School of Mathematics and Statistics

MATH 244	Differential Equations	2019

Assignment 6 due: 23:59 pm Wednesday 22 May, online

1. Use the Laplace transform to solve the following initial-value problem:

$$y'' - 2y' + y = e^{2x}, \quad y(0) = 1, \quad y'(0) = 0.$$

   Check your answer is correct.

2. Use the second translation theorem to find:

   (a) $\mathcal{L}\{x\,u(x-2)\}$ (b) $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$ (c) $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+2s+2}\right\}$

3. Use Laplace transforms to solve the initial-value problem $y' + y = f(x)$, $y(0) = 0$ where

$$f(x) = \begin{cases} 1 & 0 \le x < 1 \\ -1 & x \ge 1 \end{cases}.$$

4. Use the Laplace integral formula to find:

   (a) $\mathcal{L}\left\{\displaystyle\int_0^x \tau^3 e^{-2\tau}\, d\tau\right\}$ (b) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\}$.

Tutorial exercises Thursday 16 May – Tuesday 21 May

1. Use Laplace transforms to solve the initial-value problem

$$y'' - 2y' + 2y = xe^x, \qquad y(0) = y'(0) = 0.$$

   Check your answer is correct.

2. Use the second translation theorem to find:

$$\text{(a) } \mathcal{L}\left\{(\cos 2x)\, u\,(x-\pi)\right\} \qquad \text{(b) } \mathcal{L}^{-1}\left\{\frac{1+e^{-3s}}{s+2}\right\}$$

3. Use the Laplace integral formula for the indefinite integral of $\sin ax$ to find $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s^2+a^2)}\right\}$.

4. Use Laplace transforms to solve the integro-differential equation

$$\frac{dy}{dx} = 1 - x - \int_0^x y(\tau)\, d\tau, \quad y(0) = 0.$$

   Check your answer is correct.
