

# Time Series Analysis & Forecasting

## Class 6

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# Cross-Validation

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- Model validation technique
- Divide a dataset into a training set and a validation set
- K-fold cross validation => original data is divided into K equal size subsamples. Of the K, 1 subsample is retained as the validation set. Remaining K-1 are used as the training data
- The above is repeated K times with each K subsample used exactly once as the validation data
- The results from the validations will be combined (eg. average) to produce a single model estimation

# Cross-Validation

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- Take a rolling window for train, set forecast with  $h$  steps ahead
- Calculate accuracy
- Roll window forward by  $h$  steps and repeat
- <https://robjhyndman.com/hyndsight/tscv/>
- Challenges and nested CV: <https://towardsdatascience.com/time-series-nested-cross-validation-76adba623eb9>

## R code – TS CV

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- `data(gas)`
- `auto.arima(gas)`
- `far2 <- function(x, h){forecast(Arima(x, order=c(2,1,1), seasonal = c(0,1,1)), h=h)}`
- `e <- tsCV(gas, far2, h=3)`
- `eDf <- as.data.frame(e)`
- `matplot(eDf, type="l")`

# Regression with ARMA errors

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- Regression assumption that errors are not autocorrelated is violated

$$y_t = \beta_0 + \beta_1 x_t + u_i$$

where  $u_i$  is autocorrelated

- So  $u_i$  can be modeled as

$$\phi(B)u_i = \theta(B)e_t$$

where  $e_t$  is white noise

- ARMA can be considered as a special type of regression model => predictors are lags of the dependent variable and/or lags of the forecast error

# R code – Regression with ARMA errors

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- `x <- 1:100`
- `e <- arima.sim(model=list(ar=0.3, ma=0.9), n=100)`
- `y <- 1 + 2*x + e`
- `fit1 <- lm (y~x)`
- `summary(fit1)`
- `Plot(fit1)`
- `acf(fit1$residuals, lag=100)`
- `par(mfrow=c(1,2))`
- `qqnorm(fit1$residuals)`
- `qqline(fit1$residuals)`
- `fit2 <- auto.arima(y, xreg=x)`
- `summary(fit2)`
- `qqnorm(fit2$residuals)`
- `qqline(fit2$residuals)`
- `acf(fit2$residuals)`

# Long Memory Fractional ARIMA – ARFIMA

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- Compromise between short term memory ARMA models and the fully integrated ARIMA models
- Consider the ARIMA model

$$\Phi(B)(1 - B)^d y_t = \Theta(B)e_t$$

- Here  $|d| < 0.5$ 
  - TS  $\{y_t\}$  is stationary if  $d < 0.5$
  - TS  $\{y_t\}$  is invertible if  $d > -0.5$
- ACF goes to infinity for  $0 < d < 0.5 \Rightarrow$  long term memory
- ACF decays hyperbolically to 0 for  $-0.5 < d < 0 \Rightarrow$  intermediate memory

# R code – ARFIMA

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- `library("arfima", lib.loc=~R/win-library/3.3")`
- `data("SeriesJ")`
- `acf(SeriesJ$YJ, lag=40)`
- `library("forecast", lib.loc=~R/win-library/3.3")`
  
- `d <- fracdiff::fracdiff(SeriesJ$YJ) #get the fractional d`
- `st <- fracdiff::diffseries(SeriesJ$YJ,d$d) # do the fractional difference`
- `acf(st, lag=40)`
- `m1 <- auto.arima(st) # now the TS is stationary, run ARIMA`
- `AIC(m1)`
  
- `m2 <- forecast::arfima(SeriesJ$YJ) # does the above (fractional difference + ARIMA) in 1 step`
- `AIC(m2) # you can compare the models and choose the one with lower AIC`



# Model Uncertainty

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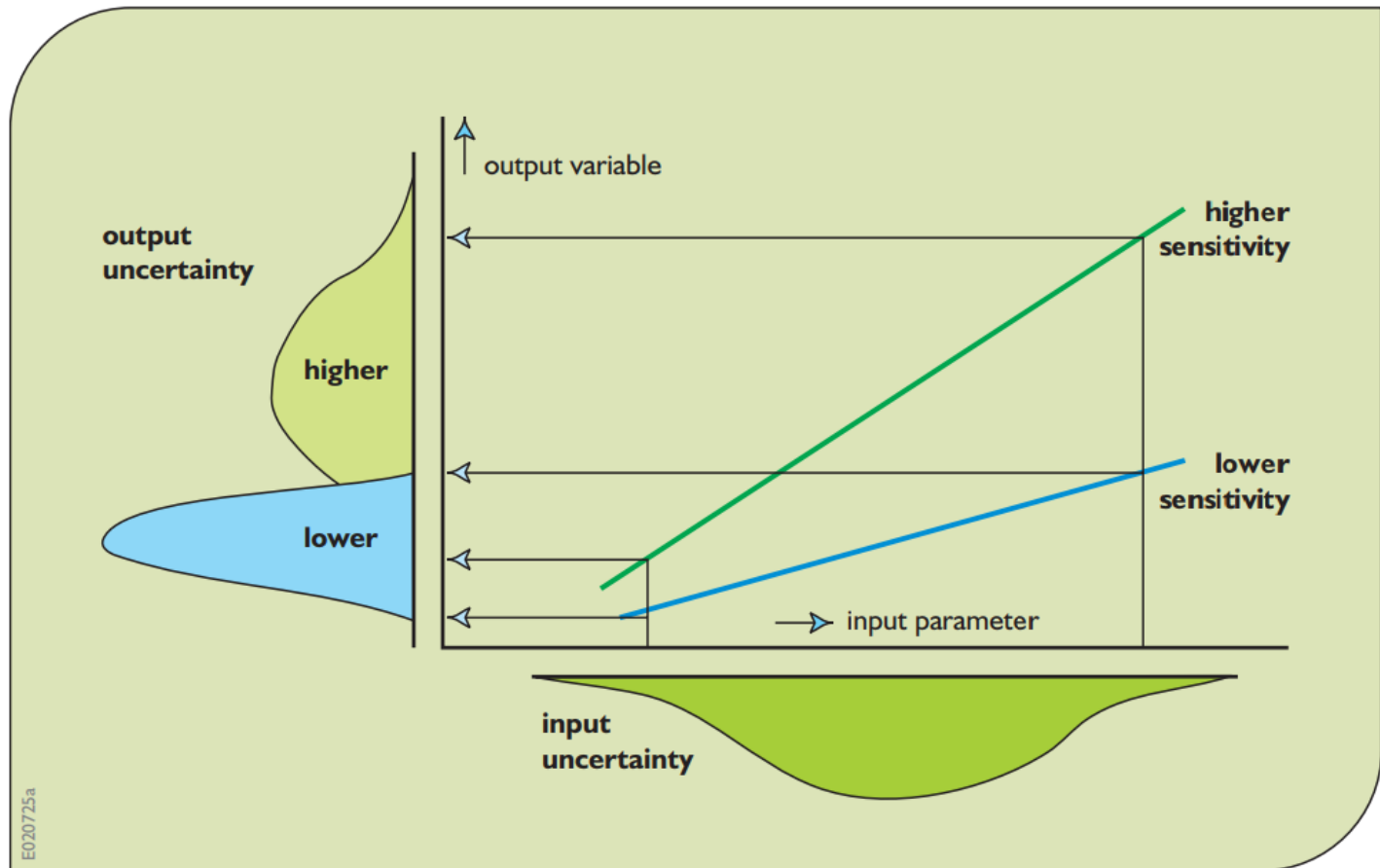
- Sequential Stability
  - Take different sample sizes and calculate the % of times same model selected
- Perturbation Stability
  - Change dataset with new additional samples and selected model should remain the same
- NOTE – Model selection method does not guarantee any forecasting performance

# Sensitivity Analysis

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- Similar to risk, sensitivity assesses how the uncertainty affects area of interest for a particular use case.
- Measures the sensitivity of a model output to changes in model input values.
- Input/output scatterplots are a simple method for sensitivity analysis.
- To calculate model output sensitivity analysis to different input parameters, use Monte Carlo simulations.
- To focus on specific regions, use Monte Carlo Filtering.

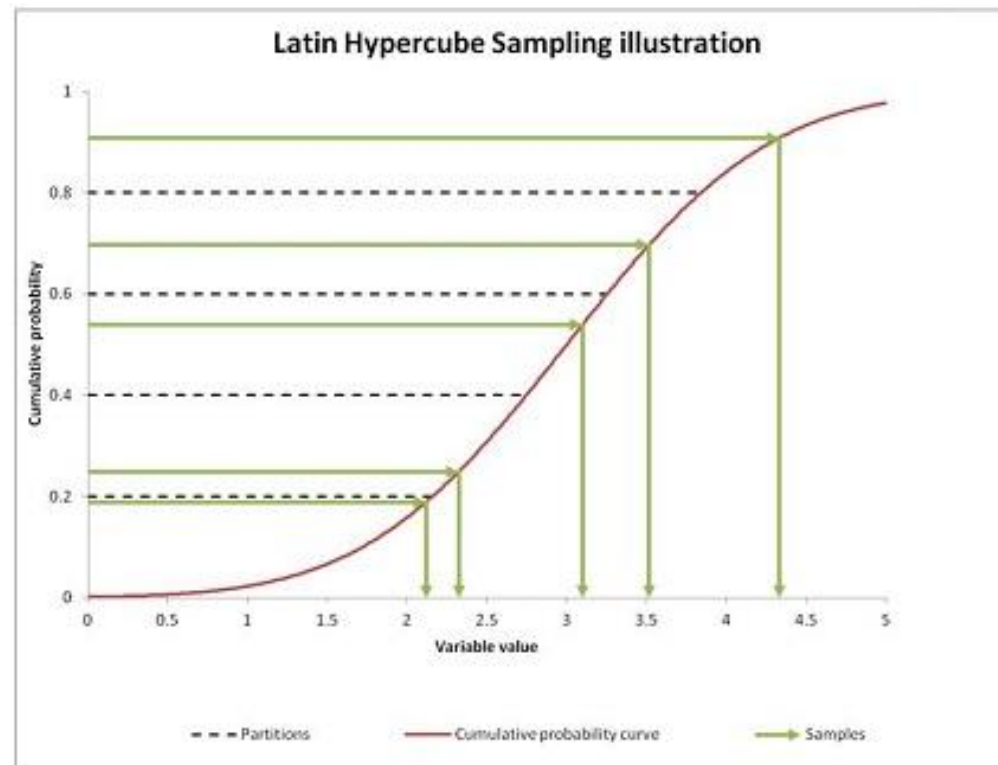
# Schematic diagram for input param uncertainty and sensitivity



LAL, W. 1995. Sensitivity and uncertainty analysis of a regional model for the natural system of South Florida. West Palm Beach, Fla., South Florida Water Management District. Draft report, November.

[https://ecommons.cornell.edu/bitstream/handle/1813/2804/09\\_chapter09.pdf;sequence=12](https://ecommons.cornell.edu/bitstream/handle/1813/2804/09_chapter09.pdf;sequence=12)

# Latin Hypercube Sampling



<http://liprof.com/blog/the-pros-and-cons-of-latin-hypercube-sampling>

# Textbook Chapters

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- Materials covered available in book:
  - FPP: Chapter 9, MKJ: Chapter 4, TSA: Chapter 9