ENGR 222
Assignment 6 Submission

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Long Questions

2. Suppose S is the subspace in $\mathbb{R}^4$ is spanned by $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$.

   Find the point P closest to $\begin{bmatrix}1\\3\\8\\2\end{bmatrix}$ (i.e. orthogonal projection).

Fo$A = \begin{bmatrix}1&1&0\\1&1&0\\1&0&1\\1&0&1\end{bmatrix}$ and $v = \begin{bmatrix}1\\3\\8\\2\end{bmatrix}, x = A^{+}\cdot v$

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A = Matrix([[1,1,0],[1,1,0],[1,0,1],[1,0,1]])
v = Matrix([1,3,8,2])
x = A.pinv()*v
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$$x = \begin{bmatrix}\frac{7}{3}\\-\frac{1}{3}\\\frac{8}{3}\end{bmatrix},\ Ax = \begin{bmatrix}2\\2\\5\\5\end{bmatrix}$$

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ is the linear transformation whose matrix is $A = \begin{bmatrix}7&1\\0&0\\5&5\end{bmatrix}$.

   The image of the circle of radius 1 with centre at $(0,0)$ under T is an eclipse with the centre at $(0,0,0)$.

   Find the points on this ellipse farthest from $(0,0,0)$ and the points closest to $(0,0,0)$.

$$U = \begin{bmatrix}\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}&0\\0&0&1\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\end{bmatrix},\ D = \begin{bmatrix}3\sqrt{10}&0\\0&\sqrt{10}\\0&0\end{bmatrix},\ V = \begin{bmatrix}\frac{2}{\sqrt{5}}&-\frac{1}{\sqrt{5}}\\\frac{1}{\sqrt{5}}&\frac{2}{\sqrt{5}}\end{bmatrix}$$

$$\text{Farthest point(s) to } (0,0,0) \to \pm 3\sqrt{10}U_1 = \begin{bmatrix}3\sqrt{5}\\0\\3\sqrt{5}\end{bmatrix}$$

$$\text{Closest point(s) to } (0,0,0) \to \pm\sqrt{10}U_2 = \begin{bmatrix}-\sqrt{5}\\0\\\sqrt{5}\end{bmatrix}$$

Blackboard Questions Resubmit

1. Least Squares:

$$A = \begin{bmatrix}-1&2\\2&-3\\-1&3\end{bmatrix},\ v = \begin{bmatrix}4\\1\\2\end{bmatrix}$$

$$via\ pinv(A) * v, x = \begin{bmatrix}3\\2\end{bmatrix},\ \therefore ans = 6$$

2. Find Distances:

$$A = \begin{bmatrix}1&1&0\\1&1&0\\1&0&1\\1&0&1\end{bmatrix},\ v = \begin{bmatrix}3\\2\\1\\1\end{bmatrix}$$

$$\text{closest point} = Ax = A\begin{bmatrix}\frac{7}{6}\\\frac{4}{3}\\-\frac{1}{6}\end{bmatrix} = \begin{bmatrix}\frac{5}{2}\\\frac{5}{2}\\1\\1\end{bmatrix}$$

$$\text{distance} = ||v - (A * x)|| = \left|\left|\begin{bmatrix}\frac{1}{2}\\-\frac{1}{2}\\0\\0\end{bmatrix}\right|\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2} \approx 0.71$$

3. Approximate Line:

$$v : \begin{bmatrix}4\\2\\1\\-1\end{bmatrix} = A : \begin{bmatrix}2&1\\3&1\\5&1\\8&-1\end{bmatrix} x : \begin{bmatrix}m\\a\end{bmatrix}$$

$$x = \begin{bmatrix}-\frac{16}{21}\\\frac{69}{14}\end{bmatrix},\ m = -0.761905$$
