

# Formative Assessment 2

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September 10, 2025

## Problem 1

**3.49** Prove that

$$\sum_{j=1}^N (X_j - 1)^2 = \sum_{j=1}^N X_j^2 - 2 \sum_{j=1}^N X_j + N.$$

**Proof:**

We want to show that

$$\sum_{j=1}^N (X_j - 1)^2 = \sum_{j=1}^N X_j^2 - 2 \sum_{j=1}^N X_j + N.$$

First, expanding the square:

$$(X_j - 1)^2 = X_j^2 - 2X_j + 1.$$

Now substituting it into the summation:

$$\sum_{j=1}^N (X_j - 1)^2 = \sum_{j=1}^N (X_j^2 - 2X_j + 1).$$

Using the linearity of summation:

$$\sum_{j=1}^N (X_j - 1)^2 = \sum_{j=1}^N X_j^2 - 2 \sum_{j=1}^N X_j + \sum_{j=1}^N 1.$$

Simplifying the last term:

$$\sum_{j=1}^N 1 = N.$$

Therefore:

$$\begin{aligned}\sum_{j=1}^N (X_j - 1)^2 &= \sum_{j=1}^N X_j^2 - 2 \sum_{j=1}^N X_j + N. \\ \sum_{j=1}^N (X_j - 1)^2 &= \sum_{j=1}^N X_j^2 - 2 \sum_{j=1}^N X_j + N \quad \blacksquare\end{aligned}$$

## Problem 2

**3.51** Two variables,  $U$  and  $V$ , assume the values  $U_1 = 3$ ,  $U_2 = -2$ ,  $U_3 = 5$ , and  $V_1 = -4$ ,  $V_2 = -1$ ,  $V_3 = 6$ , respectively.

Calculate:

(a)  $\sum UV$ :

$$U_1 V_1 = 3 \times (-4) = -12$$

$$U_2 V_2 = (-2) \times (-1) = 2$$

$$U_3 V_3 = 5 \times 6 = 30$$

Now, we sum them up:

$$\sum UV = (-12) + 2 + 30$$

$$\sum UV = -10 + 30 = 20$$

$$\boxed{\sum UV = 20}$$

(b)  $\sum(U + 3)(V - 4)$ :

$$(U_1 + 3)(V_1 - 4) = (3 + 3)(-4 - 4) = (6)(-8) = -48$$

$$(U_2 + 3)(V_2 - 4) = (-2 + 3)(-1 - 4) = (1)(-5) = -5$$

$$(U_3 + 3)(V_3 - 4) = (5 + 3)(6 - 4) = (8)(2) = 16$$

Sum the terms:

$$\sum(U + 3)(V - 4) = -48 + (-5) + 16$$

$$\sum(U + 3)(V - 4) = (-53) + 16 = -37$$

$$\boxed{\sum(U + 3)(V - 4) = -37}$$

(c)  $\sum V^2$ :

$$V_1^2 = (-4)^2 = 16, \quad V_2^2 = (-1)^2 = 1, \quad V_3^2 = 6^2 = 36$$

$$\sum V^2 = 16 + 1 + 36$$

$$\sum V^2 = 17 + 36 = 53$$

$$\boxed{\sum V^2 = 53}$$

(d)  $(\sum U)(\sum V)^2$ :

$$\sum U = 3 + (-2) + 5 = 6, \quad \sum V = -4 + (-1) + 6 = 1$$

$$(\sum U)(\sum V)^2 = 6 \times (1)^2 = 6 \times 1 = 6$$

$$\boxed{(\sum U)(\sum V)^2 = 6}$$

(e)  $\sum UV^2$ :

$$U_1 V_1^2 = 3 \times (-4)^2 = 3 \times 16 = 48$$

$$U_2 V_2^2 = (-2) \times (-1)^2 = (-2) \times 1 = -2$$

$$U_3 V_3^2 = 5 \times 6^2 = 5 \times 36 = 180$$

Sum them:

$$\sum UV^2 = 48 + (-2) + 180$$

$$\sum UV^2 = 46 + 180 = 226$$

$$\boxed{\sum UV^2 = 226}$$

(f)  $\sum(U^2 - 2V^2 + 2)$ :

$$U_1^2 - 2V_1^2 + 2 = 3^2 - 2(-4)^2 + 2 = 9 - 2 \cdot 16 + 2 = 9 - 32 + 2 = -21$$

$$U_2^2 - 2V_2^2 + 2 = (-2)^2 - 2(-1)^2 + 2 = 4 - 2 \cdot 1 + 2 = 4 - 2 + 2 = 4$$

$$U_3^2 - 2V_3^2 + 2 = 5^2 - 2(6)^2 + 2 = 25 - 2 \cdot 36 + 2 = 25 - 72 + 2 = -45$$

$$\sum(U^2 - 2V^2 + 2) = -21 + 4 + (-45)$$

$$\sum(U^2 - 2V^2 + 2) = -17 - 45 = -62$$

$$\boxed{\sum(U^2 - 2V^2 + 2) = -62}$$

$$(\mathbf{g})\sum\left(\frac{U}{V}\right):$$

$$\frac{U_1}{V_1} = \frac{3}{-4} = -\frac{3}{4}, \quad \frac{U_2}{V_2} = \frac{-2}{-1} = 2, \quad \frac{U_3}{V_3} = \frac{5}{6}$$

Add them:

$$-\frac{3}{4} + 2 + \frac{5}{6}.$$

Use common denominator 12:

$$-\frac{3}{4} = -\frac{9}{12}, \quad 2 = \frac{24}{12}, \quad \frac{5}{6} = \frac{10}{12}.$$

Therefore

$$-\frac{9}{12} + \frac{24}{12} + \frac{10}{12} = \frac{-9 + 24 + 10}{12} = \frac{25}{12}.$$

As decimal:

$$\frac{25}{12} \approx 2.08333 \dots$$

$$\sum\left(\frac{U}{V}\right) = \frac{25}{12} \approx 2.0833$$

### Problem 3

**3.90** Find the geometric mean of the sets (a) 3, 5, 8, 3, 7, 2 and (b) 28.5, 73.6, 47.2, 31.5, 64.8.

$$\text{Formula:} \quad GM = \left( \prod_{i=1}^n x_i \right)^{1/n}.$$

**(a) For the set {3, 5, 8, 3, 7, 2}:**

$$\text{Product} = 3 \times 5 \times 8 \times 3 \times 7 \times 2 = 5,040$$

Let  $n = 6$ , so we take the 6th root.

$$GM = 5040^{1/6}$$

$$\ln(5,040) \approx 8.5252, \quad \frac{1}{6} \ln(5,040) \approx 1.4209$$

$$GM = e^{1.4209} \approx 4.14$$

$$GM \approx 4.14$$

**(b) For the set**  $\{28.5, 73.6, 47.2, 31.5, 64.8\}$ :

$$\text{Product} = 28.5 \times 73.6 \times 47.2 \times 31.5 \times 64.8 = 202,092,516.864$$

Let  $n = 5$ , so we take the 5th root.

$$GM = (202,092,516.864)^{1/5}$$

$$\ln(202,092,516.864) \approx 19.1242, \quad \frac{1}{5} \ln(\text{Product}) \approx 3.8248$$

$$GM = e^{3.8248} \approx 45.83$$

$$GM \approx 45.83$$