A Crash Course on Quantal Response Equilibrium

Po-Hsuan Lin

Fall 2022 EC 106

October 12, 2022

Asymmetric Matching Pennies Game

- This is the game we played in the first class.
- Suppose $A > 0$.

	L	R
L	$A, 0$	$0, 1$
R	$0, 1$	$1, 0$

- A (mixed) strategy σ_i is a probably distribution over the action set $S_1 = S_2 = \{L, R\}$.
- A strategy profile is a (mixed) Nash equilibrium if for all i,

$$\mathbb{E}u_i(\sigma_i, \sigma_{-i}) \geq \mathbb{E}u_i(\sigma_i', \sigma_{-i}) \quad \text{for any } \sigma_i' \in \Delta(S_i).$$

Mixed Nash Equilibrium

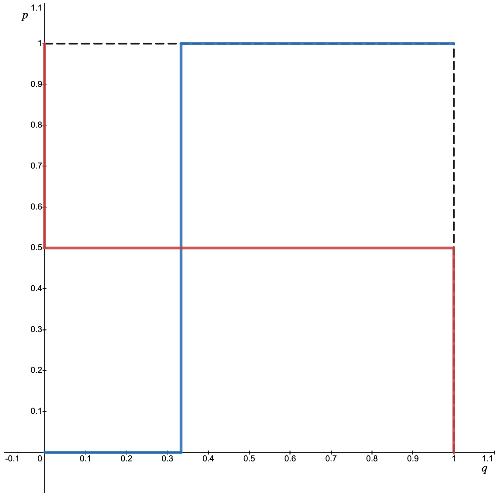
- Let $\sigma_1(L) \equiv p$ and $\sigma_2(L) \equiv q$. This describes the action set.
- Given q, player 1 would choose L if and only if

$$\mathbb{E}u_1(L, q) = Aq \geq 1 - q = \mathbb{E}u_1(R, q) \iff q \geq \frac{1}{1 + A}.$$

- Given p, player 2 would choose L if and only if

$$\mathbb{E}u_2(p, L) = 1 - p \geq p = \mathbb{E}u_2(p, R) \iff p \leq \frac{1}{2}.$$

Best Response Correspondence



- Blue (Red) is player 1 (player 2)'s BR correspondence.
- The intersection $(q^*, p^*) = (\frac{1}{1+A}, \frac{1}{2})$ is the unique mixed NE.

Quantal Response Equilibrium

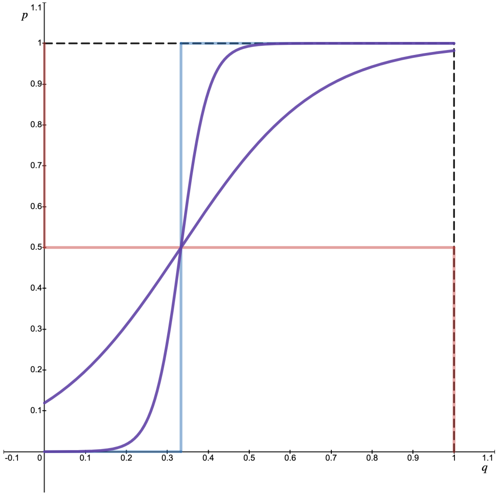
- Players don't do best response. They do **better** response.
- We consider **Logit QRE**: players' choice probabilities are determined by the logit function with the parameter λ.

- Given q, since $\mathbb{E}u_1(L,q) = Aq$ and $\mathbb{E}u_1(R,q) = 1-q$, player 1 would choose L with probability

$$p = \frac{e^{\lambda \mathbb{E}u_1(L,q)}}{e^{\lambda \mathbb{E}u_1(L,q)} + e^{\lambda \mathbb{E}u_1(R,q)}} = \frac{1}{1 + e^{\lambda[\mathbb{E}u_1(R,q) - \mathbb{E}u_1(L,q)]}}$$
$$= \frac{1}{1 + e^{\lambda[1-(1+A)q]}}.$$

- Extreme cases:
  - When $\lambda \to 0$, $p \to \frac{1}{2}$ (uniformly randomize)
  - When $\lambda \to \infty$, p converges to the BR correspondence

Player 1's Quantal Response Function



- Player 1's quantal response functions given different λ.
- When λ gets larger, QR gets closer to BR.

Quantal Response Equilibrium

- Similarly, given any p, player 2 also makes quantal response. As $\mathbb{E}u_2(p, L) = 1 - p$ and $\mathbb{E}u_2(p, R) = p$,

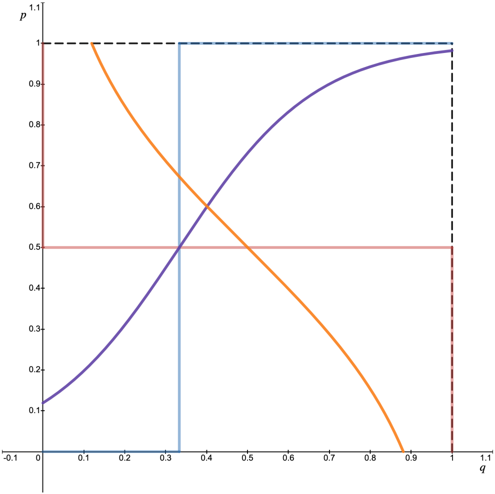
$$q = \frac{e^{\lambda \mathbb{E}u_2(p,L)}}{e^{\lambda \mathbb{E}u_1(p,L)} + e^{\lambda \mathbb{E}u_1(p,R)}} = \frac{1}{1 + e^{\lambda[\mathbb{E}u_2(p,R) - \mathbb{E}u_1(p,L)]}}$$
$$= \frac{1}{1 + e^{\lambda[2p-1]}}.$$

- QRE maintains mutual consistency. Mathematically, QRE is the solution of the following system of nonlinear equations.

$$p = \frac{1}{1 + e^{\lambda[1-(1+A)q]}}$$
$$q = \frac{1}{1 + e^{\lambda[2p-1]}}.$$

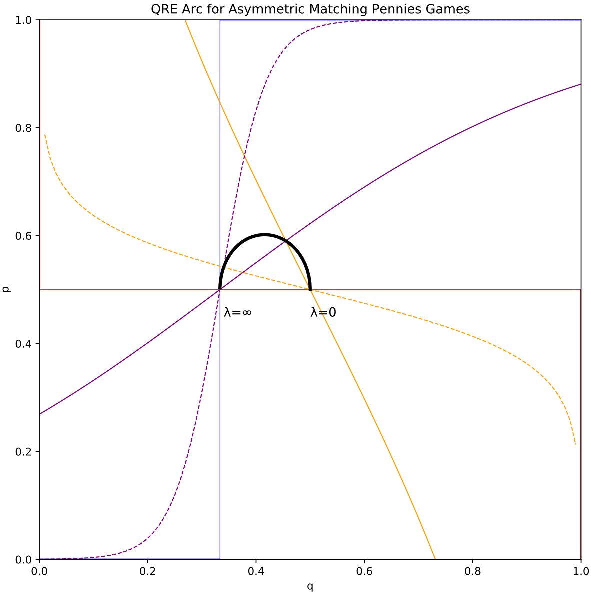
- You need to solve for numerical solutions...

Quantal Response Equilibrium



- The intersection of player 1 and player 2's quantal response functions is the QRE (given a specific λ).

QRE Arc


QRE Arc for Asymmetric Matching Pennies Games
1.0
0.8
0.6
0.4
0.2
0.0
p
λ=∞
λ=0
0.0
0.2
0.4
0.6
0.8
1.0
q