ME 02.74/740: Bio-inspired Robotics

HW 01: 2-DoF robot arm linkage

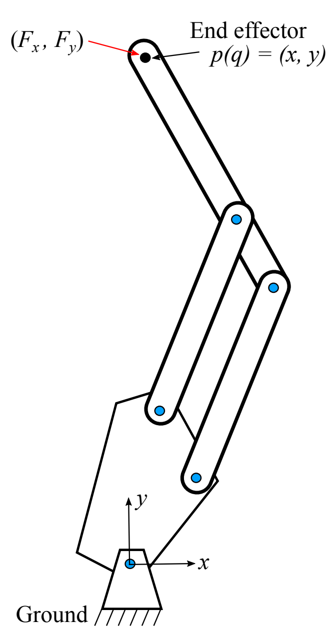
Due Online, Sunday, Sept. 18th, 11:59 PM

All questions refer to the robot arm sketched in the figure below. You should use MATLAB to perform intermediate symbolic computations. Answer the questions below with text, mathematical statements, and supporting sketches where appropriate. Include a commented copy of your MATLAB code.

1. How many degrees of freedom (n) does this mechanism possess? 2
2. Where would you attach motors to control the position of the end effector in the plane?
3. Based on your choice, define generalized coordinates $q \in \mathbb{R}^n$ and derive forward kinematic equations that relate the inputs (motor angles) to the outputs (end effector position in Cartesian coordinate, i.e. $p = (x, y)$).
4. Obtain the Jacobian matrix $J(q) = \frac{dp}{dq}$ symbolically in MATLAB.
5. Use this Jacobian to relate output forces (F_x, F_y) at the tip of the arm to statically equivalent input torques (τ_1, τ_2). $\tau = J^T F$
6. (optional) How would you verify that your Jacobian matrix is correct? Explain.
7. Repeat 3 and 4 for polar coordinates (r, θ). (What is $f(q) = (r, \theta)$? What is Jacobian matrix $\frac{df}{dq}$?)

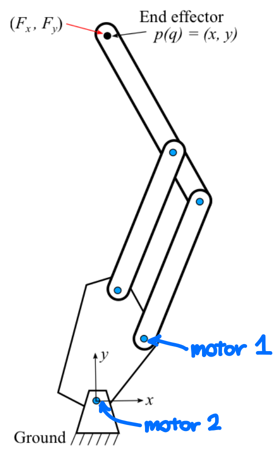
Assumptions and tips:

- You should define fixed parameters as necessary.
- You can assume that the linkage is a parallelogram.
- Consider using MATLAB functions: `syms, eval, jacobian`
- You may want to read the MATLAB Symbolic Computation Toolbox Tutorial before getting started: `http://www.mathworks.com/help/symbolic/performing-symbolic-computations.html`

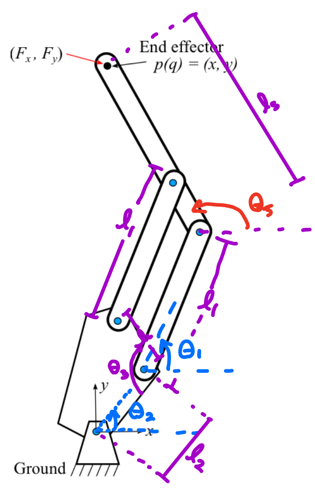


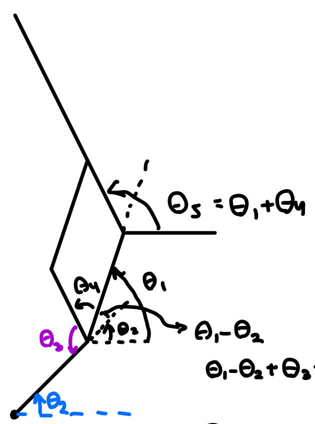
1. 2DOF

2.



3.





$\theta_1 - \theta_2 + \theta_3 + \theta_4 = \pi$

$\theta_4 = \pi - \theta_1 + \theta_2 - \theta_3$

$\theta_5 = \pi + \theta_2 - \theta_3$

defined parameters: l_1, l_2, l_5, θ_3

$q = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$

$x = x_B + l_5 \sin\theta_5$

$y = x_B + l_5 \cos\theta_5$

$x_A = l_2 \sin\theta_2$

$y_A = l_2 \cos\theta_2$

$x_B = x_A + l_1 \sin\theta_1$

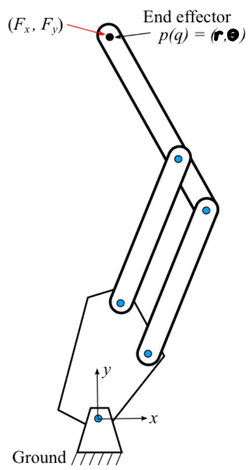
$y_B = y_A + l_1 \cos\theta_1$

$x = l_2 \sin\theta_2 + l_1 \sin\theta_1 + l_5 \sin(\pi + \theta_2 - \theta_3)$

$y = l_2 \cos\theta_2 + l_1 \cos\theta_1 + l_5 \cos(\pi + \theta_2 - \theta_3)$

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} \end{bmatrix}$$

7.



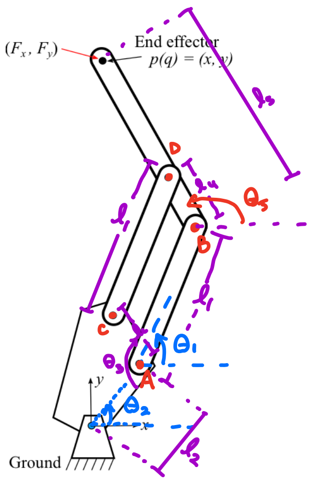
$$x = l_2 \sin\theta_2 + l_1 \sin\theta_1 + l_5 \sin(\pi + \theta_2 - \theta_3)$$
$$y = l_2 \cos\theta_2 + l_1 \cos\theta_1 + l_5 \cos(\pi + \theta_2 - \theta_3)$$

$$q = \begin{pmatrix} r \\ \theta \end{pmatrix}$$

$$(r, \theta) = \left(\sqrt{x^2+y^2}, \tan^{-1}\left(\tfrac{y}{x}\right)\right)$$

$$r = \sqrt{x^2+y^2} = \sqrt{(l_2 \sin\theta_2 + l_1 \sin\theta_1 - l_5 \sin(\theta_2-\theta_3))^2 + (l_2\cos\theta_2 + l_1\cos\theta_1 - l_5\cos(\theta_2-\theta_3))^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{l_1\cos\theta_1 + l_2\cos\theta_2 - l_5\cos(\theta_2-\theta_3)}{l_1\sin\theta_1 + l_2\sin\theta_2 - l_5\sin(\theta_2-\theta_3)}\right)$$

6.

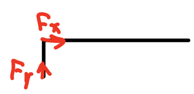


$$J = \begin{bmatrix} l_1\cos\theta_1, & l_2\cos\theta_2 - l_5\cos(\theta_2-\theta_3) \\ -l_1\sin\theta_1, & l_5\sin(\theta_2-\theta_3) - l_2\sin\theta_2 \end{bmatrix}$$

$$l_2\cos\theta_2 - l_5\cos(\theta_2-\theta_3) = 0 \Rightarrow \theta_2 = \tfrac{\pi}{2} \quad \theta_3 = 0$$
$$-l_1\sin\theta_1 = 0 \Rightarrow \theta_1 = 0$$

$$l_1\cos\theta_1 = 0 \Rightarrow \theta_1 = \tfrac{\pi}{2}$$
$$l_5\sin(\theta_2-\theta_3) - l_2\sin\theta_2 = 0 \Rightarrow \theta_2 = \theta_3 = 0$$

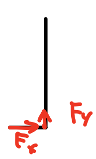
$\theta_1 = 0 \quad \theta_2 = \frac{\pi}{2} \quad \theta_3 = 0$



$$J^T = \begin{bmatrix} l_1 & 0 \\ 0 & l_5 - l_2 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} F_x l_1 \\ -F_y(l_2 - l_5) \end{bmatrix} \quad \checkmark$$

$\theta_1 = \frac{\pi}{2} \quad \theta_2 = \theta_3 = 0$



$$J^T = \begin{bmatrix} 0 & -l_1 \\ l_2 - l_5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} F_y(l_2 - l_5) \\ -F_x l_1 \end{bmatrix} \quad \checkmark$$

Problem 3

```
syms theta_1 theta_2 theta_3 l_1 l_2 l_5
% for generalized coordinates theta_1 and theta_2
% q = [x;y;], end effector position in Cartesian coordinate
x = l_2*sin(theta_2)+l_1*sin(theta_1)+l_5*sin(pi+theta_2-theta_3);
```

x = $l_1 \sin(\theta_1) + l_2 \sin(\theta_2) - l_5 \sin(\theta_2 - \theta_3)$

```
y = l_2*cos(theta_2)+l_1*cos(theta_1)+l_5*cos(pi+theta_2-theta_3);
```

y = $l_1 \cos(\theta_1) + l_2 \cos(\theta_2) - l_5 \cos(\theta_2 - \theta_3)$

Problem 4

```
% Jacobian matrix J(q)=dp/dq
J = jacobian([x,y],[theta_1,theta_2])
```

J =

$$\begin{pmatrix} l_1 \cos(\theta_1) & l_2 \cos(\theta_2) - l_5 \cos(\theta_2 - \theta_3) \\ -l_1 \sin(\theta_1) & l_5 \sin(\theta_2 - \theta_3) - l_2 \sin(\theta_2) \end{pmatrix}$$

Problem 5

```
syms F_x F_y
% relating output forces F_x, F_y to input torques tau_1, tau_2
% tau_1 and tau_2 are at the motors measured by theta_1 and theta_2
% respectively
tau=eval(transpose(J)*[F_x;F_y;]);
tau_1=tau(1,:)
```

tau_1 = $F_x \, l_1 \cos(\theta_1) - F_y \, l_1 \sin(\theta_1)$

```
tau_2=tau(2,:)
```

tau_2 = $F_x \, (l_2 \cos(\theta_2) - l_5 \cos(\theta_2 - \theta_3)) - F_y \, (l_2 \sin(\theta_2) - l_5 \sin(\theta_2 - \theta_3))$

Problem 7

```
% q = [r;theta;], end effector position in polar coordinate
r = sqrt(x^2+y^2)
```

r = $\sqrt{(l_1 \sin(\theta_1) + l_2 \sin(\theta_2) - l_5 \sin(\theta_2 - \theta_3))^2 + (l_1 \cos(\theta_1) + l_2 \cos(\theta_2) - l_5 \cos(\theta_2 - \theta_3))^2}$

```
theta = atan(y/x)
```

theta =

$$\operatorname{atan}\left(\frac{l_1 \cos(\theta_1) + l_2 \cos(\theta_2) - l_5 \cos(\theta_2 - \theta_3)}{l_1 \sin(\theta_1) + l_2 \sin(\theta_2) - l_5 \sin(\theta_2 - \theta_3)}\right)$$

```
% Jacobian matrix J(q)=dp/dq
J2 = jacobian([r,theta],[theta_1,theta_2])
```