Formative Assessment 4

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2025-10-03

GITHUB LINK: https://github.com/eivra-sm/APM1111/blob/main/FA4.md

Given Data

```
normal <- c(67,70,63,65,68,60,70,64,69,61,66,65,71,62,66,68,64,67,62,66,65,63,66,65,63,69,
            62,67,59,66,65,63,65,60,67,64,68,61,69,65,62,67,70,64,63,68,64,66,65,61,66,64)

skew_right <- c(31,43,30,30,38,26,29,55,46,26,29,57,34,34,36,40,28,26,66,63,30,33,24,35,34,40,
             24,29,24,27,35,33,75,38,34,85,29,40,41,35,26,34,19,23,28,26,31,25,22,28,34)

skew_left <- c(102,55,70,95,73,79,60,73,89,85,72,92,76,93,76,97,10,70,85,25,83,58,10,92,82,87,
            104,75,80,66,93,90,84,73,98,79,35,71,90,71,63,58,82,72,93,44,65,77,81,77)

uniform <- c(12.1,12.1,12.4,12.1,12.1,12.2,12.2,12.2,11.9,12.2,12.3,12.3,11.7,12.3,12.3,12.4,
             12.4,12.1,12.4,12.4,12.5,11.8,12.5,12.5,12.5,11.6,11.6,12.0,11.6,11.6,11.7,12.3,
             11.7,11.7,11.7,11.8,12.5,11.8,11.8,11.8,11.9,11.9,11.9,12.2,11.9,12.0,11.9,12.0,
             12.0,12.0)
```

Function to compute raw moments

```
raw_moments <- function(x) {
  c(mean(x), mean(x^2), mean(x^3), mean(x^4))
}

# ---- (a) First moments ----
first_moments <- c(
  Normal = raw_moments(normal)[1],
  Skewed_Right = raw_moments(skew_right)[1],
  Skewed_Left = raw_moments(skew_left)[1],
  Uniform = raw_moments(uniform)[1]
)

# ---- (b) Second moments ----
second_moments <- c(
  Normal = raw_moments(normal)[2],
  Skewed_Right = raw_moments(skew_right)[2],
  Skewed_Left = raw_moments(skew_left)[2],
  Uniform = raw_moments(uniform)[2]
```

```
)

# ---- (c) Third moments ----
third_moments <- c(
  Normal = raw_moments(normal)[3],
  Skewed_Right = raw_moments(skew_right)[3],
  Skewed_Left = raw_moments(skew_left)[3],
  Uniform = raw_moments(uniform)[3]
)

# ---- (d) Fourth moments ----
fourth_moments <- c(
  Normal = raw_moments(normal)[4],
  Skewed_Right = raw_moments(skew_right)[4],
  Skewed_Left = raw_moments(skew_left)[4],
  Uniform = raw_moments(uniform)[4]
)

# Displaying the results
cat("\n(a) First moments (means):\n"); print(first_moments)
```

```
##
## (a) First moments (means):

##       Normal Skewed_Right  Skewed_Left      Uniform
##     65.11538     35.45098     74.20000     12.05600
```

```
cat("\n(b) Second moments:\n"); print(second_moments)
```

```
##
## (b) Second moments:

##       Normal Skewed_Right  Skewed_Left      Uniform
##     4248.038     1432.196     5925.400      145.426
```

```
cat("\n(c) Third moments:\n"); print(third_moments)
```

```
##
## (c) Third moments:

##       Normal Skewed_Right  Skewed_Left      Uniform
##   277657.423    67724.039   489458.800     1755.158
```

```
cat("\n(d) Fourth moments:\n"); print(fourth_moments)
```

```
##
## (d) Fourth moments:

##       Normal Skewed_Right  Skewed_Left      Uniform
##  18181935.27   3749334.08  41396161.48     21194.59
```

Function to compute central moments (about the mean)

```
central_moments <- function(x, kmax = 4) {
  mean_x <- mean(x)
  cms <- sapply(1:kmax, function(k) mean((x - mean_x)^k))
  # Force very small values (close to 0) to be exactly 0
  cms[abs(cms) < 1e-10] <- 0
  return(cms)
}

# ---- (a) First central moments ----
first_central <- c(
  Normal = central_moments(normal)[1],
  Skewed_Right = central_moments(skew_right)[1],
  Skewed_Left = central_moments(skew_left)[1],
  Uniform = central_moments(uniform)[1]
)

# ---- (b) Second central moments ----
second_central <- c(
  Normal = central_moments(normal)[2],
  Skewed_Right = central_moments(skew_right)[2],
  Skewed_Left = central_moments(skew_left)[2],
  Uniform = central_moments(uniform)[2]
)

# ---- (c) Third central moments ----
third_central <- c(
  Normal = central_moments(normal)[3],
  Skewed_Right = central_moments(skew_right)[3],
  Skewed_Left = central_moments(skew_left)[3],
  Uniform = central_moments(uniform)[3]
)

# ---- (d) Fourth central moments ----
fourth_central <- c(
  Normal = central_moments(normal)[4],
  Skewed_Right = central_moments(skew_right)[4],
  Skewed_Left = central_moments(skew_left)[4],
  Uniform = central_moments(uniform)[4]
)

# Displaying the results
cat("\n(a) First central moments (exact 0):\n"); print(first_central)
```

```
##
## (a) First central moments (exact 0):

##       Normal Skewed_Right  Skewed_Left      Uniform
##            0            0            0            0
```

```r
cat("\n(b) Second central moments (variance):\n"); print(second_central)
```

```
##
## (b) Second central moments (variance):

##       Normal Skewed_Right  Skewed_Left      Uniform
##     8.025148   175.424068   419.760000     0.078864
```

```r
cat("\n(c) Third central moments:\n"); print(third_central)
```

```
##
## (c) Third central moments:

##        Normal  Skewed_Right   Skewed_Left       Uniform
## -3.563951e-01  4.513374e+03 -1.249826e+04  3.352320e-04
```

```r
cat("\n(d) Fourth central moments:\n"); print(fourth_central)
```

```
##
## (d) Fourth central moments:

##       Normal Skewed_Right  Skewed_Left      Uniform
## 1.547925e+02 2.070357e+05 9.272897e+05 1.125117e-02
```

QUESTION NUMBER 3

```r
# Moments about a constant (in this case, 75)
moments_about <- function(x, c, kmax = 4) {
  sapply(1:kmax, function(k) mean((x - c)^k))
}
```

```r
# Compute for female height (Normal) about 75
moments_about_75 <- moments_about(normal, 75, 4)

cat("\nMoments of Normal data about 75:\n")
```

```
##
## Moments of Normal data about 75:
```

```r
names(moments_about_75) <- c("First", "Second", "Third", "Fourth")
print(moments_about_75)
```

```
##        First       Second        Third       Fourth
##    -9.884615   105.730769 -1204.115385 14419.884615
```

QUESTION NUMBER 4

```r
# Extract raw and central moments for Normal data
raw_normal <- raw_moments(normal)   # m1', m2', m3', m4'
central_normal <- central_moments(normal) # m1, m2, m3, m4
```

```r
# Relation (a) : m2 = m2' - (m1')^2
lhs_a <- central_normal[2]          # left-hand side
rhs_a <- raw_normal[2] - raw_normal[1]^2   # right-hand side
```

```r
# Relation (b) : m3 = m3' - 3 m1' m2' + 2 (m1')^3
lhs_b <- central_normal[3]
rhs_b <- raw_normal[3] - 3*raw_normal[1]*raw_normal[2] + 2*raw_normal[1]^3
```

```r
# Relation (c) : m4 = m4' - 4 m1' m3' + 6 (m1')^2 m2' - 3 (m1')^4
lhs_c <- central_normal[4]
rhs_c <- raw_normal[4] - 4*raw_normal[1]*raw_normal[3] +
         6*(raw_normal[1]^2)*raw_normal[2] - 3*(raw_normal[1]^4)

cat("\n--- Verification of Moment Relations ---\n")
```

```
##
## --- Verification of Moment Relations ---
```

```r
cat("(a) m2: LHS =", lhs_a, " | RHS =", rhs_a, "\n")
```

```
## (a) m2: LHS = 8.025148  | RHS = 8.025148
```

```r
cat("(b) m3: LHS =", lhs_b, " | RHS =", rhs_b, "\n")
```

```
## (b) m3: LHS = -0.3563951  | RHS = -0.3563951
```

```r
cat("(c) m4: LHS =", lhs_c, " | RHS =", rhs_c, "\n")
```

```
## (c) m4: LHS = 154.7925  | RHS = 154.7925
```