

Formative Assessment 3

2025-02-15

Problem 1

A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95, and a transmitted 1 is correctly received with probability 0.75. Also, 70% of all messages are transmitted as a 0. If a signal is sent, determine the probability that:

- (a) a 1 was received;
- (b) a 1 was transmitted given that a 1 was received.

```
# Given Probabilities
message_transmitted_0 <- 0.70
message_transmitted_1 <- 0.30
```

```
# Given conditional probabilities

prob_received_0 <- 0.95 #P(0 received | 0 sent)
prob_received_1 <- 0.75 #P(1 received | 1 sent)
prob_0_received_1 <- 1 - prob_received_1 #P(0 received | 1 sent)
prob_1_received_0 <- 1 - prob_received_0 #P(1 received | 0 sent)
```

Printing the results

```
prob_0_received_1
```

```
## [1] 0.25
```

```
prob_1_received_0
```

```
## [1] 0.05
```

Part (a): Probability that a 1 was received

```
received <- (prob_1_received_0 * message_transmitted_0) + (prob_received_1 * message_transmitted_1)
```

Output

received

```
## [1] 0.26
```

Part (b): Probability a 1 was transmitted given that a 1 was received

```
transmitted <- (prob_received_1 * message_transmitted_1) / received
```

Output

transmitted

```
## [1] 0.8653846
```

Representation using Diagram

```
barplot(  
  c(received, transmitted),  
  names.arg = c("P(1 received)", "P(1 transmitted | 1 received)"),  
  ylim = c(0, 1),  
  col = c("lightblue", "purple"),  
  main = "Probability in a Binary Communication Channel",  
  ylab = "Probability", xlab = "")
```

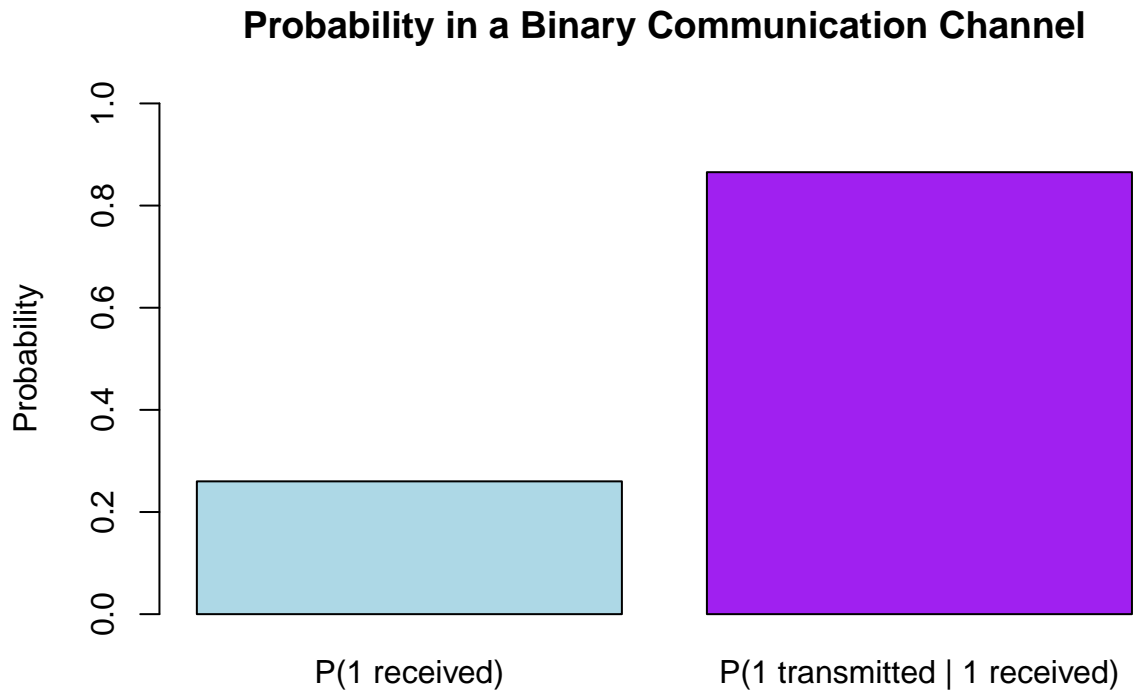


Figure 1 Barplot Representation of Probability in a Binary Communication Channel

The probability of receiving a 1 ($P(1 \text{ received})$) is 26% (0.26), which means that only 26% of all received signals are 1s. This is largely a result of the occurrence that 0s are received more frequently (70%) and received correctly 95% of the time, making it less likely to receive a 1. Conversely, if a 1 is received, there is an 86.54% chance that it was originally sent as a 1 ($P(1 \text{ transmitted} \mid \text{received})$), meaning that although noise results in some transmission errors, most received 1s were sent without error. This means that the 0s predominate the received signals due to their faster transmission rate and precision, the channel is comparatively reliable in transmitting 1s upon reception.