The Importance of Elementary Events

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Introduction

Elementary events are central to the study of probabilities; they are the primitive kind upon which it is built. In general, they are obvious, and so when Kolmogorov put forth his seminal *Foundations of the Theory of Probability*, he didn't bother to say much more than that they are "all the variants which we regard *a priori* as possible" (Kolmogorov 1950). But possibility isn't a precise concept, even in situations where the possibilities are known. What I mean is that it is frequently the case that epistemic possibility and actual possibility come apart. And in these cases it is essential that elementary events are defined and kept track of.

Given that we, as an academic tradition, have not been doing this, I am forced to say that the consequences of *doing so* are immense. Ideally, I would be able to say that the consequences of failing to do so are immense, but we are already suffering those consequences, and must actively change in order to remedy them. Some of these consequences have already been recognized as needing solving, and indeed some of them have even been solved, but until yet it has not been clear that these issues, though related, are consequences of a larger theoretical misunderstanding.

This paper will be split into two major parts. In the first, I will begin by discussing elementary events. Then I will outline a system of probabilities which treats elementary events properly, and an associated system of deducing consistent probability spaces. Then, I will discuss the implications of care about elementary events on the notion of a principle of indifference.

The second major part of this paper will be dedicated to practical consequences of this theory. In particular, I will argue that the increasingly popular notion of time-slice epistemology (see (Moss 2015) and (Hedden 2015)) falls out of this treatment of probabilities. Then I will show that my treatment of the principle of indifference also has support in the literature, in the form of David Builes' center indifference (Builes 2024). Thus, his treatment of the metaphysical and epistemological consequences will fall out neatly ((Builes 2020) and (Builes 2024)).

1 Part I

1.1 Elementary Events

Let's begin by supposing we have some stochastic process and we care about its outcomes. In this paper I will be assuming we always have a finite set of outcomes. Suppose we know what the outcomes are; we will come back to the case of assigning elementary events generally, when no outcomes are prescribed, in §1.4. An example of this might be if we were told that a fair coin was to be flipped. Call heads H and tails T, let us denote this space of outcomes $\{H^{1/2}, T^{1/2}\}$, where the respective probabilities of the outcomes are in the superscript.[1] We know that H and T are the elementary events.[2] They are the possible outcomes of the stochastic event, the basic outcomes, the elementary events.

[1]At this point, I am intentionally failing to produce a probability *triple*, consisting of a sample space, event space, and probability function. The point I want to drive home is about the elements of the sample space itself, and so if I defined an algebra over the sample space and a probability function on the algebra I would be getting ahead of myself. I urge my potentially frustrated reader to keep reading, and I assure you I will bring in the proper probability triple in §1.5.

[2]We could obviously define more events: the trivial event $H \vee T$, and the contradictory event $H \wedge T$. My point is that these are the elementary events in that any events (elements of the algebra we impose, the power algebra here) will be collections of these.

Let's complicate it somewhat, and add event A. Let's say that A happens in case of H. Say, I flip a coin to determine whether I eat a piece of candy, and I eat it iff the coin lands heads. What do we do if we want to talk about A in the probability calculus? Well, here, H and A *coincide*; the occurrence of either fixes the occurrence of the other. In this sense, they are the same event, up to coincidence. And this is how I will define an elementary event: the *a priori* possibilities that may be sampled, up to coincidence.[3] We can think of two instances of the same elementary event as *witnesses* of it.[4] Having defined equivalence, we can introduce a substitution rule on probability spaces:

(SUB): $\{A_1^{p_1}, \ldots, A_n^{p_n}\}, A_1 \longleftrightarrow B \vdash \{B^{p_1}, A_2^{p_2}, \ldots, A_n^{p_n}\}$

(*If two events coincide, we may swap them out in probability spaces.*)

I denote it as a deductive rule for reasons that will be clear in §1.3. Thus, knowing A coincides with H yields the probability space $\{A^{1/2}, T^{1/2}\}$.

Events can also coincide with probability spaces. For example, take the case where a fair coin is flipped, and a second fair coin is flipped in case of tails. We can denote the first coin flip $\{H_1^{1/2}, T_1^{1/2}\}$ and the second coin flip $\{H_2^{1/2}, T_2^{1/2}\}$. We know that this second space obtains iff the first coin lands tails, which we may denote $T_1 \longleftrightarrow \{H_2^{1/2}, T_2^{1/2}\}$. By (SUB), this yields $\{H_1^{1/2}, \{H_2^{1/2}, T_2^{1/2}\}^{1/2}\}$. In general, we will call a probability space which obtains *actual*. Our nested probability spaces suggest the need for a scaling rule:

(SCL): $\{\{B_1^{q_1}, \ldots, B_m^{q_m}\}^r, A_1^{p_1}, \ldots, A_n^{p_n}\} \vdash \{B_1^{q_1 r}, \ldots, B_1^{q_m r}, A_1^{p_1}, \ldots, A_n^{p_n}\}$

(*Me may move the events of a nested probability space into the outer probability space if we scale them all by the probability of the nested space in the outer space.*)

Which would then yield the space $\{H_1^{1/2}, H_2^{1/4}, T_2^{1/4}\}$ in our double coin flip example.

I will specify two fairly trivial rules before moving on. One allows us to eliminate zero probability events: $\{A_1^0, A_2^{p_2}, \ldots, A_n^{p_n}\} \vdash \{A_2^{p_2}, \ldots, A_n^{p_n}\}$. Another allows us to combine the probabilities of two instances of the same event: $\{A^{p_1}, A^{p_2}, B_1^{q_1} \ldots, B_n^{q_n}\} \vdash \{A^{p_1+p_2}, \ldots, A_n^{p_n}\}$.

[3]In this way, we may think of elementary events as equivalence classes modulo coincidence.

[4]The idea here is if we witness one, we witness the elementary event, and thus have knowledge of the occurrence of all which is known to coincide to it.

1.2 Probabilities

I will now turn to the rules for concluding probabilities from probability spaces. Most generally, we can conclude lower bounds:

> (P≥) $\{A_1^{p_1}, \dots, A_n^{p_n}\} \vdash P(A_i) \geq p_i$
>
> (*If an event has some probability in an actual probability space, we know its probability is at least that.*)

This accounts for cases where they may be further instances of some A_i even if it did not obtain. Here is a toy case: $\{A^{1/4}, B^{3/4}\}, B \longleftrightarrow \{A^{1/2}, C^{1/2}\}$.[5] Immediately, we can only conclude that $P(A) \geq 1/4$ using (P≥). This is to account for the fact that by applying (SUB) and (SCL), we can get another space: $\{A^{5/8}, C^{3/8}\}$. Now we can conclude that $P(A) = 5/8$.

> (P=) If there are no more instances of A_i up to (SUB) and (SCL), and the space under consideration *was not* a result of (ELIM), (IND1), or (IND2):[6]
>
> $\{A_1^{p_1}, \dots, A_n^{p_n}\} \vdash P(A_i) = p_i$
>
> (*We may conclude an exact probability when the event we wish to conclude about has no mass elsewhere.*)

Also, we will include a rule for additivity, $P(A \vee B) = P(A) + P(B)$, as is standard.

[5]This could correspond to a case such as: I have a one quarter chance of defaulting to my favorite restaurant (A), but three quarters of the time I have to think about it (B). If I think about it, I'm equally likely to go to my favorite restaurant or the closest restaurant (C). In general, we could circumvent this sort of case by using A_1 for the first A, that does not involve thought, and A_2 for the second A, after thinking about it. Then we could define A as $A_1 \vee A_2$. But I am trying to make a point about what can be concluded from spaces in general, assuming that the labeling may not have been so careful.

[6](ELIM) is the next rule we will define. (IND1) and (IND2) will be defined in 1.4. The restrictions basically say that we cannot conclude probabilities once we have changed the space of outcomes due to epistemic reasons or added to the space of outcomes via the principle of indifference.

Next, we have an elimination rule:

(ELIM) If A_n is not a witness of an elementary event with other witnesses in the probability space under consideration:[7]
$\{A_1^{p_1}, \ldots, A_n^{p_n}\}, \neg A_n \vdash \{A_1^{p_1/\Sigma_{i=1}^{n-1} p_i}, \ldots, A_{n-1}^{p_{n-1}/\Sigma_{i=1}^{n-1} p_i}\}$
(*If we can rule out an event from a probability space, the ratios of the probabilities of all remaining events must remain the same.*)

In order to avoid confusing, say, $P(A_1)$ when A_n is an option versus when we've ruled it out, we will denote probabilities we conclude after employing (ELIM) as follows:

(P_l) $\{A_1^{p_1}, \ldots, A_n^{p_n}\} \vdash P_{l:A_1,\ldots,A_n}(A_i) = p_i$
(*We may always conclude a local probability.*)

Since the remainder of the elements in the space are denoted, we do not need to concern ourselves with whether more instances of A_i can be recovered from (SUB) and (SCL). $P_{l:A_1,\ldots,A_n}(A_i)$ is read *the local probability of* A_i *among* $A_1, \ldots, A_n$. (ELIM)'s role as the equivalent of conditionalization will be clarified over the remainder of this section.[8]

1.3 Consistency

In general, we may have knowledge of some probability spaces which we would therefore take as given. Say, a probability space specified in the rules of a game. Take some set of premises, consisting of probability spaces and coincidence relations. We will call it *inconsistent* if we can deduce a contradiction using the rules laid out in the previous subsections. A simple example of this may be the spaces $\{A^{1/2}, B^{1/2}\}, \{A^{3/4}, B^{1/4}\}$ where A and B are distinct elementary events. Then we can conclude both that $P(A) = 1/2$ and $P(A) = 3/4$, and this

[7] This will be clarified in §1.4. In particular, this pertains to uses of (IND2). We will have another rule, (ELIM*), for such cases, so that if there is more than one witness of the same elementary event in a single space, they will serve to represent the event together, and so if one is ruled out, its mass should only be redistributed among its fellow witnesses, so that the event's mass remains unchanged.

[8] In an ideal case, elementary events would have been treated correctly from the get-go, and local probabilities would exactly correspond to conditionals. We will see cases where they come apart in §1.5.

is a contradiction. Thus, these spaces are inconsistent with each other. If we can derive no such contradictions, we call the premises *consistent*.

I put forth the following thesis:

> *Deductive Epistemology*: given a consistent set of premises (probability spaces and coincidence relations) an agent is committed to, and a set of constraints, the agent's rational credence that A will be A's local probability in a space: (1) they can deduce from their premises through (SUB), (SCL), (ELIM), and (ELIM*);[9] and, (2) containing none of the outcomes ruled out by the constraints and all of the outcomes consistent with them. If no such space can be deduced, the agent should withhold judgement.

Given the tame nature of the rules I've put forth thus far, this framework should not be too controversial. That being said, in the next two subsections I will show exactly where the careful treatment of elementary events comes in. Since we (in the most general sense) have not been doing this, the deductive system will yield results which are in fact impossible in epistemological frameworks using probabilities without being sensitive to elementary events, as will be illustrated in §1.5. The exception will be those frameworks which meet the time-slicing axioms, which turn out to be entailed by deductive epistemology, as will be shown in §2.1.

1.4 Indifference

The principle of indifference is not an uncontroversial thing. Paradoxes of indifference have their roots at least all the way to Keynes (Keynes 1921),[10] who put forth the example of a man who may be from France, Ireland, or Great Britain. He has no reason to prefer any, and so by the principle of indifference, he ought to think each of them equally likely. But,

[9] (ELIM*) will be introduced in §1.4. It is (ELIM) for cases in which a single event has multiple witnesses in the same probability space.

[10] I read about this in (Norton 2008).

we can also think of him as being in France or the British Isles (which is the disjunction of Ireland and Great Britain), so he should think either of *them* equally likely. Thus, he assigns France probability 1/3 and 1/2, and contradicts himself.

I will state the principle of indifference as follows:

> *Principle of Indifference*: if we have a set consisting of exclusive and exhaustive outcomes, all considered to be typical, and we have no information favoring any, then we can do no better than random, and we may represent our indifference by assuming a probability space containing each outcome with equal probabilities.

It turns out, however, that there are two distinct kinds of principles of indifference. By differentiating them, and by restricting what can be concluded from them, we may allow for the epistemic convenience of indifference without contradiction due to over-commitment to the events being actually stochastic.

The first form of the principle of indifference I will call (IND1):

> (IND1): An application of the principle of indifference over exclusive, exhaustive outcomes.

An example of this would be, for instance, rolling a die. We reason about what sorts of outcome are thought to be typical, and these will be the possible rolls, and applying (IND1) will mean assigning each a probability of 1/6 until the die begins to stabilize, even knowing full well that, for instance, a highly sophisticated physical model may have been able to predict the outcome as soon as the die was cast. This sort of reasoning circumvents the need for objective probabilities, such as may be given in game scenarios.[11] For instance, I board a flight, knowing that planes crash, because I reason that my plane is typical, and apply indifference over all flights;[12] i.e. we consider our flight to be a random flight in the same

[11] To borrow from Isaac Levi borrowing from Ian Hacking: "The agent, in effect, treats himself as in a 'chance set-up'" (Levi 1980) and (Hacking 1965).

[12] Say, if I was an aerospace engineer, perhaps I would instead consider typicality to be based off the model of the engine, and thus I would apply indifference over all of those cases

way we consider the outcome of the dice roll to be a random side. We decide what it means to be typical, and we reason as if our "sample" is typical.

When applying (IND1) to provide premises for our deduction, it will be important to denote the use of indifference. This is because (P=) is defined such that we can no longer conclude strictly non-local probabilities after using it. Doing this is important because indifference bakes in an assumption about what is typical, and so it thus depends on the reasoner, as opposed to some prescribed probability distribution. We will use a subscript IND on every element of such a space. Thus, Keynes' case would involve the following premises: $\{\mathrm{Fr}^{1/3}_{\mathrm{IND}}, \mathrm{Ir}^{1/3}_{\mathrm{IND}}, \mathrm{Br}^{1/3}_{\mathrm{IND}}\}$, $\{\mathrm{Fr}^{1/2}_{\mathrm{IND}}, \mathrm{GB}^{1/2}_{\mathrm{IND}}\}$, $\mathrm{GB} = \mathrm{Ir} \vee \mathrm{Br}$. What we can conclude is that the local probability of France when France is comparable to both Ireland and Britain is 1/3 and that the local probability of France when France is comparable to Great Britain is 1/2. Of course, such premises yield a contradiction because they involve two contradictory senses of typicality. The same thing would happen if we, for instance, applied indifference over the die outcomes 1 and $\neg 1$.[13] They are exclusive and exhausted, but this use would not be warranted. I will not go into detail in this paper about what warrants (IND1), nor about what precisely it means to be typical. My point is only that sometimes it is warranted, and sometimes it is not. In Keynes' case, it would clearly be contradictory if both uses of indifference were warranted, but either being warranted seems strange without more information; what makes a nation state, or country, or municipality, typical?[14] Regardless, we will only use local probabilities after applying (IND1), for the following reason. Suppose in Keynes' case, the man *was* warranted to apply indifference over France and Great Britain. Still, whatever probabilities we conclude are merely local. The man's location is not stochastic, there are no actual probability spaces involved, so assigning *any* value for the probability that he is in France would be incorrect. But, we use local probabilities to allow for the epistemic commitment of

[13]This example is breifly considered in (Builes 2024), but he does not appropriately distinguish between the case of indifference over all sides, where we can do no better from random, to the case of indifference over 1 and $\neg 1$, where we certainly can do better than random on typical dice rolls.

[14]As opposed to the case in the previous footnote, where it is easy to reason about what is typical because we can repeatedly sample.

reasoning as if there is some chance he is in France, without the full commitment to him, for instance, finding out he is in Great Britain and thinking that he *could* have been in France instead. Another example of this will come shortly.

The second form of indifference is (IND2):

> (IND2): An application of the principle of indifference over exclusive, exhaustive witnesses of the same elementary event.

Cases of (IND2) differ from cases of (IND1). Thus, we denote it differently. We will choose some representative of the elementary event (any of its witnesses; it does not matter so long as we are consistent) and use that representative as a subscript for all elements in the obtained probability space. Examples of this are less common, and generally involve lapses in memory or something analogous. The next subsection goes into depth on a famous example, which is the Sleeping Beauty problem. The main difference between (IND1) and (IND2) is that (IND2) rules out the use of (ELIM) in general. I will first define the new rule, and then explain it:

> (ELIM*): Take X to be some elementary event, and $X = X1, \ldots, Xn$ to be witnesses of it. Then:
> $\{X1_X^{p_1}, \ldots, Xn_X^{p_n}, A_1^{q_1}, \ldots, A_m^{q_m}\}, \neg Xn \vdash \{X1_X^{p_1/\Sigma_{i=1}^{n-1} p_i}, \ldots, X(n-1)_X^{p_{n-1}/\Sigma_{i=1}^{n-1} p_i}, A_1^{q_1}, \ldots, A_m^{q_m}\}$
> (*Ruling out one witness of an event does not change the mass of the event, and thus all redistribution from the elimination takes place among the witnesses of the event.*)

It is difficult to make intuitively clear why this matters, but how we analyze cases involving so-called self-locating belief depends heavily on this distinction. Thus, I will attempt the following illustration.

Consider a case in which we have some agent, Sally, who knows her friend to be going to a concert, where either singers A and B will each perform, individually and in that order, or singer C will perform, but Sally does not know when the concert begins.[15] Here, A singing and B singing both witness the fact that the world is such that singers A and B both perform, and likewise for singer C. There are two elementary events here: A and B, and C. The point is that knowledge of any one singer fixes the world.

Now suppose Sally thought either world equally likely, being indifferent between the two.[16] Thus, Sally thinks A and B is as likely as C. This is an application of (IND1). Then, at some point in the concert, Sally's friend texts her that the person currently singing is not singer B. Since Sally is not at the concert, she cannot differentiate among the witnesses (i.e. she does not know who is singing). How should she redistribute the probability of B?

Traditionally, the approach would have been one of the following:

- Bigen with the distribution given that someone is currently singing. Since by (IND1) Sally thinks A and B as likely as C, and since by (IND2) she should be indifferent among A and B in the first case, she should assign credence 1/4 to A, 1/4 to B, and 1/2 to C. Now she learns not B, so she redistributes its mass to the remaining options, keeping their proportions the same. And we get that she thinks C is 2/3 likely given not B.

- Begin with the distribution given that the first singer is singing (i.e. not B). Either way, a first singer sings, and so her credence is still evenly split between the two worlds. Since she thinks A and C are equally likely when not B, we can conclude that they are always equally likely, since rescaling upon eliminating B left the ratio of their probabilities the same. Similarly, Sally thinks A and B equally likely given not C by (IND2), so their ratio must remain constant as well. Working backwards, she must have had credences 1/3 for A, B, and C, knowing one of them to be singing.

[15] Thus, if she knows it to be A or B and knows that one is currently singing, she is not able to reason about how long the concert has been going on and the order they sing in. Say, for instance the advertisements only list the time at which the doors open.

[16] Of course, it was probably a marketing technique, and the producers of the concert have known which world is actual for a long time, given that they had to plan the event.

I am arguing that in either case, this is not how she should reason. Both of these treat A, B, and C as separate elementary events. But, since A and B coincide, they witness the same event. Ruling out B, she should redistribute all of its probability mass to A, and she should think A and C are equally likely given not B. Otherwise, she will be committed to thinking that B could have actually been singing instead, because of the fact that she previously considered B an option.[17] But, if A is singing, then B will sing later, so this does not make sense. The point is that Sally should think: either B has not yet sang, and that's why they aren't singing, or B is not singing at all. The first case is exactly what it is to be in the world where A and B sing, which she assigned credence 1/2, so she should assign credence 1/2 to B having yet to sing, or to A singing. This is the sort of redistribution which is possible when attention is paid to elementary events. A, B, and C singing are not individually elementary events, and so Sally should not redistribute as if they are. Instead, she should notice that A and B coincide, and though she splits her credence that both A and B sing among A and B individually under the assumption *one* is singing, she does not forget that they witness the same elementary event when it comes time to redistribute.

1.5 Sleeping Beauty

Most discussion surrounding self-locating belief over the last couple decades has at least mentioned the Sleeping Beauty problem, if the problem was not the very language in which the discussion was had. The problem is as follows (I borrow the language of (Builes 2020)):

> **Sleeping**: Beauty is a perfectly rational agent who is told that the following events will occur. On Sunday, she will be put to sleep. A fair coin will then be

[17] This is an unreasonable metaphysical commitment. Time-slice epistemology deals with this issue by saying her prior belief that A had some chance of currently singing is now irrelevant, it being a different time, and so she should calculate from scratch. In 2.1, I will argue both that deductive epistemology is time-slicing, but that stipulating the axioms of time-slice epistemology was only necessary because we were working with elementary events incorrectly. There is no issue with starting at a prior distribution and redistributing, if we are careful. Indeed, if redistributing and starting from scratch (as the time-slice epidemiologists advocate) gave us different probability spaces containing the same events, our system of probabilities would not be consistent.

> tossed. If it lands Heads, she will be awakened on Monday morning. Later, in the evening, she will be told that it is Monday, and then she will be let go. If the coin lands Tails, as before, Beauty will be awakened on Monday morning, and then she will be told that it is Monday later that evening. However, instead of being let go, she will be given a memory-loss drug that will make her forget all of her memories of Monday, and she will be put back to sleep. She will then be awakened on Tuesday, and then she will be let go. Her wakings on Monday and Tuesday will be indistinguishable. When she first awakens on Monday morning, what should her credence be that the coin landed Heads? When she is subsequently told that it is Monday, on Monday evening, what should her credence be that the coin landed Heads?

There are three camps:[18] thirders, who say that Beauty should have credence 1/3 that the coin landed heads when she awakens, but credence 1/2 knowing it be Monday (Elga 2000); single-halfers, who say that Beauty should have credence 1/2 when she awakens, but credence 2/3 knowing it be Monday (Lewis 2001); and double-halfers, who hold that the credence should be 1/2 in both cases (Meacham 2008). Single-halfing has been all but ruled out (Oesterheld and Conitzer 2024). Double-halfing, on the other hand, is not taken very seriously. It turns out that the case analyzed in the previous subsection is analogous to **Sleeping**. The first bullet point was the reasoning of single-halfers, and the second bullet point was that of thirders. In this section, I want to argue that the prevalence of these two solutions in particular is due to the mistreatment of elementary events.

Let's begin by analyzing the problem *à la* deductive epistemology. What do we know? Firstly, Beauty is awake in the game, so we have a constraint that some wake up is currently being experienced. Next, we have a probability space $\{H^{1/2}, T^{1/2}\}$. Also, we have coincidence relations—we know that the heads wake up witnesses heads, and that the two tails wake ups witness tails: $H1 \longleftrightarrow H$, $T1 \longleftrightarrow T$, $T2 \longleftrightarrow T$. Finally, we have a principle of indifference

[18]The papers cited here serve to be examplars of each which is especially relevant to the literature; there are many who hold each view.

which says that given tails, we should think either tails wake up equally likely.[19] This is a case of (IND2). Letting T represent its equivalence class up to coincidence, we get that $T \longleftrightarrow \{T1_T^{1/2}, T2_T^{1/2}\}$. By (SUB) and (SCL), we get the following space: $\{H1^{1/2}, T1_T^{1/4}, T2_T^{1/4}\}$.[20] This space contains all and only the possible outcomes which may be the one currently being experienced. Thus, $P_{l:H1,T1,T2}(H1) = 1/2$ tells us that Beauty's credence that heads should be $1/2$. Now suppose she knows it to be Monday, and rules out $T2$. We would apply (ELIM*), and the resultant space would be $\{H1^{1/2}, T1^{1/2}\}$, and Beauty's credence that heads remains $1/2$.[21]

I promised[22] I would bring in probability triples. Since there are only two elementary events in **Sleeping**, the only probability triple is that where the sample space is $\{H, T\}$, the algebra is the power set algebra, and the probability function assigns $1/2$ to both H and T. Beginning with a sample space $\{H1, T1, T2\}$ due to the fact that Beauty cannot distinguish between them as witnesses for H and T is simply inappropriate. As with the singing example in the previous subsection, such a sample space comes with assumptions. Beauty, finding out it to be Monday, thinks that it could have *instead* been Tuesday, instead of thinking that it will be Tuesday tomorrow. This is strange. But, for now, I will not spend too much time on this problem, as the literature is saturated. I defer to (Builes 2020), where

[19] This is the "highly restricted principle of indifference" used by Elga (Elga 2000). Elga did not simply apply indifference over all wake ups (though, by beginning with a sample space containing all wake ups, he makes a functionally similar assumption). I have differentiated between (IND1) and (IND2) to forbid the application of indifference over all wake ups. Either all of the outcomes we are indifferent among are exclusive, and we use (IND1), or they all coincide, and we use (IND2). There is no middle-ground which allows for indfference over all wake ups, two of which coincide and are collectively at the exclusion of the third.

[20] In (Meacham 2008), Meacham described a process similar to applying (SUB) and (SCL): "First, you take your hypothetical priors, and set the credence in every centered world incompatible with your current evidence to 0. Second, you normalize the credences in the remaining doxastic worlds; i.e., adjust the values assigned to each doxastic world such that they sum to 1, and such that the ratios between them are the same as the ratios between their priors. Finally, you normalize your credences in the remaining doxastic alternatives at each world; i.e., at each world adjust the values assigned to the alternatives so that they sum to the credence assigned to that world, and such that the ratios between them are the same as the ratios between their priors." His problem, however, is that he still considers the different wake ups given tails to be different worlds, as opposed to different witnesses of the same stochastic outcome. This will be discussed further in §2.2.

[21] Note that this space may also be directly deduced from the premises by (SUB).

[22] In a footnote.

Builes argue from the perspective of time-slicing, and for time-slicing. In the forthcoming, I will show deductive epistemology to be time-slicing, but I will argue that such stipulations as the time-slicing axioms are only necessary because of the failure to respect elementary events. This, in turn, stems from a misunderstanding of indexicals.

2 Part II

2.1 Deductive Epistemology is Time-slicing

Time-slice epistemology consists of the following two premises (Hedden 2015):

- *Synchronicity*: What attitudes you ought to have at a time does not directly depend on what attitudes you have at other times.
- *Impartiality*: In determining what attitudes you ought to have, your beliefs about what attitudes you have at other times play the same role as your beliefs about what attitudes other people have.

Trivially, deductive epistemology is time-slicing. Our rational credences at any given moment do not depend on previous beliefs; they depend only on the accepted premises and the deductive calculus.[23] In the Sleeping Beauty problem, we can deduce the probability space for the credence given that it is Monday (i.e. $\{H1^{1/2}, T1^{1/2}\}$) with no reference to Beauty's credences before finding out such a fact (i.e. $\{H1^{1/2}, T1_T^{1/4}, T2_T^{1/4}\}$). However, I hold that time-slicing is misguided. This is because if we do in fact start at such a space, add constraints, and redistribute, we end up with the same space as we would have starting from scratch. Time-slicing is only necessary in a system where elementary events are not respected as they ought to be. The issue is not Bayesian conditionalization as time-slicers wish to hold.[24] If we may represent our beliefs using probability spaces, and if we have a standard

[23]Impartiality comes into play as follows: if we have a case where a premise is that a previous belief matters, then it does, and this is also true for the beliefs of others.

[24]In (Moss 2015), Moss says: "The most obvious target of the time-slice epistemologist is the classic updating norm introduced by (Bayes 1763) and widely adopted by Bayesians, namely the claim that agents

practice for how redistribution in probability spaces works, then either this redistribution must work on our beliefs, or our beliefs should not be represented using probability spaces. The Sleeping Beauty problem is such a case where there is clearly warrant to representing beliefs using credal probabilities. Yet conditionalization still seems to fail! Anti-bayesians, including time-slicers, will take this to say that conditionalization is problematic, so as to continue using probability spaces in such cases. I hold that what is problematic is the gloss of elementary events, as argued in the first part of this paper. Solving such an issue directly solves the issues time-slicing seeks to patch using their two axioms. As long as our premises are not changing moment to moment, beyond constrains which allow for the use of our deductive rules, we should have no issue starting with our beliefs at one time and redistributing appropriately, *so long as we keep track of elementary events.*

2.2 Indifference2 yields Center Indifference

In this final section, I would like to bring in the center indifference of (Builes 2024):

> *Center Indifference*: For any two similar centered worlds c_1 and c_2, if both c_1 and c_2 are compatible with your evidence, then it is rationally required to set $\mathrm{Cr}(c_1|c_1 \text{ or } c_2) = 1/2$.

For Builes, similarity refers to the centered worlds (to be defined shortly) being associated with the same world. It is clear then that this is a case of (IND2)—c_1 and c_2 coincide, as they are in the same world. Thus, the arguments for such an indifference also hold as arguments for accepting deductive epistemology, and my definition of an elementary event which allows for differentiating the two notions of indifference. However, I have intentionally avoided the language of *centered worlds*, worlds together with a specified person and time, as introduced

should update their credences according to Conditionalization. Conditionalization demands that your later credence in a proposition should match your earlier conditional credence in that proposition, conditional on any information you have since learned. The norm is at odds with time-slice epistemology, as it says your current credences are rationally constrained by your past credences, on which your current mental states need not supervene."

by David Lewis in (Lewis 1979). In general, I think the introduction of centered worlds is what has contributed to all of the confusion surrounding self-locating belief, arguments that we live in a simulation, etc. Such a concept assumes some privileged status of the person involved—they must be uniquely specified—thus cases involving two of the same person, or the same person at different indistinguishable times, become hairy. David Lewis held that indexicals ("here, now" propositions) could not be expressed with the standard notion of a possible world. I deny this.

Say I know event X happened to me. Then I have evidence that the world is such that event X happens to a person such as me. Thus, if I am asked to choose between one world, say A, with two equivalent copies of me undergoing X and one world, say B, with only one such copy, my evidence cannot differentiate between them. Of course, there is such a thing as an anthropic principle—I can rule out worlds with no copies of me undergoing X. But this is the extent of my information. The reason for this is that in case of A, both copies of me undergoing X coincide! I should not reason about them as separate events as I reason about exclusive worlds A and B. Sure, given A, I still would not know which version of me is the me undergoing X from my perspective, but in either case there is at least a me experiencing such a thing, which is all I know. And if I am in world A, it is not that I *could have been* either person, only that I think them both typical given my evidence, and apply (IND2). My point is that I am allowed to assign local probabilities to these indistinguishable options, but we should not reason as if we are committed to these probabilities being actual in any way. The introduction of centered worlds suggests this notion of distribution of persons, which is nonsensical, especially in light of the notion of an elementary event put forth in this paper. But, I yield to Builes' discussion of the deflation of the self in (Builes 2020) and (Builes 2024). This particular discussion is beyond the scope of this paper. But I hold that correct treatment of elementary events involves not treating different centered worlds as different worlds in any capacity, beyond the potential indistinguishably among witnesses, in which case we use (IND2) and (ELIM*).

To conclude, I would like to invite those who are tempted by time-slicing and center indifference to simply accept deductive epistemology, which yields both, while still allowing for beliefs to be deduced from previous beliefs should all relevant premises remain the same, unlike time-slicing.

References

Builes, David (2020), "Time-Slice Rationality and Self-Locating Belief", *Philosophical Studies*, 177, 10, pp. 3033-3049.

— (2024), "Center indifference and skepticism", *Noûs*, 58, 3, pp. 778-798.

Elga, Adam (2000), "Self-Locating Belief and the Sleeping Beauty Problem", *Analysis*, 60, 2, pp. 143-147, DOI: 10.1111/1467-8284.00215.

Hacking, Ian (1965), *Logic of Statistical Inference*, Cambridge University Press, Cambridge, England.

Hedden, Brian (2015), "Time-Slice Rationality", *Mind*, 124, 494 (Mar. 2015), pp. 449-491.

Keynes, John Maynard (1921), *A Treatise on Probability*, Dover Publications, Mineola, N.Y.

Kolmogorov, Andrey (1950), *Foundations of the Theory of Probability*, trans. by Nathan Morrison, Chelsea Publishing Company.

Levi, Isaac (1980), *The Enterprise of Knowledge: An Essay on Knowledge, Credal Probability, and Chance*, MIT Press.

Lewis, David (1979), "Attitudes De Dicto and De Se", *The Philosophical Review*, 88, 4, pp. 513-543.

— (2001), "Sleeping Beauty: Reply to Elga", *Analysis*, 61, 3, pp. 171-76, DOI: 10.1111/1467-8284.00291.

Meacham, Christopher J. G. (2008), "Sleeping Beauty and the Dynamics of de Se Beliefs", *Philosophical Studies*, 138, 2, pp. 245-269.

Moss, Sarah (2015), "Time–Slice Epistemology and Action under Indeterminacy", in *Oxford Studies in Epistemology Volume 5*, Oxford University Press, ISBN: 9780198722762.

Norton, John D. (2008), "Ignorance and Indifference*", *Philosophy of Science*, 75, 1, pp. 45-68, DOI: 10.1086/587822.

Oesterheld, Caspar and Vincent Conitzer (2024), "Can de se choice be ex ante reasonable in games of imperfect recall? A complete analysis", *Manuscript*.