Control of a Motorised Pendulum

Lab 3: Transfer function of the pendulum and testing system open loop response.

1. Previous - Transfer function of the DC motor and prop

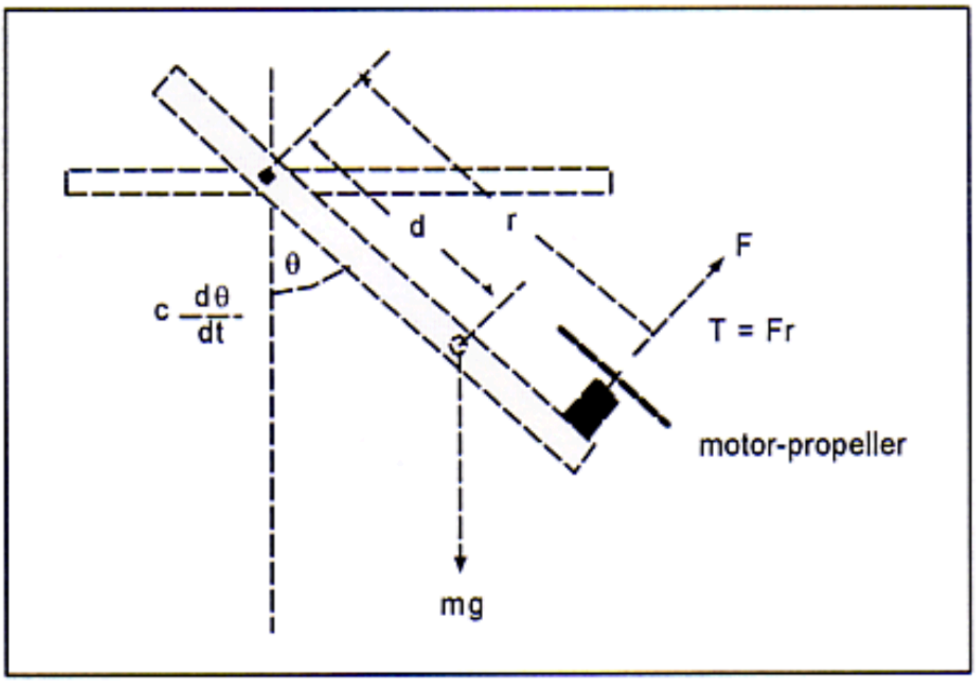
You should now have derived a transfer function that relates the rotational velocity of the prop to the input voltage.

2. Modeling the rotational velocity to thrust conversion.

Due to the rotational velocity of the prop and its aerodynamic shape a thrust (force) is obtained. Typically this is something that has been measured in the lab, however, this year we will provide you with the measured variables you require to model the system.

3. Modeling the rigid pendulum

The rigid pendulum can be modeled on the basis of the sketch below:



Write down a differential equation that will relate the angular displacement of the pendulum arm to the applied torque. How does this torque relate to the applied force?

4. System parameters for the pendulum

Your transfer function for the pendulum in 3 above should contain the following system parameters:

J_p is the moment of inertia of the pendulum arm,
c is the linear damping coefficient,
m mass of the pendulum arm ($0.168kg$),

d is the distance between the point of rotation and the centre of mass of the arm ($0.165m$)
g is the gravitational constant

Most of these are relatively easy to measure (and given above), with the exception of J_p and c. These can be estimated by considering the response of a non_driven, damped oscillator to a disturbance. The response of such a system will be given by:

$$y(t) = Ae^{-\frac{c}{2J_p}t}\cos(\omega t + \phi) = Ae^{-Bt}\cos(\omega t + \phi)$$

The term $Ae^{-\frac{c}{2J_p}t}$ represents the exponential decay envelope and the frequency of the oscillations is given by:

$$\omega = \sqrt{\frac{mgd}{J_p} - \left(\frac{c}{2J_p}\right)^2} = \sqrt{\frac{mgd}{J_p} - B^2}$$

By disturbing the pendulum and allowing it to oscillate, measuring the amplitude as it comes to rest, we can then measure the amplitude of each oscillation and plot this in order to obtain the constants A and B. From a measurement of the frequency of these oscillations the value of J_p can be calculated, after which the value of c can be calculated from the constant B.
Provide within a supplementary file on the wiki (csv file), we have a table showing some measured results for the disturbed pendulum, create a plot and use it to calculate J_p and c. You should now be able to write down the transfer function for the pendulum and model the step response of this subsystem.

5. System Simulation.

Combine all the different blocks yield a system transfer function for the driven pendulum as:

$$G(s) = \frac{\Theta(s)}{V(s)}$$

Sketch the pole-zero map of your system. What do you predict the response to look like? Simulate what the response of the system will look like to input voltages of 3, 4 and 5 V. Can you identify any physical aspects of the pendulum system that we have not modelled? How might these alter the real world response?

6. Lab report

Write a detailed lab report on your results of the first three laboratories, detailing the open loop response of the system. This should be handed in by the date provided in the course outline.