Lab Section: Tues PM

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Lab 3 – P and PI control of flywheel velocity

Prelab

a) $$\frac{\Omega(s)}{\Omega_r(s)} = \frac{\left(K_p + \frac{K_i}{s}\right)\left(\frac{K}{\tau s+1}\right)}{1+\left(K_p + \frac{K_i}{s}\right)\left(\frac{K}{\tau s+1}\right)}$$

b) $K_{dc} = 8.78 \quad \tau = 0.15 \quad K = 0.172$

$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{\left(K_p + \frac{K_i}{s}\right)\left(\frac{0.172}{0.15s+1}\right)}{1+\left(K_p + \frac{K_i}{s}\right)\left(\frac{0.172}{0.15s+1}\right)}$$

c) $K_i = 0$ $\quad 0 = 1 + K_p\left(\frac{0.172}{0.15s+1}\right)$

$$s = -\frac{20}{3} - 1.1467K_p$$

@ $K_p = 5$: $\quad s = -\frac{62}{5} = -12.4$

@ $K_p = 10$: $\quad s = -18\frac{2}{15} = -18.13$

@ $K_p = 15$: $\quad s = -23\frac{13}{15} \approx -23.87$

d) $K_i = 0 \quad \Omega_r(s) = \frac{1}{s}$

$$e_{ss} = 1 - \lim_{s\to 0} s \cdot \frac{\Omega(s)}{\Omega_r(s)} \cdot \frac{1}{s} = 1 - (1 + 0.172K_p)$$
$$= -0.172K_p$$

@ $K_p = 5$: $e_{ss} = -0.86$

@ $K_p = 10$: $e_{ss} = -1.72$

@ $K_p = 15$: $e_{ss} = -2.58$

Experiment #1. Implement PID code

error = set_point - filt_vel;

d_error = (error-error_pre)/loop_time;

error_pre = error;

*sum_error = sum_error + error*loop_time;*

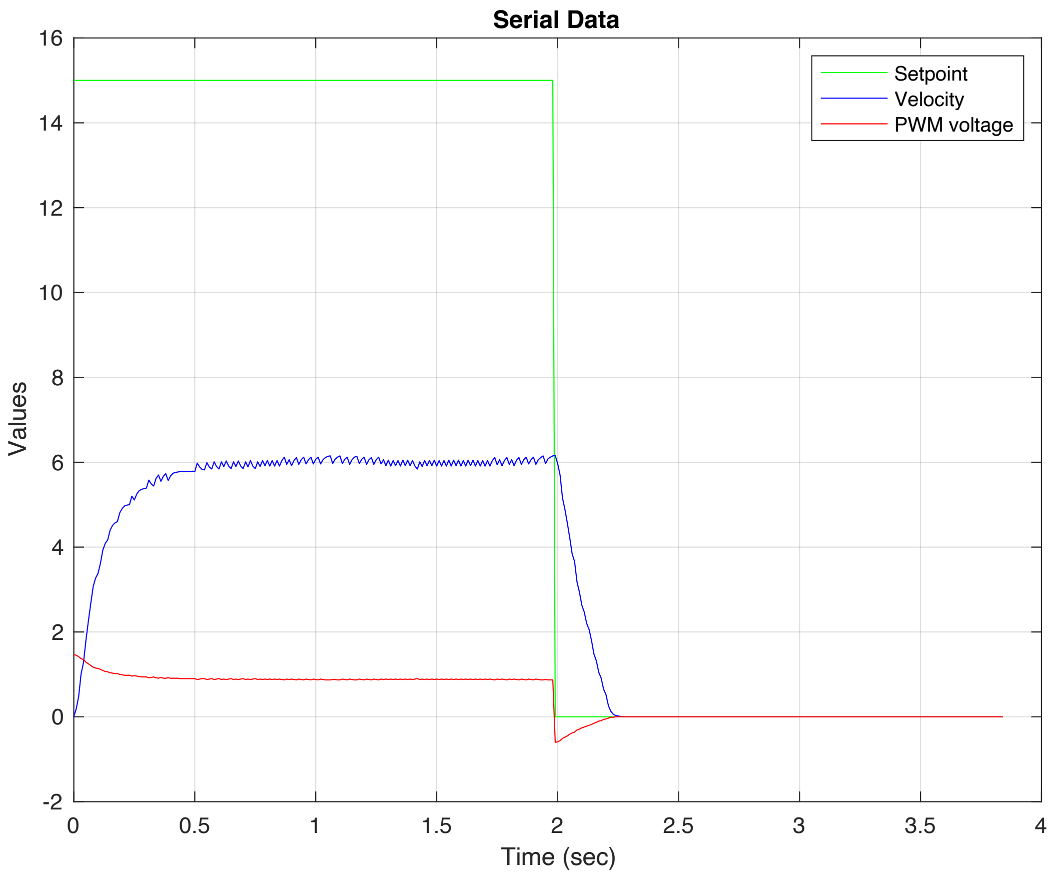
*Pcontrol = Kp*error;*

*Icontrol = Ki*sum_error;*

*Dcontrol = Kd*d_error;*

Experiment #2. P control

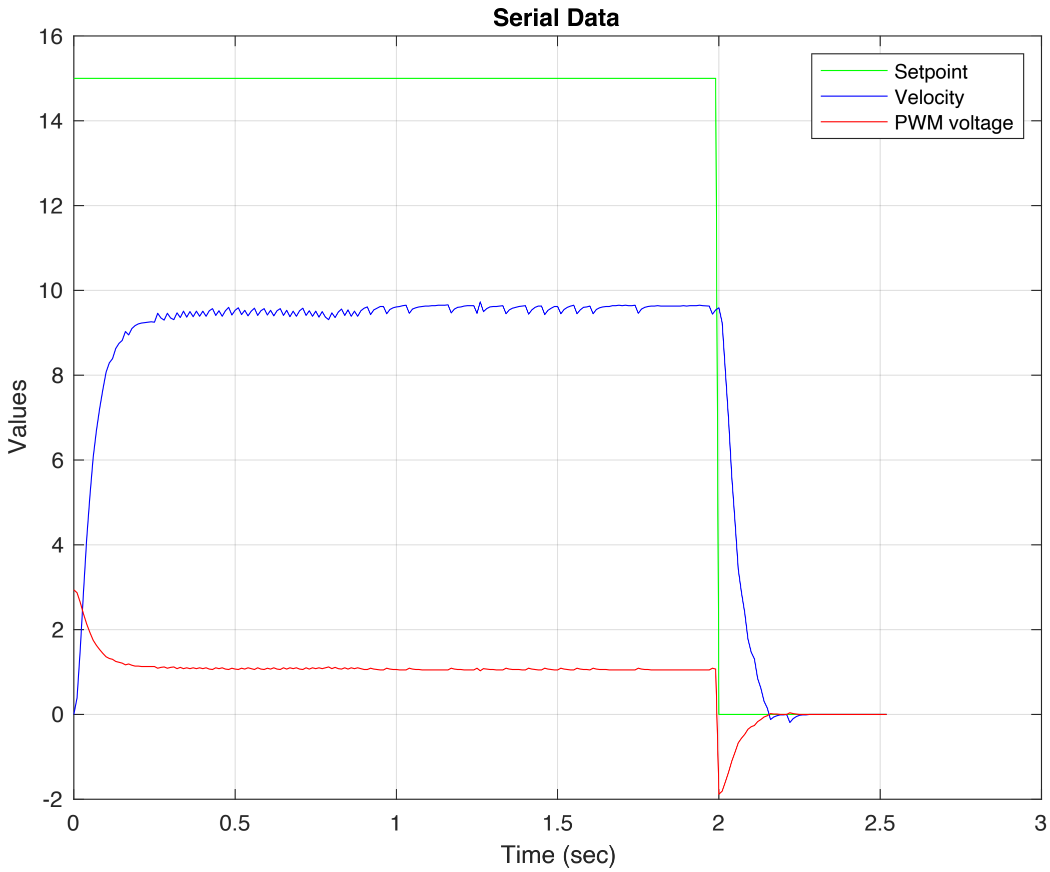
1. Set the desired velocity to 15 rad/s and run the controller with Kp = 5. Select a relevant section of the trace and estimate the time constant τ and the fractional steady-state error.

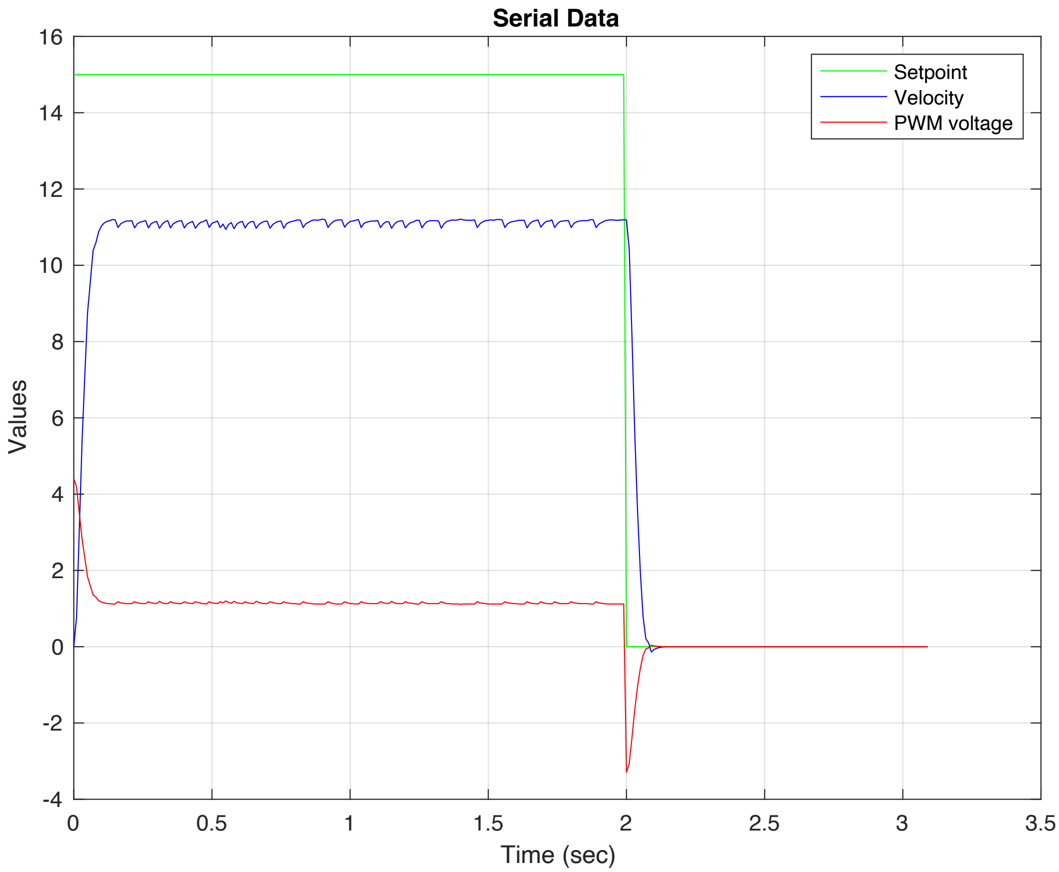


$\tau = 0.12$

$\Delta = \frac{\Omega_r - \Omega_{ss}}{\Omega_r}$ =(15-6)/15 = 0.6

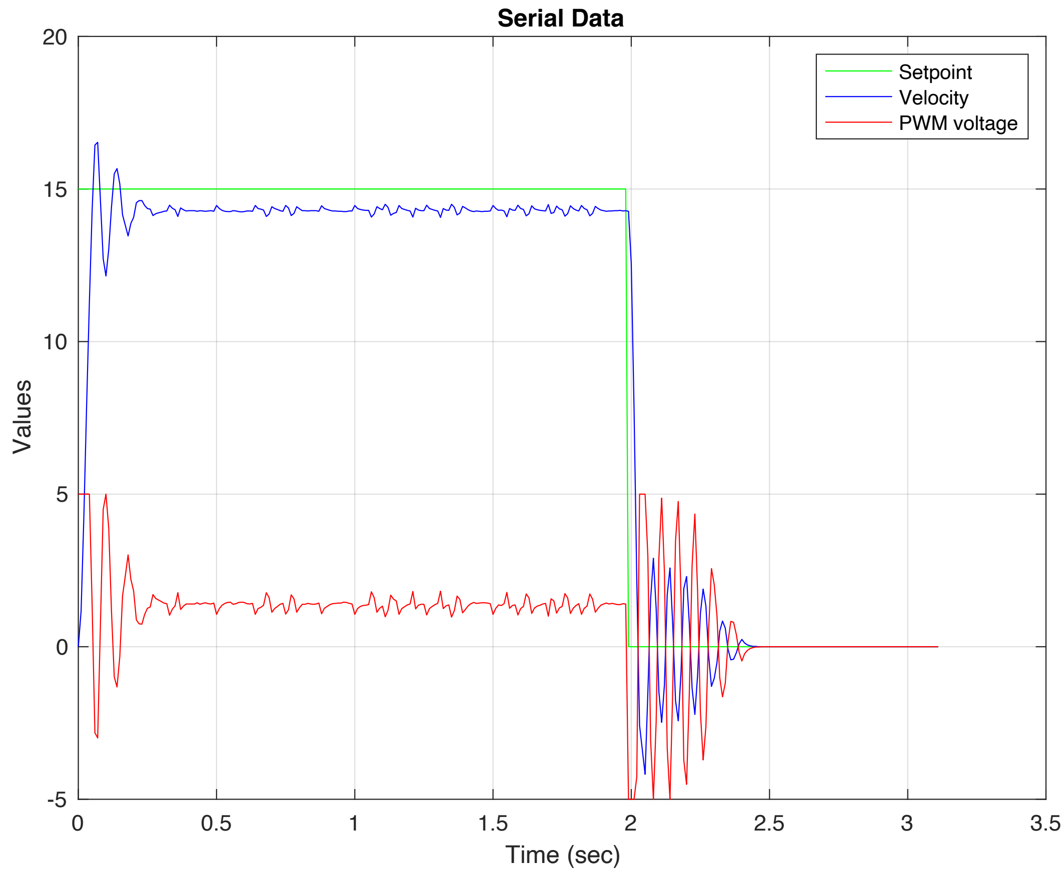
2. Repeat the experiment with Kp = 10 and 15, and describe the effect of Kp on τ and Δ.





As Kp increases, tau and delta both decrease.

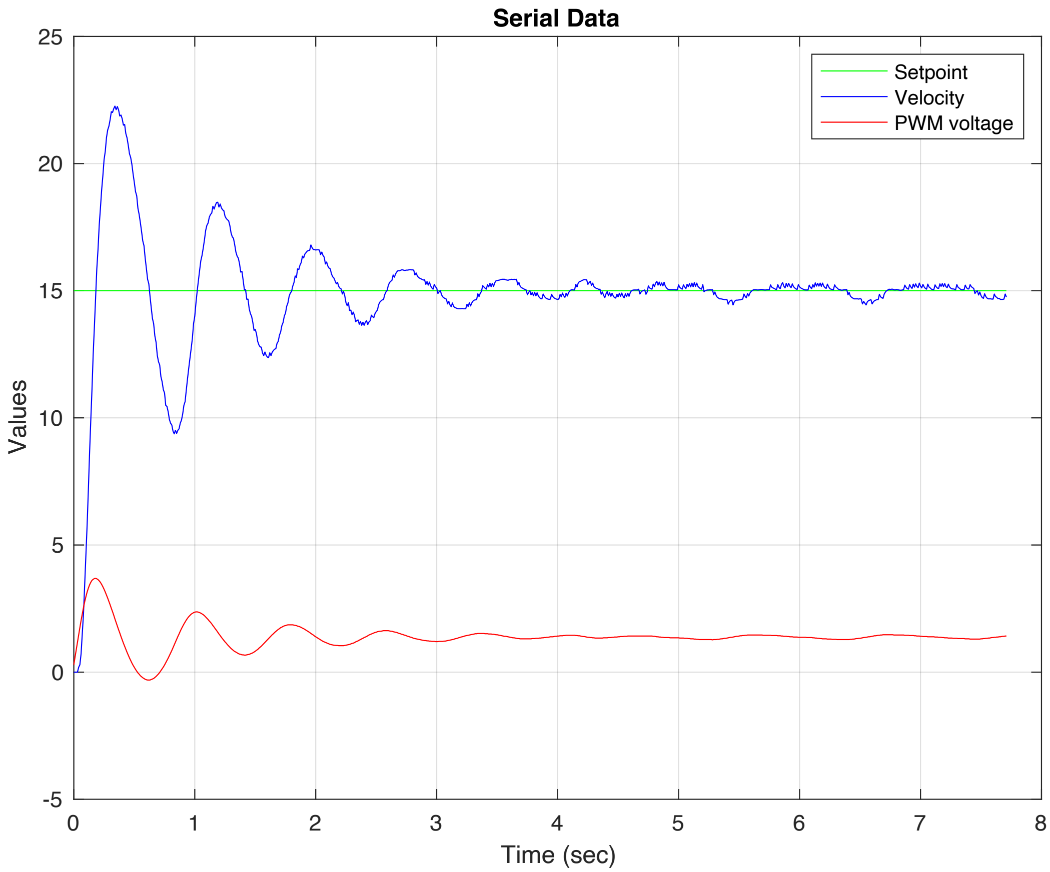
3. Comment on the response when Kp = 100.



The wheel appears to spin faster but spaz a little when the input is shut off. The graph shows a large overshoot.

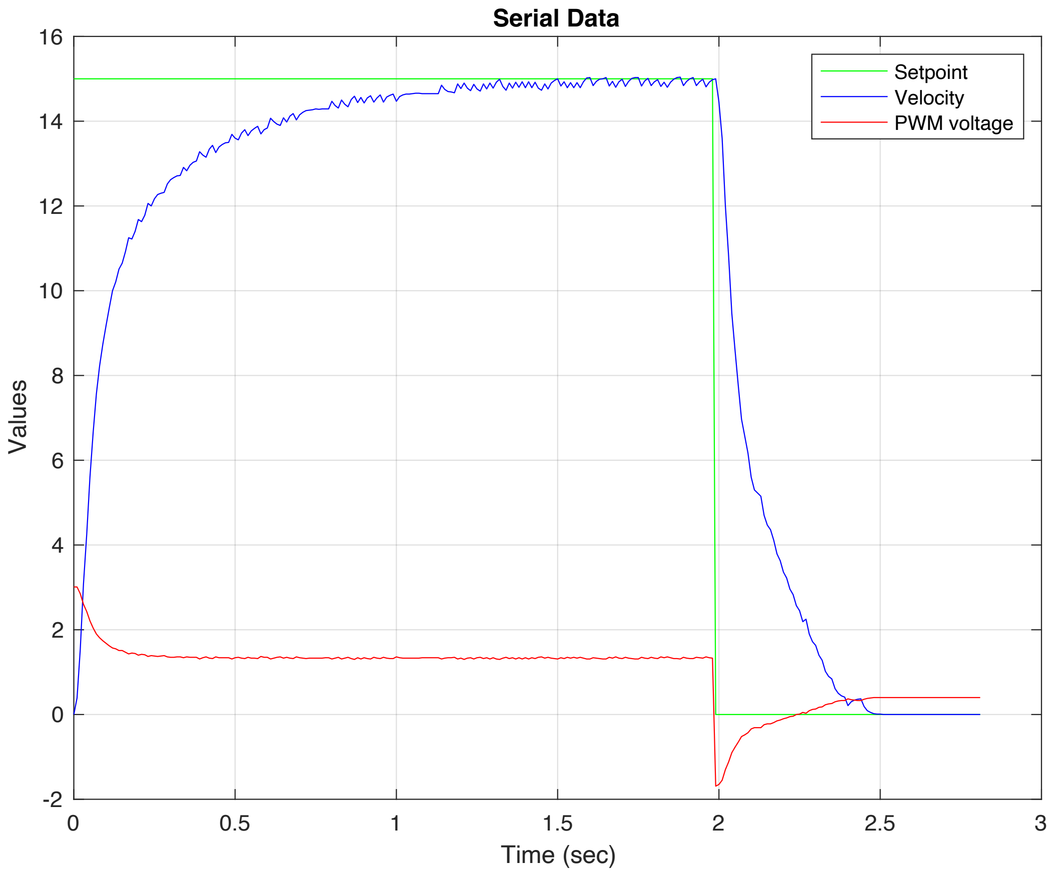
Experiment #3. PI control

1. Set Kp = 0, Ki = 100 (pure integral control) and record system response when driven by a constant velocity setpoint. What can you say about steady-state of the response? Is the response acceptable in terms of settling time?



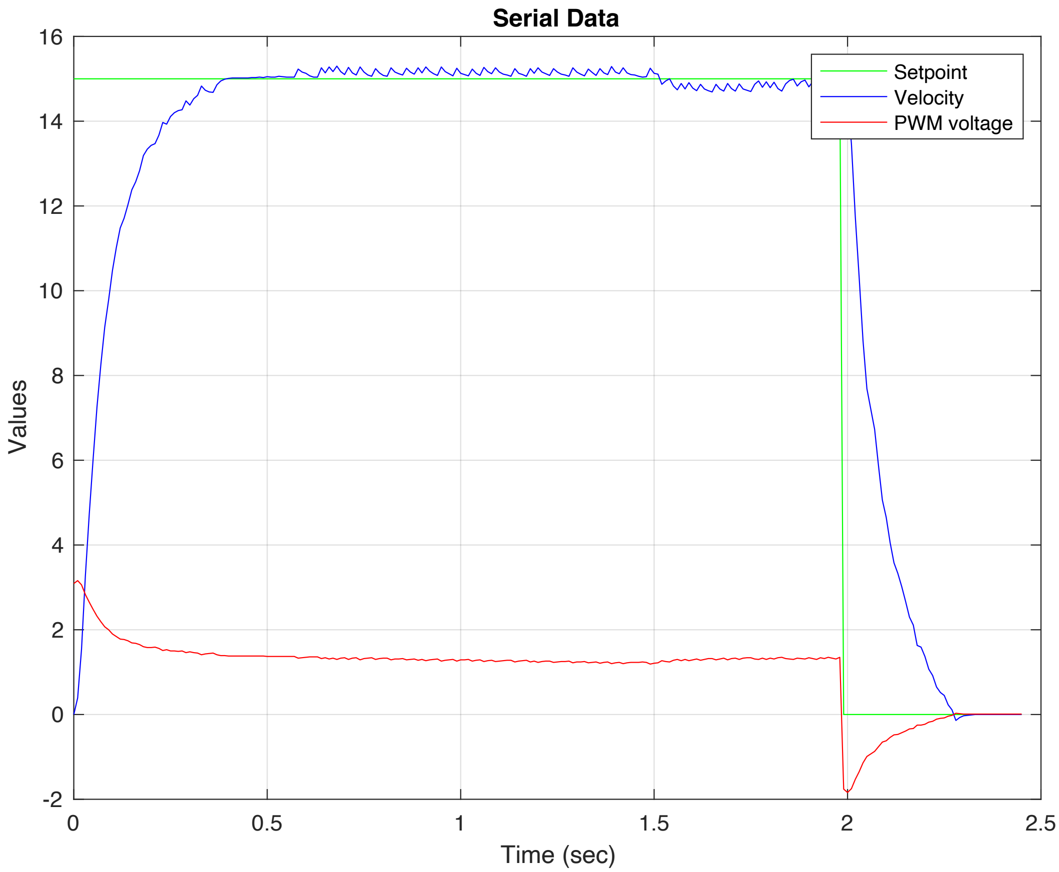
The steady state error appears to be 0. The settling time appears >5seconds, which is a really long time – probably unacceptably long.

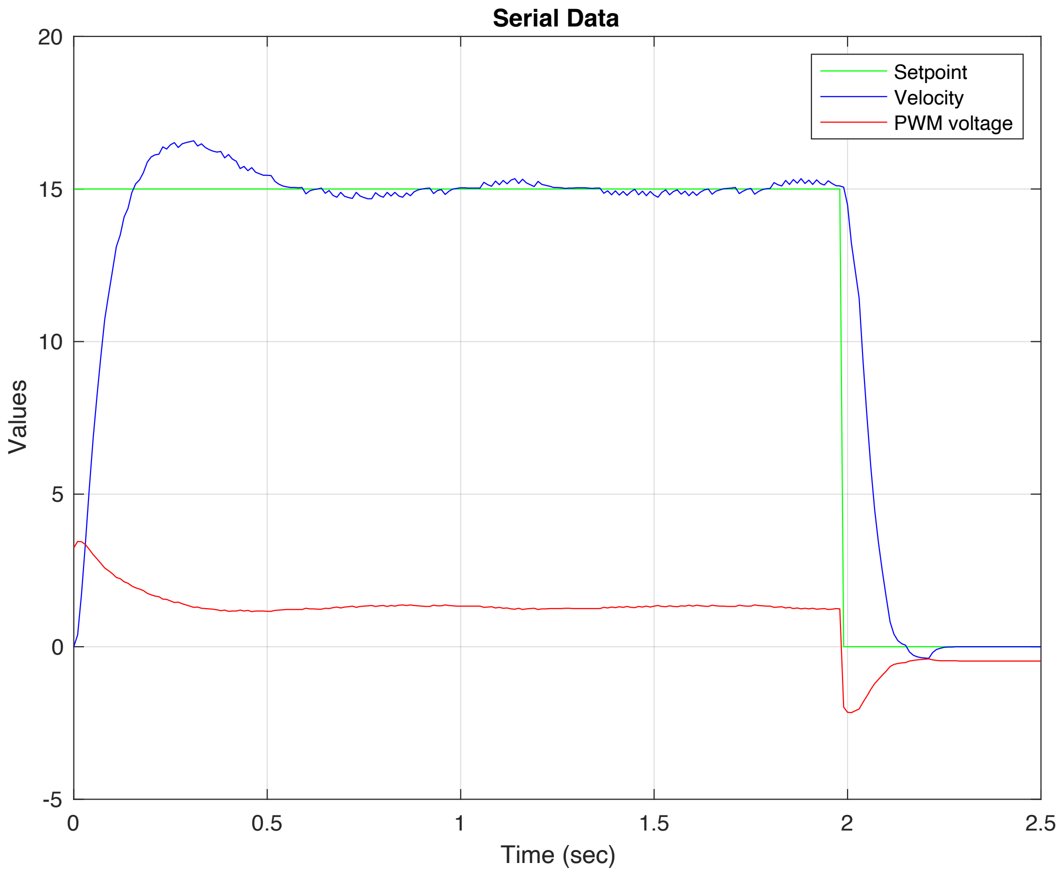
2. Set Kp = 10, Ki= 25 and record your response data. Measure steady-state error. Compare this result with pure integral control and pure proportional control. What effect does the integrator



Compared to the pure integral control, the response has a smaller settling time. Compared to proportional control, the response has a smaller steady state error of 0, similar to the pure integral control. However, the settling time is much larger than that of proportional control.

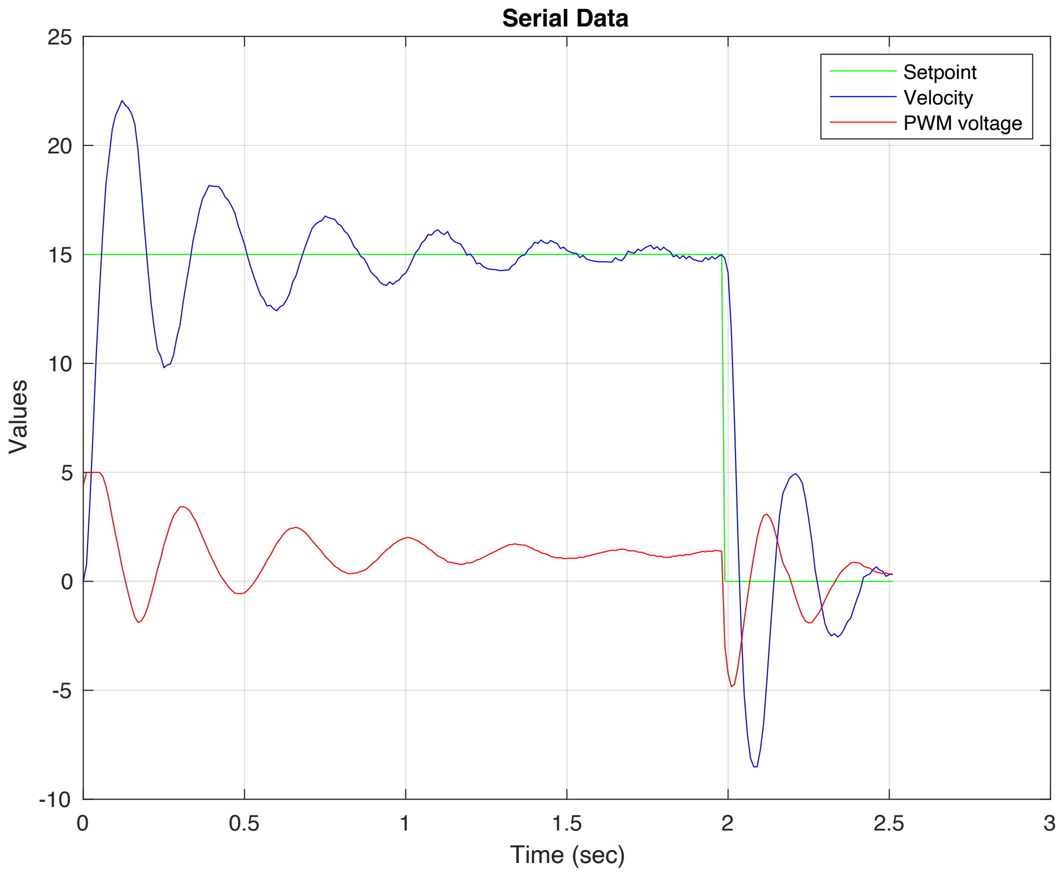
3. Keep Kp = 10 but change Ki to 50, and 100 and repeat the experiment. Describe the effect of Ki on the transient part of the response such as settling time, peak time, and percent overshoot.


Serial Data
Setpoint
Velocity
PWM voltage
Values
Time (sec)
16
14
12
10
8
6
4
2
0
-2
0
0.5
1
1.5
2
2.5



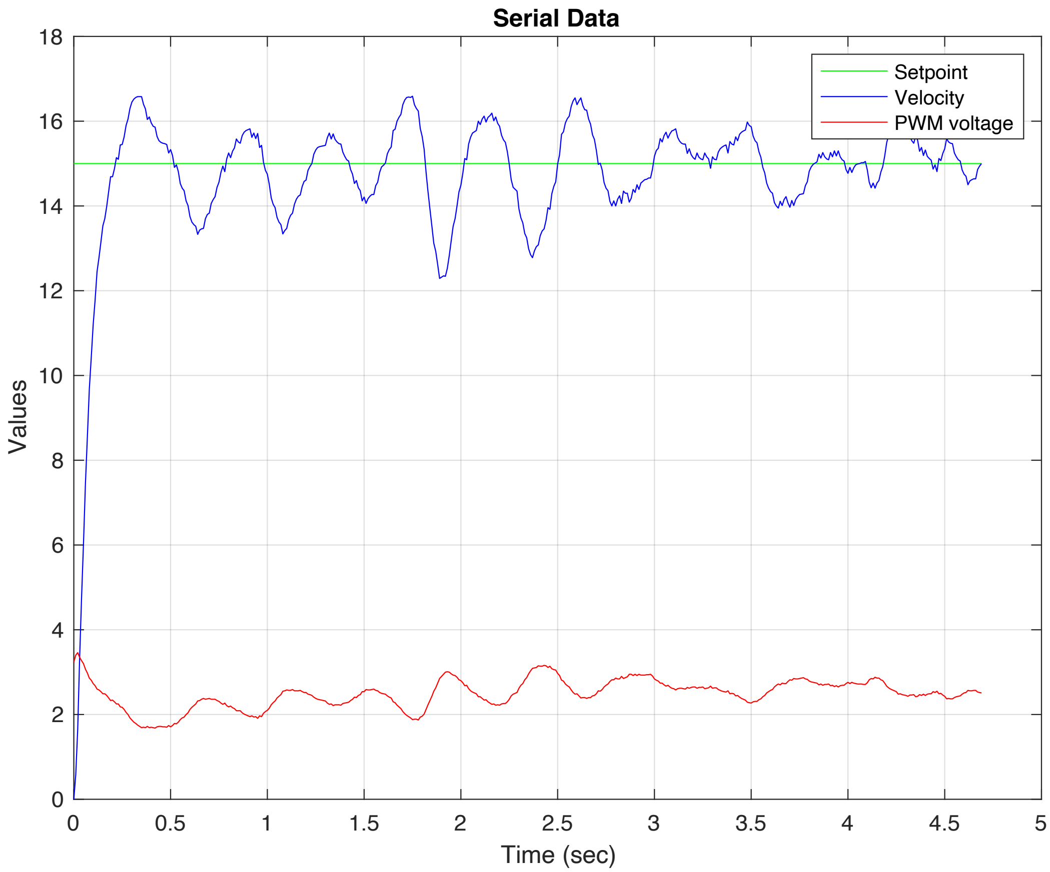
As Ki increases, peak time decreases while settling time increases, and percent overshoot increases.

4. Comment on the response when Kp = 10 and Ki = 500.



There is a very large overshoot at a smaller peak time, and the settling time is significantly larger.

5. Use Kp = 10 and Ki = 100, and disable square wave. Press a finger onto the wheel to create a *__constant__* *disturbance torque. Observe the* *__controller output__* *and make comments.*



The controller output appears to have a wave-like shape.

Experiment #4. PI Controller Design

1. Find gains K_p and K_i such that the closed loop system is critically damped with a natural frequency ω_n = 10 rad/s. Do you see overshoot? Why or why not?
(Graph)

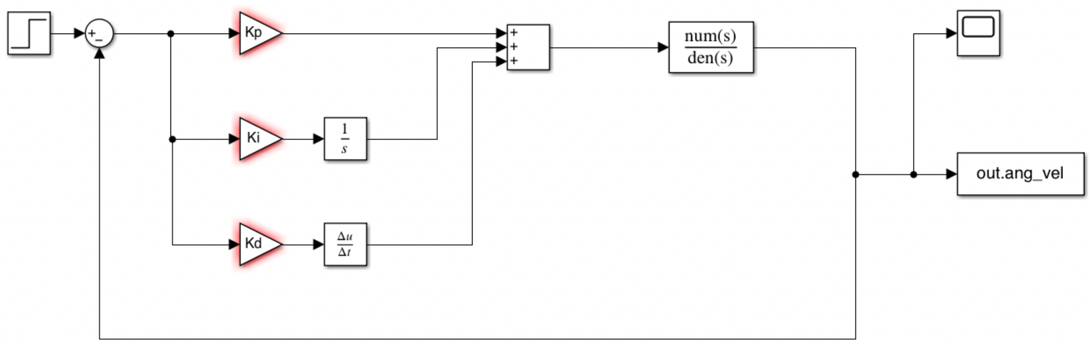
$$K_p = 11.63$$
$$K_i = 87.21$$

2. Determine the desired angular velocity of the motor to produce the stroboscopic effect when viewing the motion of the spokes with a video camera.

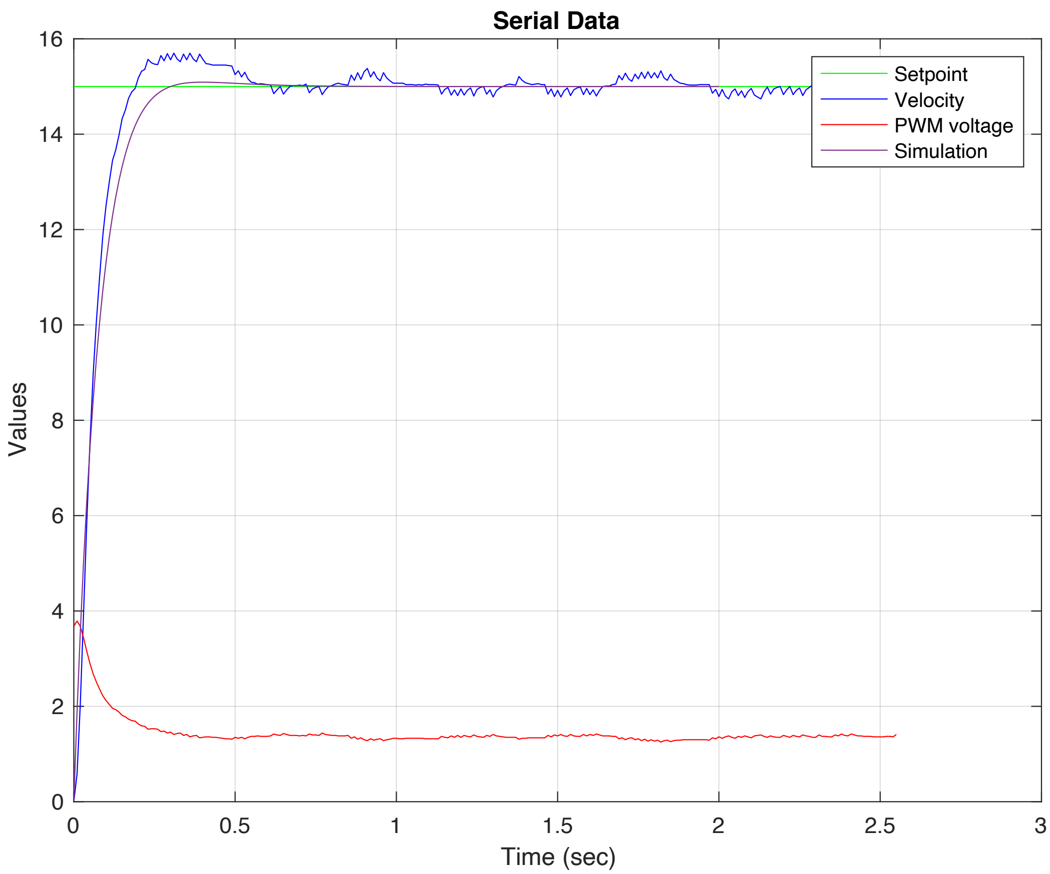
$$\Omega_r = 18.85$$

Extra Credit Task: Simulink Simulation

1. *Screenshot of the Simulink model.*



2. *Use K_p and K_i values from Experiment 4.1 to perform simulation, and compare the simulated response to the actual velocity response. Note any differences.*



The actual response has a greater overshoot, larger settling time, and smaller peak time than the simulated response.

$$G(s) = \frac{K_{dc} \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n = 10 \text{ rad/s}$
$\zeta = 1$
$K_{dc} = 8.78$

$$= \frac{878}{s^2 + 20s + 100}$$

from intro

$$G_{cl}(s) = \frac{K(K_p s + K_i)}{\tau s^2 + (1+K_p K)s + K_i K}$$

$$den = s^2 + \frac{(1+K_p K)}{\tau}s + \frac{K_i K}{\tau}$$

$$den = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$K = .172 \quad \tau = 0.15$
$\omega_n = 10 \quad \zeta = 1$

$$2\zeta\omega_n = \frac{1+K_p K}{\tau} \qquad \omega_n^2 = \frac{K_i K}{\tau}$$

$$20 = \frac{1 + .172K_p}{.15} \qquad 100 = \frac{.172K_i}{.15}$$

$$K_p = 11.63 \qquad K_i = 87.21$$

$$f_{motor} = f_{strobe/m} = 30/10 = 3$$

$$\Omega_{motor} = 2\pi f_{motor} \simeq 18.85$$