

# ECEN321: Engineering Statistics

## Assignment 7

Due: 9:00 a.m., Wednesday 27 May 2020

### Central Limit Theorem

1. (Navidi 4.11.14) The concentration of particles in a suspension is 30 per mL.
  - (a) What is the probability that a 2 mL sample will contain more than 50 particles? [3 marks]
  - (b) Ten 2 mL samples are drawn. What is the probability that at least 9 of them contain more than 50 particles? [2 marks]
  - (c) One hundred 2 mL samples are drawn. What is the probability that at least 90 of them contain more than 50 particles? [3 marks]
2. (Navidi 4.11.16) A battery manufacturer claims that the lifetime of a certain type of battery has a population mean of 40 hours and a standard deviation of 5 hours. Let  $\bar{X}$  represent the mean lifetime of the batteries in a simple random sample of size 100.
  - (a) If the claim is true, what is  $P(\hat{X} \leq 36.7)$ ? [1 mark]
  - (b) Based on the answer to part (a), if the claim is true, is a sample mean lifetime of 36.7 hours unusually short? [1 mark]
  - (c) If the sample mean lifetime of the 100 batteries were 36.7 hours, would you find the manufacturer's claim to be plausible? Explain. [1 mark]
  - (d) If the claim is true, what is  $P(\hat{X} \leq 39.8)$ ? [2 marks]
  - (e) Based on the answer to part (d), if the claim is true, is a sample mean lifetime of 39.8 hours unusually short? [1 mark]
  - (f) If the sample mean lifetime of the 100 batteries were 39.8 hours, would you find the manufacturer's claim to be plausible? Explain. [1 mark]

### Confidence Intervals

3. (Navidi 5.1.2) Find the levels of the confidence intervals that have the following values of  $z_{\alpha/2}$ :
  - (a)  $z_{\alpha/2} = 1.96$
  - (b)  $z_{\alpha/2} = 2.17$  [1 mark]
  - (c)  $z_{\alpha/2} = 1.28$  [1 mark]
  - (d)  $z_{\alpha/2} = 3.28$  [1 mark]
4. (Navidi 5.1.4) The article "Modeling Arterial Signal Optimization with Enhanced Cell Transmission Formulations" (Z. Li, *J. Transportation Engineering* 2011:445–454) presents a new method for timing traffic signals in heavily traveled intersections. The effectiveness of the new method was evaluated in a simulation study. In 50 simulations, the mean improvement in traffic flow in a particular intersection was 654.1 vehicles per hour, with a standard deviation of 311.7 vehicles per hour.

- (a) Find a 95% confidence interval for the improvement in traffic flow due to the new system. [1 mark]
- (b) Find a 98% confidence interval for the improvement in traffic flow due to the new system. [1 mark]
- (c) A traffic engineer states that the mean improvement is between 581.6 and 726.6 vehicles per hour. With what level of confidence can this statement be made? [2 marks]
- (d) Approximately what sample size is needed so that a 95% confidence interval will specify the mean to within  $\pm 50$  vehicles per hour? [1 mark]
- (e) Approximately what sample size is needed so that a 98% confidence interval will specify the mean to within  $\pm 50$  vehicles per hour? [1 mark]
5. (Navidi 5.1.6) The article “Application of Surgical Navigation to Total Hip Arthroplasty” (T. Ecker and S. Murphy, *J. Engineering in Medicine*, 2007:699–712) reports that in a sample of 123 hip surgeries of a certain type, the average surgery time was 136.9 minutes with a standard deviation of 22.6 minutes.
- (a) Find a 95% confidence interval for the mean surgery time for this procedure. [1 mark]
- (b) Find a 99.5% confidence interval for the mean surgery time for this procedure. [1 mark]
- (c) A surgeon claims that the mean surgery time is between 133.9 and 139.9 minutes. With what level of confidence can this statement be made? [2 marks]
- (d) Approximately how many surgeries must be sampled so that a 95% confidence interval will specify the mean to within  $\pm 3$  minutes? [1 mark]
- (e) Approximately how many surgeries must be sampled so that a 99% confidence interval will specify the mean to within  $\pm 3$  minutes? [1 mark]
- (f) Find a 98% lower confidence bound for the mean time. [1 mark]
- (g) Someone says that the mean time is greater than 134.3 minutes. With what level of confidence can this statement be made? [2 marks]