Time Series Analysis & Forecasting

Class 5

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Time Series Decomposition

- Additive Model

$$y_t = S_t + T_t + E_t$$

y_t - data at period t

S_t - seasonal component at period t

T_t - trend-cycle component at period t

E_t - error component at period t

- Seasonal Model

$$y_t = S_t \times T_t \times E_t$$

STL Decomposition

1. Can handle any type of seasonality (not just monthly or daily)
2. Useful for business cycles where every cycle may not have the same period
3. Seasonal component can change over time
4. Robust to outliers- occasional unusual observations do not affect the estimates
5. Can handle non-stationary data
6. TS repeated patterns
    1. Seasonal – fixed known period that is associated with calendar (seasonal ARMA models)
    2. Cyclic – data has ups and downs with no fixed period (ARMA models)

Yule-Walker Equations

- Consider AR(2) process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$$

- Multiply both sides with Y_{t-k} and take expectations to get autocovariance

$$\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2}, for\ k = 1, 2, 3, \ldots$$

And dividing by γ_0 we get autocorrelation defined by

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$$

- The above autocovariance and autocorrelation equations are called the Yule-Walker equations.
- Use sample moments to recursively solve for AR parameter estimates.

ARIMA Point Forecasting

- Expand ARIMA equation with Y_t on the left hand side and all other terms on the right
- Rewrite the equation replacing t with $t + l$, where l is the forecasting horizon
- On the right hand side of the equation, replace future observations by their forecasts, future errors by zero, and past errors by the corresponding residuals

ARIMA Forecast Updates

ARIMA Process $\phi(B)Y_t = \theta(B)e_t$

1. Directly in terms of the difference equation of previous Y's and current and previous white noise error e's

$$Y_{t+l} = \phi_1 Y_{t+l-1} + \ldots + \phi_{p+d}\, Y_{t+l-p-d} - \theta_1 e_{t+l-1} - \ldots - \theta_q e_{t+l-q} - e_{t+l}$$

2. Infinite weighted sum of current and previous shocks e's

$$Y_{t+l} = \sum_{j=0}^{\infty} \psi_j e_{t+l-j}$$

3. Infinite weighted sum of previous observations plus current shock e

$$Y_{t+l} = \sum_{j=0}^{\infty} \pi_j Y_{t+l-j} + e_{t+l}$$

Multi-step Forecasting Strategies

	Pros	Cons	Computational time needed
Recursive	Suitable for noise-free time series (e.g. chaotic)	Accumulation of errors	+
Direct	No accumulation of errors	Conditional independence assumption	+ + ++
DirRec	Trade-off between Direct and Recursive	Input set grows linearly with H	+ + + + +
MIMO	No conditional independence assumption	Reduced flexibility: same model structure for all the horizons	++
DIRMO	Trade-off between total dependence and total independence of forecasts	One additional parameter to estimate	+ + +

Table 3: A Summary of the Pros and Cons of the Different Multi-step Forecasting Strategies

Source: S. B. Taieba, G. Bontempia, A. Atiyac, A. Sorjamaa, *A review and comparison of strategies for multi-step ahead time series forecasting based on the NN5 forecasting competition*, https://arxiv.org/pdf/1108.3259.pdf, 2011.

Prediction Interval

- Take a linear regression

$$y_i \; = \; \beta_0 + \; \beta_t x_i$$

- Distance value = $\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum(x_i - \bar{x})^2}$

  where x_0 is a particular value of x_i

- Assume S to be the residual standard error

- Then $100(1 - \alpha)\%$ prediction interval for an individual value of y_i when $x_i = x_0$ is

$$\widehat{y_i} \pm t * S * \sqrt{1 + Distance\ value}$$

  where t is the t-value at $^{\alpha}/_{2}$ level of significance for $n - 2$ DOF

Forecast Evaluation

- Forecast error at time t

$$e_t = y_t - \widehat{y_t}$$

- Absolute Deviation

$$|e_t| = |y_t - \widehat{y_t}|$$

- Mean Absolute Deviation

$$MAD = \frac{\sum_1^n |e_t|}{n}$$

- Mean Squared Error

$$MSE = \frac{\sum_1^n e_t^2}{n}$$

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Forecast Evaluation

- Absolute Percentage Error

$$APE = \frac{|e_t|}{y_t} \times 100\%$$

- Mean Absolute Percentage Error

$$MAPE = \frac{\sum_1^n APE}{n}$$

- Symmetric Mean Absolute Percentage Error

$$sMAPE = \frac{1}{n}\sum_1^n \frac{|e_t|}{y_t + \widehat{y_t}}$$

Textbook Chapters

- Materials covered available in book:
  - MJK: Chapter 2, TSA: Chapters 7, 8, 9

- Web: http://robjhyndman.com/hyndsight/tscvexample/