Clarifying Elementary Events

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The goal of this paper is to provide a brief clarification on elementary events. The importance to recent philosophical literature will be made apparent, and in future writing I will present a formal system which takes this clarification into account in order to consider the implications for epistemology and the philosophy of science.

Elementary events are central to the study of probabilities; they are the primitive kind upon which it is built. In general, they are obvious, and so when Kolmogorov (1933/1950) put forth his seminal *Foundations of the Theory of Probability*, he didn't bother to say much more than that they are 'all the variants which we regard *a priori* as possible'. But possibility isn't a unified concept, even in situations where the possibilities are known. What I mean is that it is frequently the case that epistemic possibility and actual possibility come apart. And in these cases it is essential that elementary events are defined and kept track of. The overall thesis will be that, given some stochastic process, or something which may at least be treated as such, epistemic considerations, which occur at the level of descriptions, cannot create new events.

Generally, an elementary event is one among an exclusive and exhaustive set of the most basic possibilities. These possibilities may be thought of as possible worlds; then, events, or propositions, may be thought of as collections of possible worlds which can be logically defined from the elementary events. There is no unified interpretation of probabilities, but the argument which follows will not depend on a particular interpretation. The argument will merely hinge on the fact that, in probabilistic contexts, whichever elementary event *is the case* occurs at the expense of the remainder. Different elementary events never coincide. By coincidence, I mean that the occurrence of one entails the occurrence of the other, and vice versa. There may be more than one description of a given elementary event: two outcomes (to mean, generally, an elementary event or a description of one) which coincide stochastically are the same elementary event. Then, we can think of elementary events as equivalence classes over coincidence (that coincidence is an equivalence relation is trivial).

To illustrate an example, take a fair coin flip (supposing such a thing may exist, or that we may, at least, be warranted in reasoning as if a coin flip is fair). Here, the elementary events are clearly heads and tails. Actually, these are descriptions which coincide with the elementary events, but since elementary events are equivalence classes, we may take heads and tails as representatives of their classes, and so we will gloss heads and tails as the elementary events themselves. The simplest example in which epistemic and actual possibility come apart involves self-locating belief. Consider the Sleeping Beauty problem, as stated in Builes 2020:

> Beauty is a perfectly rational agent who is told that the following events will occur. On Sunday, she will be put to sleep. A fair coin will then be tossed. If it lands Heads, she will be awakened on Monday morning. Later, in the evening, she will be told that it is Monday, and then she will be let go. If the coin lands Tails, as before, Beauty will be awakened on Monday morning, and then she will be told that it is Monday later that evening. However, instead of being let go, she will be given a memory-loss drug that will make her forget all of her memories of Monday, and she will be put back to sleep. She will then be awakened on Tuesday, and then she will be let go. Her wakings on Monday and Tuesday will be indistinguishable. When she first awakens on Monday morning, what should her credence be that the coin landed Heads? When she is subsequently told that it is Monday, on Monday evening, what should her credence be that the coin

landed Heads?

Call heads and Monday $H1$, tails and Monday $T1$, and tails and Tuesday $T2$. Adam Elga (2000) presents a proof that Beauty's credence in each, upon waking up on Monday morning, should be 1/3. Necessary to his proof is that $H1$, $T1$, and $T2$ are the elementary events under consideration. From a first glance, this is convincing; upon waking up, Beauty has three options to choose among, all of which seem possible, and the possibilities are exclusive and exhaustive. But, stochastically, we know that there are only two elementary events in our set up (a coin flip). Indeed, heads coincides with $H1$, and tails coincides with *both* $T1$ *and* $T2$. That is, the considerations which distinguish $T1$ and $T2$ are descriptive and depend on Beauty: time and memory. They are not stochastic. $T1$ and $T2$ are distinguished because Beauty cannot experience more than one point in time simultaneously and because Beauty cannot distinguish first from second. That being said, knowledge of either $T1$ or $T2$ implies knowledge of the other and of the coin having landed tails. In this way, we may think of members of an elementary event's equivalence class as *witnesses* of it. What matters here is that, in the Sleeping Beauty problem, there are only two elementary events.

The importance of this clarification lies in how the probabilities are then treated. Thinking of $H1$, $T1$, and $T2$ as distinct elementary events means that we treat them as if they all occur at the expense of the rest. We know, of course, that this is not the case; $T1$ and $T2$ coincide. $T1$ does not occur at the expense of $T2$ – one occurs iff the other does – but one can rightly say that *witnessing* $T1$ occurred at the expense of *witnessing* $T2$. Epistemically, they are both options before Beauty knows better. Stochastically, they coincide.

Since the coin is fair, we assign probability 1/2 to either elementary event. Then, since we want to consider $T1$ and $T2$ as distinct epistemic possibilities, we allow them both to stand in for tails, and we must distribute its probability, somehow, among $T1$ and $T2$. Thus, Beauty should answer 1/2 on Monday morning. Then, on Monday evening, having ruled out $T2$, Beauty should not redistribute its probability to both $H1$ and $T1$; this treats them as distinct elementary events. Given not $T2$, it will no longer be the case that $T1$ and $T2$ together stand in for tails, but that only $T1$ does. That is, $T2$'s probability should be redistributed entirely to $T1$; the probability of tails does not change, but we have epistemically relevant information about which witnesses we think possible. Thus, Beauty should answer 1/2 on Monday evening.

In this way, we end up at the double-halfer position advocated for in Builes 2020 without appealing to time-slice epistemology.[1] Time-slice epistemology is unsatisfactory in that it does not allow for redistributing past probabilities to obtain new ones, even with all else held equal (i.e. other than ruling out $T2$). Having clarified elementary events, we may indeed think of Beauty as redistributing her probabilities, and not needing to restart from scratch at any given moment in time, which is preferable. Furthermore, the clarification provided herein suggests that a principle of indifference can be applied either over a partition of stochastic (or stochastic-like) outcomes *or* the equivalence class of a single elementary event.[2] As a bonus, the latter form of indifference implies the notion of center indifference discussed in Builes 2024. I will treat all of these ideas more fully in later writing.

My analysis of elementary events also makes it clear where other solutions to the Sleeping Beauty problem have gone wrong. Adam Elga, advocating for the thirder position (1/3 on Monday morning and 1/2 on Monday evening) correctly takes the premisses that: (1) given Monday, heads and tails are equally likely; and (2), given tails, Monday and Tuesday are equally likely. However, since he treats $H1$, $T1$, and $T2$ as different elementary events, he combines these premisses in a way that changes the probabilities of heads and tails! In particular, he assumes that the ratios between the probabilities of the events must be preserved when the events are combined into a space containing all three of them. Similarly, David Lewis (2001) advocates for the single-halfer position (1/2 on Monday morning and

[1] There are other double halfers in the literature. Generally, they advocate the use of a different redistribution rule, i.e. not Bayesian conditionalization. I am advocating Bayesian conditionalization with a slight clarification as to how redistribution works.

[2] The first case is like assigning equal probabilities to a coin flip, not guaranteed to be fair, given that we can do no better. The second case is like the indifference used by many authors to say that given tails, Monday and Tuesday are equally likely. We cannot apply indifference over the wake ups since they do not qualify as either case.

2/3 on Monday evening) by beginning with the same premiss (2) along with (3): that upon waking up, heads and tails should be equally likely. Since he also treats them as distinct elementary events, (3) is incompatible with (1) and (2), and so he throws out (1). Both Elga and Lewis are right about one thing, and wrong about how they treat elementary events, and so both are forced to accept one wrong thing (Elga's conclusion and Lewis's disposal of (1)). It turns out that (1), (2), and (3) all hold.

A final point of emphasis is that the Sleeping Beauty problem only contains two possible worlds, heads and tails. It does make sense for Beauty to say something like 'the coin flip could have landed otherwise', since we know that heads and tails each occur at the expense of the other. It does not make sense for Beauty to say something like, 'it is Monday, but it could have instead been Tuesday'. Beauty, upon realizing that $T2$ was not a candidate possibility, does not think the Monday wake up she currently experiences was at its expense. Instead, she merely thinks: 'if it is a tails world, then I will *also* wake up on Tuesday'. Beauty's lack of an ability to distinguish Monday and Tuesday does not render them different events. Because of this, the standard appeal to Bayes' rule along the lines of 'if Tuesday is evidence for tails then Monday must be evidence for heads' fails. This is because Monday is not the complement of Tuesday; since we only have two elementary events, we can only say that heads, or only a Monday wake up, is the complement of tails, or waking up on both Monday and Tuesday. Both Elga and Lewis rely on such an argument, but I have already stated that they treat the wake ups as elementary events.

It seems that the main reason for the confusion here was David Lewis's introduction of the notion of a 'centered world' in Lewis 1979. Lewis thought that the ability to say something about 'here' and 'now' required more than the standard toolkit of propositional logic, for the agent and time must be specified, along with the world, to know whether such a statement holds. This seems to be a misunderstanding; why do agents need a privileged status? Does it not suffice to say, simply, that when one uses such words as 'here' and 'now,' they simply mean that such a thing as they just experienced happened to such a one as them? Treating the experiences of different agents at different times as different worlds simply begs the sort of error I have tried to clarify in this paper – that of treating epistemic uncertainty as generating new elementary events.

References

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