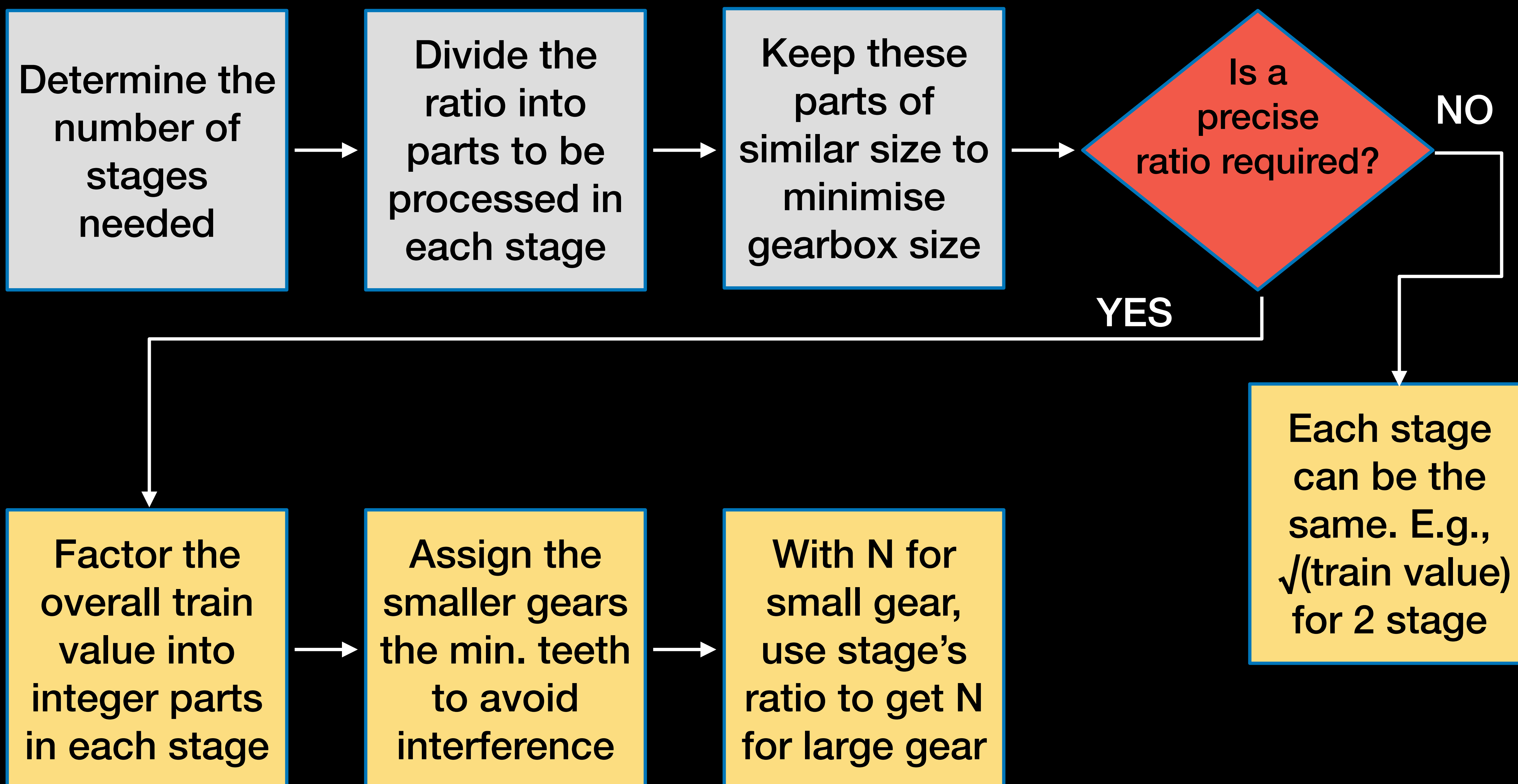
GEAR TRAIN DESIGN

- For gear train values of approx. 10 to 1, a single pair of gears (optionally with an idler to flip direction) can be used.
- For ratios up to about 100 to 1, a two stage compound gear train should be used.
  - This reduces size and stress (no need for a single massive gear).



GEAR TRAIN DESIGN

- Goal: Design a gear train to realise a particular train value.



GEAR TRAIN PROBLEM I

- A client's mechanised crayfish pot hoist needs to rotate at 20 rpm (+/- 5%) in order to raise the cray pots at the desired speed. The client's existing gearmotor operates at 1 rpm and is too slow. You must create a gearbox to speed up the cray pot raising. The entire assembly will be mounted on the client's boat, so you must minimise the overall size of the gearbox. What tooth numbers (with ϕ of 20°) of the gears are needed to realise this?

Ratio (20:1) is more than 10:1 but less than 100:1 - a two-stage compound gear train will work.

Since an approximate final speed is okay, we can simply divide the compound train into two equal sections.

Since we're trying to reduce the gearbox size, we determine smallest that pinions can be to avoid interference

We have pinions with 17 teeth. The driving gears should have 17×4.472 = 76.02 ≈ 76 teeth

We have the driving teeth values (N = 76) and the driven teeth values (N = 17). From this we can find *e*

Amount of reduction per stage:

$$\sqrt{20} = 4.472$$

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

$$= 16.73 \approx 17$$

$$e = \left(\frac{76}{17}\right)\left(\frac{76}{17}\right) = 19.99$$

GEAR TRAIN PROBLEM II

- A gearbox must realise an exact 30:1 speed increase. This gearbox must have a minimal size, $\phi = 20^\circ$. What are the tooth numbers in the gearbox?

Ratio (30:1) is more than 10:1 but less than 100:1 - a two-stage compound gear train will work.

The gearbox's total ratio should be factored into two integer portions.

The driven gears (N_2 and N_4) are the smallest gears. These should be chosen to be as small as possible.

We now have N for the small gears. We can use this to set N for the larger gears.

We can then check the overall train value, verifying that it is exact.

e **= 30 = (6)(5)**
N_1/N_2 = 6
N_3/N_4 = 5

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

$$\approx 16 \text{ teeth}$$

$$N_1 = 6(16) = 96 \text{ teeth}$$

$$N_3 = 5(16) = 80 \text{ teeth}$$

$$e = \left(\frac{96}{16}\right)\left(\frac{80}{16}\right) = 30$$