Time Series Analysis & Forecasting

Class 2

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Trend Stationary

- Deterministic Trend

$$y_t = \beta t + \varepsilon_t, \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

$$\mu = E[y_t] = \beta t$$

$$Var(y_t) = E[(y_t - \mu)^2] = E[\varepsilon_t^2] = \sigma^2$$

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Difference Stationary

- Stochastic Trend

$$y_t = \beta + y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

$$\mu = E[y_t] = \beta t$$

$$Var(y_t) = E[(y_t - \mu)^2] = E[t\varepsilon_t^2] = t\sigma^2$$

R code – Trend & Difference Stationary

- # Trend
- epsilon <- rnorm(500)
- for (i in 1:length(epsilon))
-   y <- 3 * i + epsilon
- adf.test(y)
- adf.test(y, alternative = "explosive")
- kpss.test(y)
- kpss.test(y, null="Trend")
- # Difference – Random Walk with Drift
- wn <- rnorm(500)
- rw <- c(0)
- rw[1] <- wn[1]
- for (i in 2:length(wn))
- rw[i] <- 0.9 + rw[i-1] + wn[i]
- ts.plot(rw)
- adf.test(rw)
- adf.test(rw, alternative="explosive")

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Exponential Smoothing

- A method to smooth time series using weight $0 \leq \alpha \leq 1$ where the recent observations are weighted more than the less recent ones.
- Mathematically represent smooth TS as level

$$l_t = \alpha x_t + (1 - \alpha)\, l_{t-1}$$

- Forecast

$$\hat{y}_{t+\tau}(t) = l_t$$

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$$
\begin{aligned}
S_t &= \alpha y_t + (1-\alpha) S_{t-1} \\
&= \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha) S_{t-2}\right] \\
&= \alpha\left[y_t + (1-\alpha) y_{t-1}\right] + (1-\alpha)^2 S_{t-2} \\
&= \alpha\left[y_t + (1-\alpha) y_{t-1}\right] + (1-\alpha)^2\left[\alpha y_{t-2} + (1-\alpha) S_{t-3}\right] \\
&\;\;\vdots \\
&= \alpha\Big[\underbrace{y_t + (1-\alpha) y_{t-1} + (1-\alpha)^2 y_{t-2} + \cdots}_{}\Big] + (1-\alpha)^t S_0
\end{aligned}
$$

Holt-Winters (Additive)

HW is a smoothing algorithm for a TS $\{y_t\}$ that exhibits linear trend and seasonality with smoothing constants $0 \leq \alpha, \beta, \gamma \leq 1$ and periodicity L

- Additive model to be used when the seasonal variation is additive in nature – toy sales increase by $1 million every Dec.

Level

$$l_t = \alpha(y_t - s_{t-L}) + (1-\alpha)(l_{t-1} + b_{t-1})$$

Trend

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$

Seasonality

$$s_t = \gamma(y_t - l_{t-1}) + (1-\gamma)s_{t-L}$$

Forecast

$$\hat{y}_{t+\tau}(t) = l_t + \tau b_t + s_{t+\tau-L}$$

Holt-Winters (Multiplicative)

- Multiplicative model to be used when the seasonal variation is multiplicative in nature – toy sales increase by 42% every Dec.

Level

$$l_t = \alpha(y_t/s_{t-L}) + (1-\alpha)(l_{t-1} + b_{t-1})$$

Trend

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$

Seasonality

$$s_t = \gamma(y_t/l_{t-1}) + (1-\gamma)s_{t-L}$$

Forecast

$$\hat{y}_{t+\tau}(t) = (l_t + \tau b_t)s_{t+\tau-L}$$

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Regression

- Introduced by Sir Francis Galton in “Family Likeness in Stature” 1886.
- Estimate or forecast the average value of one variable (dependent) on the basis of fixed values of other variables (independent)
- Dependent ≡ Explained ≡ Predictand ≡ Regressand ≡ Response ≡ Endogeneous ≡ Outcome ≡ Controlled
- Explanatory ≡ Independent ≡ Predictor ≡ Regressor ≡ Stimulus ≡ Exogeneous ≡ Covariate ≡ Control

Regression – Assumptions

Mathematical representation $y_t = \widehat{\beta_0} + \widehat{\beta_1} x_t + u_t$

1. Linear in the parameters
2. X values are fixed in repeated sampling
3. Zero mean value of the disturbance u_t
4. Homoscedasticity or equal variance of u_t
5. No autocorrelation in u_t
6. Zero covariance between u_t and x_t
7. Number of observations must be greater than number of parameters to be estimated
8. X values in a given sample must vary
9. No specification bias in the model
10. No perfect linear relationships among the explanatory variables

Regression – what about error normality assumption ?

- Probability distribution of the parameters depend on the assumption of the probability distribution of u_t
- NOTE – OLS makes no assumption about the probability of u_t
- Advantage of normal distribution of u_t, i.e. normal distribution of the parameters allow the use of t, F and χ^2 distributions
- If normal, then OLS and Maximum Likelihood (ML) estimators are identical.

Multiple Regression

Mathematical representation $y_t = \widehat{\beta_0} + \widehat{\beta_1} x_{1t} + \widehat{\beta_2} x_{2t} + u_t$

- Pearson correlation among the variables: $r_{yx1}, r_{yx2}, r_{x1x2}$
- These correlation coefficients are known as zero-order correlations because they do not control for interconnection amongst variables.
- Multiple coefficient of determination is R^2 that represents how much of the variability in y can be explained by the predictor variables.

Multicollinearity

- If present,
  - OLS estimators will have large variances and covariances
  - the confidence intervals of the estimators tend to be wider – readily accept the null hypothesis
  - Estimators will be sensitive to small changes in data – high standard error
  - Low t values for the estimators

- To detect, use the Variation Inflation Factor denoted by $VIF = \frac{1}{1 - r_{x1x2}^2}$

- VIF indicates how the variance of the estimator is inflated by multicollinearity

$$y_t = \beta_1 x_{1t} + \beta_2 x_{2t}$$

$X_{1t} \quad X_{2t} \quad \ldots \quad X_{Kt}$

$$v_t + \lambda_1 X_{1t} + \lambda_2 X_{2t} + \ldots + \lambda_K X_{Kt} = 0$$

$$\lambda_1 X_{1t} + \lambda_2 X_{2t} = 0$$

$\lambda_1, \lambda_2 \ldots$ not all are zero simultaneously

$$X_{1t} = -\frac{\lambda_2}{\lambda_1} X_{2t}$$

multicollinearity is a feature of your data

R code

- *require(graphics)*
- *## Seasonal Holt-Winters*
- *(m <- HoltWinters(co2))*
- *plot(m)*
- *plot(fitted(m))*
- *(m <- HoltWinters(AirPassengers, seasonal = "mult"))*
- *plot(m)*

- *## Non-Seasonal Holt-Winters*
- *x <- uspop + rnorm(uspop, sd = 5)*
- *m <- HoltWinters(x, gamma = FALSE)*
- *plot(m)*

- *## Exponential Smoothing*
- *m2 <- HoltWinters(x, gamma = FALSE, beta = FALSE)*
- *lines(fitted(m2)[,1], col = 3)*

R code (continued)

```
require(car)
fit <- lm(prestige ~ income + education, data=Duncan)
fit
vif(fit)

x <- c(1,2,3,4,5)
y <- c(11,20,25,24,29)
mod <- lm (y ~ x)
mod0 <- lm(y ~ x + 0)
summary(mod)
summary(mod0)
plot(x, y, ylim=c(0, 30))
abline(mod, col="blue")
abline(mod0, col="red")
```

Textbook Chapters

- Materials covered available in book:
  - FPP: Chapters 4 – 7, MJK: Chapters 3 – 4, TSA: Chapter 3