ECEN 220
Lab Report 4
The Discrete Time Fourier Transform

Daniel Eisen
300447549

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1 DTFT

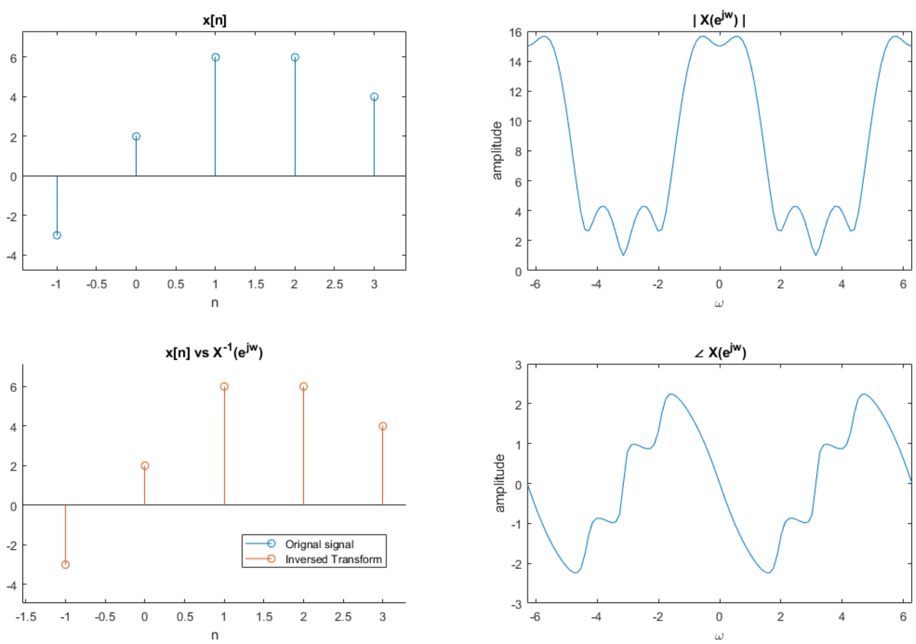
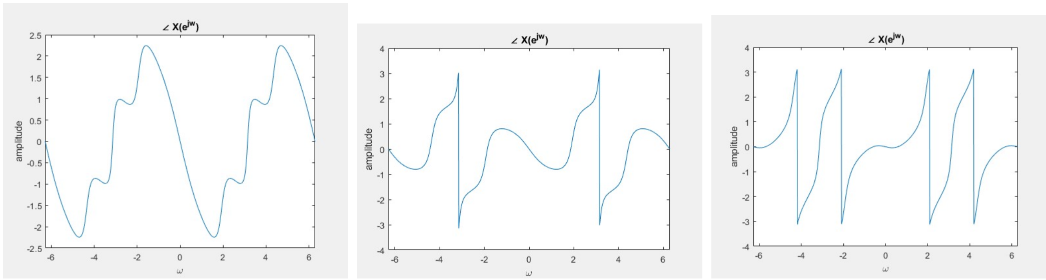


Figure 1: DTFT (and Inverse) of x[n]

1.1 q.1b

A reasonable amount of samples (M) was anything over 100, even then some resolution was lost in the transform. Down at ten all but the largest features of frequency were lost. $X(e^{jw})$ is complex and thus magnitude and phase have been plotted separately (RHS of Fig. 1).

1.2 q.1c



(a) shift by 1 (b) shift by 2 (c) shift by 3

Figure 2: x[n] shifted by different integer values

As x[n] has been shifted in 'n' the transforms gets corresponding phase change.

$$x[n - N_0] \Rightarrow e^{-j\omega N_0} X(j\omega)$$

As Figure 2 shows as x is shifted at different N_0 the phase of $X(j\omega)$ changes correspondingly.

2 Inverse DTFT

2.1 q.2b

As seen Fig 1 shows (subplot 3), the result of the inverse DTFT overlaid with the original sequence shows no difference.
The resulting sequence does have complex components but these are not significant as they are all real part + 0j. This is probably due to the transform vector being of complex doubles and MATLAB's matrix operation hence result in a complex vector sequence.

3 Filtering

3.1 q.3a

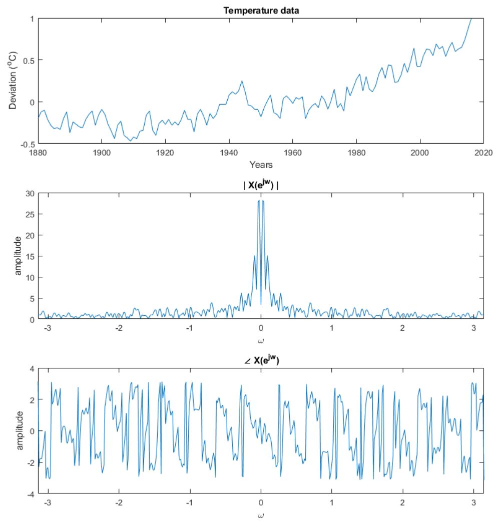


Figure 3: Imported Temperature Data and Transform

Figure 3 above shows show the plotted Temperature data and the Mag/Phase plots of this transform. As shown the data has highly contributing low frequency and lots of smaller contributing high frequency oscillations.

3.2 q.3b

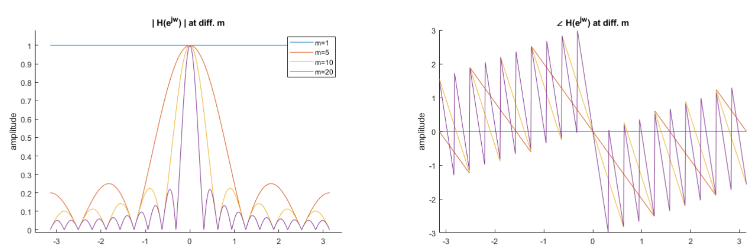


Figure 4: $H(e^{j\omega}$, moving-average filter at diff. m)

Fig. 4 Above is the plotted Mag/Phase of the moving average filter implementation. As seen the increasing m value narrows the high amplitude bandwidth in ω.

3.3 q.3c

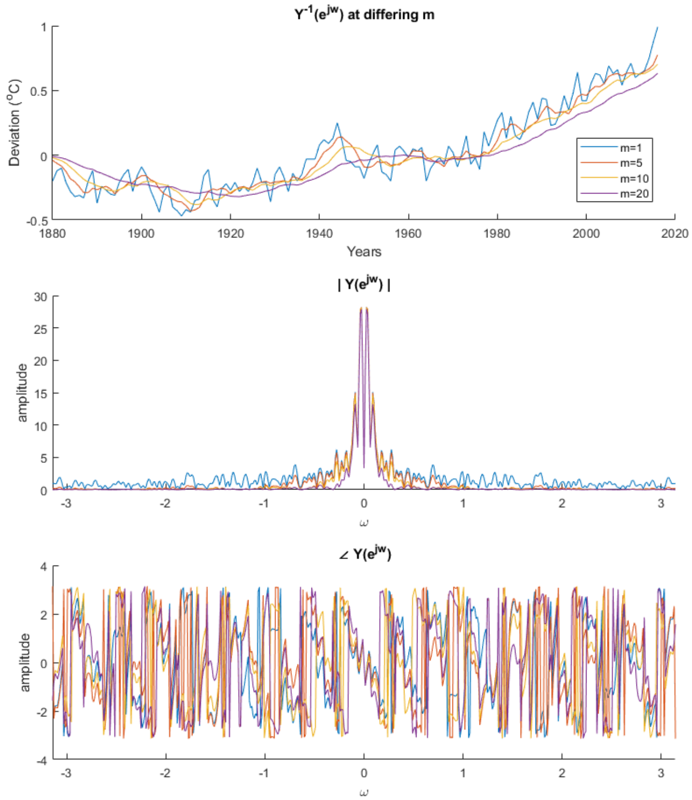


Figure 5: Filter applied to data signal at diff. m

The moving-average acts as a low pass filter with its cutoff determined by m. As Fig. 5 shows m=1 is an unmodified signal (see Fig 4, m=1 $|H(e^{j\omega})| = 1$, $\angle H(e^{j\omega}) = 0$) and as m increases the filtered signal is low pass filtered to a greater and greater extent. As previously shown $H(e^{j\omega})$ "sharpens" around 0 as m increases and this increases the strength of the low pass filtering.

Appendix

DTFT

```
function [X] = dtft(x,n, omega)
V = exp(-1j*omega*n');
X=V*x;
end
```

Inverse DTFT

```
function [x] = invdtft(X,n,omega)
V = exp(-1j*omega*n');
x = V\X;
end
```

Q1 / Q2

```
clc
clear
%% 1a
M = 1000;
omega0=-2*pi;
omegaM=2*pi;
k=(0:M)';
omega=omega0+(omegaM-omega0)*k/M;
%% 1b
sig = subplot(2,2,1);
x = [-3,2,6,6,4]';
n = [0:length(x)-1]'-3;
stem(n,x);
title("x[n]");
xlabel("n");

X = dtft(x,n,omega);
mag = subplot(2,2,2);
plot(omega, abs(X));
xlim([-2*pi 2*pi])
title("| X(e^{jw}) |");
xlabel("\omega")
ylabel("amplitude")

phs = subplot(2,2,4);
plot(omega, angle(X));
xlim([-2*pi 2*pi])
title("\angle X(e^{jw})");
xlabel("\omega")
ylabel("amplitude")
%% 2a
reconstr_sig = subplot(2,2,3);
x_inv = invdtft(X, n, omega);
hold on;
stem(n,x_inv);
stem(n,x);
hold off;
title("x[n] vs X^{-1}(e^{jw})");
xlabel("n");
legend("Orignal signal","Inversed Transform")
```

Q3

```
clc
clear

%% 3a
figure(1)
subplot(3,1,1);
data = importdata('Temperature.txt');
n = data(:,1);
x = data(:,2);
plot(n,x);
title("Temperature data")
xlabel("Years")
ylabel("Deviation (^{o}C)")

M = 500;
k=(0:M)';
omega=-pi+(pi--pi)*k/M;

X = dtft(x,n,omega);
subplot(3,1,2);
plot(omega, abs(X));
xlim([-pi pi])
title("| X(e^{jw}) |");
xlabel("\omega")
ylabel("amplitude")
subplot(3,1,3);
plot(omega, angle(X));
xlim([-pi pi])
title("\angle X(e^{jw})");
xlabel("\omega")
ylabel("amplitude")

%% 3b

Havs = [];
for m = [1,5,10,20]
    Hav = (1/m)*exp(-1j.*omega.*(m-1)/2).*(sin(omega.*m/2)./sin(omega./2));
    Hav((length(omega)-1)/2+1) = 1;
    Havs = [Havs Hav];
end
figure(2)
subplot(1,2,1);
hold on;
for i=1:length(Havs(1,:))
    Hav = Havs(:,i);
    plot(omega, abs(Hav));
    xlim([-pi pi])
end
hold off;
title("| H(e^{jw}) | at diff. m");
xlabel("\omega")
ylabel("amplitude")
legend("m=1","m=5","m=10","m=20");
subplot(1,2,2);
hold on;
```

```
for i=1:length(Havs(1,:))
    Hav = Havs(:,i);
    plot(omega, angle(Hav));
    xlim([-pi pi])
end
hold off;
title("\angle H(e^{jw}) at diff. m");
xlabel("\omega")
ylabel("amplitude")

%% 3c
figure(3)
subplot(3,1,1);
hold on;
for i=1:length(Havs(1,:))
    Hav = Havs(:,i);
    Y = Hav.*X;
    y = invdtft(Y, n, omega);
    plot(n,y);
end
hold off;
title("Y^{-1}(e^{jw}) at differing m")
xlabel("Years")
ylabel("Deviation (^{o}C)")
legend("m=1","m=5","m=10","m=20");
subplot(3,1,2)
hold on;
for i=1:length(Havs(1,:))
    Hav = Havs(:,i);
    Y = Hav.*X;
    y = invdtft(Y, n, omega);
    plot(omega, abs(Y));
    xlim([-pi pi]);
end
hold off;
title("| Y(e^{jw}) |");
xlabel("\omega")
ylabel("amplitude")
subplot(3,1,3)
hold on;
for i=1:length(Havs(1,:))
    Hav = Havs(:,i);
    Y = Hav.*X;
    y = invdtft(Y, n, omega);
    plot(omega, angle(Y));
    xlim([-pi pi])
end
hold off;
title("\angle Y(e^{jw})");
xlabel("\omega")
ylabel("amplitude")
```