Ann Eitrheim Time Series HW #1 due 4/11

2.1 $E(x)=2$ $E(y)=0$ $corr(y,x)=.25$
$var(x)=9$ $var(y)=4$

find a) $var(x+y) = var(x) + var(y) + 2\,cov(x,y)$
$var(x) + var(y) + 2(corr(x,y) \times std(x) \times std(y))$
$9 + 4 + (2 \times .25 \times 3 \times 2) = 16$

b) $cov(x, x+y) = cov(x,x) + cov(x,y)$
$= \sigma_x \sigma_x \rho_{xx} + \sigma_x \sigma_y \rho_{xy}$
$= (3 \times 3 \times 1) + (3 \times 2 \times .25) = 9 + 1.5 = 10.5$

c) $corr(x+y, x-y) = \left[\dfrac{cov(x+y, x-y)}{\sigma_{x+y} \times \sigma_{x-y}}\right]$
$= \left[\dfrac{cov(x,x) - \cancel{cov(x,y) + cov(y,x)}^{\,0} - cov(y,y)}{\sqrt{16} \times (9+4-(2 \times .25 \times 3 \times 2))}\right]$
$= \left[\dfrac{cov(x,x) - cov(y,y)}{\sqrt{16 \times 10}}\right]$
$= \left[\dfrac{var(x) - var(y)}{4\sqrt{10}}\right]$
$= \left[\dfrac{9 - 4}{4\sqrt{10}}\right] = \dfrac{5}{4\sqrt{10}} = \dfrac{1.25}{\sqrt{10}} = .395$

2.2. if X and Y are dependent but $Var(x) = Var(y)$, find $cov((x+y),(x-y))$

$= cov(x,x) \cancel{- cov(x,y) + cov(y,x)}^{\,0} - cov(y,y)$

$= cov(x,x) - cov(y,y)$
$= \sigma_x \sigma_x \cancel{\rho_{xx}}^{\,1} - \sigma_y \sigma_y \cancel{\rho_{yy}}^{\,1}$
$= var(x) - var(y) = 0$

2.5 suppose $Y_t = 5 + 2t + X_t$ where $\{X_t\}$ is a zero mean stationary series w/ autocovariance function γ_k.

find a) the mean function for $\{Y_t\}$ $E(Y_t) = 5 + 2t$

b) the auto covariance function for $\{Y_t\}$

$cov(Y_t, Y_s) = cov(\cancel{5}+2t+X_t, \cancel{5}+2s+X_s)$ #can remove constants

$= \cancel{cov(2t, 2s)} + \cancel{cov(2t, X_s)} + \cancel{cov(X_t, 2s)} + cov(X_t, X_s)$

$= cov(X_t, X_s) = \gamma_k$ #same autocov function as $\{X_t\}$

c) is $\{Y_t\}$ stationary? why or why not?

not stationary because mean is not constant; it is based on time

2.6. let $\{X_t\}$ be a stationary ts, & define $Y_t = \begin{cases} X_t & \text{for } t \text{ odd} \\ X_t + 3 & \text{for } t \text{ even} \end{cases}$

a) show that $cov(Y_t, Y_{t-k})$ is free of t for all lags k

for odd lags: $cov(Y_t, Y_{t-1}) = cov(X_t + 3, X_{t-k}) = cov(X_t, X_{t-k})$ ← autocov for $\{X_t\}$

$\{X_t\}$ is stationary so it is not dependent on time & covariance is not affected by adding constants (ie +3)

for even lags: $cov(Y_t, Y_{t-2}) = cov(X_t, X_{t-2})$

same reasoning for odd lags

thx for the help on this one, wendy!

b) is $\{Y_t\}$ stationary? no because the mean isnt constant w/ time

2.7 Suppose that $\{Y_t\}$ is a stationary w/ autocovariance function γ_k.

a) show that $W_t = \nabla Y_t = Y_t - Y_{t-1}$ is stationary by finding the mean & auto covariance function for $\{W_t\}$

$E(W_t) = E(Y_t) - E(Y_{t-1}) = 0$ because $\{Y_t\}$ is stationary & therefore its mean is constant

& these equal each other

$cov(W_t, W_{t-1}) = cov(Y_t - Y_{t-1}, Y_{t-1} - Y_{t-1-k})$

$= cov(Y_t, Y_{t-1}) - cov(Y_t, Y_{t-1-k}) - cov(Y_{t-1}, Y_{t-1}) + cov(Y_{t-1}, Y_{t-1-k})$

$\gamma_k - \gamma_{k+1} - \gamma_{k-1} + \gamma_k$

b) show that $U_t = \nabla^2 Y_t = \nabla[Y_t - Y_{t-1}] = Y_t - 2_{t-1} + Y_{t-2}$ is stationary

you don't need to find mean & auto cov for $\{U_t\}$.

in a we found the diff between two stationary processes was also stationary. so the difference here would also be stationary

2.8 suppose that $\{y_t\}$ is stationary w/ autocovariance function γ_k. Show that for any fixed positive integer n & any constants $c_1, c_2, \dots c_n$, the process $\{W_t\}$ defined by $W_t = c_1 y_t + c_2 y_{t-1} + \dots + c_n y_{t-n+1}$ is stationary.

mean of W_t is constant because we know the mean of $\{y_t\}$ is constant over time and multiplying those by constants, which are not dependent on time, will result in a mean which is also not dependent on time $E(W_t) = E(y_t)(c_1 + c_2 + \dots c_n)$

the autocovariance is a product of $\{y_t\}$'s & therefore results in $\{W_t\}$ being stationary. With the autocovariance, we can make the constants being moved to the front using this property of covariance:

$$\text{cov}(\cancel{a} + bx, \cancel{c} + dy) = bd \cdot \text{cov}(x,y)$$

$$\text{cov}(W_t, W_{t-k}) = \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j \,\text{Cov}[y_{t-j}, y_{t-i-k}] = \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j \gamma_{j-k-i}$$

2.11 suppose $\text{cov}(X_t, X_{t-k}) = \gamma_k$ is free of t but $E(X_t) = 3t$

a) is $\{X_t\}$ stationary? no because the mean is not constant w/ time

b) let $y_t = 7 - 3t + X_t$. is $\{y_t\}$ stationary?

$E(y_t) = 7 - 3t + X_t = 7 - 3t + 3t = 7$ mean is constant w/ time

$$\begin{aligned} \text{cov}(y_t, y_{t-k}) &= \text{cov}(7 - 3t + X_t, 7 - 3(t-k) + X_{t-k}) \\ &= \text{cov}(X_t, X_{t-k}) \quad \text{\#drop constants} \\ &= \gamma_k \end{aligned}$$

mean is constant & the autocovariance is constant w/ k

yes it is stationary because the

2.12 Suppose that $Y_t = e_t - e_{t-12}$. Show that $\{Y_t\}$ is stationary & that for $k>0$, its auto correlation function is nonzero only for lag $k=12$.

$E(Y_t) = e_t - e_{t-12} = 0 - 0 = 0$ mean is stable w/ time

$cov(e_t - e_{t-12}, e_{t-k} - e_{t-k-12})$

$= cov(e_t, e_{t-k}) - cov(e_t, e_{t-k-12}) - cov(e_{t-12}, e_{t-k}) + cov(e_{t-12}, e_{t-k-12})$

$= 0 - 0 - cov(e_{t-12}, e_{t-k}) + 0$

since e's are independent of each other we can eliminate them when not paired w/ itself

$= -cov(e_{t-12}, e_{t-k})$

if $k=12$ then $= -cov(e_{t-12}, e_{t-12})$

$= -var(e_{t-12})$

otherwise autocovariance $= 0$

2.14 Evaluate the mean & covariance function for each of the following processes. In each case, determine whether or not the process is stationary.

a) $Y_t = \theta_0 + te_t$ $E(Y_t) = \theta_0$ # because e_t is expected to be 0 & cancels out t

$cov(Y_t, Y_{t-k}) = cov(\cancel{\theta_0} + te_t, \cancel{\theta} + (t-k)e_{t-k})$ # drop constants

$= cov(te_t, te_{t-k} - \cancel{ke_{t-k}})$

$= cov(te_t, te_{t-k})$

$= t^2 cov(e_t, e_{t-k})$

$= t^2 \sigma_e^2$

it is NOT stationary because autocovariance varies with time

b) $W_t = \nabla Y_t$ where Y_t is from part a.

$E(W_t) = \theta_0 + te_t - (\theta_0 + (t-1)e_{t-1})$

$= \cancel{\theta_0} + te_t - \cancel{\theta_0} - te_{t-1} + e_{t-1}$ # θ cancel eachother out

$= 0$ # e's are expected to be Ø

$cov(W_t, W_{t-k}) = cov(\cancel{\theta_0} + te_t - \cancel{\theta_0} - (t-1)e_{t-1}; \cancel{\theta_0} + (t-k)e_{t-k} - \cancel{\theta_0} - (t-k-1)e_{t-k-1}$

$= cov(te_t - (t-1)e_{t-1}, (t-1)e_{t-1} - (t-1-k)e_{t-k-1})$

$= cov(te_t, (t-1)e_{t-1}) - cov(te_t, (t-1-k)e_{t-k-1})$

$- cov((t-1)e_{t-1}, (t-1)e_{t-1}) + cov((t-1)e_{t-1}, (t-1-k)e_{t-k-1})$

$= [t^2 + (t-1)]\sigma^2$

it is Not stationary because covariance varies with time

c) $y_t = e_t e_{t-1}$

$E(y_t) = 0$ since es have a mean of 0

$\text{cov}(y_t, y_{t-k}) = \text{cov}(e_t e_{t-1}, e_{t-k} e_{t-k-1})$

$= E[(e_t e_{t-1} - \mu_t^2)(e_{t-k} e_{t-k-1} - \mu_t^2)]$

$= 0$

it is stationary

2.15 suppose that X is a random variable w/ zero mean. Define a time series by $y_t = (-1)^t X$

a) find the mean function for $\{y_t\}$ $E(y) = (-1)^t X$

$= 0$

we know: $E(X) = 0$

b) find covariance function for $\{y_t\}$

$\text{cov}((-1)^t X, (-1)^{t-k} X) = (-1^t)(-1^{t-k}) \text{cov}(x,x)$

$= (-1^{2t-k}) \text{var}(x)$ # $-1^{2t} = -1$ so can remove

$= (-1^{-k}) \sigma^2 = (-1)^k \sigma^2$

c) is $\{y_t\}$ stationary? yes as the mean & covariance are free of time