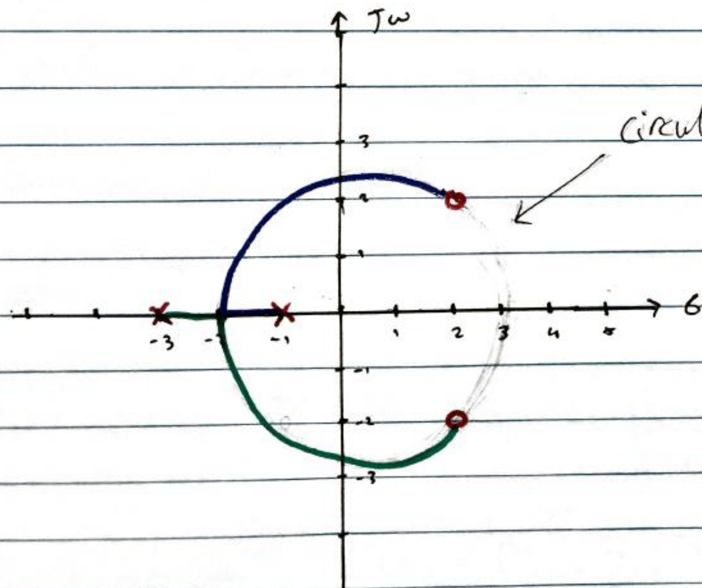
Ecen 315. Test 2. Daniel [illegible]

1. a) $G(s) = \dfrac{s^2 - 4s + 8}{s^2 + 4s + 3}$

Z: $2 \pm 2i$

P: $-1, -3$, $P - Z = \phi \Rightarrow$ No assymtopes



circular sketch path fit to points

$$G(s) = \frac{s+4}{s(s+6)(s+3)(s+1)}$$

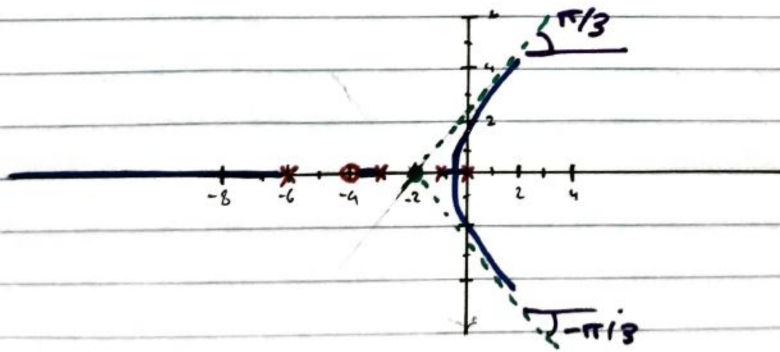
Z : -4

P : $0, -6, -3, -1$, $P - Z = 3$

Assymtops:

$$\sigma_a = \frac{\Sigma P - \Sigma Z}{3} = -2$$

$$\theta_a = \frac{(2k+1)\pi}{3}$$, $k=0$: $\pi/3$, $k=1$: π , $k=-1$: $-\pi/3$



1 b) $G(s) = \dfrac{s^2 + 10s + 24}{s^2 + 3s + 2}$

Z: $-4, -6$

P: $-1, -2$, $P - Z = 0$

Breakaway & Break in point along $\mathbb{R}$

$1 + KG(s) = 0$, $K = -\frac{1}{G(s)}$, ie find point of max gain on $\mathbb{R}$ (Break away)

solve $\frac{dK}{ds} = 0$

$K = -\dfrac{s^2 + 3s + 2}{s^2 + 10s + 24} \Rightarrow -\dfrac{P(s)}{Z(s)}$ (know s is $\mathbb{R}$)

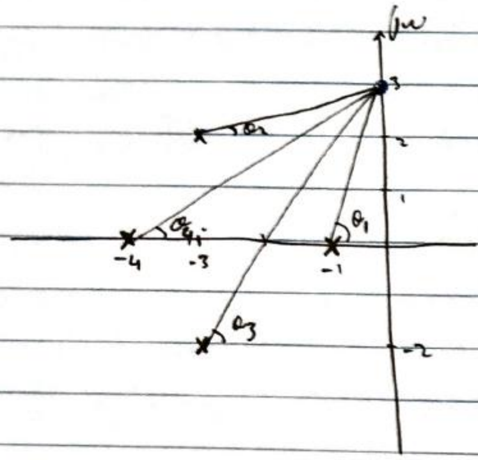
$K' = -\dfrac{(P(s) Z'(s) - P'(s) Z(s))}{(Z s)^2}$

$= -\dfrac{7s^2 + 44s + 52}{(s^2 + 10s + 24)^2}$

$-7s^2 + 44s + 52 = \phi \Rightarrow s = -1.5779, -4.707$

Break away Break In

1c) P: $-1, -3 \pm 2i, -4$



θ_1: $3i - (-1)) = 3.16 \angle 71.56°$

θ_2 : $3i - (-3+2i) = 3.16 \angle 18.4°$

θ_3 : $3i - (-3-2i) = 5.83 \angle 59.04°$

θ_4 : $3i - (-4) = 5 \angle 36.87°$

Sum = 185.87 @ $3i$

Try for $2.6i$: θ_1: $2.6i - (-1) \Rightarrow 68.96°$
θ_2: " $- (-3+2i) \Rightarrow 11.31°$
θ_3: " $- (-3-2i) \Rightarrow 56.89°$
" $- (-4) \Rightarrow 33.02°$

Sum = 170.18

Try 2.8 : $\theta_1 = 70.35$ $\theta_2 = 14.93$ $\theta_3 = 57.99$ $\theta_4 = 34.99$
Sum = 178.262

$\therefore$ $j\omega$ intercept is $\sim$ 2.8j (~~still~~ just about maybe 2.85j)

2.85j $\Rightarrow$ intercept gives 180.72°

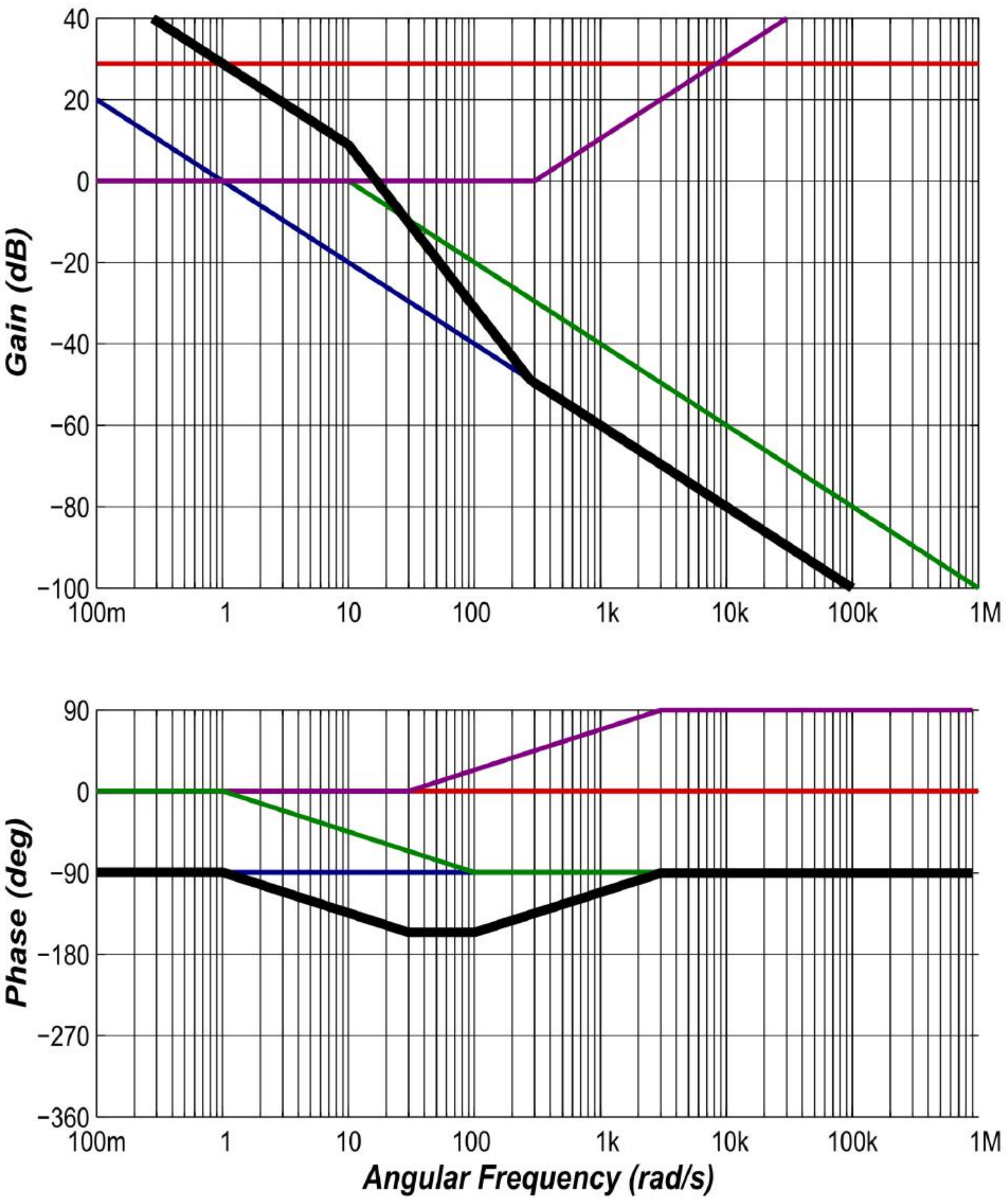
Unstable Gain : ie for $s = 0 \pm 2.85j$

$$K = -\frac{1}{G(s)} = -\frac{(s+1)(s+4)(s+(3+2j))(s+(3-2j))}{1}$$

sub in $s = \pm 2.85j$, $\boxed{K = 263.7}$ (taking only $\mathbb{R}$ part)

2. a) $G(s) = \frac{s+300}{s^2+10s} = \frac{s+300}{s(s+10)} = \frac{300}{10} \cdot \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{10}+1\right)} \cdot \left(\frac{s}{300}+1\right)$

$300/10 = 30 \Rightarrow 20\log(30) = 29.54\,dB$



2(b)

P: -3, -8, -100

$$\Rightarrow \cancel{K_s} \frac{1}{(\frac{s}{3}+1)} \cdot \frac{1}{(\frac{s}{8}+1)} \cdot \frac{1}{(\frac{s}{100}+1)} \cdot \frac{1}{3\times 8\times 100} \cdot K$$

$$\frac{K}{2400} = 10^{\frac{20dB}{20}} \text{ (ie } 10), \quad K = 24000$$

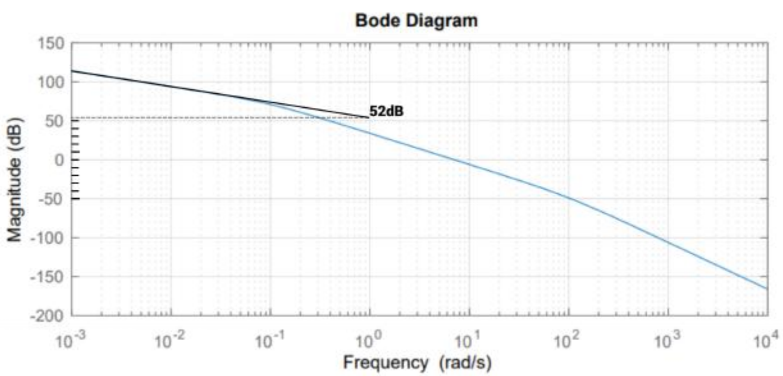
$$TF = \frac{24000}{(s+3)(s+8)(s+100)}$$

2(c) Initial Slope at -20dB/decade

∴ Type 1.

Step: Zero error, Parabolic: Infinite error

Ramp: $\frac{1}{K_v}$

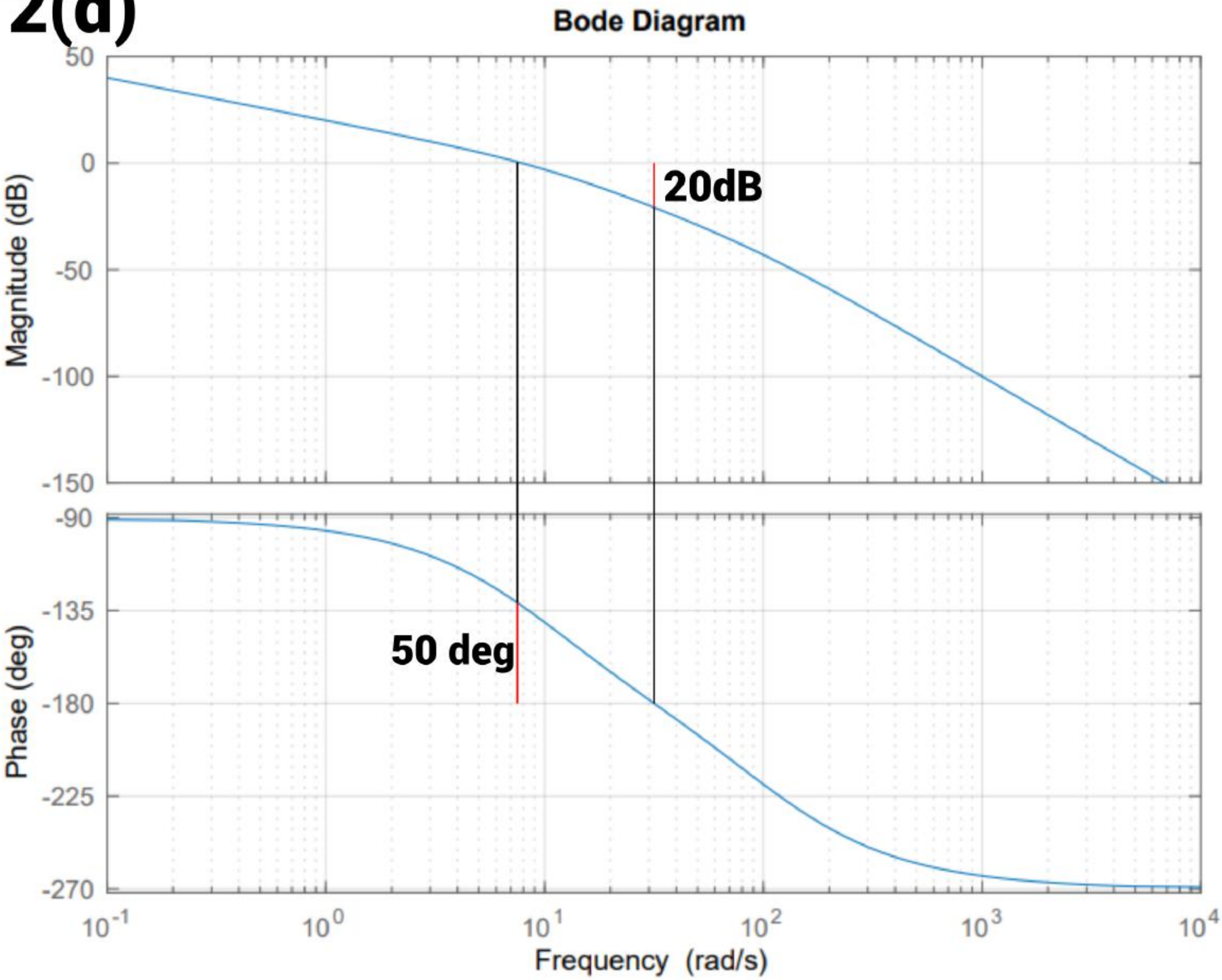


$\Rightarrow 52dB \Rightarrow \cancel{18}\ 15$

$K_v = 10^{52/20} = 398.1$

error $= 1/K_v = \underline{2.51 \times 10^{-3}}$

2(d)



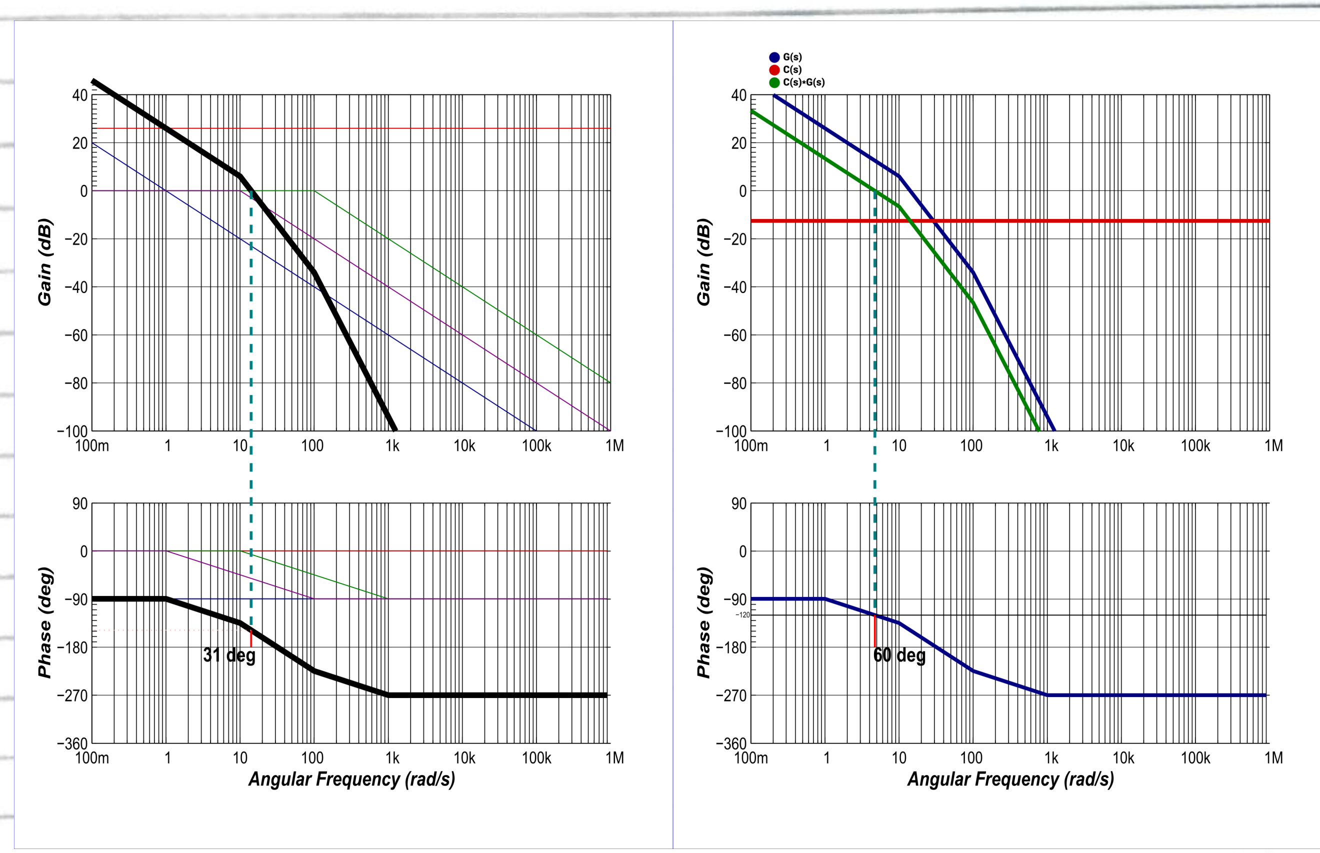
For $\zeta < 0.65$ we can use the approximation $\phi_{\mathrm{m}} = 100\zeta$.

Therefore, for a phase margin of ~50, the damping factor is ~0.5

2e) $G(s) = \dfrac{20000}{s(s+100)(s+10)} = 20 \cdot \dfrac{1}{s} \cdot \dfrac{1}{\left(\frac{s}{100}+1\right)} \cdot \dfrac{1}{\left(\frac{s}{10}+1\right)}$

$20 \cdot \log(20) = \underline{26.02\ dB}$

Given that ϕ_m is the only requirement a $K_p < 1$



$\approx -12\ dB \Rightarrow K_p = 0.25$