The Gaussian Model of Wind-Generated Waves

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Lecture within the framework of the project:
Inversion of radar remote sensing images and deterministic prediction of ocean waves.
University of Oslo (UiO) and Research Council of Norway (RCN) grant 214556/F20.

Contents

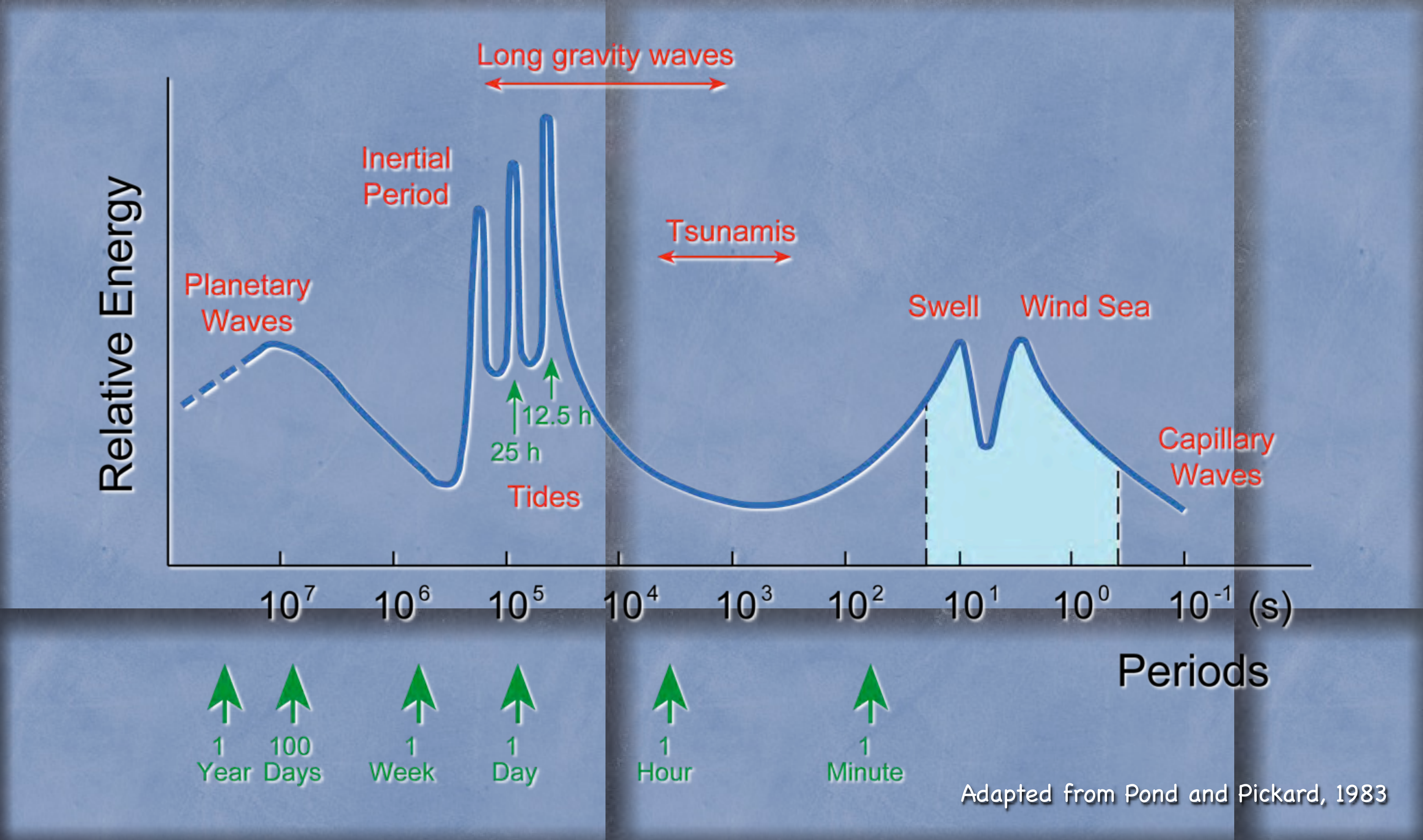
Wind-Generated Waves

Wind-Generated Waves

- Ocean (gravity) waves are wind-generated waves.
- That occur on the free surface of oceans, seas, lakes, etc.
- When directly being generated and affected by the local winds, a wind wave system is called wind sea.
- When these waves propagate to other locations wind waves are called swell.
- A wave field can be the result of a superposition of a win sea and several swell systems.

Wind-Generated Waves

- Wind waves are not tsunamis, nor tidal waves, nor internal waves,....

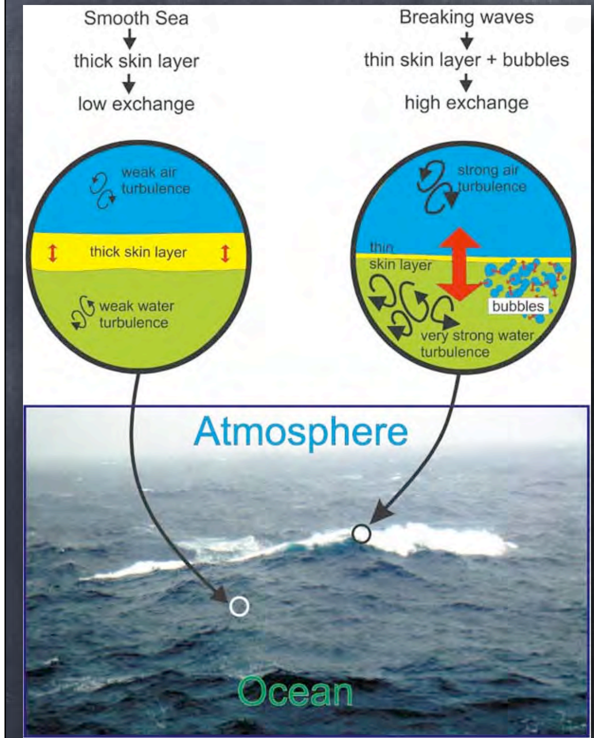


Adapted from Pond and Pickard, 1983

Wind-Generated Waves

- Wind waves play an important role in the energy transference mechanisms between the atmosphere and the ocean.
- Not all the hydrodynamical mechanisms of wind waves are well understood.
- They can be extremely nonlinear.
- To improve the wave forecast.
- To improve the design of vessels, off and onshore structures
  - Oil platforms.
  - Breakwaters.
  - Coastal protection.

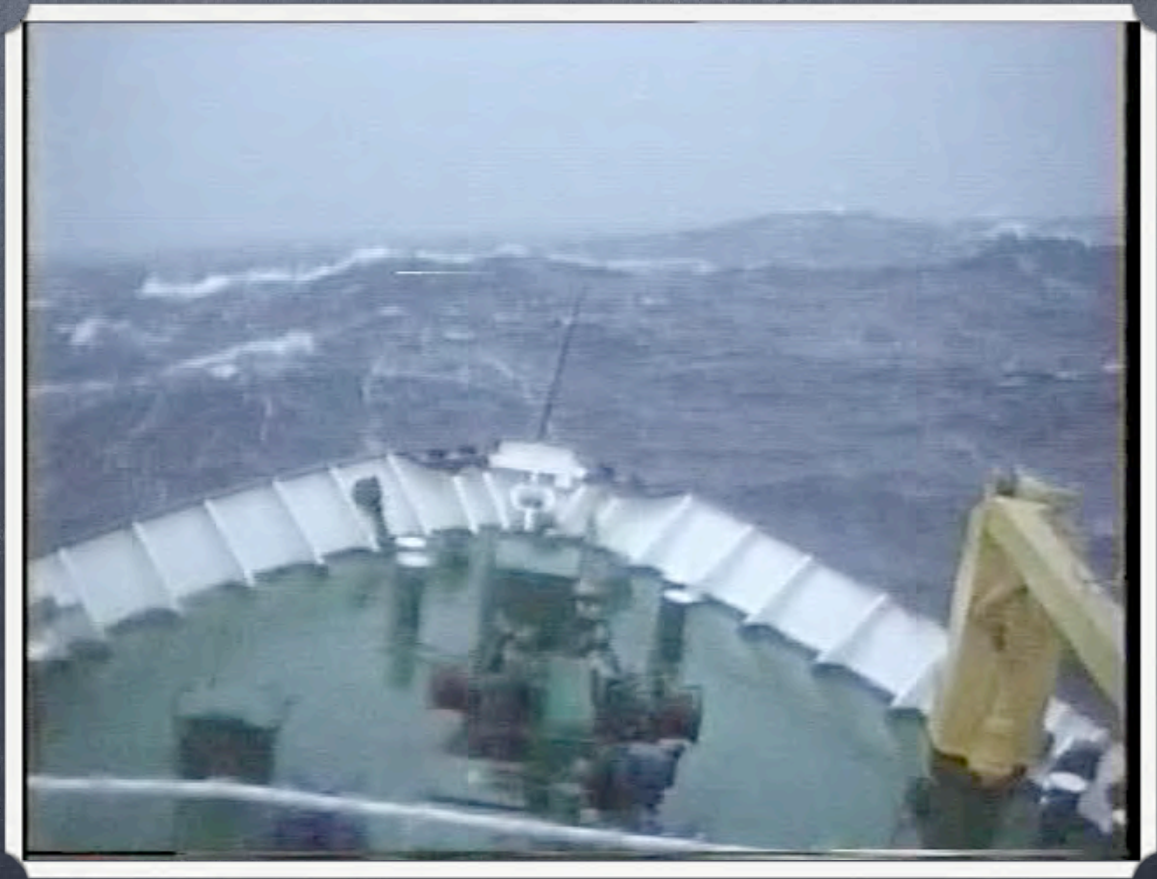
There are still many open questions on the study of wave generation and propagation!!



How wind waves look like?

- Oceanographic cruise of the German vessel **FSS Gauss** in 1992.
- Part of the mission was to carry out field experiments to measure wind waves with marine radars during severe storm conditions.

Filmed by Dr. Friedwart Ziemer
(GKSS Research Centre, Germany)



Spectral Description of Wave Fields

Mathematical Description of Ocean Waves

- Ocean surface waves are caused by the wind.
    - Interaction of two fluids: atmosphere-ocean
- Ocean Waves are described by the spatio-temporal evolution of the vertical elevation of the free sea surface over the sea level

$$\eta(\mathbf{r}, t)$$

Horizontal sea surface coordinates: $\mathbf{r} = (x, y)$

Time: t

Vertical coordinate of the sea surface: $z = \eta(\mathbf{r}, t)$ ⟵ This is the information we try to find or to measure!!

Linear solutions for the wind-generated waves

- Linearizing the hydrodynamic equations that describe the elevation of the sea surface:
  - The sea surface can be described as linear superpositions of several monochromatic waves
  - Those monochromatic waves (e.g. wave spectral components) take the form of
    - sinusoidal waves
    - cosinusoidal waves
    - complex exponential waves

Equivalent descriptions

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

Monochromatic wave in time

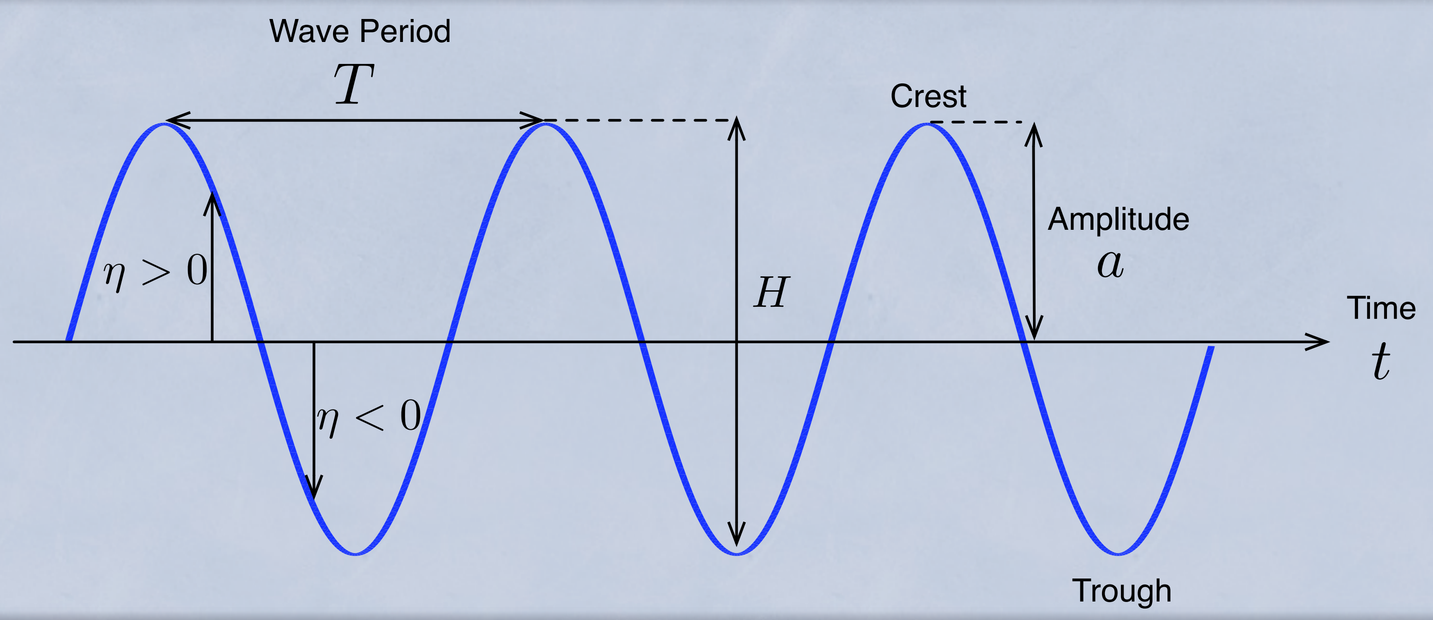
$$\boxed{\eta(t) = a\cos(\omega t + \varphi)}$$

Amplitude [m]: a

Angular frequency [rad/s]: $\omega = 2\pi f$

Wave period [s]: $T = \dfrac{1}{f}$

Phase [rad]: φ



Monochromatic wave in space (1D)

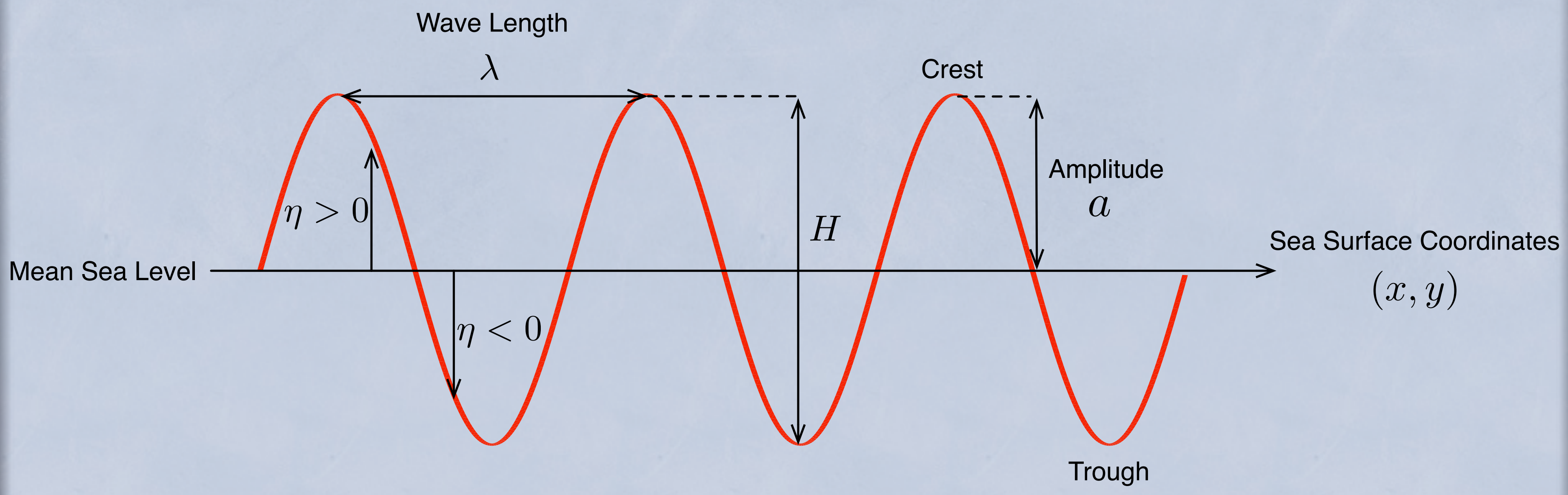
$$\boxed{\eta(x) = a\cos(kx + \varphi)}$$

Amplitude [m]: a

Phase [rad]: φ

Wave number [rad/m]: $k = \dfrac{2\pi}{\lambda}$

Wave length [m]: λ

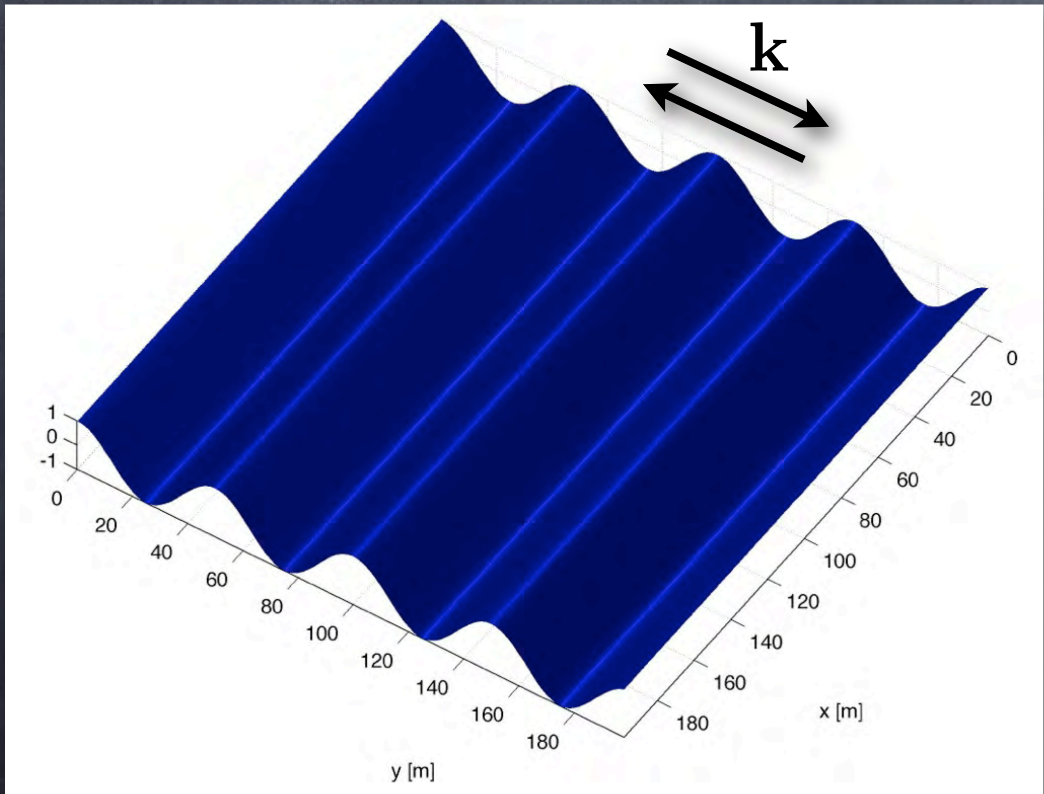


Monochromatic wave in space (2D)

$$\eta(x, y) = a\cos(\mathbf{k}\cdot\mathbf{r} + \varphi) = a\cos(k_x x + k_y y + \varphi)$$

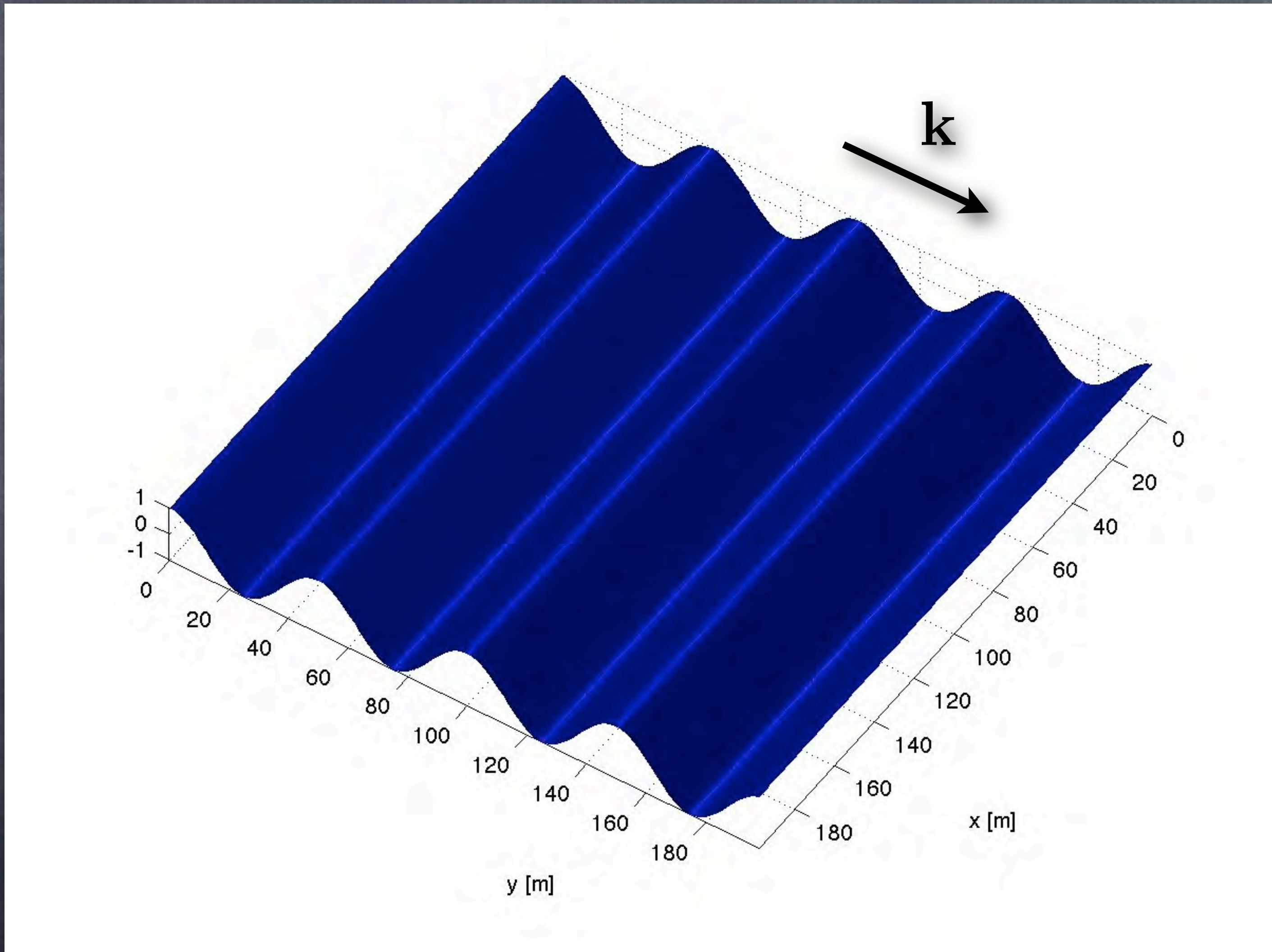
Wave number vector [rad/m]: $\mathbf{k} = (k_x, k_y)$

$$k = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2} = \frac{2\pi}{\lambda}$$



Monochromatic wave in space and time (3D)

$$\eta(x, y, t) = a\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi)$$



Monochromatic wave in space and time (3D) (complex notation)

$$\eta(x,y,t) = a\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi)$$

$$\cos\alpha = \frac{1}{2}\left(e^{i\alpha} + e^{-i\alpha}\right)$$

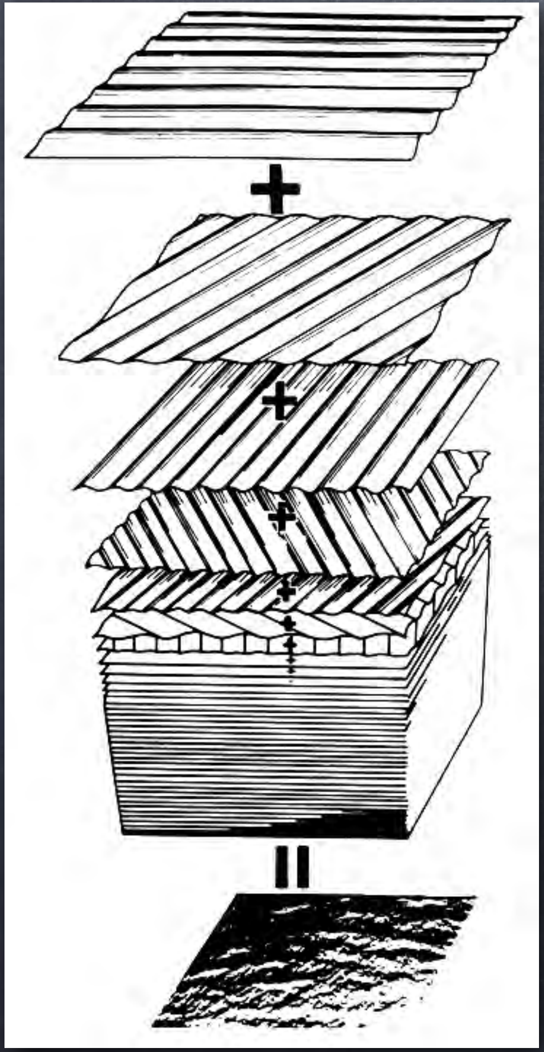
$$\boxed{\eta(x,y,t) = c\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} + \text{c.c.}}$$

$$c = \frac{a}{2}\, e^{i\varphi}$$

Complex amplitude

General solutions of the linear wave theory

- Under the frame of the linear theory, the wave elevation of the free sea surface can be expressed as a liner superposition of different monochromatic waves



(Pierson et al. 1985)

- Each wave component is characterized by its:
  - Wave number vector (e.g. wave length and propagation direction)

$$\mathbf{k} = (k_x, k_y) \qquad \lambda = \frac{2\pi}{k}$$

$$\theta = \tan^{-1}\left(\frac{k_y}{k_x}\right)$$

  - Frequency ω
  - Amplitude a
  - Phase φ

General solutions of the linear wave theory

- Notations (I):

$$\eta(\mathbf{r},t) = \sum_n a_n \cos(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t + \varphi_n)$$

$$\eta(\mathbf{r},t) = \sum_n c_n e^{i(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t)} + \mathrm{c.c.}$$

General solutions of the linear wave theory

- Notations (II):

$$\eta(\mathbf{r},t) = \sum_{k_x}\sum_{k_y}\sum_{\omega} a(k_x,k_y,\omega)\cos\left[\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi(k_x,k_y,\omega)\right]$$

$$\eta(\mathbf{r},t) = \sum_{k_x}\sum_{k_y}\sum_{\omega} c(k_x,k_y,\omega)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \mathrm{c\ .c.}$$

$\begin{pmatrix} a(k_x,k_y,\omega) \\ \varphi(k_x,k_y,\omega) \end{pmatrix}$ and $c(k_x,k_y,\omega)$ are estimated from the Fourier Transform

General solutions of the linear wave theory

- Notations (III): continuous notation

This expression normally includes the complex conjugates

$$\eta(\mathbf{r}, t) = \int_{\Omega_{\mathbf{k},\omega}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} dZ(\mathbf{k}, \omega)$$

Complex amplitude: $dZ(\mathbf{k}, \omega)$

- Spectral domain where ocean waves are defined: $\Omega_{\mathbf{k},\omega} = \Omega_{\mathbf{k}} \times \Omega_{\omega}$
- In practice the spectral domain is limited by the resolution of the sensor in space and time

$$\Omega_{\mathbf{k}} = [-k_{x_c}, k_{x_c}) \times [-k_{y_c}, k_{y_c})$$

$$\Omega_{\omega} = [-\omega_c, \omega_c)$$

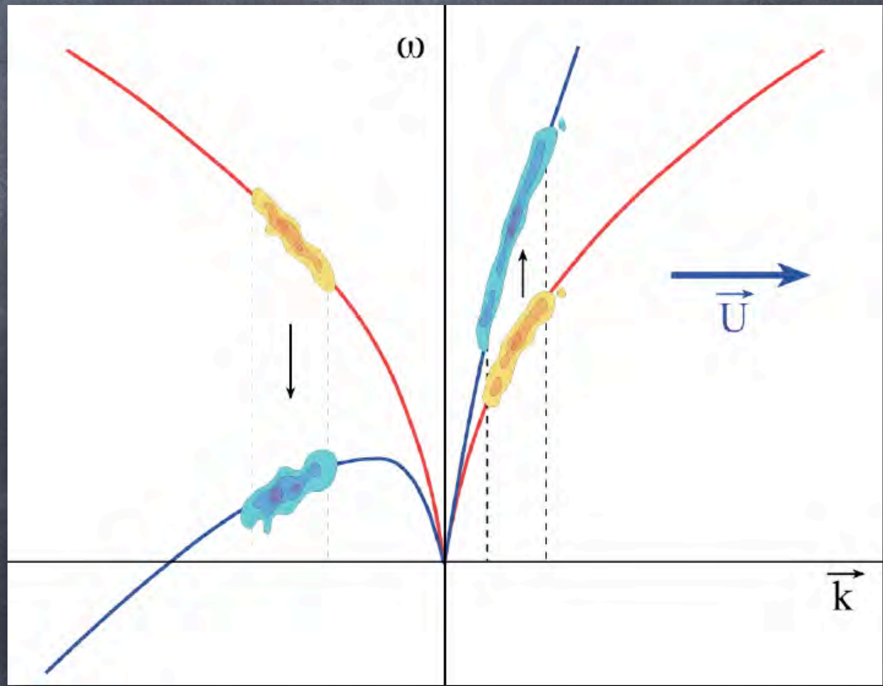
Dispersion Relation

- Ocean waves are dispersive.
- The dispersion relation is given by

$$\omega = \sqrt{gh\tanh(kh)} + \mathbf{k}\cdot\mathbf{U}$$

Current of encounter: $\mathbf{U} = (U_x, U_y)$

Water depth: h



Dispersion Relation

- General Case:
  - Phase velocity: $v_p = \dfrac{\omega}{k}$
  - Group velocity: $v_g = \dfrac{d\omega}{dk}$

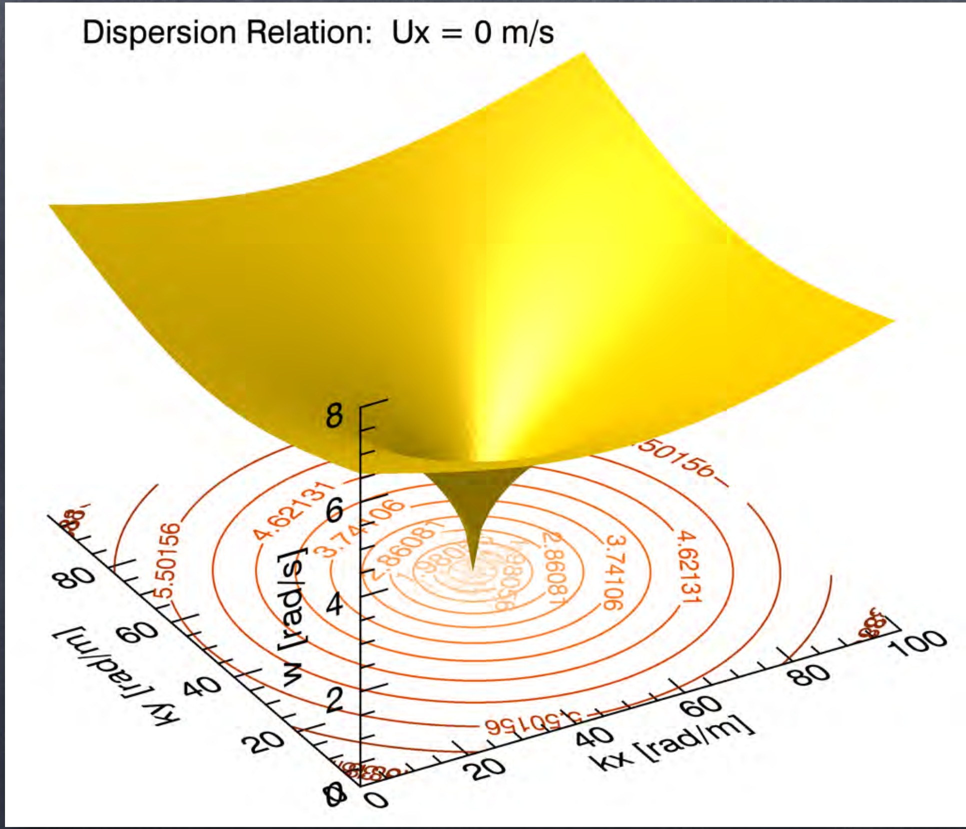
$$v_p \neq v_g$$ (dispersive)

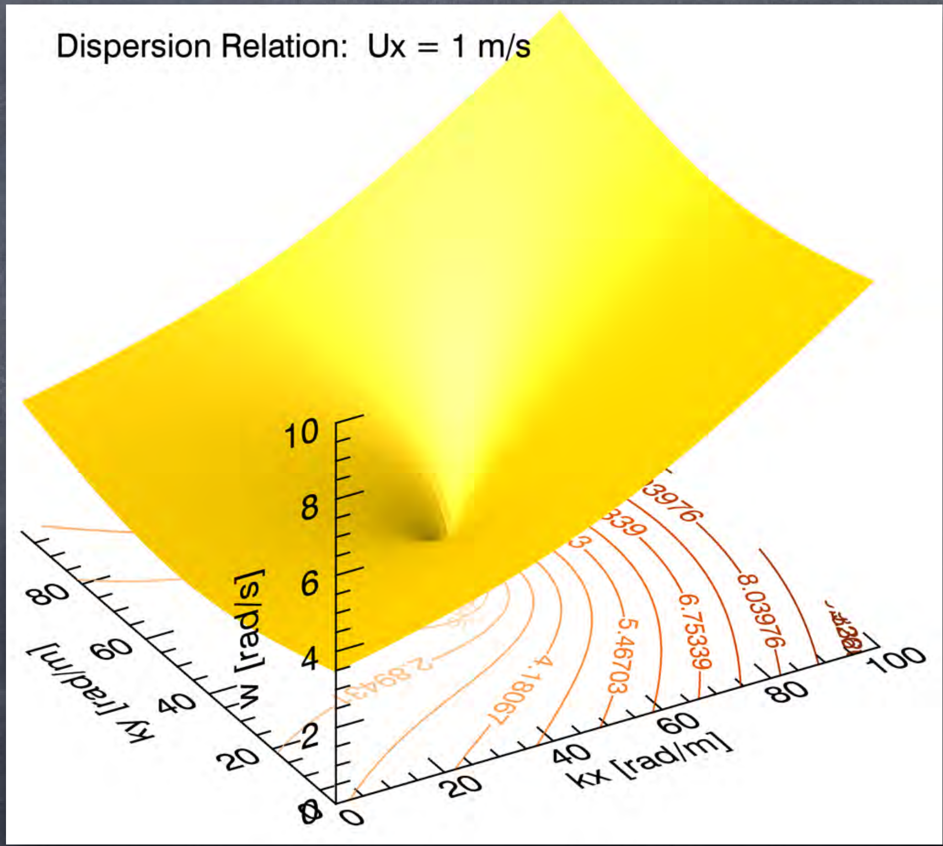
- Approximations (without current of encounter):
  - Deep water: $kh \gg 1 \Longrightarrow \tanh(kh) \approx 1 \Longrightarrow \omega \approx \sqrt{gk}$

    (dispersive)
  - Shallow water: $kh \ll 1 \Longrightarrow \tanh(kh) \approx kh \Longrightarrow \omega \approx k\sqrt{gh}$

    (non dispersive)

Dispersion Relation





Dispersion Relation

- Considering the dispersion relation the linear wave field can be expressed as

$$\eta(\mathbf{r},t) = \sum_{k_x}\sum_{k_y}\sum_{\omega} a(k_x,k_y,\omega)\cos\left[\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi(k_x,k_y,\omega)\right]$$

$$\Big\downarrow\ \omega(\mathbf{k})$$

$$\eta(\mathbf{r},t) = \sum_{k_x}\sum_{k_y} a(k_x,k_y)\cos\left[\mathbf{k}\cdot\mathbf{r} - \omega(\mathbf{k}) + \varphi(k_x,k_y)\right]$$

Sea State (I)

- The movement of the ocean free surface are complex, even assuming the linear wave theory.
- A way to improve the the sea surface description given by the linear wave theory is to assume that the wave elevation presents a stochastic behavior.
  - The parameters if the linear wave solutions are considered as random variables
  - The statistical properties of those parameters depend on the meteorological and geophysical conditions
- Under these considerations, the concept of sea state is defined from:
  - Temporal domain where the wave field is statistically stationary.
  - Area of the ocean where the wave field is statistically homogeneous.

Sea State (II)

- Random parameters for different linear wave theory notations:

$$\eta(\mathbf{r},t) = \sum_n a_n \cos(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t + \varphi_n)$$

$$\eta(\mathbf{r},t) = \sum_n c_n e^{i(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t)} + \text{c .c.}$$

$$\eta(\mathbf{r},t) = \int_{\Omega_{\mathbf{k},\omega}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} dZ(\mathbf{k},\omega)$$

η is a stochastic process

Gaussian Sea States

- The Gaussian sea is the simplest stochastic model to describe sea surface variability
  - It considers different components that are statistically independent (uncorrelated).
  - Statistical symmetry between crests and troughs.
  - η is a zero-mean Gaussian stochastic process: $\mathsf{E}[\eta] = 0$
  - Variance: $\mathsf{Var}[\eta] = \sigma^2$

a_n	is Rayleigh distributed
φ_n	is uniformly distributed in $[-\pi,\, \pi)$
c_n	is complex-Gaussian distributed
$dZ(\mathbf{k}, \omega)$	is complex-Gaussian distributed

Gaussian Sea States

$\mathsf{E}[\eta] = 0 \qquad \mathsf{Var}[\eta] = \sigma^2$

- Considering that the spectral components are statistically independent:

$$\sigma^2 = \mathsf{E}[\eta^2] = \frac{1}{2}\sum_n \mathsf{E}[a_n]^2$$

$$\sigma^2 = \mathsf{E}[\eta^2] = 2\sum_n \mathsf{E}[|c_n|]^2$$

$$\sigma^2 = \mathsf{E}[\eta^2] = \int_{\Omega_{\mathbf{k},\omega}} \mathsf{E}[|dZ(\mathbf{k},\omega)|]^2$$

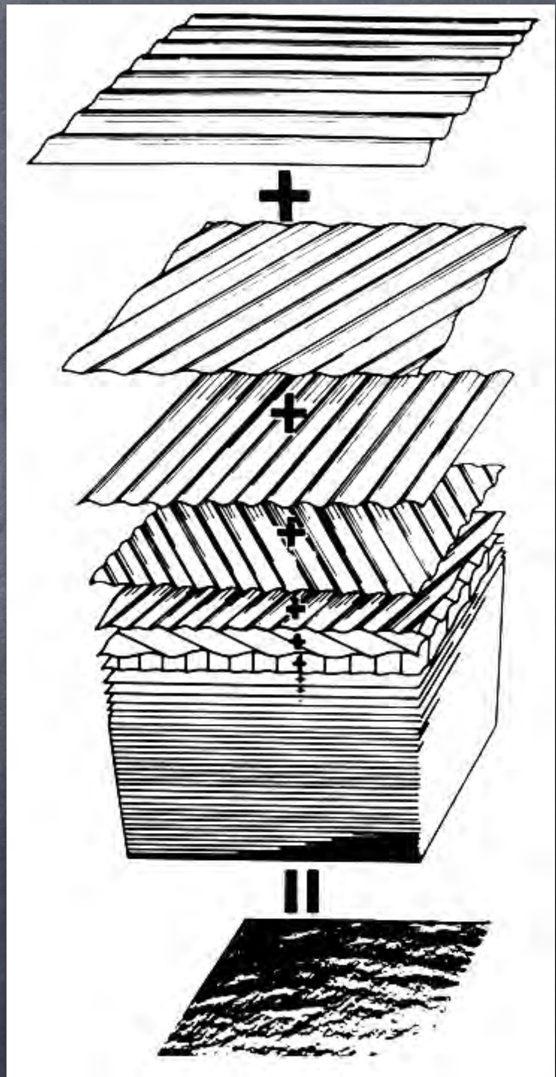
Spectral Representation of Sea States

- Using the continuos representation of sea states:

(it could be done with the discrete notations as well)

$$\eta(\mathbf{r},t) = \int_{\Omega_{\mathbf{k},\omega}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \underbrace{dZ(\mathbf{k},\omega)}_{\text{Spectral random measure}}$$

Spectral random measure



(Pierson et al. 1985)

Spectral Representation of Sea States

- This spectral representation corresponds to the **Eulerian** description of the sea surface.

- Different individual wave components are uncorrelated (statistically independent).

$$\mathsf{E}\left[dZ(\mathbf{k},\omega)dZ^*(\mathbf{k}',\omega')\right] = 0$$

$$\forall \omega \neq \omega' \quad \forall \mathbf{k} \neq \mathbf{k}'$$

$$\mathsf{E}\left[dZ(\mathbf{k},\omega)\right] = 0$$

$$\mathsf{E}\left[\eta\right] = 0$$

Zero-mean Gaussian process
(for the Eulerian description)

Three-dimensional Wave Spectrum

$$F^{(3)}(\mathbf{k},\omega)d^2kd\omega = \mathsf{E}\left[\left|dZ(\mathbf{k},\omega)\right|^2\right]$$

$$F^{(3)}(\mathbf{k},\omega) = F^{(3)}(-\mathbf{k},-\omega)$$

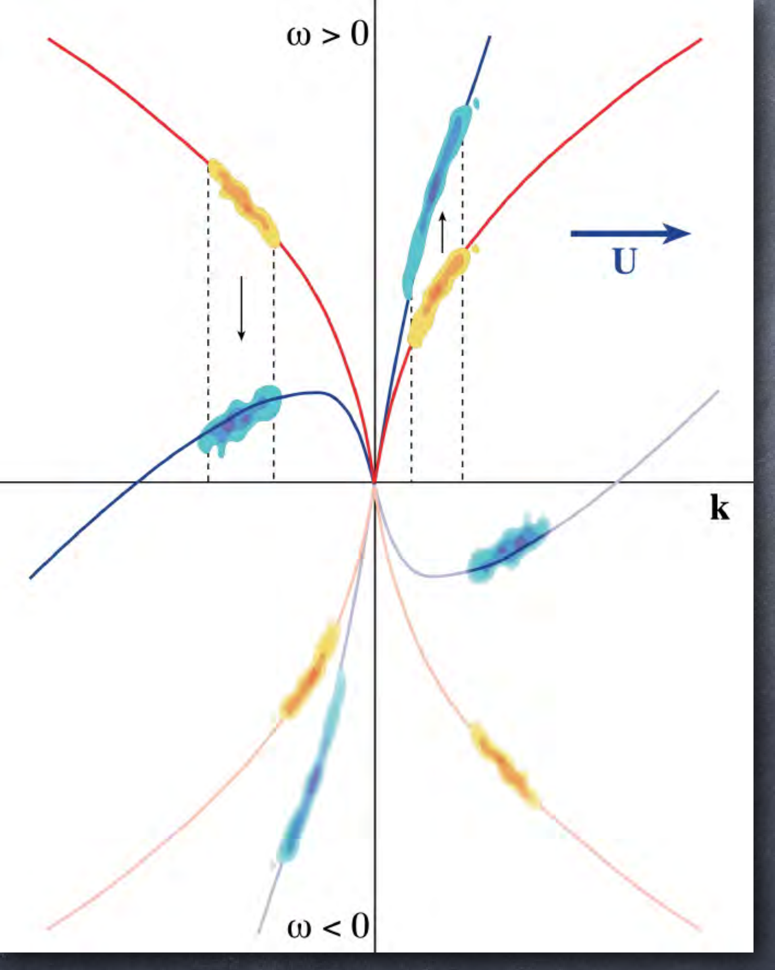
Variance of the sea surface

Zeroth-order moment

$$\sigma^2 = \mathsf{Var}\left[\eta\right] = \int_{\Omega_{\mathbf{k},\omega}} F^{(3)}(\mathbf{k},\omega)d^2kd\omega = m_0$$

Structure of the 3D Wave Spectrum

$$F^{(3)}(\mathbf{k}, \omega) = F^{(3)}(-\mathbf{k}, -\omega)$$



Alternative Wave Spectral Descriptions

- Different spectral density functions can be derived by integrating the 3D wave spectrum over different subsets of the spectral domain.
- All these spectral density functions must preserve the total energy (e.g. the variance of the wave elevation process).
- The transformations of the spectral density functions assume:
  - The dispersion relation.
  - The wave field is statistically homogeneous in space.
  - The wave field is statistically stationary in time.
    - The sea surface elevation is assumed to be an Ergodic process.

Alternative Wave Spectral Descriptions: Frequency Spectrum

- This spectral density is obtained integrating over all the wave number domain.
- It represents the spectrum obtained from a point measurement.
  - F.e. a record of a buoy moored at a fixed ocean location.

$$S(\omega) = \int_{\Omega_{\mathbf{k}}} F^{(3)}(\mathbf{k}, \omega) d^2 k$$

$$S(\omega) = S(-\omega)$$

$$S(\omega) \longmapsto 2 \cdot S(\omega) \quad , \quad \forall \omega > 0$$

Alternative Wave Spectral Descriptions: Directional Wave Number Spectrum

- Integrating over all the frequency domain:

$$F^{(2)}(\mathbf{k}) = \int_{\Omega_\omega} F^{(3)}(\mathbf{k}, \omega) d\omega$$

- This spectrum presents symmetric dependence on the wave propagation direction

$$F^{(2)}(\mathbf{k}) = F^{(2)}(-\mathbf{k})$$

- There is an ambiguity of 180 degrees.

Alternative Wave Spectral Descriptions: Unambiguous Directional Wave Number Spectrum

- Integrating over the positive frequency domain:

$$F_{+}^{(2)}(\mathbf{k}) = 2\int_{0}^{\omega_c} F^{(3)}(\mathbf{k},\omega)d\omega$$

- This spectrum resolves the directional ambiguity

$$F_{+}^{(2)}(\mathbf{k}) \neq F_{+}^{(2)}(-\mathbf{k})$$

Alternative Wave Spectral Descriptions: 3D spectrum from 2D spectrum

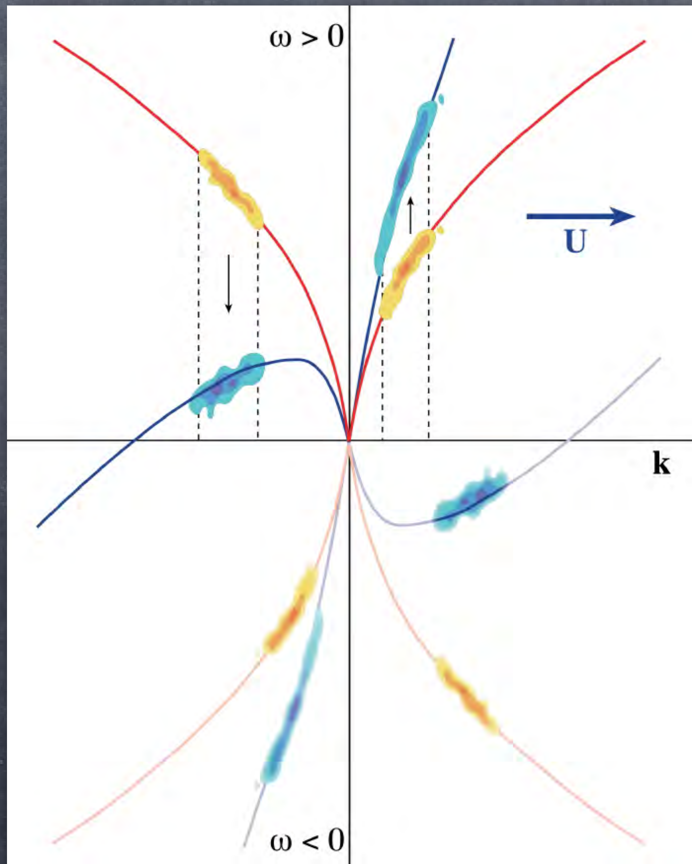
- Assuming the dispersion relation the three-dimensional wave spectrum can be obtained from the unambiguous wave number spectrum as

$$F^{(3)}(\mathbf{k},\omega)d^2kd\omega = \left\{ \frac{F_+^{(2)}(\mathbf{k})}{2}\delta[\omega-\varpi(\mathbf{k})] + \frac{F_+^{(2)}(-\mathbf{k})}{2}\delta[\omega+\varpi(\mathbf{k})] \right\} d^2kd\omega$$

$$F^{(2)}(\mathbf{k}) = \frac{1}{2}\left[F_+^{(2)}(\mathbf{k}) + F_+^{(2)}(-\mathbf{k})\right]$$

Alternative Wave Spectral Descriptions: 3D spectrum from 2D spectrum

$$F^{(3)}(\mathbf{k},\omega)d^2kd\omega = \left\{ \frac{F_+^{(2)}(\mathbf{k})}{2}\delta[\omega - \varpi(\mathbf{k})] + \frac{F_+^{(2)}(-\mathbf{k})}{2}\delta[\omega + \varpi(\mathbf{k})] \right\} d^2kd\omega$$



Alternative Wave Spectral Descriptions: Directional Spectrum (I)

- Transforming the coordinate system from Cartesian to polar coordinates

$$(k_x, k_y) \longmapsto (k, \theta)$$

$$k = \sqrt{k_x^2 + k_y^2} \qquad \theta = \tan^{-1}\left(\frac{k_y}{k_x}\right)$$

Wave propagation direction

$$\tilde{F}^{(2)}(k, \theta) = F_+^{(2)}[\mathbf{k}(k, \theta)] \cdot k$$

Jacobian from Cartesian to polar coordinates

Alternative Wave Spectral Descriptions: Directional Spectrum (II)

- Transforming from wave number to frequency:
- The dispersion relation is assumed

$$(k, \theta) \longmapsto (\omega, \theta)$$

$$E(\omega, \theta) = \tilde{F}^{(2)}[k(\omega), \theta] \cdot \frac{dk}{d\omega}$$

Jacobian:
Group Velocity

Alternative Wave Spectral Descriptions: Directional Spectrum (III)

- The frequency-direction spectrum is factorized as

$$E(\omega, \theta) = S(\omega) D(\omega, \theta)$$

Directional spreading function: $D(\omega, \theta)$; $\displaystyle\int_{-\pi}^{\pi} D(\omega, \theta) d\theta = 1$

$$S(\omega) = \int_{-\pi}^{\pi} E(\omega, \theta) d\theta$$

Frequency Spectrum

Alternative Wave Spectral Descriptions

$$F^{(3)}(\mathbf{k}, \omega)$$

$$S(\omega) = \int_{\Omega_{\mathbf{k}}} F^{(3)}(\mathbf{k}, \omega) d^2 k$$

$$F_+^{(2)}(\mathbf{k}) = 2 \int_0^{\omega_c} F^{(3)}(\mathbf{k}, \omega) d\omega$$

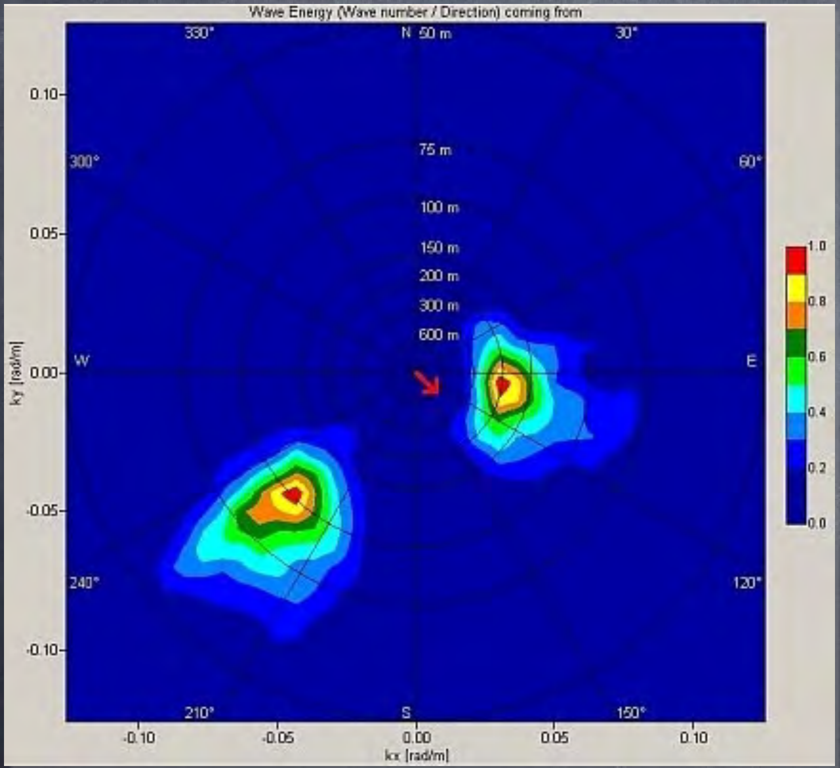
$$\tilde{F}^{(2)}(k, \theta) = F_+^{(2)}[\mathbf{k}(k, \theta)] \cdot k$$

$$E(\omega, \theta) = \tilde{F}^{(2)}[k(\omega), \theta] \cdot \frac{dk}{d\omega}$$

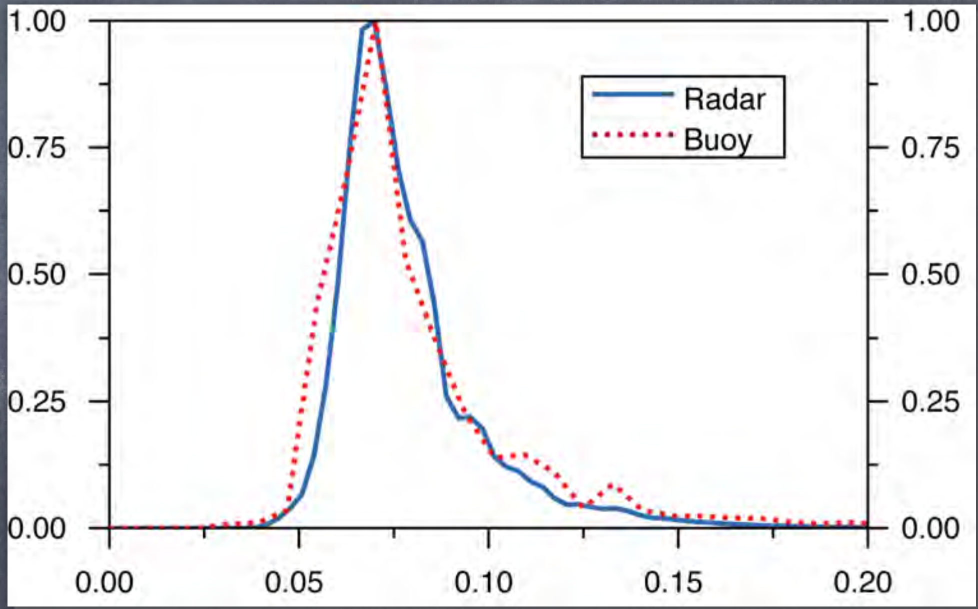
$$S(\omega) = \int_{-\pi}^{\pi} E(\omega, \theta) d\theta$$

Examples of Wave Spectra

Wave number spectrum $F_{+}^{(2)}(\mathbf{k})$



Frequency spectrum $S(f)$



wave frequency [Hz]

Sea state parameters derived from the wave spectra

- Spectral moments: $\omega = 2\pi f$

$$m_j = \int f^j S(f) df \qquad (j = \ldots,\ -1,\ 0,\ 1,\ 2,\ \ldots)$$

- Significant wave height $H_s = 4\sqrt{m_0}$

- Mean period estimations:

$$T_e = \frac{m_{-1}}{m_0}$$

$$T_{m01} = \frac{m_0}{m_1}$$

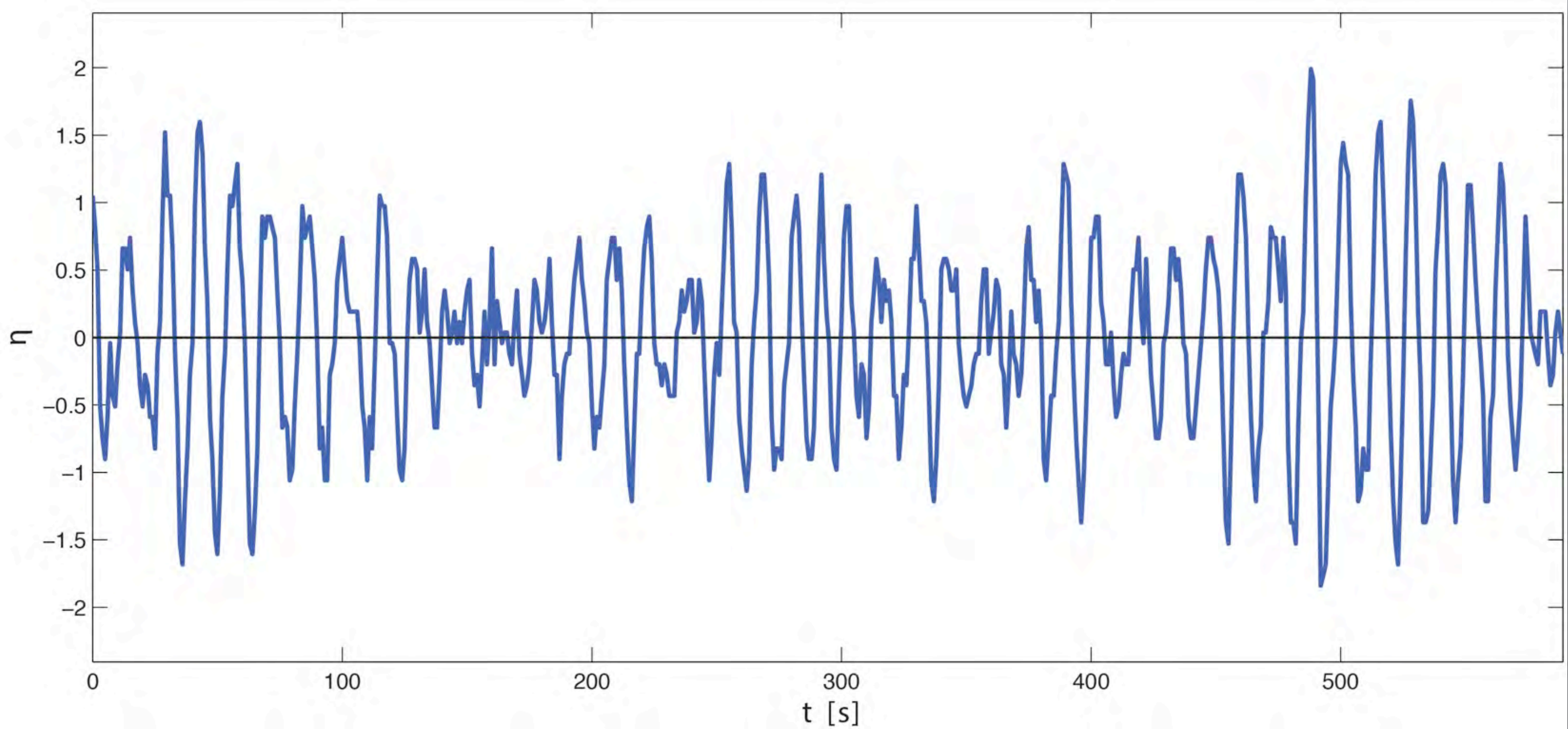
$$T_{m02} = \sqrt{\frac{m_0}{m_2}}$$

Sea state parameters derived from the wave spectra

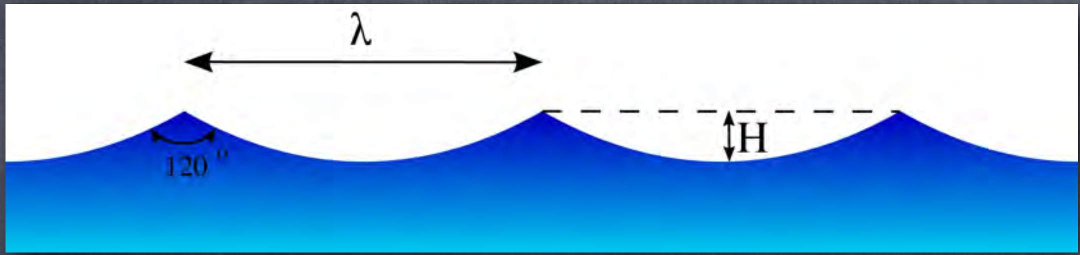
- Other parameters can be derived as
  - Peak period,
  - Significant steepness,
  - Stability parameters,
  - Wave grouping parameters,
  - etc.
- From the de directional spectra (wave number, etc.) additional parameters are:
  - Mean, peak wave length,
  - Mean, peak wave propagation direction,
  - etc.

Examples of Gaussian waves

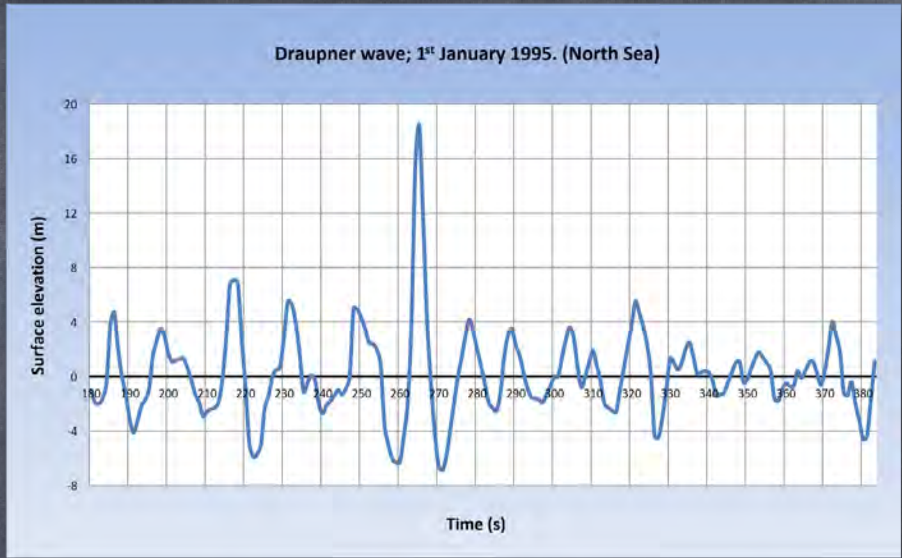
- Example of wave record measured by a buoy deployed in the Northern coast of Spain (Bay of Biscay)



Examples of non Gaussian waves



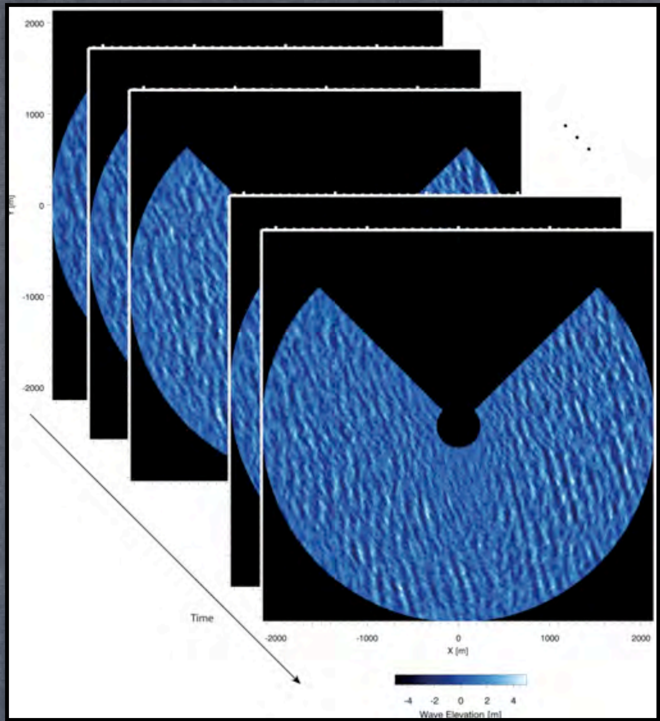
Stokes waves



Freak wave (New Year Wave)

Breaking wave

FFT Estimation of the Three-Dimensional Spectrum



Spectral Analysis Techniques Applied to Image Time Series

Sampling of an Image Time Series

- Consider time series of images sampled in space and time

$$\xi_{mnl} = \xi(x_m, y_n, t_l)$$

$$x_m = m \cdot \Delta x \quad ; \quad m = 0, \ldots, N_x - 1$$

$$y_n = n \cdot \Delta y \quad ; \quad n = 0, \ldots, N_y - 1$$

$$t_l = l \cdot \Delta t \quad ; \quad l = 0, \ldots, N_t - 1$$

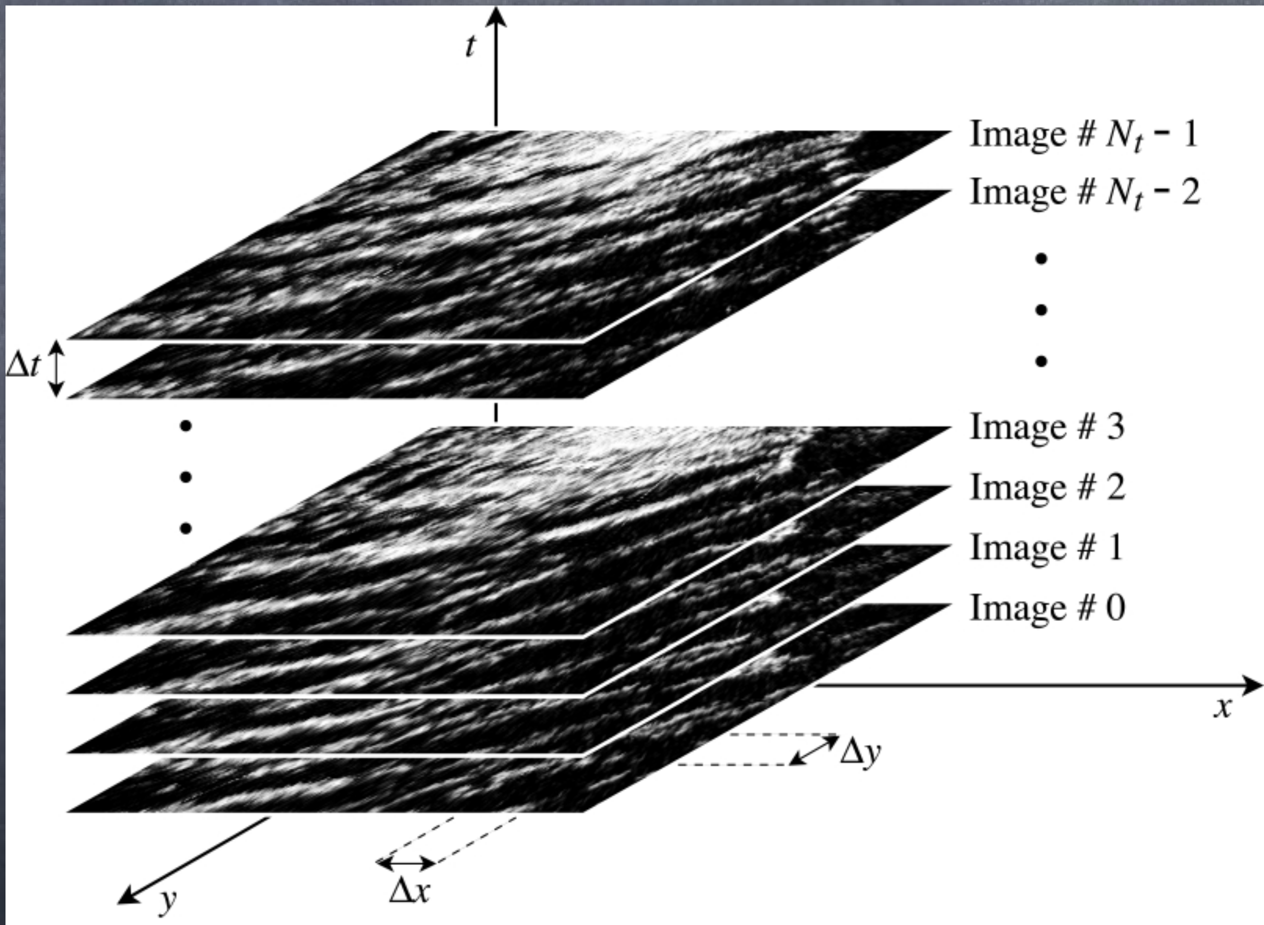
Sampling time: Δt

Spatial resolution along X-axis: Δx

Spatial resolution along Y-axis: Δy

Sampling of an Image Time Series

- Example: temporal sequence of X-band marine radar images



3D Discrete Fourier Transform (3D-DFT)

- The three-dimensional Fourier coefficients of $\xi_{mnl} = \xi(x_m, y_n, t_l)$

$$\Xi_{m'n'l'} = \frac{1}{N_x N_y N_t} \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} \sum_{l=0}^{N_t-1} \xi_{mnl}\, e^{-i2\pi(mm'+nn'+ll')/(N_x N_y N_t)}$$

$$m' = 0, \ldots, N_x - 1$$

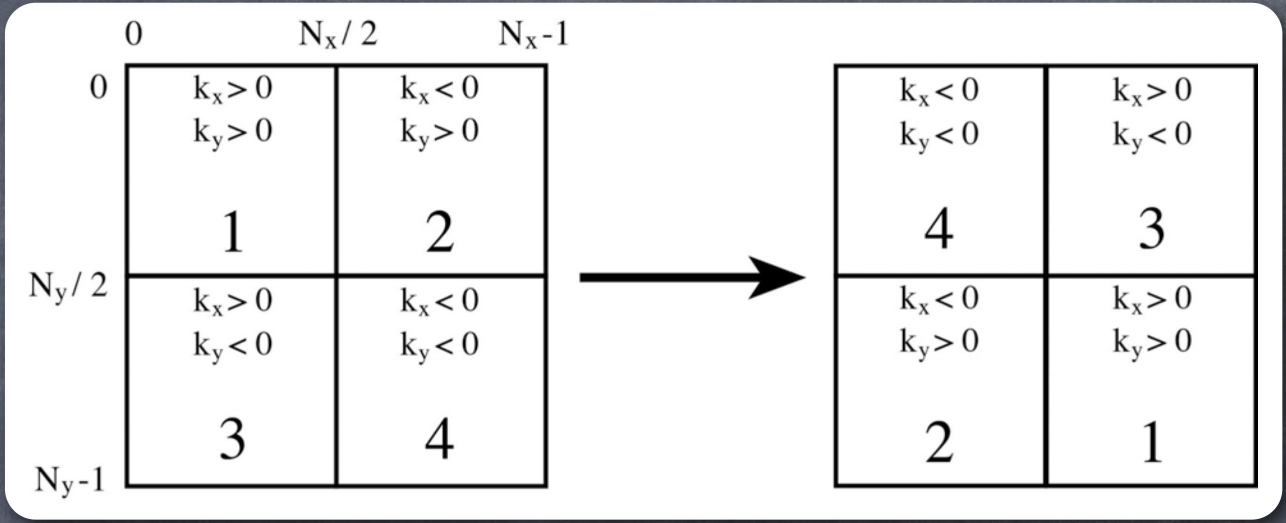
$$\Xi_{m'n'l'} = \Xi(k_{x_{m'}}, k_{y_{n'}}, \omega_{l'}) \qquad n' = 0, \ldots, N_y - 1$$

$$l' = 0, \ldots, N_t - 1$$

- In practice, the DFT is computed using the Fast Fourier Transform (FFT) algorithm.
- The output of a FFT function has to be reordered to have the negative branch of each spectral variable before the positive branch.

3D Discrete Fourier Transform (3D-DFT)

- Two-dimensional example of data reordering (half-period shift)



- Most of the data analysis softwares include a specific function for that:
- Examples:
  - IDL: **SHIFT** function.
  - Matlab: **fftshift** function.

3D Discrete Fourier Transform (3D-DFT)

- Scheme of the application of the 3D FFT:

$$\xi(x_m, y_n, t_l) \xrightarrow{\text{3D FFT}} \Xi(k_{x_{m'}}, k_{y_{n'}}, \omega_{l'}) \xrightarrow{\text{half-period shift}} \Xi(k_{x_{m'}}, k_{y_{n'}}, \omega_{l'})$$

3D Discrete Fourier Transform (3D-DFT) Spectral Variables

- Once the half-period shift is carried out, the sampled spectral variables are

$$k_{x_{m'}} = -k_{x_c} + m' \cdot \Delta k_x \qquad ; \qquad m' = 0, \ldots, N_x - 1$$

$$k_{y_{n'}} = -k_{y_c} + n' \cdot \Delta k_y \qquad ; \qquad n' = 0, \ldots, N_y - 1$$

$$\omega_{l'} = -\omega_c + l' \cdot \Delta\omega \qquad ; \qquad l' = 0, \ldots, N_t - 1$$

$$k_{x_c} = \frac{\pi}{\Delta x} \qquad \Delta k_x = \frac{2\pi}{N_x \Delta x}$$

$$k_{y_c} = \frac{\pi}{\Delta y} \qquad \Delta k_y = \frac{2\pi}{N_y \Delta y}$$

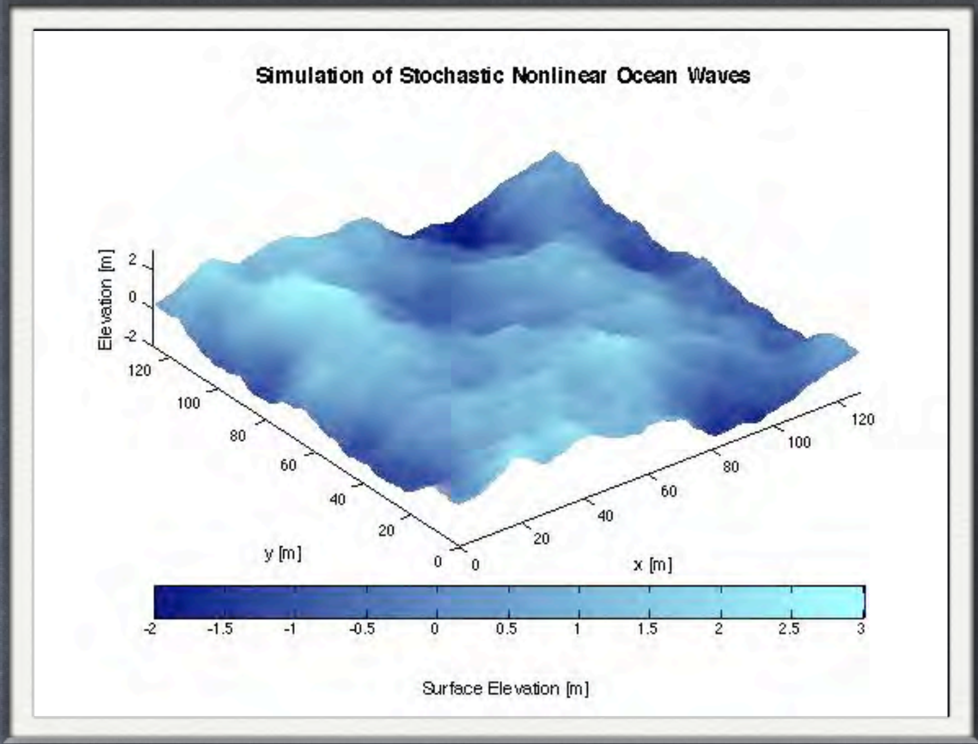
$$\omega_c = \frac{\pi}{\Delta t} \qquad \Delta\omega = \frac{2\pi}{N_t \Delta t}$$

3D Discrete Fourier Transform (3D-DFT) Spectral Estimation

- The spectral estimation using the DFT is called **periodogram**.

$$F^{(3)}(m', n', l') = \frac{1}{\Delta k_x \Delta k_y \Delta \omega} \left| \Xi(m', n', l') \right|^2$$

$$F^{(3)}(m', n', l') \equiv F^{(3)}(k_{x_{m'}}, k_{y_{n'}}, \omega_{l'})$$



3D Discrete Fourier Transform (DFT) for wave analysis

3D Discrete Fourier Transform (DFT) for wave analysis

- The DFT permits to estimate the complex Fourier coefficients

$$\eta(x, y, t) \longrightarrow \boxed{\text{3D DFT}} \longrightarrow c(k_x, k_y, \omega)$$

- Or the amplitude and phase of the spectral components:

$$a(k_x, k_y, \omega) \qquad \varphi(k_x, k_y, \omega)$$

- The computation of the DFT from its definition needs extremely large CPU time
  - Fast Fourier Transform (FFT) algorithms permit to estimate the DFT in short CPU time

Examples

- Three different examples to illustrate ow the FFT works in 1D, 2D and 3D:
  - 1D (t): Time series of wave elevations measured by a buoy at a fixed location.
  - 2D (x, y): Sea surface elevations to estimate the 2D wave number spectra:
    - Three cases: wind sea, swell, bimodal sea state.
  - 3D (x, y, t): Time series of sea surface elevations to estimate:
    - 3D wave number-frequency spectrum
    - 2D wave number spectrum
    - Three cases: wind sea, swell, bimodal sea state.

Example 1: Wave elevation time series

- Read the data file WaveTimeSeries.dat
- Estimate the variance of the time series.
- Estimate the spectral density using the function fft.
- Plot the spectral density for each frequency.
  - What do you see in the plot?
- Compute the variance from the spectral estimation.
- Proposed exercise:
  - Compute the autocorrelation function from the spectral density.

Example 2: 2D Sea surface elevations (I)

- Read the data files for each case (Matlab data format):
  - Wind sea, files: WindSea2D_1.dat, WindSea2D_2.dat and WindSea2D_3.dat
  - Swell, files: Swell2D_1.dat, Swell2D_2.dat and Swell2D_3.dat
  - Bimodal sea state, files: Bimodal2D_1.dat, Bimodal2D_2.dat and Bimodal2D_3.dat
- Estimate the variance of the surfaces
- Plot the spectra
  - Compare the different obtained the spectra for each case.
    - Are they equal? Why?
  - Do they present even-symmetry on the wave numbers? Why?
- Estimate the variance from the spectral estimation.

Example 2: 2D Sea surface elevations (II)

- Way to proceed:
  - Load the data in the Matlab workspace
  - Compute the 2D FFT by using the Matlab functions fft2 or fftn.
  - shift a half-period in the wave number-frequency domain using the function fftshift
  - compute the 32D spectral estimation (wave number space)

Example 3: spatio-temporal evolution of a wave field (I)

- Purpose: analyze a spatio-temporal wave field evolution by using a 3D FFT.
- Data:
  - Matlab data files:
    - Wind sea: WindSea.mat
    - Swell: Swell.mat
    - Bimodal sea state: Bimodal.mat

Example 3: spatio-temporal evolution of a wave field (II)

- Way to proceed (I):
  - Load the data in the Matlab workspace
  - Estimate the variance from the wave elevation field
  - Compute the 3D spectral estimation (wave number and frequency space)
    - Compute the 3D FFT using the function fftn
    - Shift a half-period in the wave number-frequency domain using the function fftshift
    - Estimate the 3D spectrum.

Example 3: spatio-temporal evolution of a wave field (II)

- Way to proceed (II):
  - Identify the dispersion relation by applying different transects on the wave number-frequency domain.
  - Estimate the variance of the wave field from the 3D spectrum
  - compute the 2D wave number spectrum, by integrating over all the positive frequencies.
    - Is this wave number spectrum symmetric? Why?
  - Estimate the variance of the wave field from the 2D spectrum

Thanks