Formative Assessment 2

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Github Link: Latex Code Here

Problem 1

3.49 Prove that

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N.$$

Proof:

We want to show that

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N.$$

First, expanding the square:

$$(X_j - 1)^2 = X_j^2 - 2X_j + 1.$$

Now substituting it into the summation:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} \left(X_j^2 - 2X_j + 1\right).$$

Using the linearity of summation:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + \sum_{j=1}^{N} 1.$$

Simplifying the last term:

$$\sum_{j=1}^{N} 1 = N.$$

Therefore:

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N.$$

$$\sum_{j=1}^{N}(X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N \quad \blacksquare$$

Problem 2

3.51 Two variables, U and V, assume the values $U_1 = 3$, $U_2 = -2$, $U_3 = 5$, and $V_1 = -4$, $V_2 = -1$, $V_3 = 6$, respectively.

Calculate:

(a) $\sum UV$**:**

$$U_1V_1 = 3 \times (-4) = -12$$
$$U_2V_2 = (-2) \times (-1) = 2$$
$$U_3V_3 = 5 \times 6 = 30$$

Now, we sum them up:

$$\sum UV = (-12) + 2 + 30$$
$$\sum UV = -10 + 30 = 20$$
$$\boxed{\sum UV = \ 20}$$

(b)$\sum(U+3)(V-4)$**:**

$$(U_1 + 3)(V_1 - 4) = (3 + 3)(-4 - 4) = (6)(-8) = -48$$
$$(U_2 + 3)(V_2 - 4) = (-2 + 3)(-1 - 4) = (1)(-5) = -5$$
$$(U_3 + 3)(V_3 - 4) = (5 + 3)(6 - 4) = (8)(2) = 16$$

Sum the terms:

$$\sum(U + 3)(V - 4) = -48 + (-5) + 16$$
$$\sum(U + 3)(V - 4) = (-53) + 16 = -37$$
$$\boxed{\sum(U + 3)(V - 4) = \ -37}$$

(c)$\sum V^2$**:**

$$V_1^2 = (-4)^2 = 16, \qquad V_2^2 = (-1)^2 = 1, \qquad V_3^2 = 6^2 = 36$$
$$\sum V^2 = 16 + 1 + 36$$
$$\sum V^2 = 17 + 36 = 53$$
$$\boxed{\sum V^2 = \ 53}$$

(d)$(\sum U)(\sum V)^2$**:**

$$\sum U = 3 + (-2) + 5 = 6, \qquad \sum V = -4 + (-1) + 6 = 1$$
$$(\sum U)(\sum V)^2 = 6 \times (1)^2 = 6 \times 1 = 6$$
$$\boxed{(\sum U)(\sum V)^2 = \ 6}$$

(e)$\sum UV^2$**:**

$$U_1V_1^2 = 3 \times (-4)^2 = 3 \times 16 = 48$$

$$U_2V_2^2 = (-2) \times (-1)^2 = (-2) \times 1 = -2$$
$$U_3V_3^2 = 5 \times 6^2 = 5 \times 36 = 180$$

Sum them:

$$\sum UV^2 = 48 + (-2) + 180$$
$$\sum UV^2 = 46 + 180 = 226$$
$$\boxed{\sum UV^2 = 226}$$

(f) $\sum(U^2 - 2V^2 + 2)$**:**

$$U_1^2 - 2V_1^2 + 2 = 3^2 - 2(-4)^2 + 2 = 9 - 2 \cdot 16 + 2 = 9 - 32 + 2 = -21$$
$$U_2^2 - 2V_2^2 + 2 = (-2)^2 - 2(-1)^2 + 2 = 4 - 2 \cdot 1 + 2 = 4 - 2 + 2 = 4$$
$$U_3^2 - 2V_3^2 + 2 = 5^2 - 2(6)^2 + 2 = 25 - 2 \cdot 36 + 2 = 25 - 72 + 2 = -45$$
$$\sum(U^2 - 2V^2 + 2) = -21 + 4 + (-45)$$
$$\sum(U^2 - 2V^2 + 2) = -17 - 45 = -62$$
$$\boxed{\sum(U^2 - 2V^2 + 2) = -62}$$

(g) $\sum\left(\frac{U}{V}\right)$**:**

$$\frac{U_1}{V_1} = \frac{3}{-4} = -\frac{3}{4}, \qquad \frac{U_2}{V_2} = \frac{-2}{-1} = 2, \qquad \frac{U_3}{V_3} = \frac{5}{6}$$

Add them:

$$-\frac{3}{4} + 2 + \frac{5}{6}.$$

Use common denominator 12:

$$-\frac{3}{4} = -\frac{9}{12}, \quad 2 = \frac{24}{12}, \quad \frac{5}{6} = \frac{10}{12}.$$

Therefore

$$-\frac{9}{12} + \frac{24}{12} + \frac{10}{12} = \frac{-9 + 24 + 10}{12} = \frac{25}{12}.$$

As decimal:

$$\frac{25}{12} \approx 2.08333\ldots$$
$$\boxed{\sum\left(\frac{U}{V}\right) = \frac{25}{12} \approx 2.0833}$$

Problem 3

3.90 Find the geometric mean of the sets (a) 3, 5, 8, 3, 7, 2 and (b) 28.5, 73.6, 47.2, 31.5, 64.8.

$$\text{Formula:} \qquad GM = \left(\prod_{i=1}^{n} x_i\right)^{1/n}.$$

(a) For the set $\{3, 5, 8, 3, 7, 2\}$:

$$\text{Product} = 3 \times 5 \times 8 \times 3 \times 7 \times 2 = 5{,}040$$

Let n = 6, so we take the 6th root.

$$GM = 5040^{1/6}$$
$$\ln(5,040) \approx 8.5252, \qquad \frac{1}{6}\ln(5,040) \approx 1.4209$$
$$GM = e^{1.4209} \approx 4.14$$
$$\boxed{GM \approx 4.14}$$

(b) For the set $\{28.5, 73.6, 47.2, 31.5, 64.8\}$:

$$\text{Product} = 28.5 \times 73.6 \times 47.2 \times 31.5 \times 64.8 = 202{,}092{,}516.864$$

Let n = 5, so we take the 5th root.

$$GM = (202{,}092{,}516.864)^{1/5}$$
$$\ln(202{,}092{,}516.864) \approx 19.1242, \qquad \frac{1}{5}\ln(\text{Product}) \approx 3.8248$$
$$GM = e^{3.8248} \approx 45.83$$
$$\boxed{GM \approx 45.83}$$