School of Mathematics and Statistics
Te Kura Mātai Tatauranga

ENGR 222 **Assignment 1** **Due: Thursday 04 March 11:59pm**

1 Consider the parametric equations

$$(x, y) = (8\sin(t), 2t - \sin(2t))$$

over the interval $0 \leq t \leq 2\pi$.

(a) Determine the location at $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$ and use this to draw a rough sketch of the curve.

(b) Find the unit tangent vector to the curve when $t = \pi/6$.

(c) Determine an equation describing the tangent line at $t = \pi/6$.

(d) Determine an equation describing the normal line at $t = \pi/6$.

(e) Calculate the arc length over the interval $0 \leq t \leq 2\pi$.
Hint: you may need the double angle formula $\cos(2x) = 2\cos(x)^2 - 1$.

2 Consider the curve described by the vector valued function

$$\mathbf{r}(t) = \frac{1}{4}\left(e^{2t} - 2t\right)\mathbf{i} + e^t\mathbf{j}$$

for $t \in \mathbb{R}$.

(a) Find a point on the curve for which $\mathbf{r}(t) \cdot \mathbf{j} = 2$.

(b) Determine the unit tangent vector to the curve (for arbitrary t).

(c) Determine the principal unit normal vector to the curve (for arbitrary t).
(Calculate this via formula involving the derivative of the unit tangent vector.)

(d) Determine the curvature of the curve (for arbitrary t).

(e) Determine the arc length of the curve over $0 \leq t \leq 3$.

3 Quick questions:

(a) Determine the arc length parametrisation of

$$(x, y, z) = (3t, -2 + t, 7 - 5t)$$

with $t = 0$ as the starting/reference point.

(b) Determine the arc length parametrisation of the curve described by

$$\mathbf{r}(t) = (-3 + 5\cos(t))\mathbf{i} + (2 - 5\sin(t))\mathbf{j}$$

using $t = 0$ as the starting/reference point.

(c) Find the unit tangent vector to

$$\mathbf{r}(t) = \sqrt{2}\cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \sin(t)\mathbf{k}$$

at $t = \pi/3$.

(d) An alternative formula for the binormal vector is

$$\mathbf{B}(t) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}.$$

Use this to find the binormal vector to

$$\mathbf{r}(t) = t\mathbf{i} - t^3\mathbf{j} + t^2\mathbf{k}$$

at $t = 1$.

(e) Find the minimum and maximum curvature for the curve described by

$$\mathbf{r}(t) = (2 + 3\sin(t))\mathbf{i} + (1 + 2\cos(t))\mathbf{j}$$

4 Suppose a roller coaster follows a path described by

$$\mathbf{r}(t) = \frac{1}{5}t(20-t)\mathbf{i} + \frac{1}{50}t^2(20-t)\mathbf{j} + \frac{1}{50}t(10-t)(20-t)\mathbf{k}$$

over $t \in [0, 20]$.

(a) Determine the velocity vector of the roller coaster (for arbitrary t).

(b) Determine the speed when $t = 5$.

(c) Determine the acceleration vector of the roller coaster (for arbitrary t).

(d) Determine the curvature of curve followed by the roller coaster when $t = 5$.

(e) Determine the normal scalar component of acceleration when $t = 5$.
Comparing this with $\|\mathbf{a}(5)\|$, what can you say about the tangential scalar component of acceleration when $t = 5$?