~~integrating factor $\exp(\int p\,dt)$~~

2a) require V of free falling partical.

$N^{2d} \Rightarrow$ force $=$ ~~ma~~ $= mg$ $\quad m=$ mass, $g = 9.81\,ms^{-1}$

$a = g$, $\frac{dv}{dt} = g$, $V = gt + V_0$ $\quad$ assume $V_0 = \emptyset$

$\boxed{V = gt}$ $\quad V = \frac{dz}{dt} = gt \quad z = \frac{1}{2}gt^2 + z_0$

$z(0) = 0 \quad z = \frac{1}{2}gt^2$

$t = \sqrt{2\frac{z}{g}}$

To fall heigh (h) , $z = h \Rightarrow t = \sqrt{2h/g}$

w/ $v = gt \quad v = g\sqrt{2h/g} = \sqrt{2gh}$

$V = \sqrt{2gh}$

b) $A(h)\cdot\frac{dh}{dt} = -a\sqrt{2gh}$

outflow through area A (m^2) is m^3/s ie. $A(m^2) \times V(m/s) \rightarrow AU = m^3/s$

$\Rightarrow av = a\sqrt{2gh}$

the r of volume change in tank is $v \cdot A$,

where v is the rate of change of the height ie $\frac{dh}{dt}$

$A(h)\frac{dh}{dt}$, as $\frac{dh}{dt}$ is positive but h is decreasing, the outflow must be negative ie

$A(h)\frac{dh}{dt} = -a\sqrt{2gh}$

(IVP) c) $h = 3$, $r_{tank} = 1$, $A = \pi$, $a = 0.01\pi$

$A\cdot\frac{dh}{dt} = -a\sqrt{2gh}$ $\quad g = 9.8$

$\pi h' = -0.01\pi\sqrt{19.6 \cdot h}$

$\int \frac{1}{\sqrt{h}}\,dh = \int -0.01\sqrt{19.6}\,dt$

$\Rightarrow \int h^{-1/2}\,dh = -0.01\sqrt{19.6}\cdot\int dt$

$\Rightarrow 2h^{\frac{1}{2}} = -0.01\sqrt{19.6}\;t + C$

$2\sqrt{h} = -0.01\sqrt{19.6}\cdot t + C$

at $t=0$, $h=3$ ie $2\sqrt{3}=C$

$\Rightarrow 2\sqrt{h} = 2\sqrt{3} - 0.01\sqrt{19.6}\,t$

$h=0$ (tank empty) when $2\sqrt{3} - 0.01\sqrt{19.6}\,t = 0$

$\Rightarrow 2\sqrt{3} = 0.01\sqrt{19.6} = t$

$t = 2\sqrt{3} / 0.01\sqrt{19.6}$

$t \simeq 78$ seconds

(3)