TYPES OF GEARS

SPUR GEARS: Teeth parallel to the axis of rotation. Simplest type (and the type we'll mostly be studying)

HELICAL GEARS: Teeth inclined to axis of rotation: gradual engagement of teeth means they're quieter. Thrust loads develop.

BEVEL GEARS: Teeth on a conical surface. For transmitting power between perpendicular/ intersecting shafts

WORM GEARS: A screw-like worm's direction of rotation dictates the direction of rotation of a pinion / worm-wheel. Locking

Spur Gears

- Spur gears are used to transmit motion between parallel shafts
- They are usually cylindrical (flat); the teeth project radially.

Spur Gears

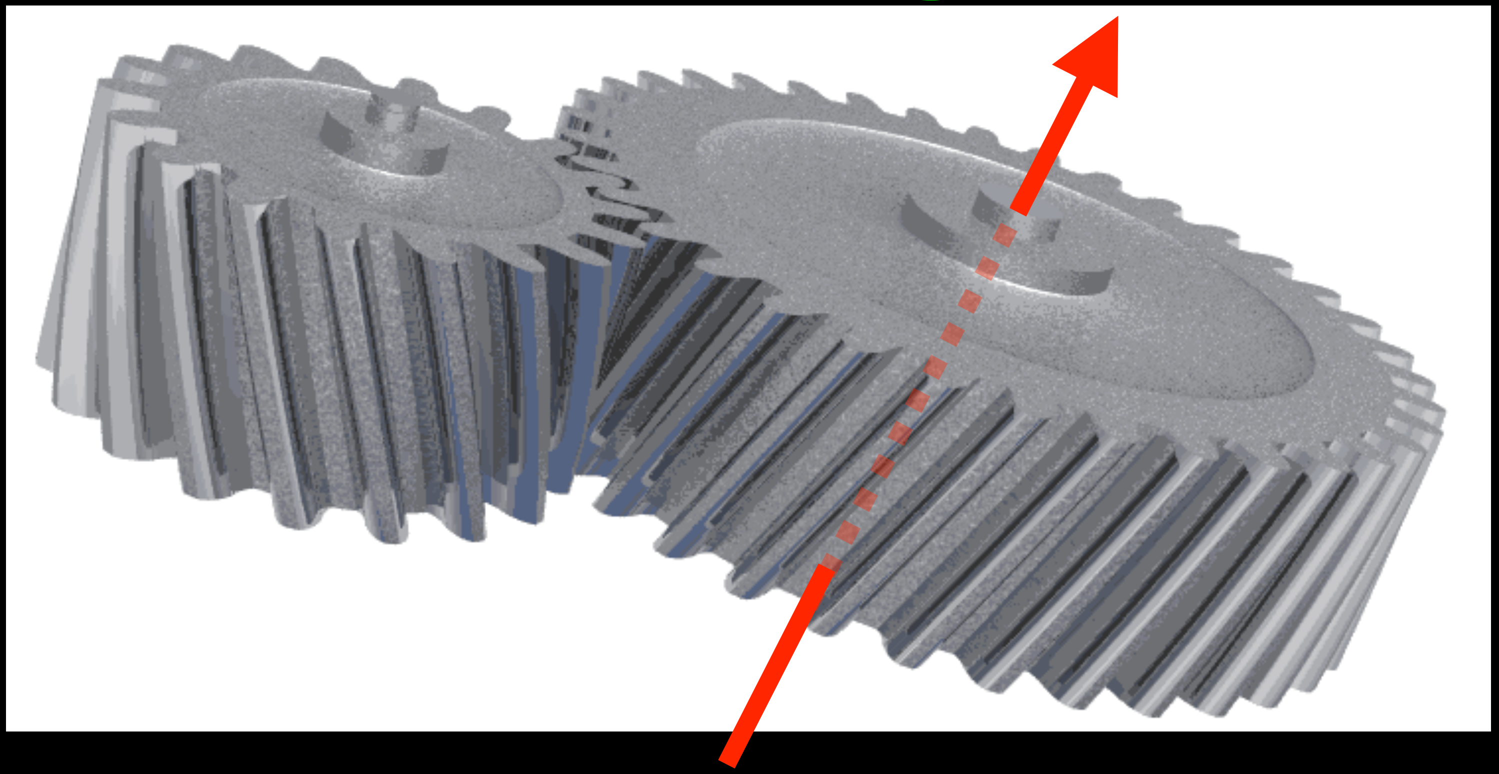
Spur gear with keyway and grub screw.

- Spur gears are prone to noise.
- If they aren't manufactured with high precision, gear meshing can be rough.

Helical gears

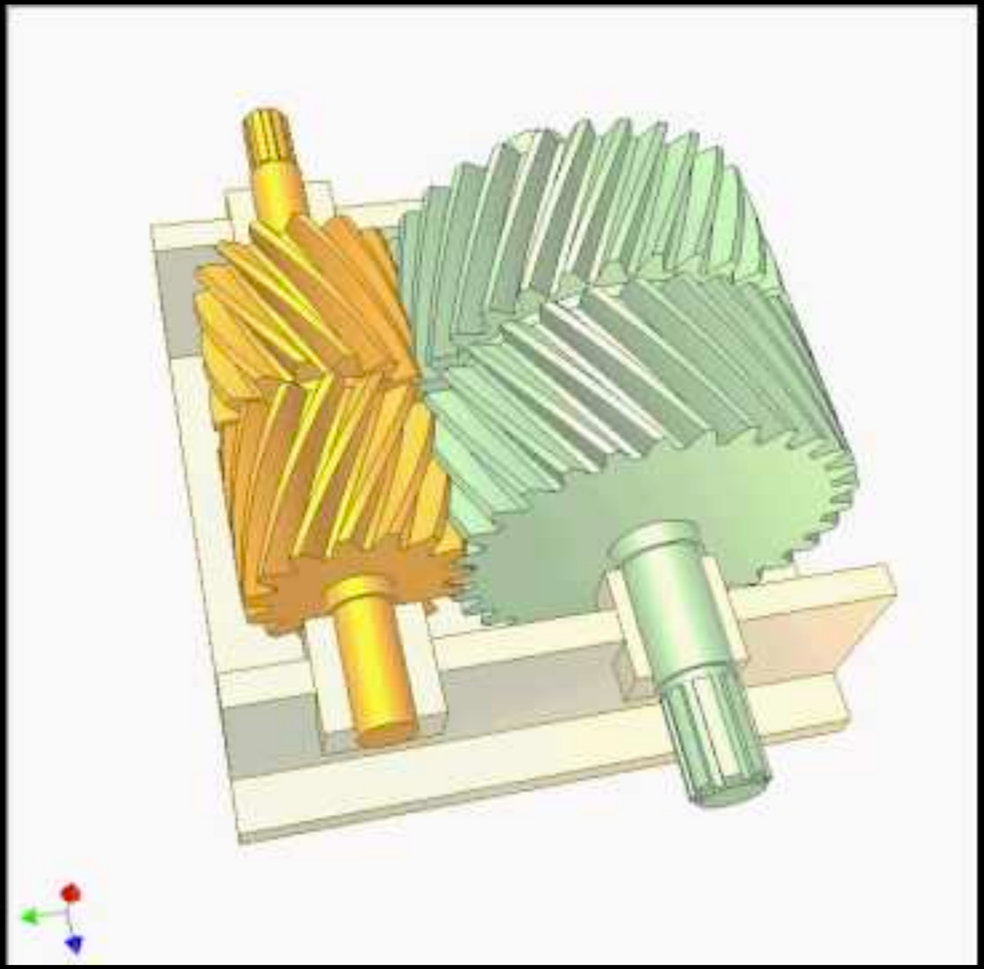
- Helical gears are an improvement on spur gears.
- The teeth are cut at an angle, allowing for a more gradual (and smoother) meshing between gears.
  - This allows for greater speed and reduced noise.
  - Also good for high torque applications: more tooth area in contact.

Helical gears



- Somewhat more difficult to manufacture than spur gears.
- When turning, tooth angle results in a thrust force along the axis of the gear.
  - Must be compensated for with an adequate thrust bearing.

Double helical gears

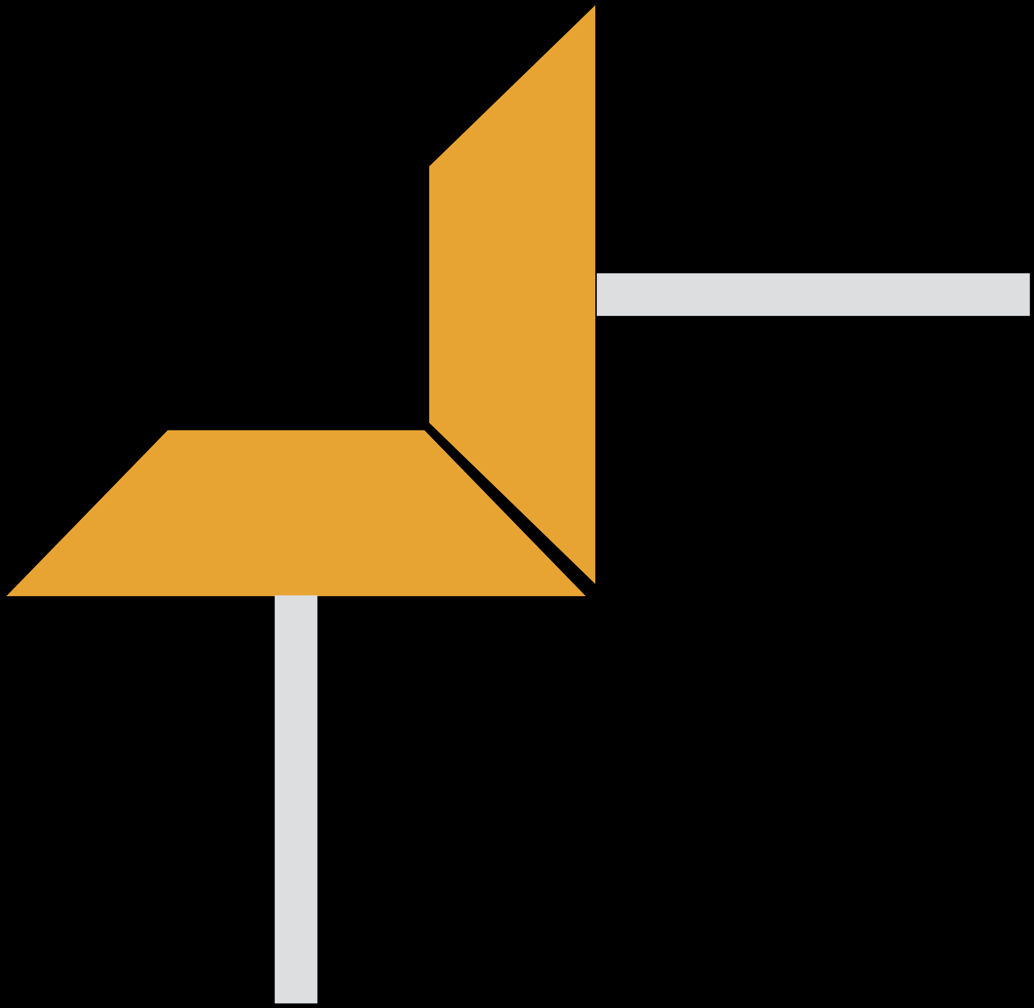


- Also known as herringbone gears.
- The teeth are ‘V’ shaped (a reflected helical gear).
  - These angles create opposing thrust forces, canceling each other out.

High speed & right angles with Helical gears

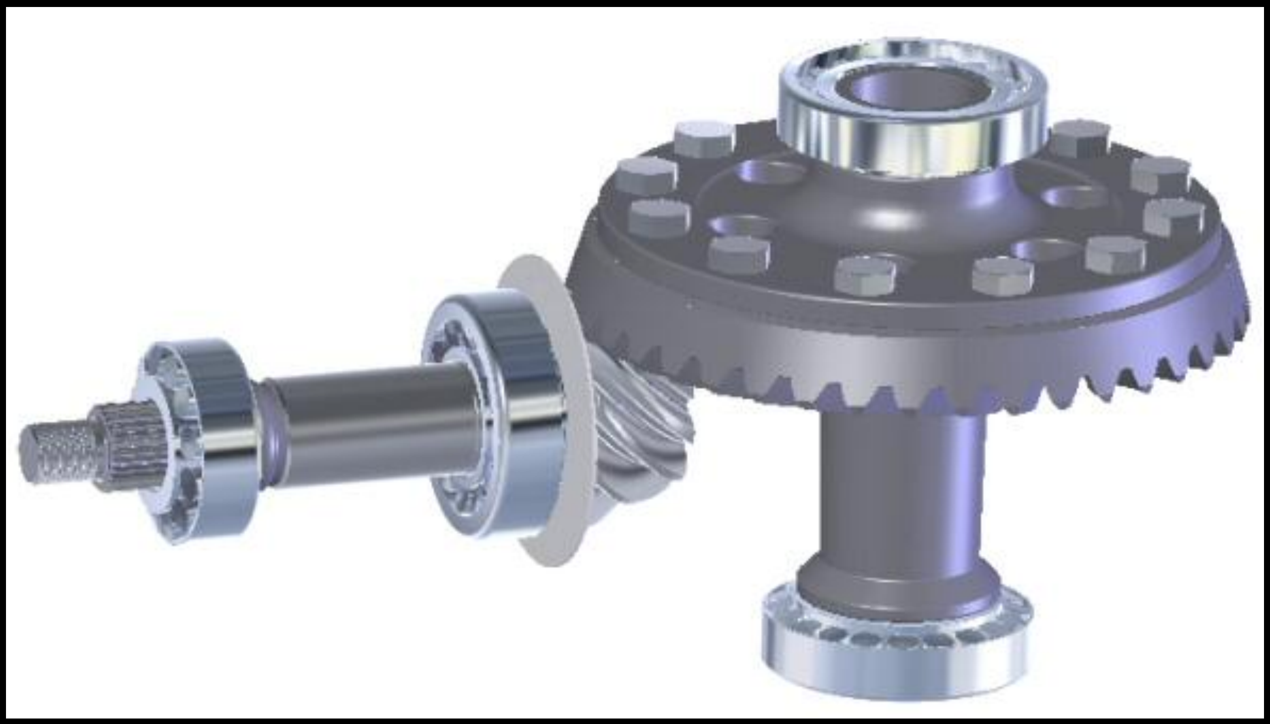
- With correct angles of gear teeth, helical gears can be used to adjust the rotation angle by 90 degrees.
- Helical gears: popular in high speed applications.
  - High speed: pitch line velocity > 1500 m/min; rotational speed of pinion > 3600 rpm

Bevel gears



- Bevel gears allow for changing the rotation direction between shafts.
  - 90 degrees is common, though other angles are available.

Helical/Spiral Bevel gears



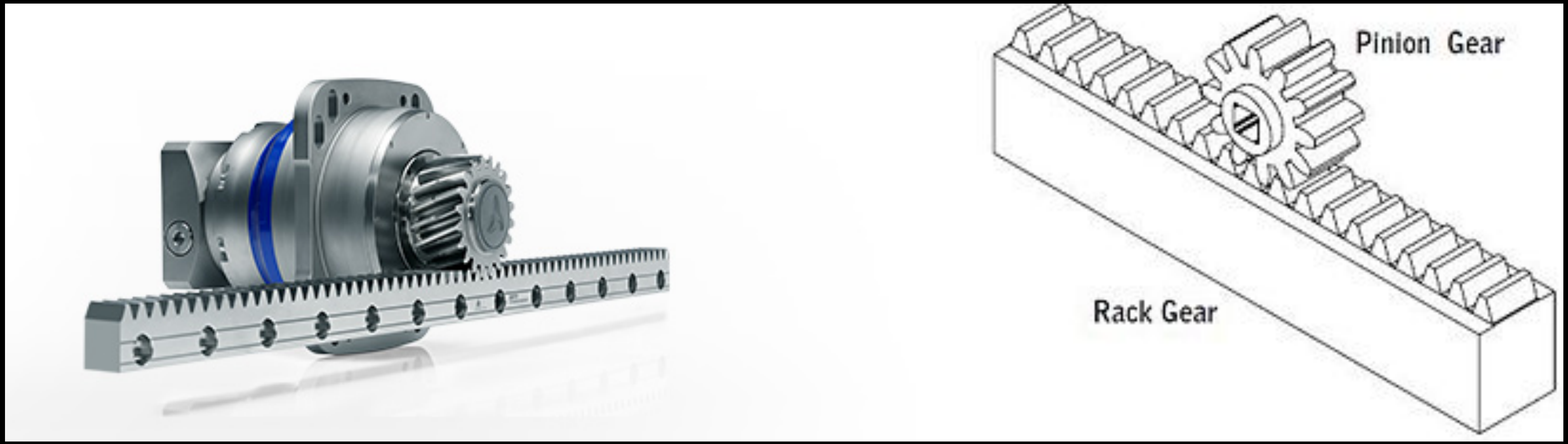
- For high speed applications, spiral (helical) bevel gears are used.
- Display “handedness” (Right hand = clockwise inclination of tooth from axis through tooth midpoint)
  - High-torque, high-speed applications (e.g., turboshafts on helicopters)

Worm gears



- Worm gears also allow for right angle power transmission between shafts.
  - The worm is a screw-like device with helical threads.
- Very compact size allows for high gear ratios.
- Worm gear can turn the spur gear.
  - Spur gear can't (easily) turn the worm gear.
  - This allows spur shaft to be locked into place unless worm is being driven.
- Advantages: Low noise, high power transmission
- Disadvantages: Increased friction; requires higher starting torque.

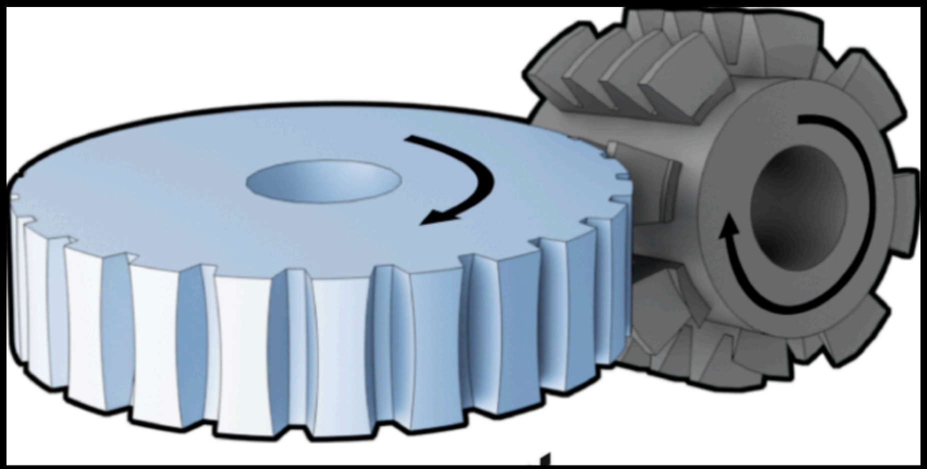
Rack and Pinion Gears



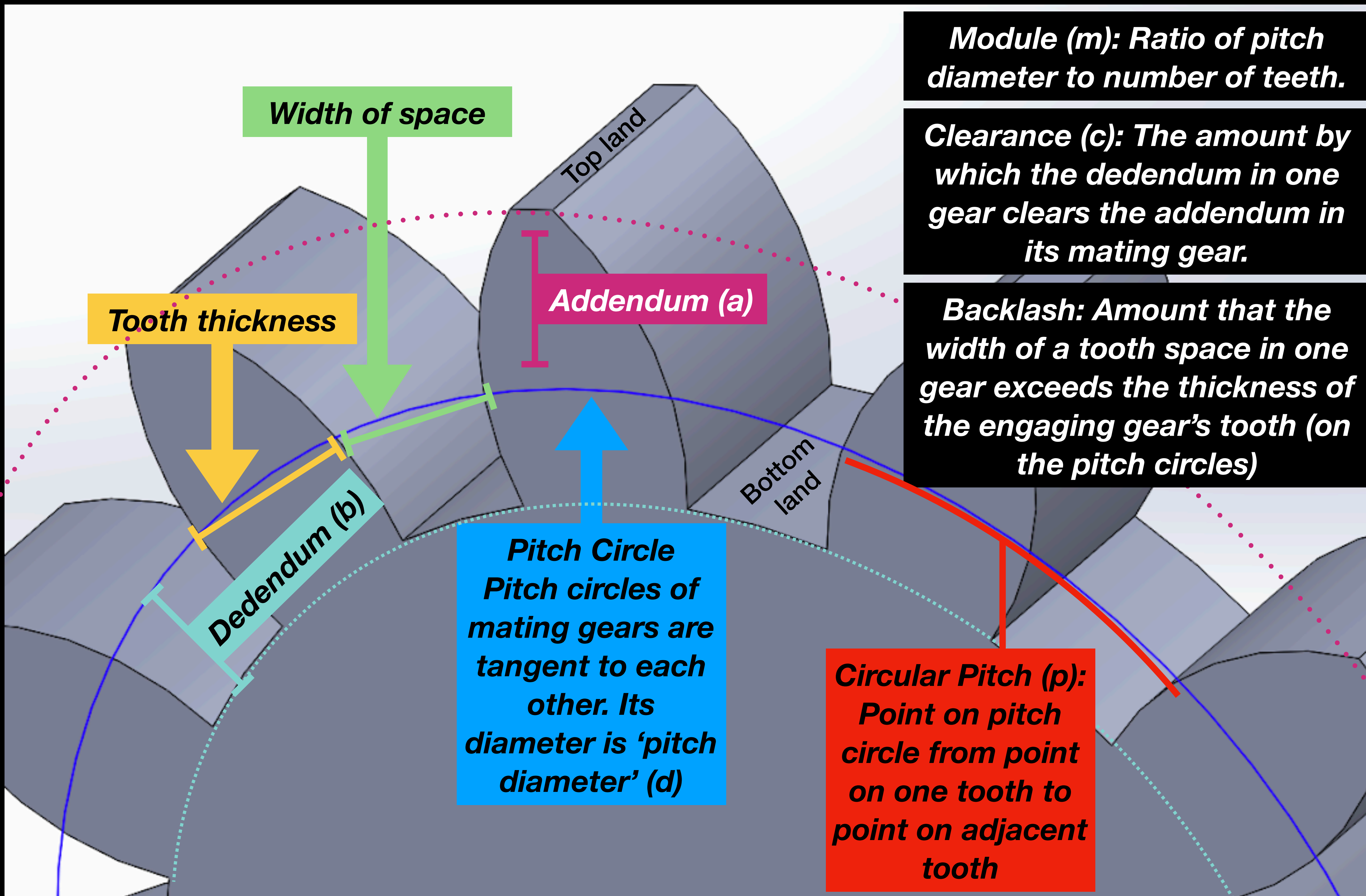
- One way of converting rotational motion into linear motion
- The circular pinion engages the teeth on the flat bar - the rack.
  - The rack is an 'unrolled' spur gear.
- Found in the steering mechanism of many cars.
- Must have some form of mechanical limit built in, to prevent rack from slipping past pinion.

Gear Manufacture

- Hobbing:
  - Fast to manufacture (multiple teeth formed at once). Very accurate profiles.
  - Can not produce ‘exotic’ gear tooth shapes
- Milling:
  - Dedicated cutter is used, teeth are cut one at a time.
  - Cheap tooling, relatively slow.
- Forming:
  - Metal is ‘pressed’ (moved away through plastic deformation)
    - Cheap but inaccurate - might require post-processing machining steps (e.g., turning to diameter)



GEAR NOMENCLATURE



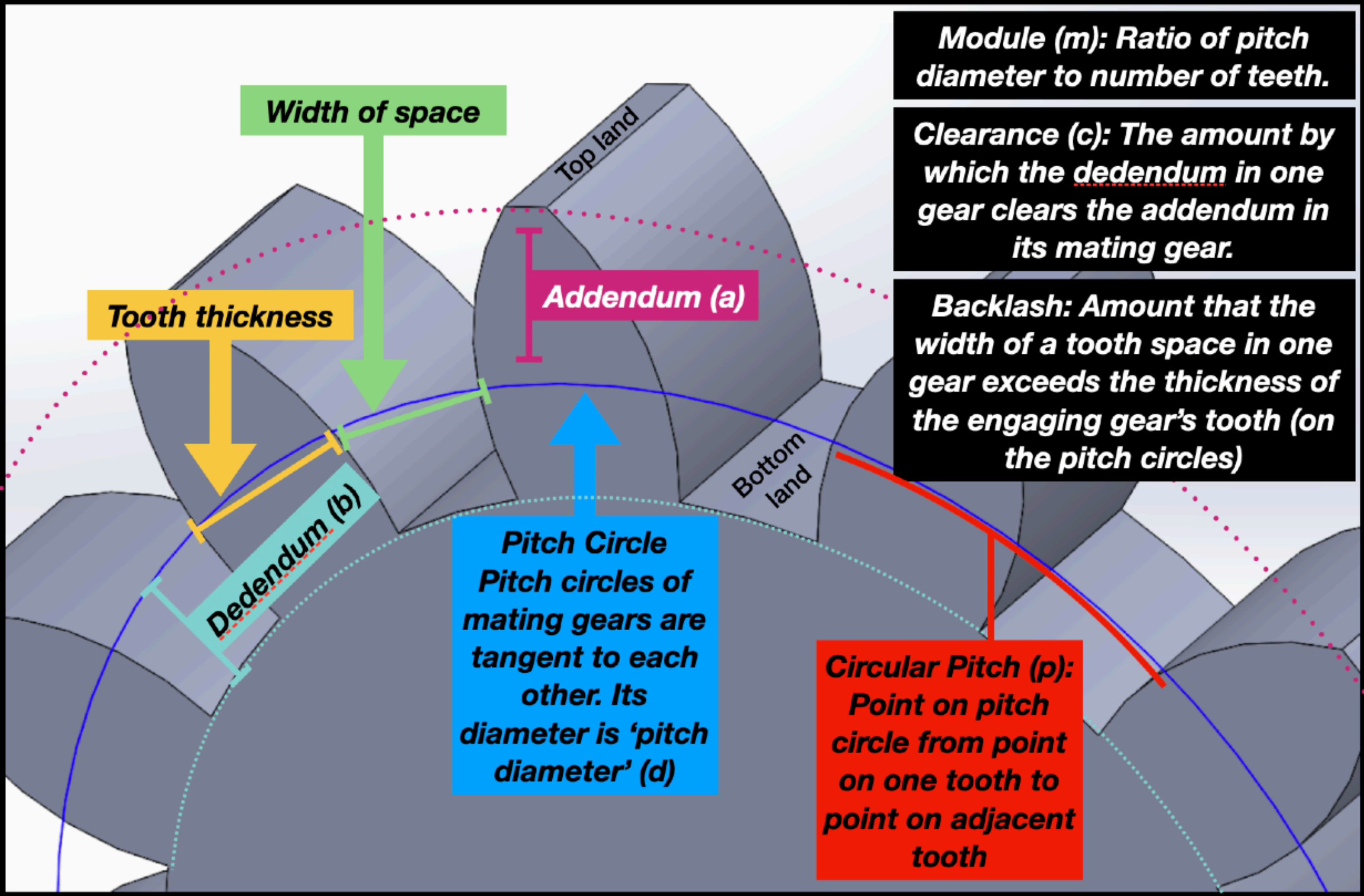
BASIC GEAR RELATIONS

$$m = \frac{d}{N}$$

A bigger module: larger teeth.

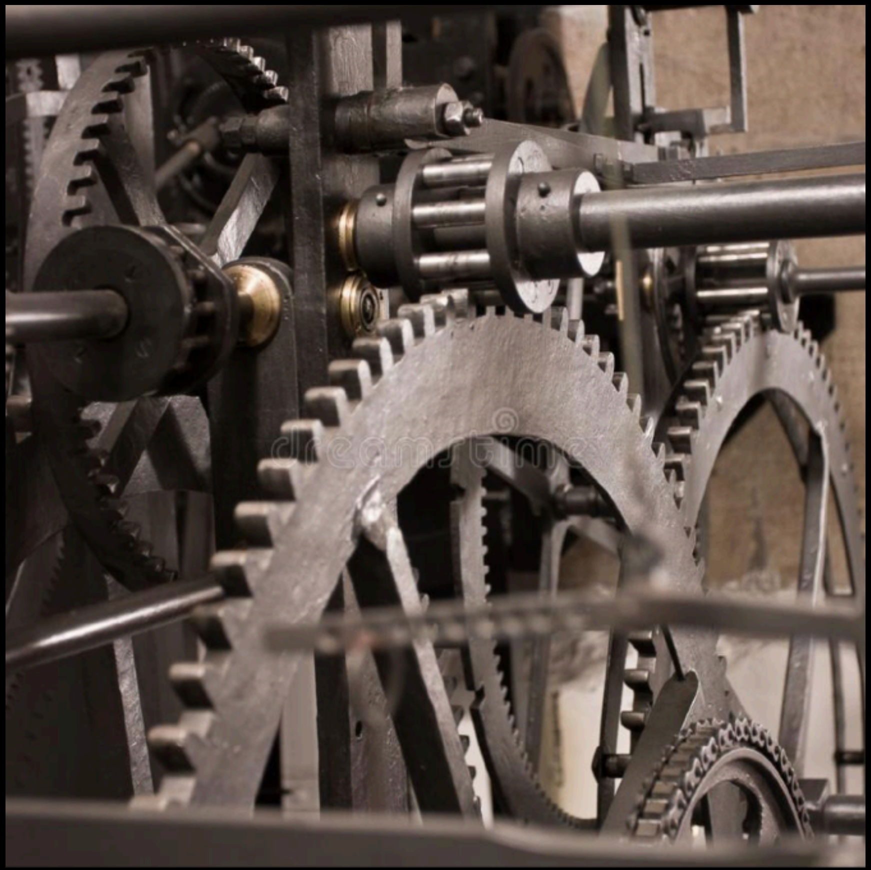
$$p = \frac{\pi d}{N} = \pi m$$

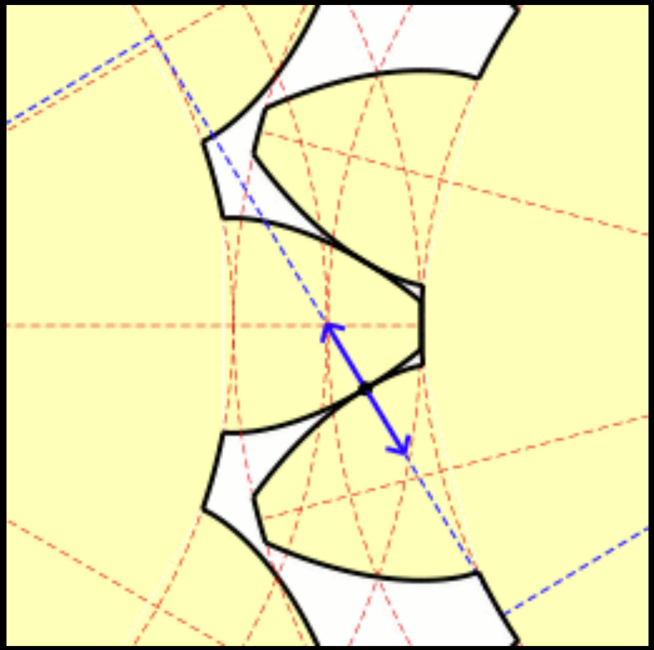
N = number of teeth



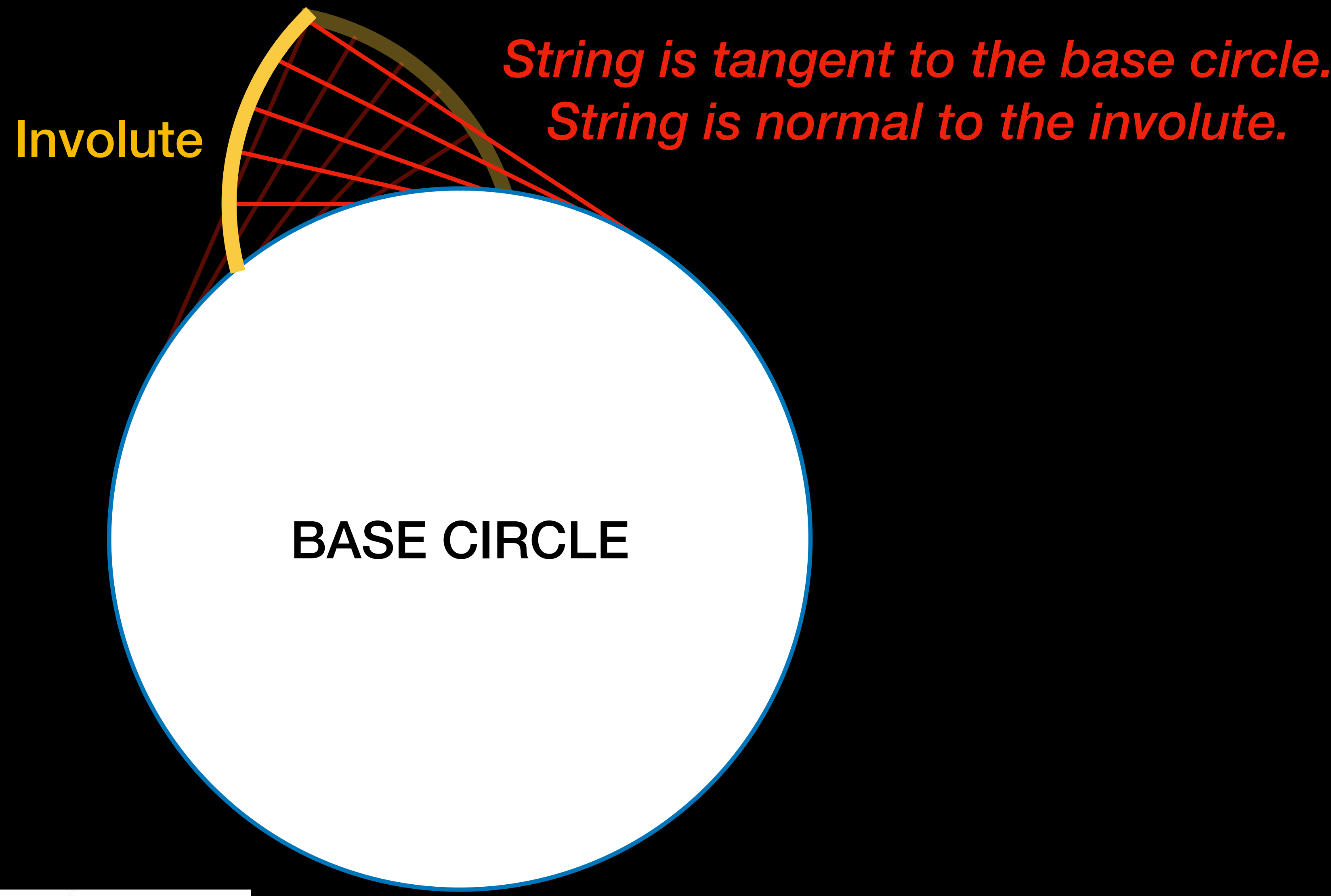
INVOLUTE PROFILE

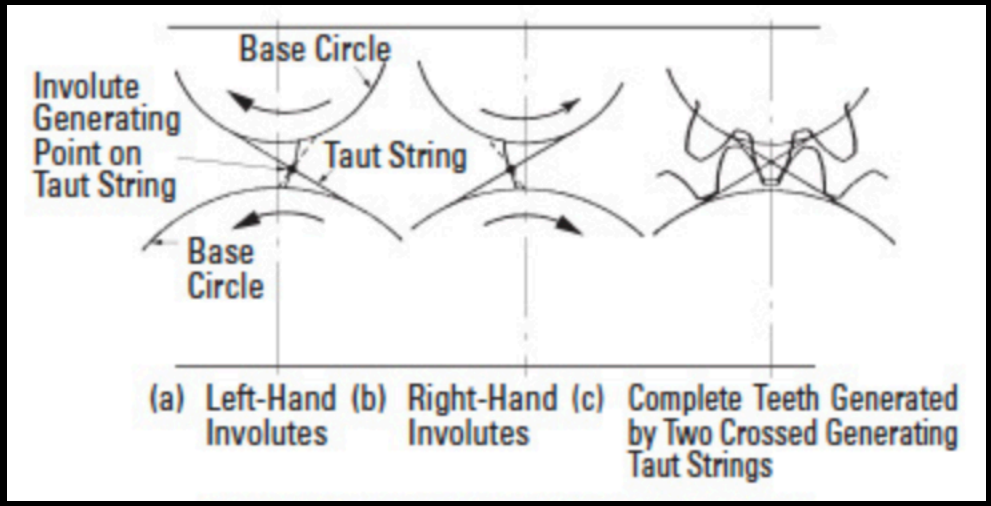
- While many gear profiles can be designed that mesh, they often don't allow "conjugate action"
  - Conjugate action: Constant angular velocity ratio during meshing.
  - This is essential in high-speed, high-load applications.
- The most common gear tooth profile that allows conjugate action is the involute profile.
  - A guide to drawing involute profiles (using Unwin's Construction approximation) is on Blackboard
  - *(SolidWorks toolbox gears: approximate involute profiles)*
- Involute: smooth meshing, resistant to errors in gear centre-distance spacing



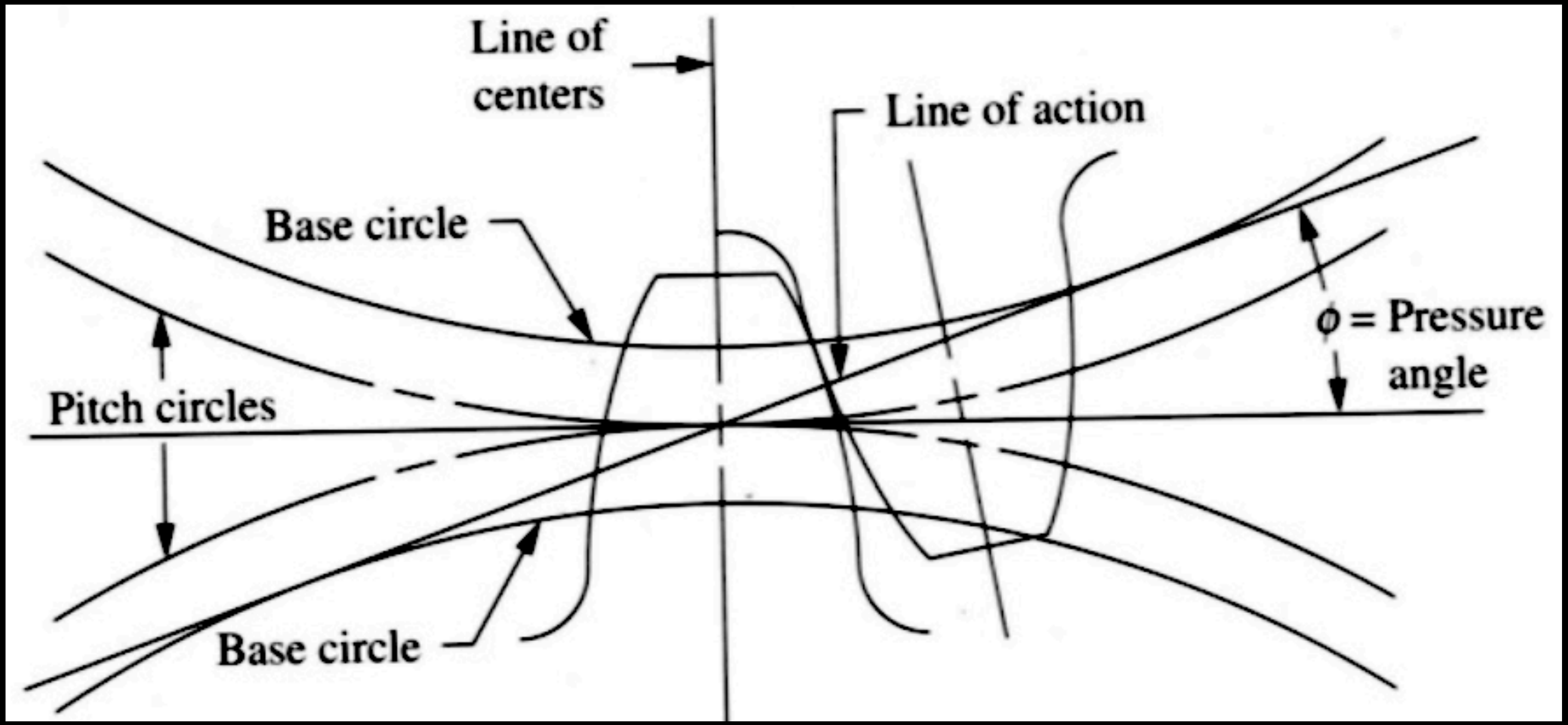


INVOLUTE PROFILE





INVOLUTE NOMENCLATURE



LINE OF CENTRES: The line connecting two meshing gears' centres.

BASE CIRCLE: Circle from which the involute profile is formed (e.g., by unwinding a taut string from it)

POINT OF CONTACT: In an involute gear, a single (sliding) point of contact exists between both gears.

LINE OF ACTION: A line tangent to both base circles that follows the point of contact

PRESSURE ANGLE: The angle formed between a line normal to the line connecting gear centres and the line of action

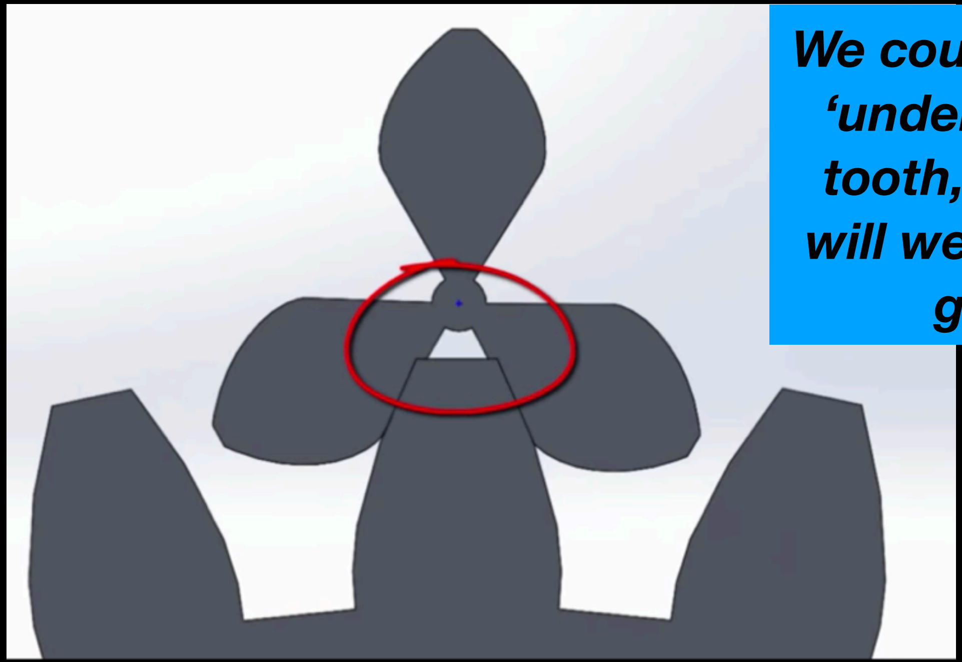
GEAR STANDARDS

- Module is the ratio of pitch circle to number of teeth.
  - In order to mesh, two gears must have the same module.
- Involute gears with different pressure angles won’t correctly mesh.
- Since modules and pressure angles must be shared between mating gears , there are some commonly used standards, called ‘tooth systems.’
  - Standard modules for spur gears: 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
  - A pressure angle of 20° is typical for modern spur gears. Less commonly, pressure angles of 22.5° and 25° are used.
  - Historically, a 14.5° pressure angle was common; now considered obsolete.

$$m = \frac{d}{N}$$

INTERFERENCE

- With involute gears, if the gear's base circle is too small, a mating gear's teeth may intrude with it.
  - Both gears' base circles need to be large enough to allow the mating gears' teeth to move with conjugate action.
  - The practical upshot is that, for a given gear size, there is a minimum number of teeth below which there will be interference.
  - It can be important to know this minimum number, in order to design the most compact gearboxes possible.



We could further 'undercut' the tooth, but this will weaken the gear.

https://www.youtube.com/watch?v=s1V9-RFuf0I

For a spur gear with a one-to-one ratio (equal teeth on both gears), the smallest number of teeth w/out interference N_p:

$$N_p = \frac{2}{3sin^2\phi}(1 + \sqrt{1 + 3sin^2\phi})$$

Where ϕ is the pressure angle *(Often 20º)*

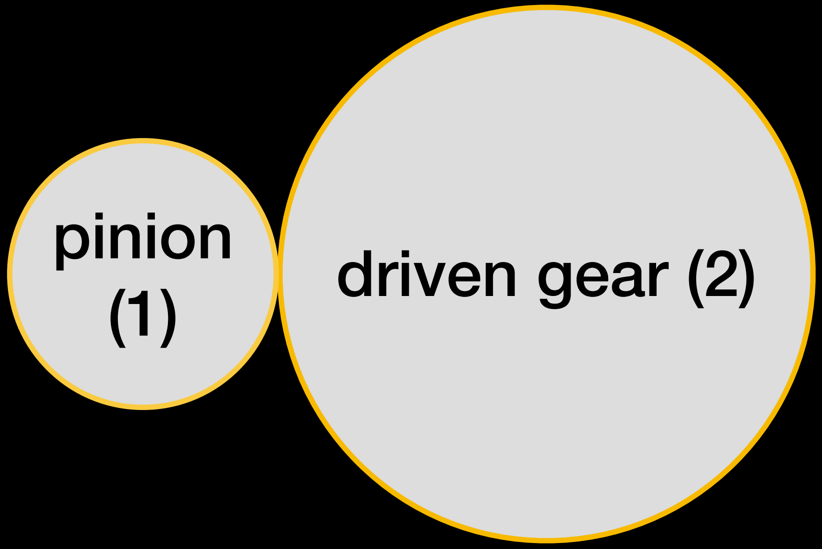
If a gear has more teeth than its pinion, the smallest number of teeth w/out interference N_p:

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

Where ϕ is the pressure angle and m is the the ratio of gear teeth over pinion teeth (N_G/N_P)

GEAR TRAINS I

Given a pinion (1) driving a gear (2): the speed of the driven gear (n_2, in rpm) can be determined if: the speed of the pinion (n_1) and the numbers of teeth (N1 and N2) or the pitch diameters (d1 and d2) are known.

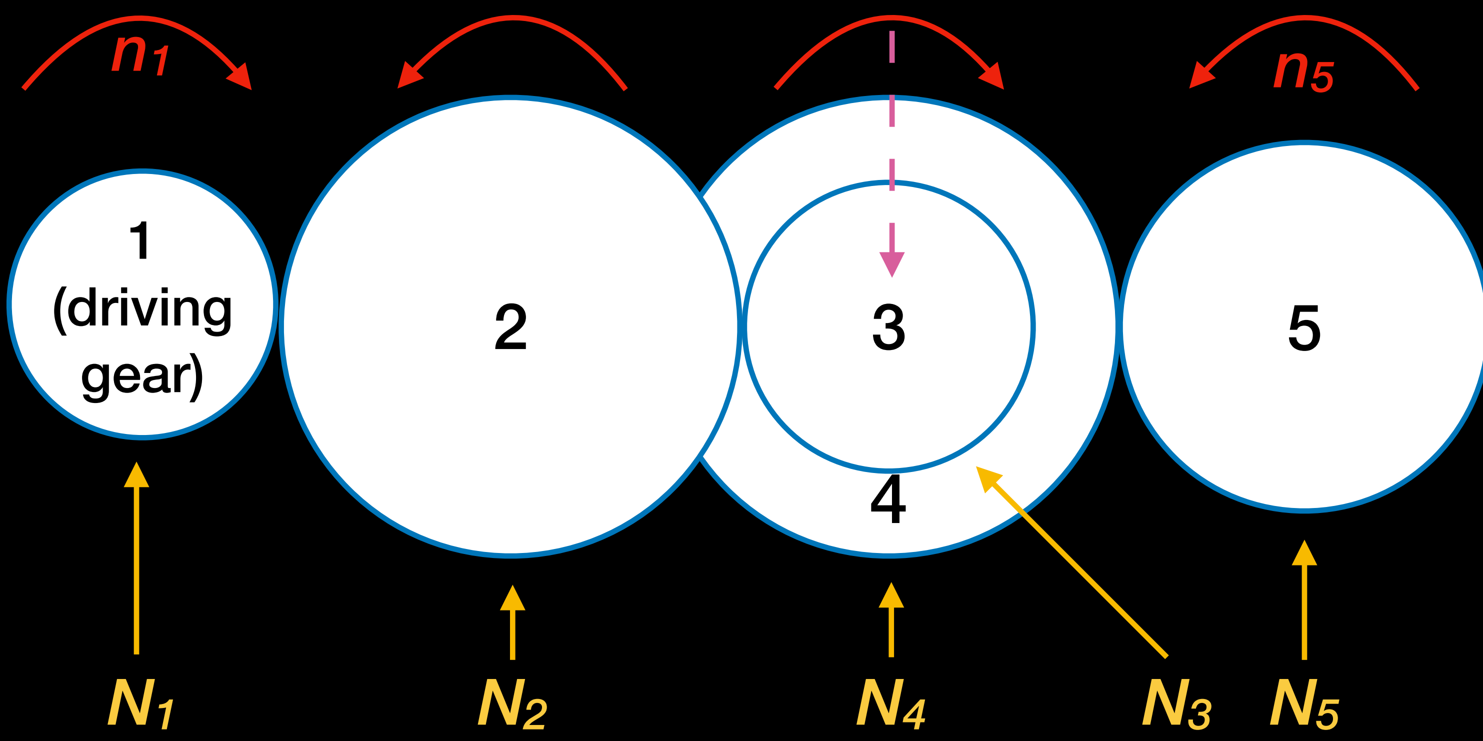


$$n_2 = \left| \frac{N_1}{N_2} n_1 \right| = \left| \frac{D_1}{D_2} n_1 \right|$$

This equation works for any gear type: spur, helical, bevel, etc.

Absolute value signs allow for any rotation direction to be used.

GEAR TRAINS II



Without this 'double reduction gear' (both fixed to axle), the interim gear 2 & 4 will act only as idlers.

If we know the speed of the driving gear (n_1), then what is the speed of gear 5 (n_5)?

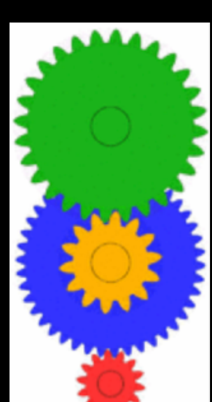
Driver gears

Driven gears

$$\frac{N_1}{N_2} \qquad \frac{N_2}{N_3} \qquad \frac{N_4}{N_5}$$

$$n_5 = -\frac{N_1}{N_2}\frac{N_2}{N_3}\frac{N_4}{N_5}n_1$$

Negative: direction of rotation is opposite to input



Double reduction gear

TRAIN VALUE

Driver gears N_1 N_2 N_4

Driven gears N_2 N_3 N_5

$$n_5 = -\frac{N_1}{N_2}\frac{N_2}{N_3}\frac{N_4}{N_5}n_1$$

We can abstract the ratio of the products of driving tooth numbers (or pitch diameters) to driven tooth numbers (or pitch diameters) as *e*, the train value

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

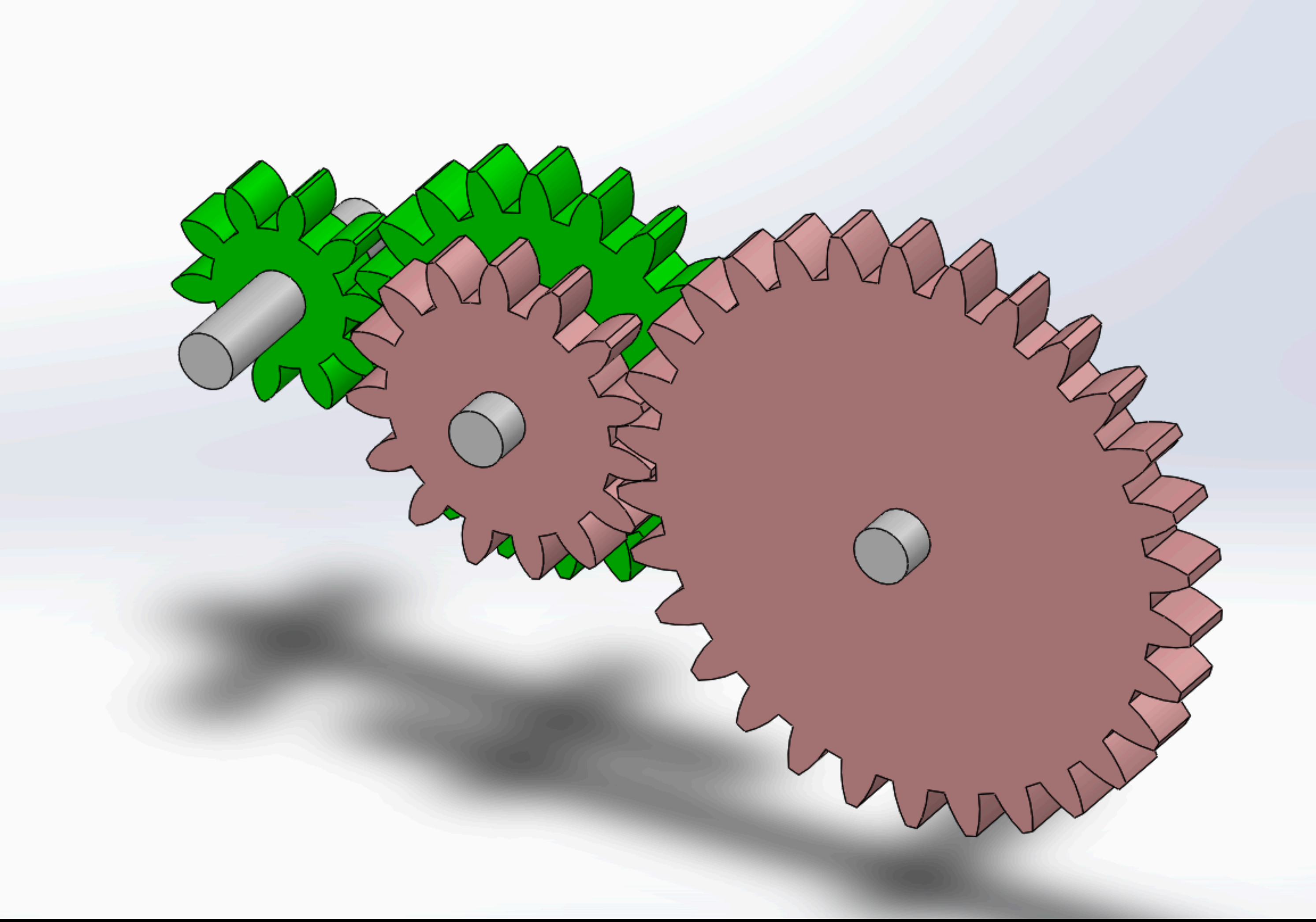
Pitch diameters may be substituted for tooth numbers.

e **is positive if the last gear rotates in the same direction/sense as the first; it's negative (as in the example above) if it rotates in the opposite sense**

Speed of last gear in the train → $n_L = en_F$ ← **Speed of first gear in the train**

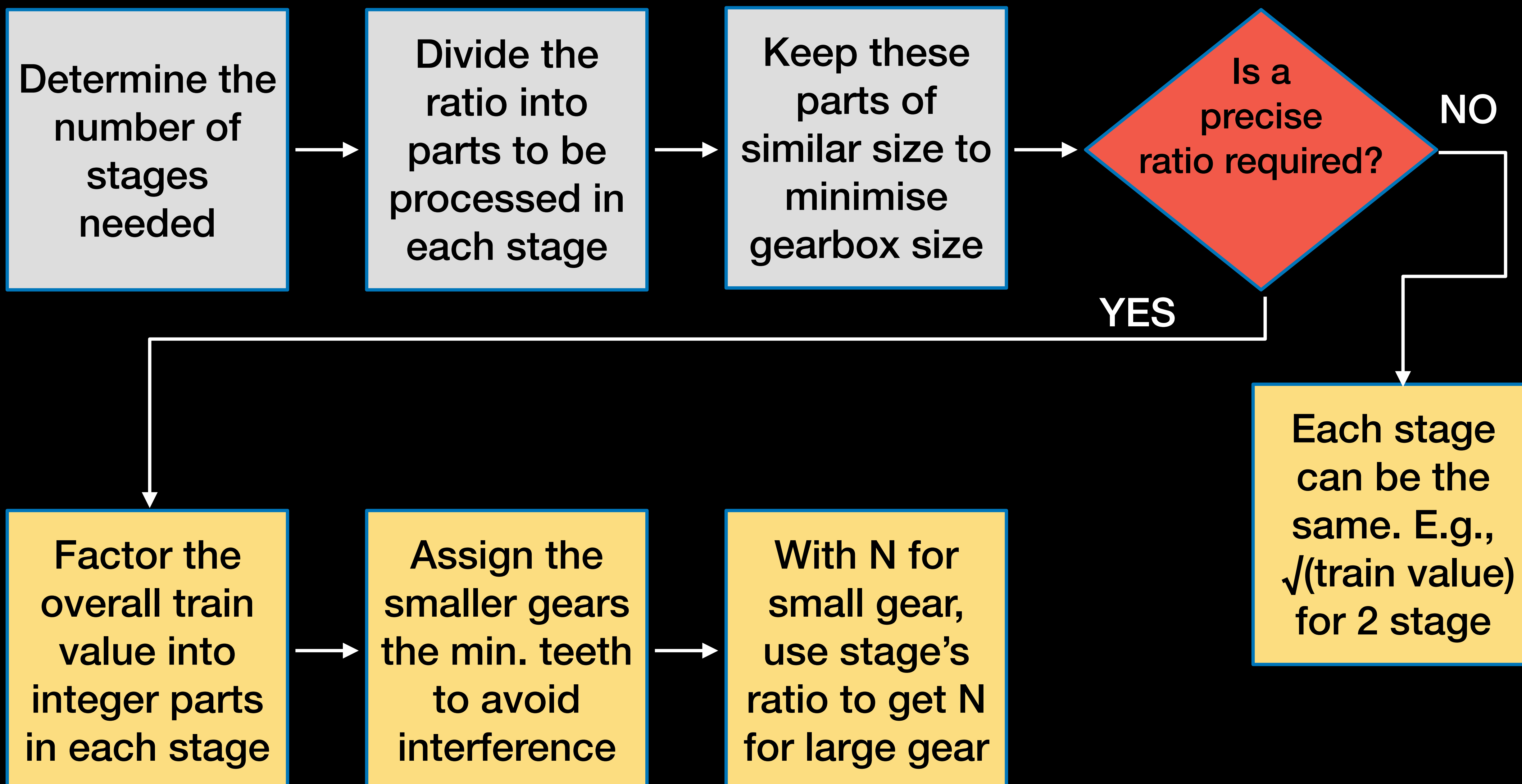
GEAR TRAIN DESIGN

- For gear train values of approx. 10 to 1, a single pair of gears (optionally with an idler to flip direction) can be used.
- For ratios up to about 100 to 1, a two stage compound gear train should be used.
  - This reduces size and stress (no need for a single massive gear).



GEAR TRAIN DESIGN

- Goal: Design a gear train to realise a particular train value.



GEAR TRAIN PROBLEM I

- A client's mechanised crayfish pot hoist needs to rotate at 20 rpm (+/- 5%) in order to raise the cray pots at the desired speed. The client's existing gearmotor operates at 1 rpm and is too slow. You must create a gearbox to speed up the cray pot raising. The entire assembly will be mounted on the client's boat, so you must minimise the overall size of the gearbox. What tooth numbers (with ϕ of 20°) of the gears are needed to realise this?

Ratio (20:1) is more than 10:1 but less than 100:1 - a two-stage compound gear train will work.

Since an approximate final speed is okay, we can simply divide the compound train into two equal sections.

Since we're trying to reduce the gearbox size, we determine smallest that pinions can be to avoid interference

We have pinions with 17 teeth. The driving gears should have 17×4.472 = 76.02 ≈ 76 teeth

We have the driving teeth values (N = 76) and the driven teeth values (N = 17). From this we can find *e*

Amount of reduction per stage:

$$\sqrt{20} = 4.472$$

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

$$= 16.73 \approx 17$$

$$e = \left(\frac{76}{17}\right)\left(\frac{76}{17}\right) = 19.99$$

GEAR TRAIN PROBLEM II

- A gearbox must realise an exact 30:1 speed increase. This gearbox must have a minimal size, $\phi = 20^\circ$. What are the tooth numbers in the gearbox?

Ratio (30:1) is more than 10:1 but less than 100:1 - a two-stage compound gear train will work.	The gearbox's total ratio should be factored into two integer portions.	The driven gears (N_2 and N_4) are the smallest gears. These should be chosen to be as small as possible.	We now have N for the small gears. We can use this to set N for the larger gears.	We can then check the overall train value, verifying that it is exact.

$e = 30 = (6)(5)$
$N_1/N_2 = 6$
$N_3/N_4 = 5$

$$N_p = \frac{2}{(1+2m)sin^2\phi}(m + \sqrt{m^2 + (1+2m)sin^2\phi})$$

$$\approx 16 \text{ teeth}$$

$$N_1 = 6(16) = 96 \text{ teeth}$$

$$N_3 = 5(16) = 80 \text{ teeth}$$

$$e = \left(\frac{96}{16}\right)\left(\frac{80}{16}\right) = 30$$