1. ARGUMENT

We will now describe how to construct $\operatorname{soc} D\Gamma$ given $\underline{r}\Gamma$, and the other way around.

Suppose that we know $\underline{r}\Gamma$. Then we also know $\operatorname{Top}\Gamma$, and we have a canonical projection

$$\Gamma \xrightarrow{p} \operatorname{Top}\Gamma.$$

Applying D we get a map

$$D\operatorname{Top}\Gamma \xrightarrow{p^*} D\Gamma,$$

and $\operatorname{im} p^* = \operatorname{Soc} D\Gamma$ reference .

Conversely, suppose that we know $\operatorname{Soc} D\Gamma$. This is a simple right Γ-module reference . So if $f \in \operatorname{Soc} D\Gamma$ is any non-zero element, we obtain a short exact sequence of right Γ-modules

$$0 \longrightarrow M \longrightarrow \Gamma \xrightarrow{\phi} \operatorname{Soc} D\Gamma \longrightarrow 0,$$

where $\phi(a) = f \cdot a$ and $M = \ker \phi$. Since $\operatorname{Soc} D\Gamma$ is simple, M is a maximal right ideal. But Γ has only one right maximal ideal reference , so $M = \underline{r}\Gamma$.