Assignment 4

Due: 23:59pm Wednesday 8 May, after Easter break, online

Extra time has been given, to do this larger assignment. Nearly two full weeks of tutorials are intended to cover the tut questions.

Submit early, check the file that you uploaded after it goes up: one pdf, is it the right file?

Autonomous DEs, Exact DEs. Second Order DEs with constant coefficients, superposition, existence, uniqueness, Wronskians, repeated roots, complex roots. Nonhomogeneous DEs. Boyce & DiPrima, Ch 2 sections 2.5, 2.6, and Ch 3, sections 3.1, 3.2, 3.3, 3.4, 3.5.

1. For the autonomous DE $y' = y(y-1)(y-2)$, sketch a phase diagram, find the equilibrium points and determine if they are asymptotically stable or unstable.

2. Determine if each of these differential equations is exact, and if so, solve it. Make sure you put the DE into standard differential form first.

   (a) $(\sin y - y\sin x)\,dx + (\cos x + x\cos y - y)\,dy = 0$

   (b) $x\,dy/dx = 2xe^x - y + 6x^2$

3. Make this differential equation exact by finding an integrating factor μ, and then solve it:
$$(2y^2 + 3x)\,dx + (2xy)\,dy = 0.$$

4. Show that any separable DE
$$M(x) + N(y)y' = 0$$
is also exact.

5. Use the characteristic equation to find the general solution of these constant coefficient linear homogeneous DEs:

   (a) $y'' - 36y = 0$ (b) $y'' - 36y' = 0$

   (c) $y'' - 3y' + 2y = 0$

6. (a) Show that $y = c_1 + c_2t^2$ is a two-parameter family of solutions to $ty'' - y' = 0$ on the interval $(-\infty, \infty)$.

(b) Find values of c_1, c_2 that give a solution to the IVP $y(1) = 0$, $y'(1) = 1$.

(c) Then show that constants c_1 and c_2 cannot be found for initial conditions $y(0) = 0$, $y'(0) = 1$. Explain why this does not violate Theorem 3.2.3 for solutions of a linear second-order equation.

7. Find an interval centered on $x = 0$ for which the initial value problem

$$y'' + (\tan x)y = e^x \ , \quad y(0) = 1 \ , \ \ y'(0) = 0$$

has a unique solution, using Theorem 3.2.1.

8. Can $y = t^4$ be a solution on an interval containing $t = 0$ for the DE

$$y'' + p(t)y' + q(t)y = 0$$

if the coefficient functions are continuous everywhere? Explain your answer.

9. Use the characteristic equation (also known as the auxiliary equation) to find the general solution of these constant coefficient linear homogeneous DEs:

(a) $y'' - 10y' + 25y = 0$ (b) $2y'' + 2y' + y = 0$

10. Find the general solution of these constant coefficient linear nonhomogeneous DEs:

(a) $y'' - 10y' + 25y = e^t$ (b) $2y'' + 2y' + y = t$

11. Use reduction of order to find a second solution to $t^2y'' + 3ty' + y = 0$, $y_1 = 1/t$.

See next page for Tutorial Questions

MATH244 Tutorial Exercises: 12 April and 2–7 May

Extra time for this extra big tutorial set.

1. What can you say about the stability of the equilibrium point $y = 1$ for the autonomous DE $y' = (y-1)^2$?

2. Determine if each of these differential equations is exact, and if so, solve it. Make sure you put the DE into standard differential form first.

   (a) $(5y - 2x)y' - 2y = 0$

   (b) $\left(\frac{1}{t} + \frac{1}{t^2} - \frac{y}{t^2+y^2}\right) dt + \left(ye^y + \frac{t}{t^2+y^2}\right) , dy = 0$

3. Make the differential equation exact by first finding an integrating factor μ, and then solve it:
$$y(x+y+1)\ dx + (x+2y)\ dy = 0.$$

4. Make the differential equation exact by first finding an integrating factor μ, and then solve it:
$$y' = e^{2x} + y - 1.$$

5. Use the characteristic equation to find the general solution of these constant coefficient linear homogeneous DEs:

   (a) $y'' - y' - 6y = 0$

   (b) $y'' + y' = 0$

   (c) $y'' - 9y = 0$

6. Find the largest interval centered on $x = 0$ for which the initial value problem
$$(x-2)y'' + 3y = x\ , \quad y(0) = 0\ , \ \ y'(0) = 1$$
has a unique solution, using Theorem 3.2.1.

7. Can $y = \sin(t^2)$ be a solution on an interval containing $t = 0$ for the DE
$$y'' + p(t)y' + q(t)y = 0$$
if the coefficient functions are continuous everywhere? Explain your answer.

8. Use the characteristic equation to find the general solution of these constant coefficient linear homogeneous DEs:

   (a) $y'' - 4y' + 4y = 0$

   (b) $y'' + y' + y = 0$

9. Find the general solution of these constant coefficient linear nonhomogeneous DEs:

   (a) $y'' - 4y' + 4y = e^t$

   (b) $y'' + y' + y = t^2$

10. Use reduction of order to find a second solution to $t^2y'' + 2ty' - 2y = 0$, $y_1 = t$.