Daniel Eisen 30044549, Engr 222 Test 2

1. $3x + 3y - 1z - 2w = 0$
$2x + 2y + 1z + 2w = 5$
$1x + 1y + 0z + 0w = 1$

$$\Rightarrow \left(\begin{array}{cccc|c} 3 & 3 & -1 & -2 & 0 \\ 2 & 2 & 1 & 2 & 5 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

row reduce: $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1-y \\ y \\ 3-2w \\ w \end{pmatrix}$

2. $T(x,y,z) = (2x+3y, y-4z)$

$T(1,0,0) = (2,0)$
$T(0,1,0) = (3,1)$
$T(0,0,1) = (0,-4)$

$$, A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & -4 \end{pmatrix}$$

3. rotate 50° scale x3

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$= \begin{pmatrix} \cos(50) & -\sin(50) \\ \sin(50) & \cos(50) \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 3\cos 50 & -\sin 50 \\ 3\sin 50 & 5\cos 50 \end{pmatrix}$$

$$= \begin{pmatrix} 1.928 & -2.298 \\ 2.298 & 1.928 \end{pmatrix}$$

4. $(x\ y\ z\ w) \mapsto (y\ z)$

$\Rightarrow x: \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \mapsto \begin{pmatrix}0\\0\end{pmatrix}$, $y\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \mapsto \begin{pmatrix}1\\0\end{pmatrix}$, $z\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \mapsto \begin{pmatrix}0\\1\end{pmatrix}$, $w = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \mapsto \begin{pmatrix}0\\0\end{pmatrix}$

$$A = \left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

5. $3x + 2y - 6z = -1$
$-2x + 5y + 4z = 7$, $A = \begin{pmatrix}3 & 2 & -6\\ -2 & 5 & 4\\ 4 & -3 & -8\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-1\\7\\2\end{pmatrix} = V$
$4x - 3y - 8z = 2$

$$\left(A^TA \,\middle|\, A^TV\right) = \left(\begin{array}{ccc|c} 29 & -16 & -58 & -9 \\ -16 & 38 & 32 & 27 \\ -58 & 32 & 116 & 18 \end{array}\right)$$

row reduce: $\left(\begin{array}{ccc|c} 1 & 0 & -2 & 5/47 \\ 0 & 1 & 0 & 71/94 \\ 0 & 0 & 0 & 0 \end{array}\right)$, $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}z + 5/47\\ 71/94\\ z\end{pmatrix}$

6. $A=(0,0,1)$, $B=(1,x,2)$, $C=(t,t,-2+2t)$

$$\vec{AB}=\begin{pmatrix}1\\x\\2\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\x\\1\end{pmatrix},\quad \vec{BC}=\begin{pmatrix}t\\t\\-2-2t\end{pmatrix}-\begin{pmatrix}1\\x\\2\end{pmatrix}=\begin{pmatrix}t-1\\t-x\\2t-4\end{pmatrix}$$

$$\vec{BC}=k\vec{AB},\ \begin{pmatrix}t-1\\t-x\\2t-4\end{pmatrix}=k\begin{pmatrix}1\\x\\1\end{pmatrix},\ \begin{matrix}t-1=k\\2t-4=k\end{matrix},\ \left(\begin{array}{cc|c}1&-1&1\\2&-1&4\end{array}\right)\Rightarrow\begin{pmatrix}t\\k\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$$

$t-x=kx \Rightarrow 3-x=2x$, $x=1$

7. $A=\begin{pmatrix}1&0\\1&-1\\2&1\end{pmatrix}$, $v=\begin{pmatrix}-2\\1\\4\end{pmatrix}$

$$x=A^{+}v=\begin{pmatrix}1\\1\end{pmatrix},\quad Ax=\begin{pmatrix}1\\0\\3\end{pmatrix}$$

8. $A \underset{0.1}{\overset{0.3}{\rightleftarrows}} B$,

$A(n+1) = 0.7A(n) + 0.1B(n)$

$B(n+1) = 0.3A(n) + 0.9B(n)$

$$\begin{pmatrix}A(n+1)\\B(n+1)\end{pmatrix}=\begin{pmatrix}0.7&0.1\\0.3&0.9\end{pmatrix}\begin{pmatrix}A(n)\\A(n)\end{pmatrix},\quad A+B=4000$$

eigenvalues $=\lambda_1: 1,\ \lambda_2: \frac{3}{2}$

λ_1 has eigenvetor $\begin{pmatrix}1/3\\1\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$, $A=1000$ gallons, $B=3000$ gallons