Victoria University of Wellington
School of Mathematics and Statistics

MATH 244 — Modelling with Differential Equations — 2019

# Assignment 2

Due: 11:59pm Wednesday 27 March online

Integrating factors, separation of variables. Boyce & DiPrima, Ch 2 sections 2.1, 2.2

1. For the following DEs, check they are linear first-order by writing in standard form. Then use an integrating factor to find the general solution. In each case write down the largest interval $I$ over which the general solution is defined.
   (a) $y' + 3x^2 y = x^2$
   (b) $x\dfrac{dy}{dx} - y = x^2 \sin x$
   (c) $\cos x \dfrac{dy}{dx} + (\sin x)\, y = 1$
   (d) $\dfrac{dr}{d\theta} + r \sec\theta = \cos\theta$
2. Solve the following separable DEs:
   (a) $(1 + t^4)\frac{dy}{dt} = \frac{t^3}{y}$
   (b) $e^{-y} \sin t - y' \cos^2 t = 0$
3. Find the general solution to the DE $y' = y/2t$ in implicit form. Sketch several solution curves corresponding to different values of the constant of integration. Find an explicit form of the solution $y = y(t)$ satisfying the initial condition $y(-1) = 2$. For what values $t$ is this solution defined?
4. (a) Solve the Verhulst DE
   $$\frac{dy}{dt} = 0.5y\left(1 - \frac{y}{5}\right)$$
   with initial condition $y(0) = 1$.
   (b) Differentiate the DE with respect to $t$ to find $d^2y/dt^2$ (you will need implicit differentiation on the right-hand side) and hence show that the graph of the solution has an inflection point when $y = 2.5$. At what time $t$ does this inflection point occur?
   (c) Plot the graph of the solution—you can use a graphical calculator or Maple if you like. Show the inflection point and indicate what happens as $t \to +\infty$.

# Tutorial Exercises: 21–26 March

1. For the following DEs, check they are linear first-order by writing in standard form. Then use an integrating factor to find the general solution. In each case write down the largest interval $I$ over which the general solution is defined.

   (a) $\dfrac{dy}{dx} + 2xy = x^3$

   (b) $xy' + 3y = 2$

   (c) $\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x)\, y = 0$

   (d) $(x^2 - 1)\dfrac{dy}{dx} + 2y = (x+1)^2$

2. Solve the following separable DEs:

   (a) $\frac{dy}{dt} = 2(1 + y^2)t$

   (b) $y' + ty = 0$

3. Find the general solution to the DE $y' = -t/y$ in implicit form. Sketch the solution curves corresponding to the constant of integration $C$ taking values $1, 2, 3, 4$. Find an explicit form of the solution $y = y(t)$ satisfying the initial condition $y(1) = -\sqrt{3}$. For what values $t$ is this solution defined?