Data Analysis for Earth, Marine, and Environmental Sciences

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Preface

This is a Quarto book.

To learn more about Quarto books visit https://quarto.org/docs/books.

1 Introduction

This is a book created from markdown and executable code.

See Knuth (1984) for additional discussion of literate programming.

Part I

Time Series

This section is about time series analysis.

2 Fourier Analysis

This section is an introduction to Fourier Analysis. We will cover a variety of topics including

- Complex Numbers Review
- Series Expansions: exp, cosine, sine Euler's Formulae
- Definition of Fourier Transform (Continuous) Fourier Transform Pairs Amplitude and Phase
- Frequency, Period, Sampling
- Nyquist Frequency
- Convolution vs. Correlation Periodogram
- Leakage and Tapering

2.1 Fourier Basic Idea

How would you describe this signal?:

The signal can be represented as a sum of different sinusoids:

- Signal S1: .2 Hz amplitude 10
- Signal S2: 3 Hz amplitude .4
- Signal S3 = S1 + S2

One could convey all the information with 6 numbers:

- dt, length
- 10, .2
- .4, 3

2.2 Discrete Sampling

- Red: Period of Sine Wave (s)
- Blue: Amplitude of Sine Wave
- δt : sampling rate (s)

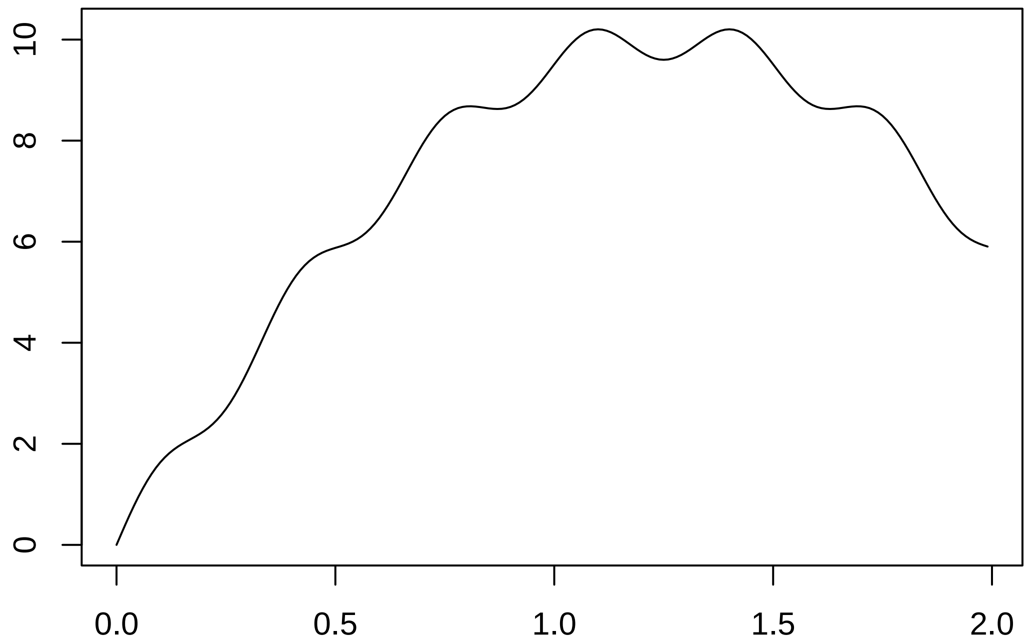

Figure 2.1: A single time series

2.3 Cycles, Phase and Frequency

The rotating pen height in Figure 2.4 represents the signal.

$$Y_i = A \sin\left(2\pi x_i / X + \phi\right)$$

$$\alpha_i = \left(2\pi x_i / X + \phi\right)$$

2.4 Time Series: Basics

- Signal Characteristics:
  - period = T/cycle
  - frequency $f = 1/T$ cycles/s
- Sampling
  - sample rate $=\Delta t$
  - Sampling Frequency $= \frac{1}{\Delta t} = f_{sampling}$

Units:
if $y = \cos(\theta) = \cos(\omega t)$

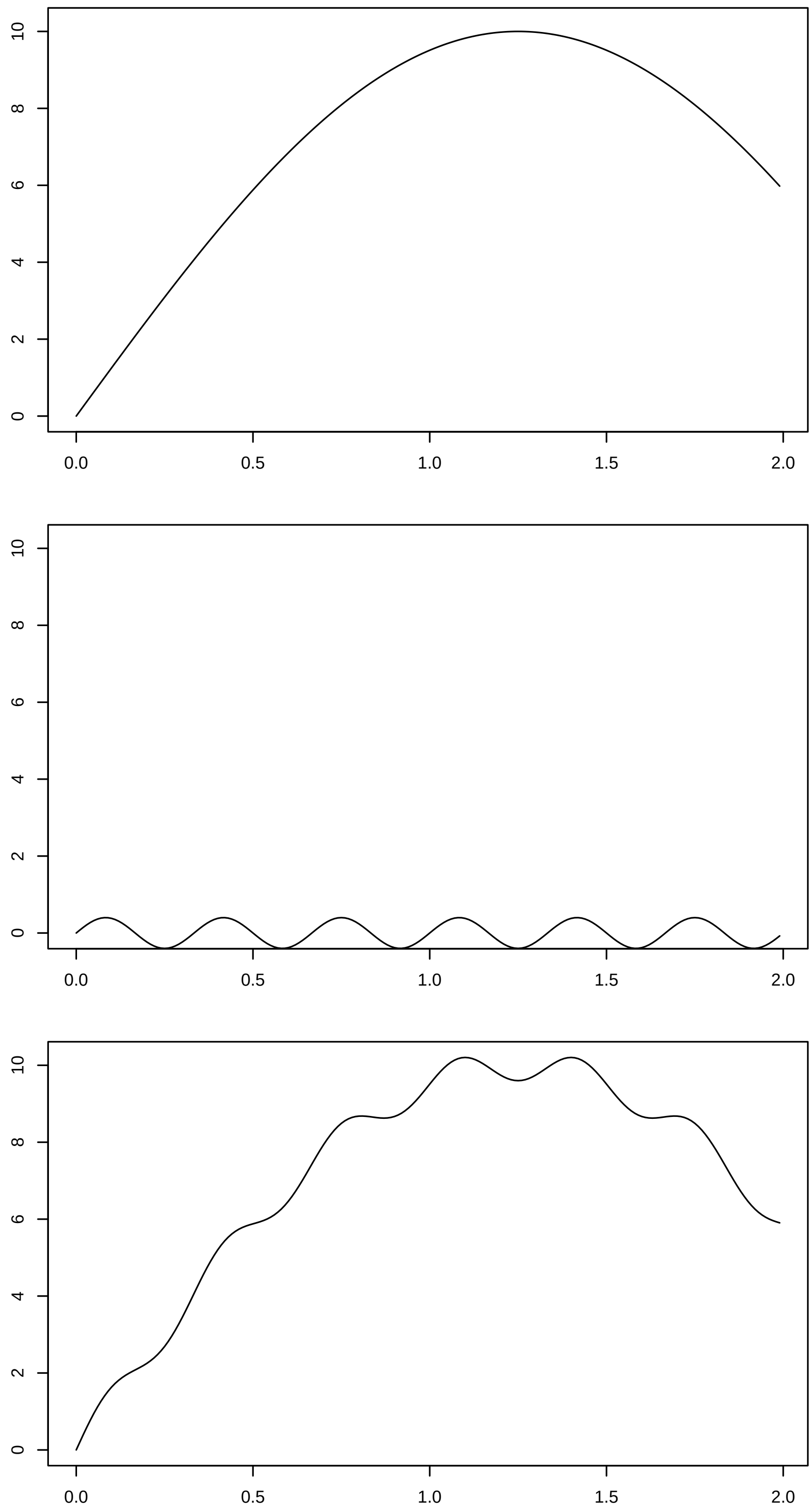

Figure 2.2: The same signal can be represented as a sum of sinusoids.

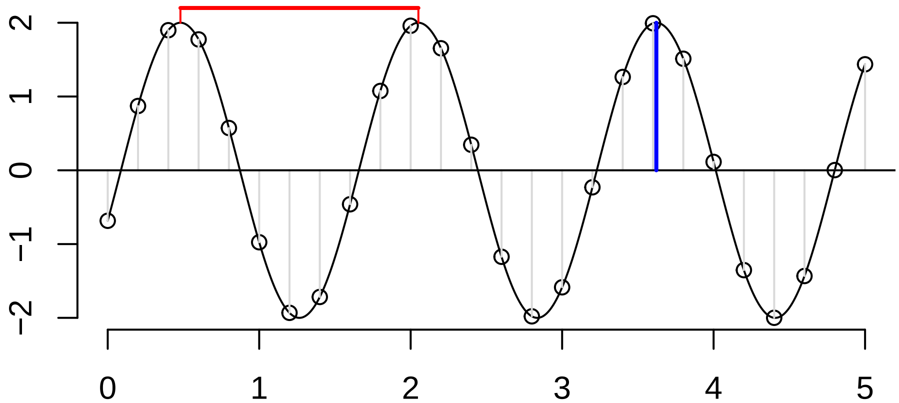

Figure 2.3: A wave is sampled at discrete points in time.

- θ is in units of radians $= 2\pi f t$
- there are 2π radians per cycle
- we define the angular frequency $\omega = 2\pi f$ radians/sec
- time t is defined as $t = i \cdot \Delta t$ where i is the sample
- $T = \sum i \cdot \Delta t$ is the total time

$$y_k = A\cos(\omega t - \phi)$$

where ϕ is the phase and ω is the frequency.

$$\begin{aligned}
y_k &= A\cos(\omega t - \phi) \\
&= A\cos(\omega t)\cos\phi + A\sin(\omega t)\sin\phi \\
&= \alpha_k\cos(\omega t) + \beta_k\sin(\omega t)
\end{aligned}$$

Trig Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

To draw one complete wave form, the pen must revolve completely around the disk, moving through 360° or 2π radians. Suppose we start the device operating with the pen initially resting at an arbitrary location on the paper that we will call 0. The angle α between the pen, the center of the disk, and the center line on the paper is some value ϕ. These are shown in Figure 4.51. If we allow the device to operate for a distance x_i down the record and then stop, the pen will be resting

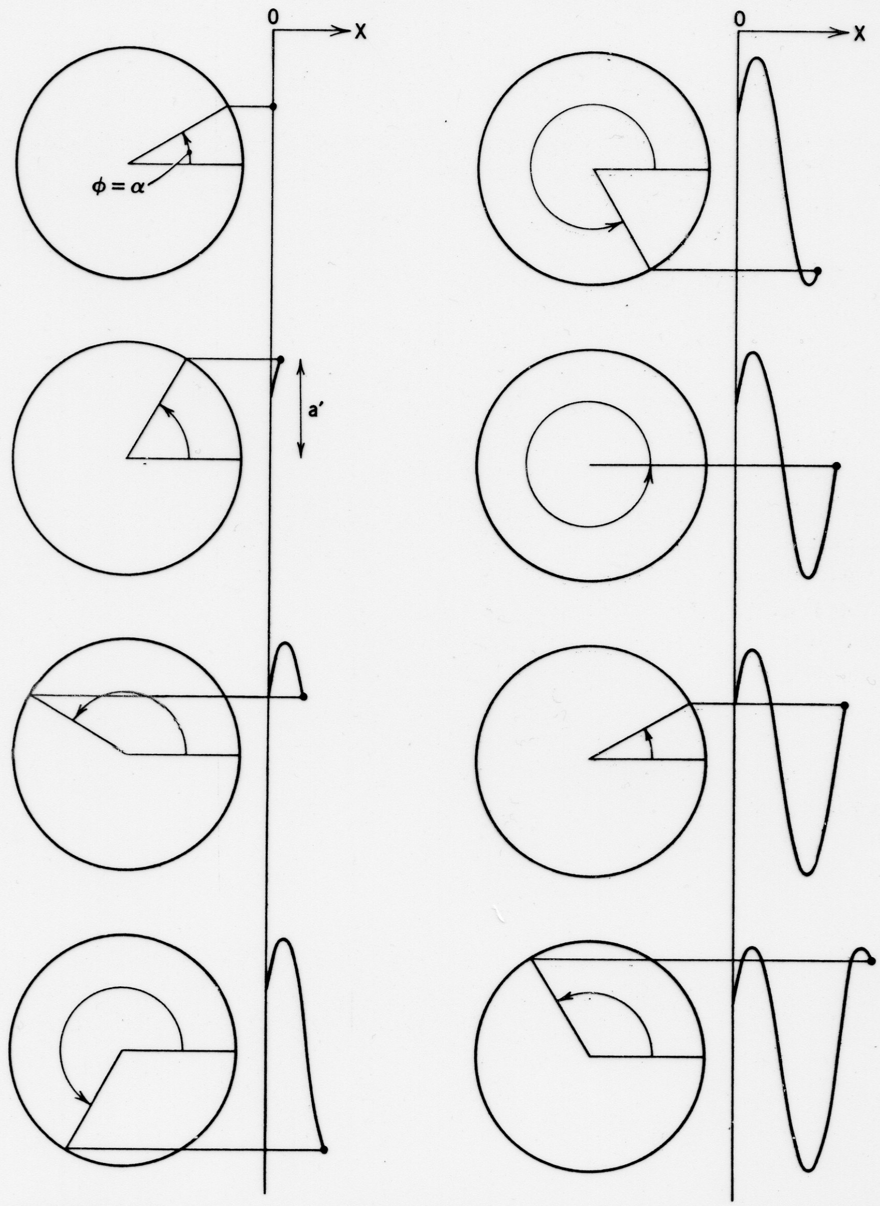

FIGURE 4.51 Progressive changes in phase angle from an initial value ϕ. At successive values of x_i, $Y_i = A \sin(2\pi x_i/X + \phi)$ and $\alpha_i = (2\pi x_i/X + \phi)$.

Figure 2.4

3 Summary

In summary, this book has no content whatsoever.

References

Knuth, Donald E. 1984. "Literate Programming." *Comput. J.* 27 (2): 97–111. https://doi.org/10.1093/comjnl/27.2.97.