ECEN 220
Lab Report 1
Signals and LTI Systems

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Contents

1 Periodicity

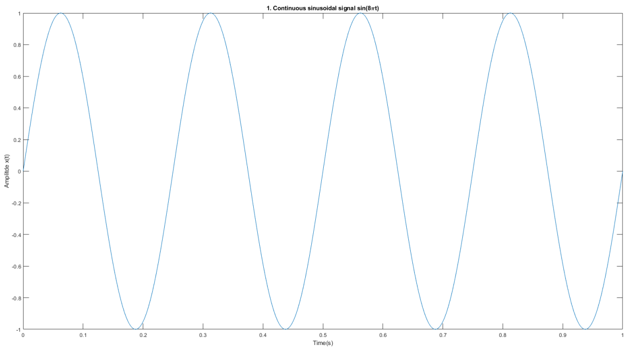

Figure 1: Continuous time signal $f(x) = sin(2\pi f_0 t)$

1.1 Plotting Sampled Signal

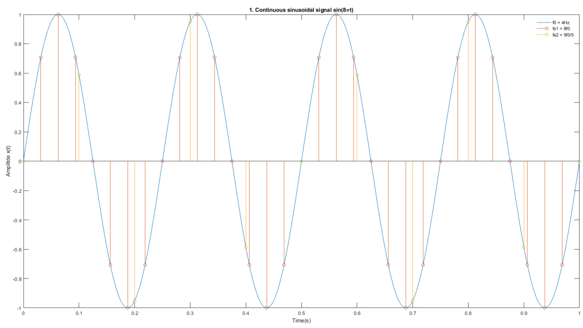

Figure 2: Sampled continuous signals

The above shows the signal $f(x) = sin(2\pi f_0 t)$ sampled in discrete time space at $8f_0$ and $\frac{5}{2}f_0$, ie at 32 and 10 sample respectively over a 1 second interval.

1.2 Discrete Periodicity Comparison

$$P = \frac{2\pi}{\omega_0}$$

The sampled signal at $32s^{-s}$ has 8 sample per period (of CT) and thus as a period of $\frac{1}{4}$ ie the same as the CT signal The sampled signal at $10s^{-s}$ has 4 sample per period (of CT) and thus as a period of $\frac{1}{2}$ ie the twice that of the CT signal

2 Linearity

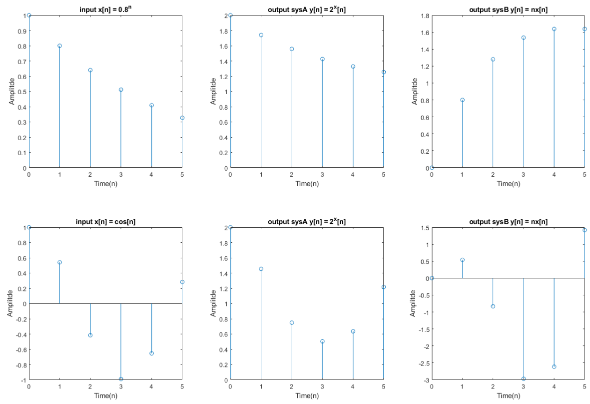

Figure 3: Inputs and outputs for systems (A, B)

For systems A $= y[n] = 2^{x[n]}$ and B $= y[n] = nx[n]$ the above graphs plot the outputs to the respective input signals $x[n] = 0.8^n, cos(n)$ over the interval $0 \leq n \leq 5$

2.1 Test by summation

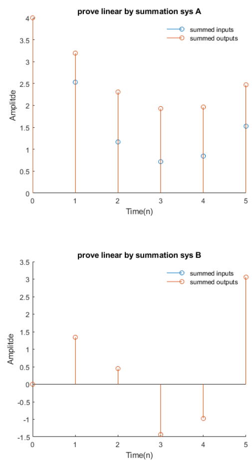

Figure 4: Summed inputs vs summed outputs (A, B)

First test of linearity is to invoke the rule of summation. The output of the summed input signals (as an input to the system), if system is linear, should equal the summed outputs of the input signals individually.

$$y(x_1 + x_2) = y(x_1) + y(x_2)$$

The plot above shows this test for both systems A, and B in superposition. This shows that System a is confirmed to be non-linear by summation and B is still a candidate for linearity by summation.

2.2 Test by scaling

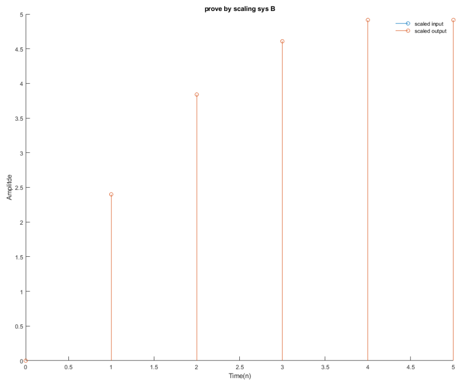

Figure 5: Scaled input vs scaled output (B)

As B is still a candidate for linearity, the next text is that of scaling. The output for an input scaled be some α should equal the output (of unscaled input) scaled by the same α.

$$\alpha y(x) = y(\alpha x)$$

The plot above shows the outputs for this test (again in superposition) thus proving the system B's linearity.

3 Convolution

3.1 Matlab Convolution

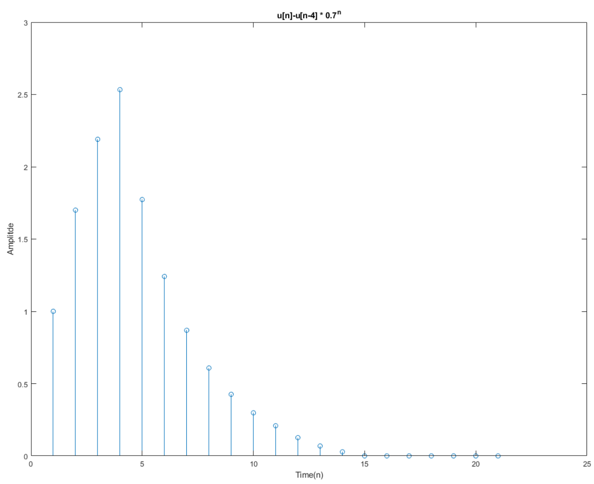

Figure 6: Computational Convolution

The above plots the convolution of the input sequence $x[n] = u[n] - u[n-4]$ with the (DT) systems impulse response of $h[n] = 0.7^n : 0 \leq n \leq 10$

3.2 Manual Convolution

Case 1,5: No Overlap

$n < 0, n > 13 : y[n] = 0$

Case 2: Partial Overlap Approach

$$0 \le n < 3$$

$$\begin{aligned} y[n] &= \sum_{k=0}^{n} 0.7^k \\ &= \frac{1 - 0.7^{n+1}}{1 - 0.7} \end{aligned}$$

$$y[n] = \frac{1 - 0.7^{n+1}}{0.3} : 0 \le n < 3$$

Case 2: Complete Overlap

$$n \le 10, n - 3 \ge 0 \Rightarrow 0 \le n \le 10$$

$$\begin{aligned} y[n] &= \sum_{k=n-3}^{n} 0.7^k \\ &= \sum_{k=0}^{n} 0.7^k - \sum_{k=0}^{n-4} 0.7^k \\ &= \frac{1 - 0.7^{n+1}}{0.3} - \frac{1 - 0.7^{n-3}}{0.3} \end{aligned}$$

$$y[n] = \frac{0.7^{n-3} - 0.7^{n+1}}{0.3} : 3 \le n \le 10$$

Case 2: Partial Overlap Departure

$$n > 10, n - 3 \le 10 \Rightarrow 10 < n \le 13$$

$$\begin{aligned} y[n] &= \sum_{k=n-3}^{10} 0.7^k \\ &= \sum_{k=0}^{10} 0.7^k - \sum_{k=0}^{n-4} 0.7^k \\ &= \frac{1 - 0.7^{11}}{0.3} - \frac{1 - 0.7^{n-3}}{0.3} \end{aligned}$$

$$y[n] = \frac{0.7^{n-3} - 0.7^{11}}{0.3} : 10 < n \le 13$$

Piecewise Defined

$$y[n] = \begin{cases} 0 & n < 0 \\ \dfrac{1 - 0.7^{n+1}}{0.3} & 0 \le n < 3 \\ \dfrac{0.7^{n-3} - 0.7^{n+1}}{0.3} & 3 \le n \le 10 \\ \dfrac{0.7^{n-3} - 0.7^{11}}{0.3} & 10 < n \le 13 \\ 0 & n > 13 \end{cases}$$

4 Appendix

Figure 1, 2:

```
%1A continuous signal

f0 = 4;                          %signal frequency
fs0=1000000;             %sample freq (very high to emu contin)
t = 0:1/fs0:1;           %spit timespace
x = sin(2*pi*f0*t);      %cont signal

%1B
fs1 = (8*f0);            %x1 sample freq
t1 = 0:1/fs1:1;
x1 = sin(2*pi*f0*t1);

fs2= (5*f0)/2;           %x2 sample freq
t2 = 0:1/fs2:1;
x2 = sin(2*pi*f0*t2);

plot(t,x);
hold on;
stem(t1,x1);
stem(t2,x2);
hold off;

title('1. Continuous sinusoidal signal sin(8\pit)');
xlabel('Time(s)');
ylabel('Amplitde x(t)');
h = legend('f0 = 4Hz','fs1 = 8f0','fs2 = 5f0/5');
set(h, 'Box', 'off')
```

Figure 3:

```
%2A
n = 0:5;
x1 = 0.8.^n;
x2 = cos(n);

%input 1 sys a,b
subplot(2,3,1);
stem(n,x1);
title('input x[n] = 0.8^n');
xlabel('Time(n)');
ylabel('Amplitde');

%sys a
subplot(2,3,2);
stem(n,2.^x1)
title('output sysA y[n] = 2^x[n]');
xlabel('Time(n)');
ylabel('Amplitde');
%sys b

subplot(2,3,3);
stem(n,n.*x1)
title('output sysB y[n] = nx[n]');
xlabel('Time(n)');
ylabel('Amplitde');

%input 2 sys a,b
subplot(2,3,4);
stem(n,x2);
title('input x[n] = cos[n]')
xlabel('Time(n)');
ylabel('Amplitde');

%sys a
```

```
subplot(2,3,5);
stem(n,2.^x2)
title('output sysA y[n] = 2^x[n]');
xlabel('Time(n)');
ylabel('Amplitde');

%sys b
subplot(2,3,6);
stem(n,n.*x2)
title('output sysB y[n] = nx[n]')
xlabel('Time(n)');
ylabel('Amplitde');
```

Figure 4:

```
n = 0:5;
x1 = 0.8.^n;
x2 = cos(n);

%prove linear by summation
%ie y(x1+x2) = y1+y2

%sysA
subplot(2,1,1);
hold on;
stem(n,2.^(x1+x2));
stem(n,(2.^x1) + (2.^x2));
hold off;
title('prove linear by summation sys A');
set(legend('summed inputs', 'summed outputs'),'Box','off');
xlabel('Time(n)');
ylabel('Amplitde');
%A is non-linear according to summation

%sysB
subplot(2,1,2);
hold on;
stem(n,n.*(x1+x2));
stem(n,(n.*x1) + (n.*x2));
hold off;
title('prove linear by summation sys B');
set(legend('summed inputs', 'summed outputs'),'Box','off');
xlabel('Time(n)');
ylabel('Amplitde');
%B is linear according to summation
```

Figure 5:

```
n = 0:5;
x1 = 0.8.^n;
x2 = cos(n);

%prove by scaling
%ie y(Mx1) = Mx1

%sysB
hold on;
stem(n,n.*(3.*x1));
stem(n,3.*(n.*x1))
hold off;
title('prove by scaling sys B')
xlabel('Time(n)');
ylabel('Amplitde');
set(legend('scaled input', 'scaled output'),'Box','off');

%B in linear occording to scaling
```

Figure 6:

```
syms x
u(x) = piecewise(x<0, 0, x>=0, 1);

n = 0:10;
h = 0.7.^n;
x = (n>=0) - ((n-4)>=0);
c = conv(h,x);
stem(c);
title('u[n]-u[n-4] * 0.7^n');
xlabel('Time(n)');
ylabel('Amplitde');
```