ECEN321: Engineering Statistics

Assignment 6

Due: 9:00 a.m., Wednesday 20 May 2020

Normal Distribution

1. (Navidi 4.5.4) If $X \sim \mathcal{N}(2, 9)$, compute

   (a) $P(X \geq 2)$

   [1 mark]

   (b) $P(1 \leq X < 7)$

   [2 marks]

   (c) $P(-2.5 \leq X < -1)$

   [2 marks]

   (d) $P(-3 \leq (X - 2) < 3)$

   [2 marks]

2. (Navidi 4.5.22) The molarity of a solute in solution is defined to be the number of moles of solute per litre of solution (1 mole $= 6.02 \times 10^{23}$ molecules). If X is the molarity of a solution of sodium chloride (NaCl), and Y is the molarity of a solution of sodium carbonate (Na_2CO_3), the molarity of sodium ion (Na^+) in a solution made of equal parts NaCl and Na_2CO_3 is given by $M = 0.5X + Y$. Assume X and Y are independent and normally distributed, and that X has mean 0.450 and standard deviation 0.050, and Y has mean 0.250 and standard deviation 0.025.

   (a) What is the distribution (class and parameter(s)) of M?

   [3 marks]

   (b) Calculate $P(M > 0.5)$

   [1 mark]

Exponential Distribution

3. (Navidi 4.7.10) The distance between consecutive flaws on a roll of sheet aluminium is exponentially distributed with mean distance 3 m. Let X be the distance, in metres, between flaws.

   (a) What is the mean number of flaws per metre?

   [1 mark]

   (b) What is the probability that a 5 m length of aluminium contains exactly two flaws?

   [2 marks]

Estimation

4. (Navidi 4.9.2) Choose the best completion. The variance of an estimator measures...

   (a) how close the estimator is to the true value

   (b) how close repeated values of the estimator are to each other

   (c) how close the mean of the estimator is to the true value

   (d) how close repeated values of the mean of the estimator are to each other

   [1 mark]

P.T.O.

5. (Navidi 4.9.4) Let $X_1, \ldots, X_N$ be a simple random sample from a $\mathcal{N}(\mu, \sigma^2)$ population. For any constant $k > 0$, define $\hat{\sigma}_k^2 = \left(\sum_{i=1}^{N}(X_i - \bar{X})^2\right)/k$. Consider σ_k^2 as an estimator of σ^2.

   (a) Compute the bias of $\hat{\sigma}_k^2$ in terms of k. [Hint: The sample variance s^2 is unbiased, and $\hat{\sigma}_k^2 = (N-1)s^2/k$.]

   [2 marks]

   (b) Compute the variance of $\hat{\sigma}_k^2$ in terms of k. [Hint: $\sigma_{s^2}^2 = 2\sigma^4/(N-1)$, and $\hat{\sigma}_k^2 = (N-1)s^2/k$.]

   [2 marks]

   (c) Compute the mean squared error of $\hat{\sigma}_k^2$ in terms of k.

   [2 marks]

   (d) For what value of k is the mean squared error of $\hat{\sigma}_k^2$ minimised?

   [2 marks]

6. (Navidi 4.9.8) Let $X_1, \ldots, X_n$ be a random sample from a $\mathcal{N}(\mu, 1)$ population. Find the MLE of μ.

   [4 marks]