Intelligent Robots Practice

Wheeled Mobile Robots

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Contents

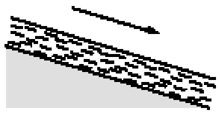
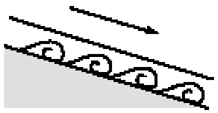
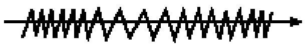
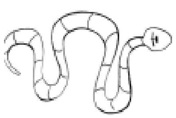
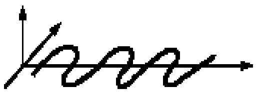
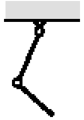
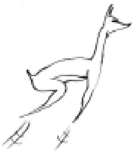
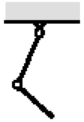
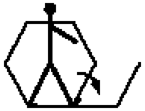
- Locomotion
- Wheels
- Mobile Robot Kinematics

Locomotion

(physical interaction between the vehicle and its environment)

Locomotion Concepts

Principles Found in Nature

Type of motion	Resistance to motion	Basic kinematics of motion
Flow in a Channel 	Hydrodynamic forces	Eddies 
Crawl 	Friction forces	 Longitudinal vibration
Sliding 	Friction forces	 Transverse vibration
Running 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Jumping 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Walking 	Gravitational forces	Rolling of a polygon (see figure 2.2) 

- Concepts found in nature: difficult to imitate technically
- Rolling is most efficient, but not found in nature
- However, the movement of a walking biped is **close to rolling**

Characterization of locomotion concept

- Locomotion
  - Generated by the **Mechanisms** and **Actuators**
- The most important issues in locomotion
  - Stability
    - number of contact points
    - center of gravity
    - static/dynamic stabilization
    - inclination of terrain
  - Characteristics of contact
    - contact point or contact area
    - angle of contact
    - friction
  - Type of environment
    - structure
    - medium (water, air, soft or hard ground)

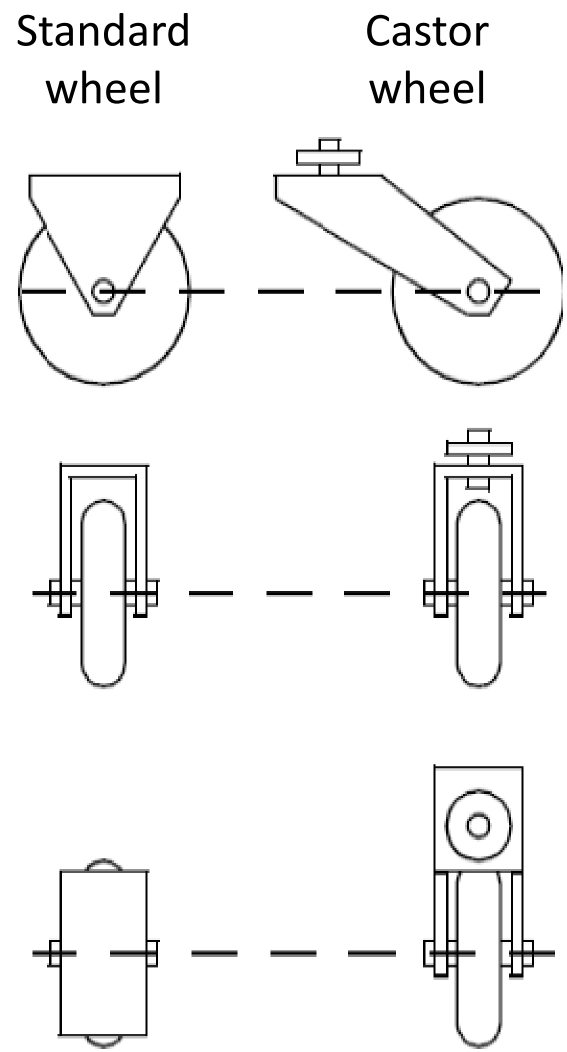
Wheels

Mobile Robots with Wheels

- Wheels
  - The most appropriate solution for most applications
  - Three wheels are sufficient and to guarantee stability
  - With more than three wheels a flexible suspension is required
  - Selection of wheels depends on the application

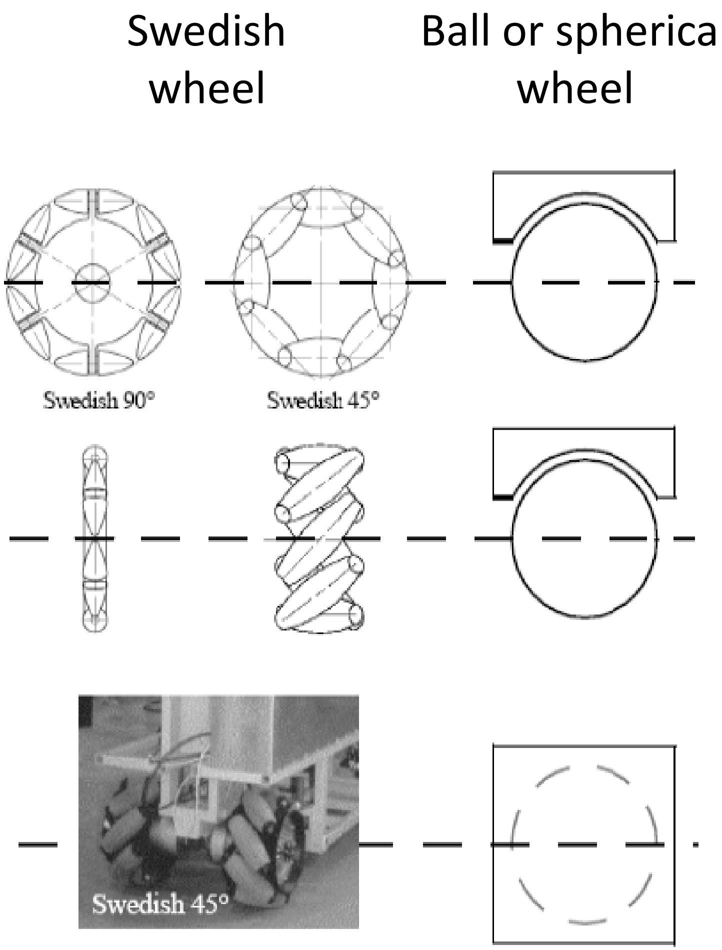
Four Basic Wheels Types

- Standard wheel
  - Two degrees of freedom
    - rotation around the (motorized) wheel axle and the contact point
- Castor wheel
  - Three degrees of freedom
    - rotation around the wheel axle, the contact point and the castor axle



Four Basic Wheels Types

- Swedish wheel
  - Three degrees of freedom
    - rotation around the (motorized) wheel axle, around the rollers and around the contact point
- Ball or spherical wheel
  - Suspension technically not solved

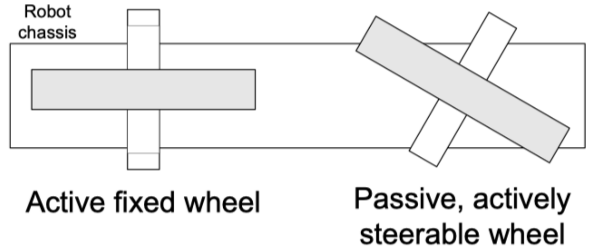


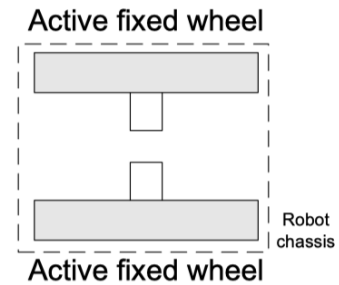
Characteristics of Wheeled Robots

- Stability
  - guaranteed with **3 wheels**
    - If center of gravity is within the triangle which is formed by the ground contact point of the wheels.
    - Stability is improved by 4 and more wheel
      - however, this arrangements are hyper static and require a flexible suspension system.
- Bigger wheels allow to overcome higher obstacles
  - but they require higher torque or reductions in the gear box.
- Most arrangements are **nonholonomic**
- Combining actuation and steering on one wheel makes the design complex and adds additional errors for odometry.

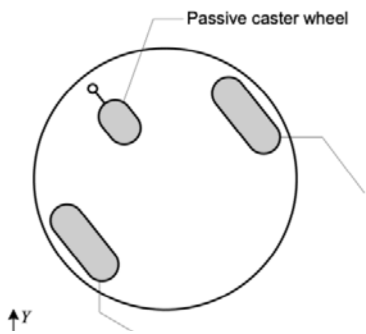
Different Arrangements of Wheels

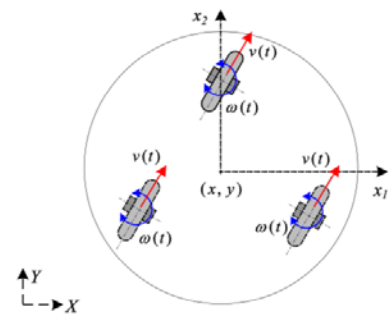
- Two Wheels

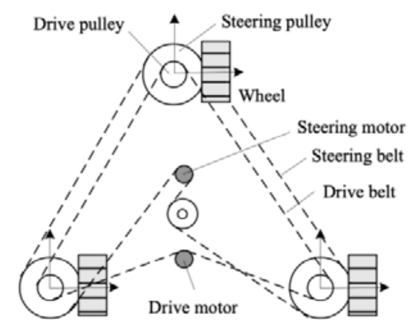


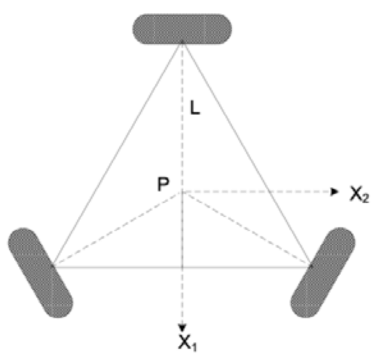


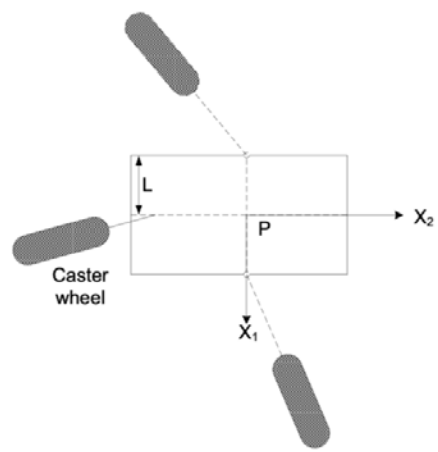
- Three Wheels

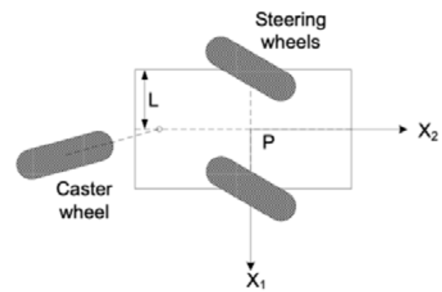






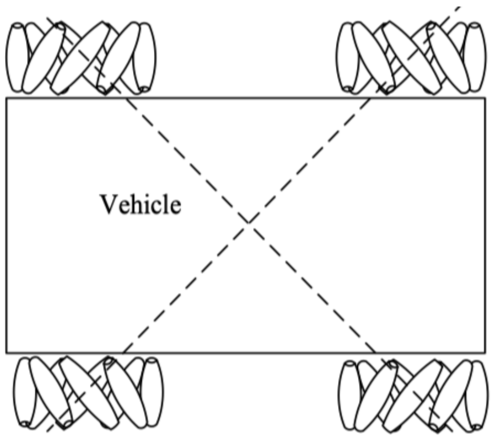


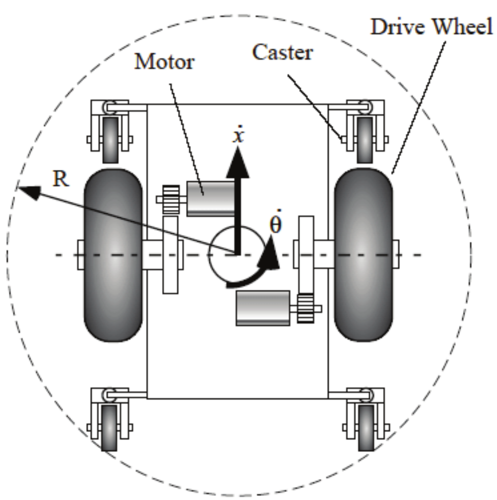


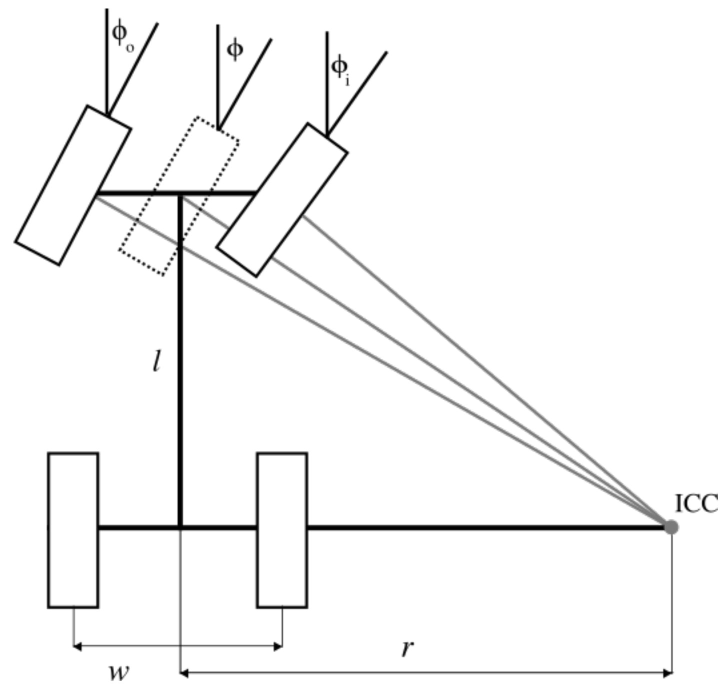


Different Arrangements of Wheels

- Four Wheels or more

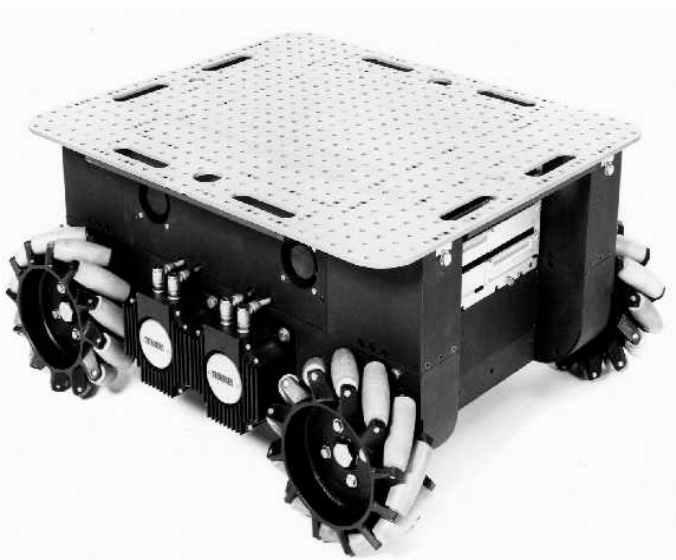


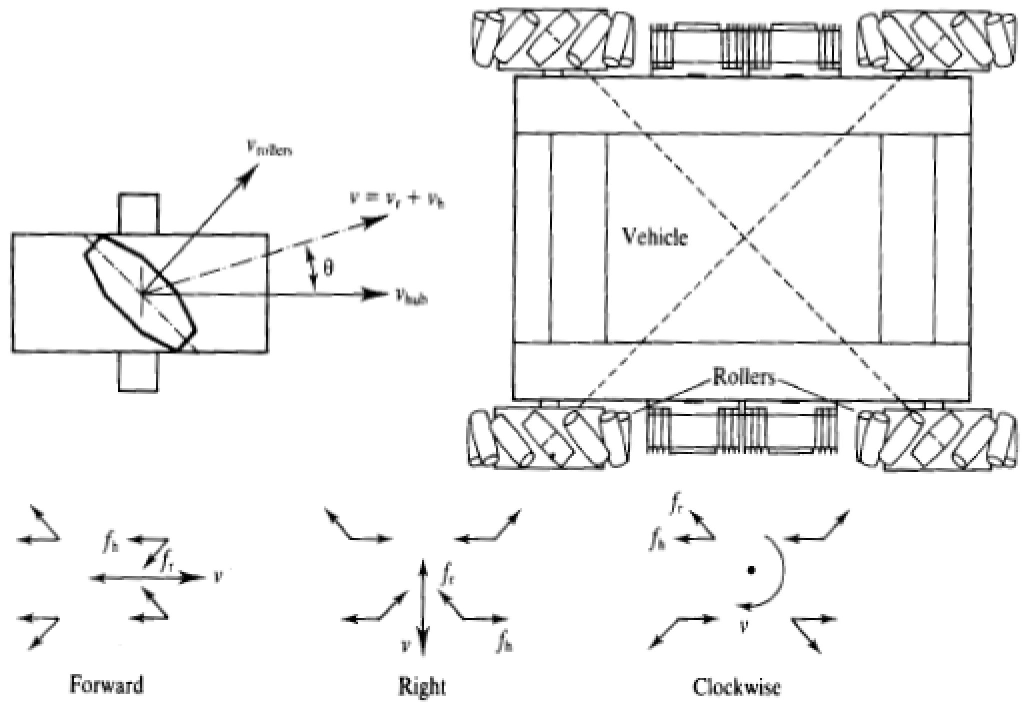




Wheeled Mobile Robots (applications)

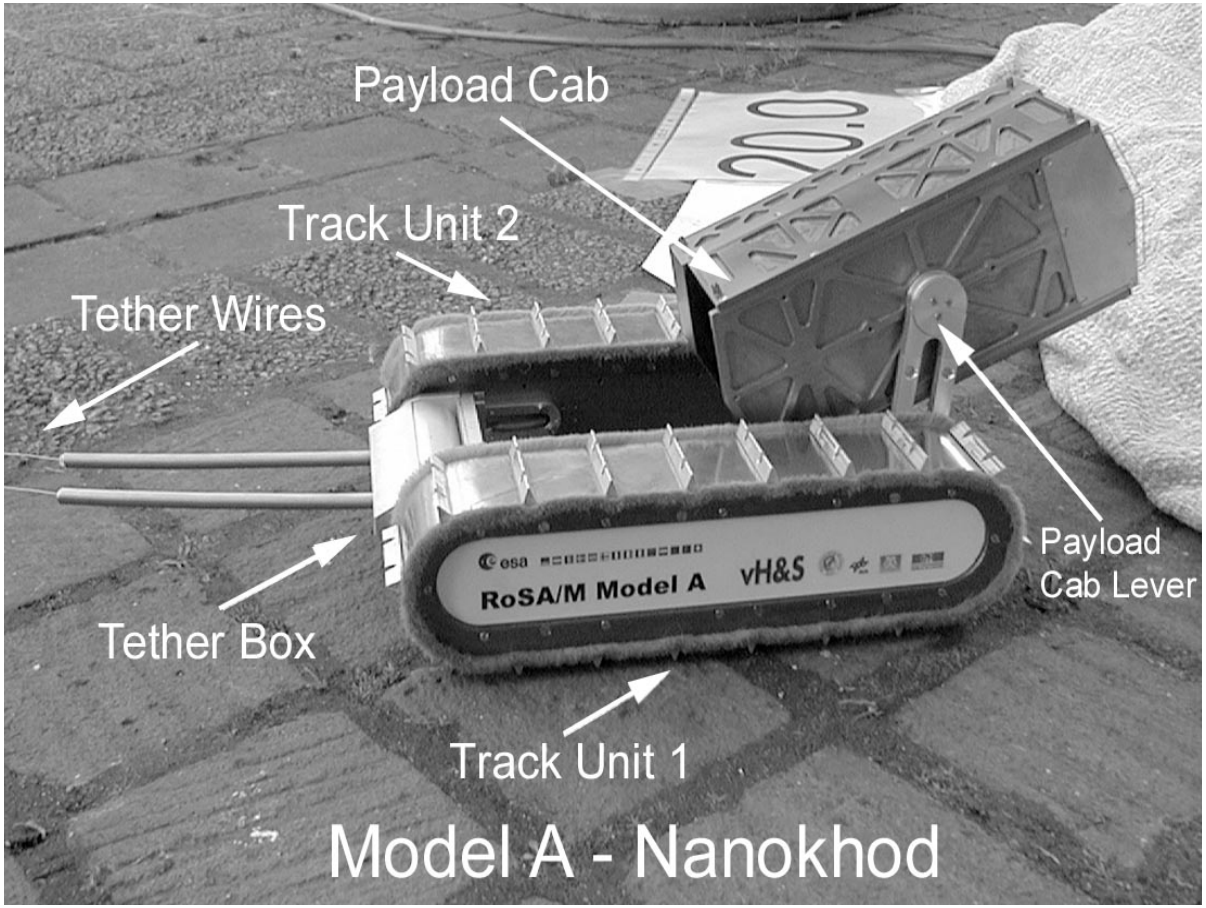
- Uranus, CMU: Omnidirectional Drive with 4 Wheels
  - Movement in the plane has 3 DOF
    - thus only three wheels can be independently controlled





Wheeled Mobile Robots (applications)

- The NANOKHOD II: Caterpillar
  - developed by von Hoerner & Sulger GmbH and Max Planck Institute, Mainz



Wheeled Mobile Robots (applications)

- SpaceCat: Stepping / Walking with Wheels
  - micro-rover for Mars, developed by Mecanex Sa and EPFL for the European Space Agency (ESA)

Wheeled Mobile Robots (applications)

- SHRIMP (EPFL)
  - Mobile Robot with Excellent Climbing Abilities
  - Passive locomotion concept

  - 6 wheels
    - two boogies on each side
    - fixed wheel in the rear
    - front wheel with spring suspension
  - Characteristics
    - highly stable in rough terrain
    - overcomes obstacles up to 2 times its wheel diameter

Mobile Robot Kinematics

Mobile Robot Kinematics

- Kinematics
  - The subfield of Mechanics dealing with motions of bodies
  - Forward kinematics
    - Given is a set of actuator positions
    - Determine corresponding reference pose
  - Inverse kinematics
    - Given is a desired reference pose
    - Determine corresponding actuator positions

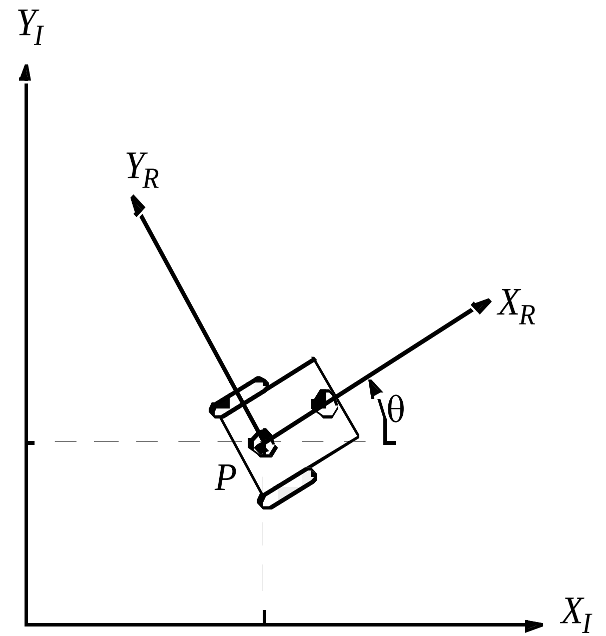
Mobile Robot Kinematics

Representing Robot Position

- Initial frame: $\{X_I, Y_I\}$
- Robot frame: $\{X_R, Y_R\}$
- Robot position: $\xi_I = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$
- Mapping between the two frames

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta) \cdot \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



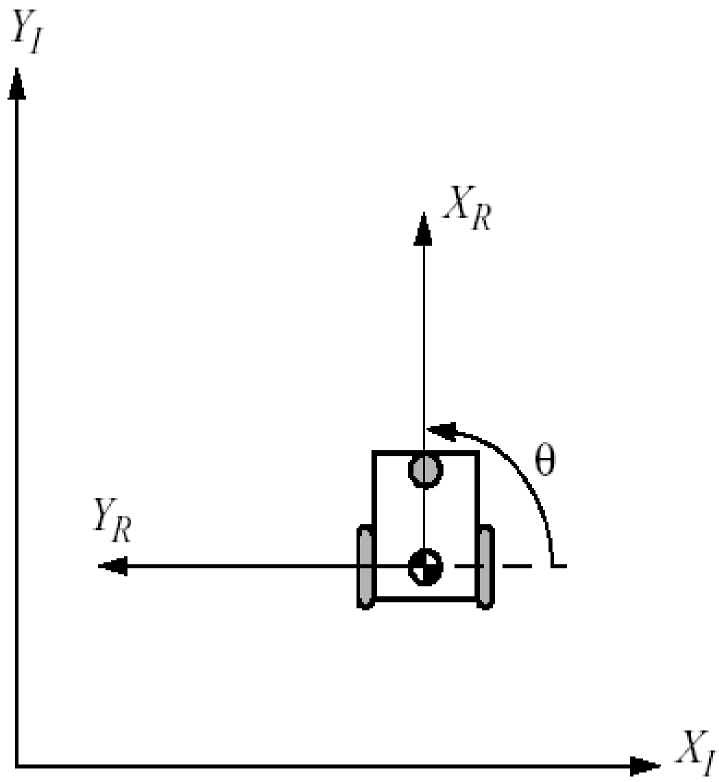
→ Rotation matrix representing the orientation of the moving frame relative to the reference frame

Mobile Robot Kinematics

- Representing Robot Position
  - Mapping between the two frames
    - Example: Robot aligned with Y_I

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

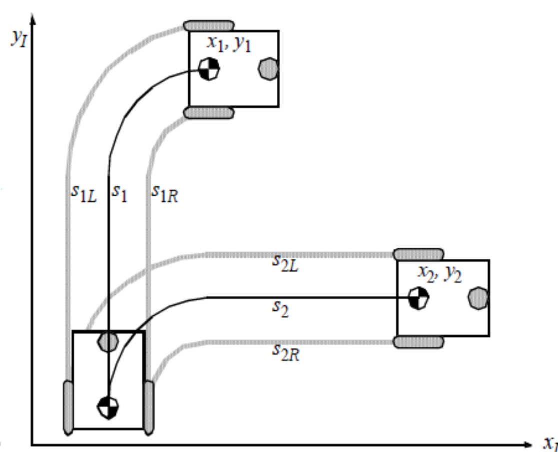


Mobile Robot Kinematics

- Kinematics
  - Holonomic systems
    - Diff. eqn. of $\dot{\xi}_I$ are **integrable** to the final position
    - the measure of the traveled distance of each wheel is sufficient to calculate the final position of the robot
  - Non-holonomic systems
    - Diff. eqn. of $\dot{\xi}_I$ are **not integrable** to the final position
    - The measure of the traveled distance s of each wheel is not sufficient to calculate the final robot position
    - Knowledge of the movement as a function of time becomes necessary

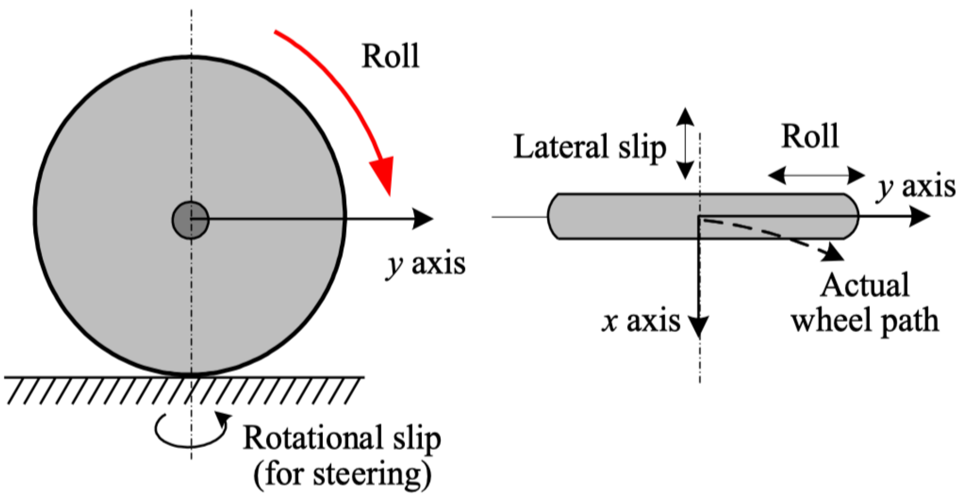
$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$

$$x_1 \neq x_2, y_1 \neq y_2$$



Mobile Robot Kinematics

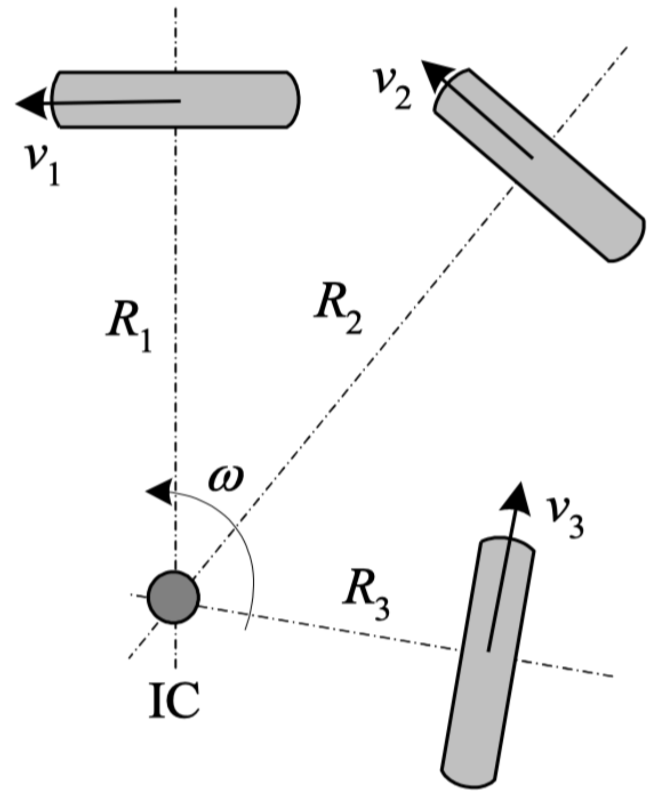
- Kinematics of wheel motion
  - Wheel motion model
    - Roll
    - Lateral slip: small at low velocities
    - Rotational slip → steering
    - An ideal wheel moves only along the roll direction. The actual motion of the wheel deviates off the roll direction to some extent.
    - A rolling wheel model is reasonable for low velocities.



Mobile Robot Kinematics

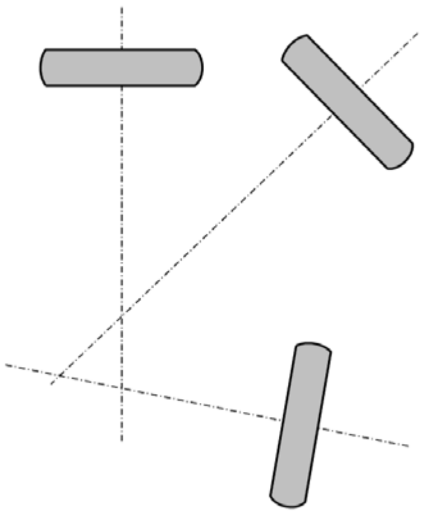
- Instantaneous Center of Rotation (IC or ICR)
  - Case 1: For a wheeled mobile robot to exhibit rolling motion
    - Each wheel on the vehicle follows a circular course about the IC.
      - The IC is at the intersection of the roll axis of each wheel.
    - Each wheel's velocity must be consistent with rotation of the vehicle.

$$v_1 = R_1\omega, v_2 = R_2\omega, v_3 = R_3\omega$$



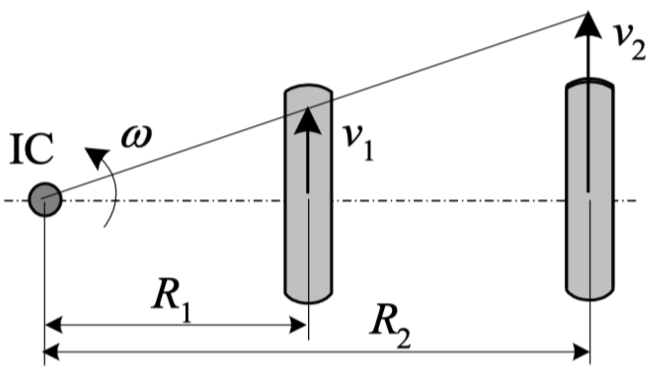
Mobile Robot Kinematics

- Instantaneous Center of Rotation (IC or ICR)
  - Case 2: No IC
    - The wheel exhibits rolling and slipping during motion



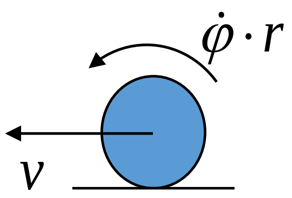
  - Case 3

$$v_1 = R_1\omega, v_2 = R_2\omega$$



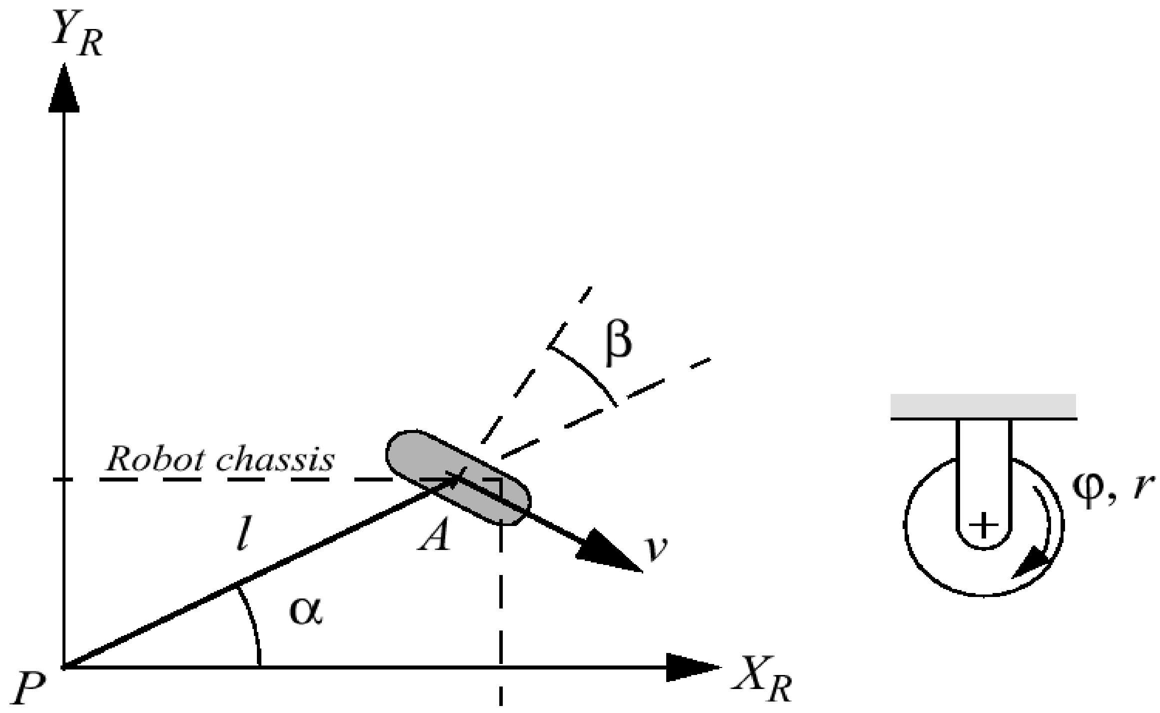
Mobile Robot Kinematics

- Wheel Kinematic Constraints
  - Assumptions
    - Movement on a horizontal plane
    - Point contact of the wheels
    - Wheels not deformable
    - Pure rolling
    - No slipping, skidding or sliding
    - No friction for rotation around contact point
    - Steering axes orthogonal to the surface
    - Wheels connected by rigid frame (chassis)



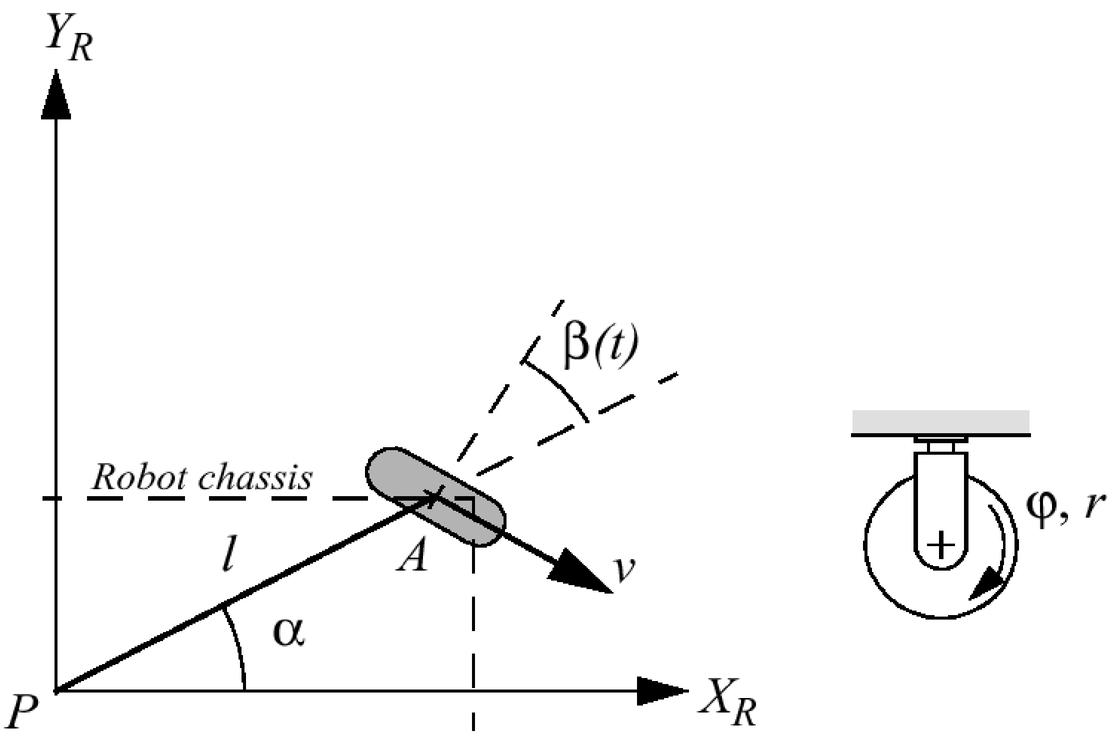
Mobile Robot Kinematics

- Wheel Kinematic Constraints
  - Fixed Standard Wheel
    - A standard wheel provides a directional constraint of velocity



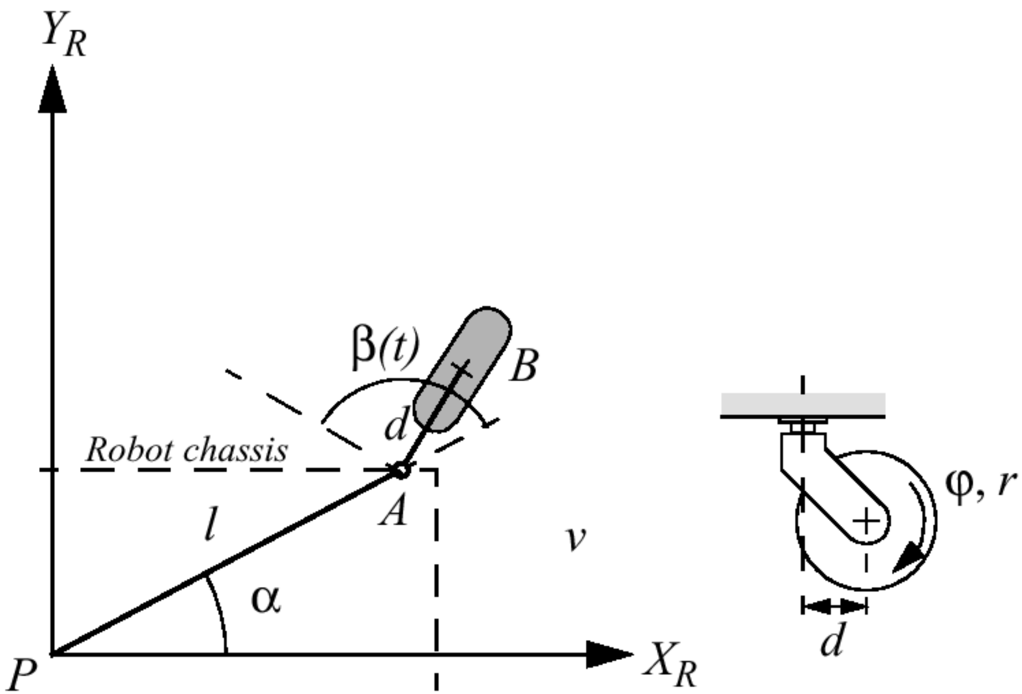
Mobile Robot Kinematics

- Wheel Kinematic Constraints
  - Steered Standard Wheel
    - A steerable standard wheel can be aligned by steering actuation



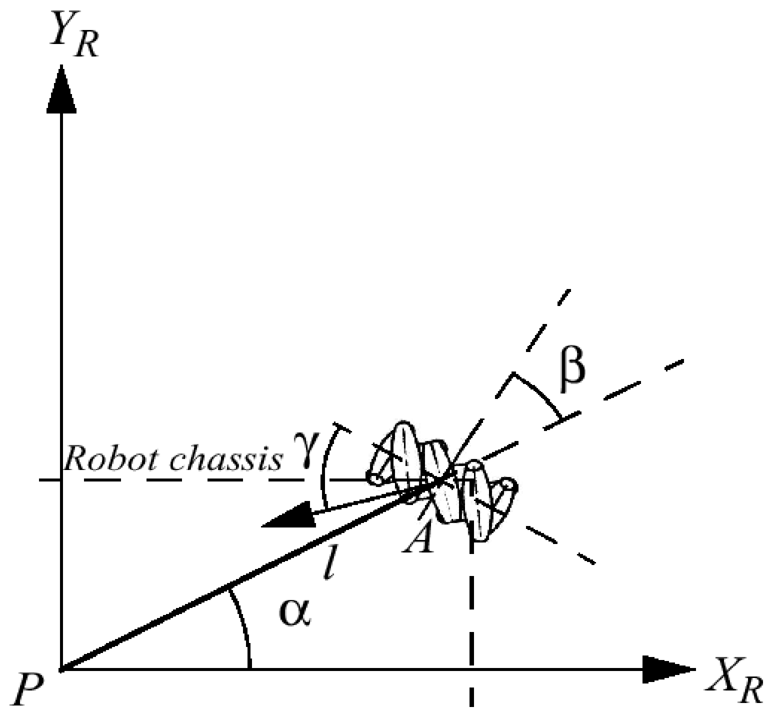
Mobile Robot Kinematics

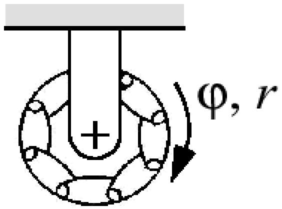
- Wheel Kinematic Constraints
  - Castor Wheel
    - An offset caster wheel allows two orthogonal linear velocities at the connecting point



Mobile Robot Kinematics

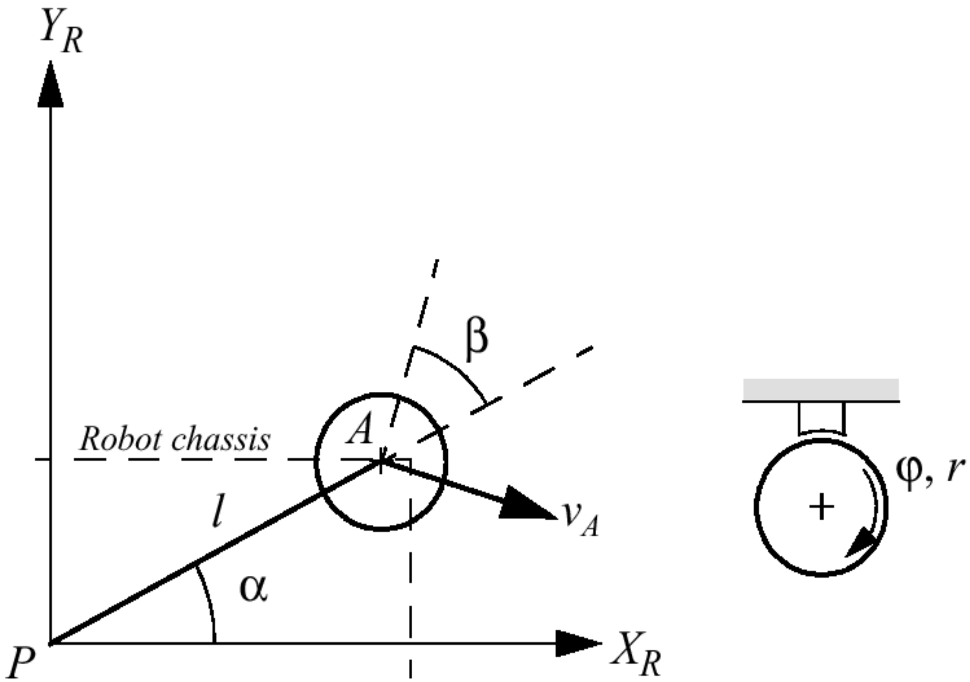
- Wheel Kinematic Constraints
  - Swedish Wheel
    - Standard+1 DOF





Mobile Robot Kinematics

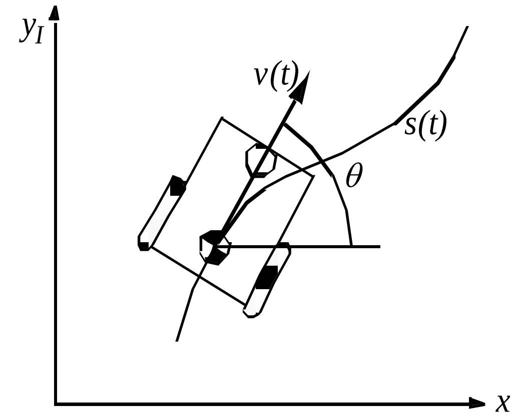
- Wheel Kinematic Constraints
  - Spherical Wheel
    - No direct constraints on motion
    - Omnidirectional



Mobile Robot Kinematics

- Kinematics Model
  - Goal
    - establish the robot speed $\dot{\xi} = [\dot{x} \quad \dot{y} \quad \dot{\theta}]^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i, steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (configuration coordinates)
  - Forward kinematics

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots \dot{\varphi}_n, \beta_1, \ldots \beta_m, \dot{\beta}_1, \ldots \dot{\beta}_m)$$

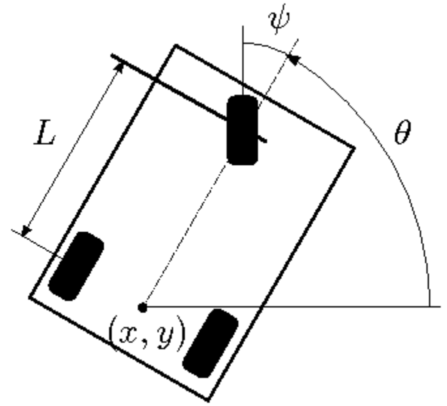


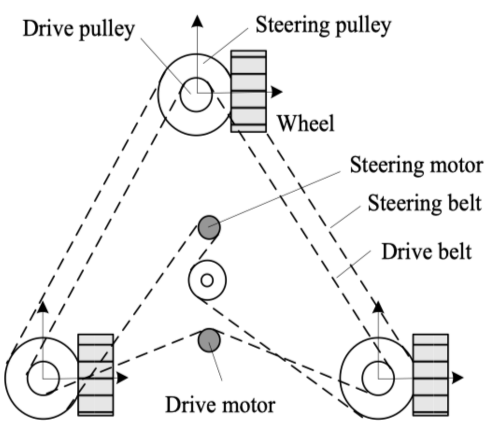
  - Inverse kinematics

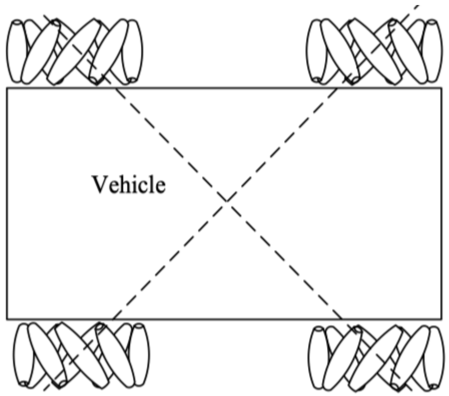
$$\begin{bmatrix} \dot{\varphi}_1 & \cdots & \dot{\varphi}_n & \beta_1 & \ldots & \beta_m & \dot{\beta}_1 & \ldots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

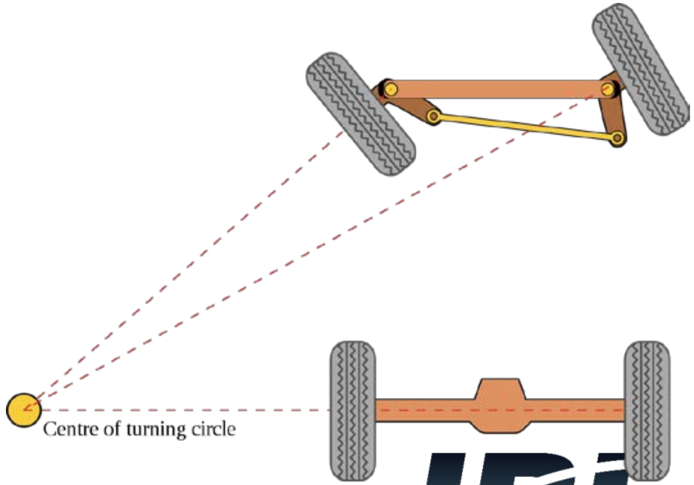
Mobile Robot Kinematics

- Mobile Robot Locomotion
  - Differential Drive
  - Steered wheels (tricycle, bicycles, wagon)
  - Synchronous Drive
  - Omni-directional
  - Car Drive (Ackerman Steering)
  - etc









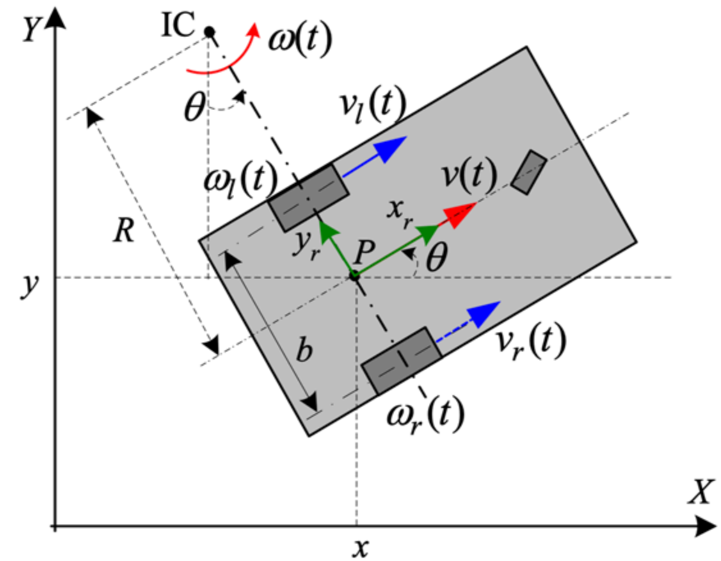
Mobile Robot Kinematics

- Differential drive robots
  - Differential drive mobile robots
    - Two wheels are mounted on a common axis and controlled by separate motors.
    - Simplest, but the most popular drive mechanism.
    - For each wheel to exhibit rolling motion, the robot must rotate about the IC lying on the common axis.
    - The IC changes depending on the relative velocity of two wheels.

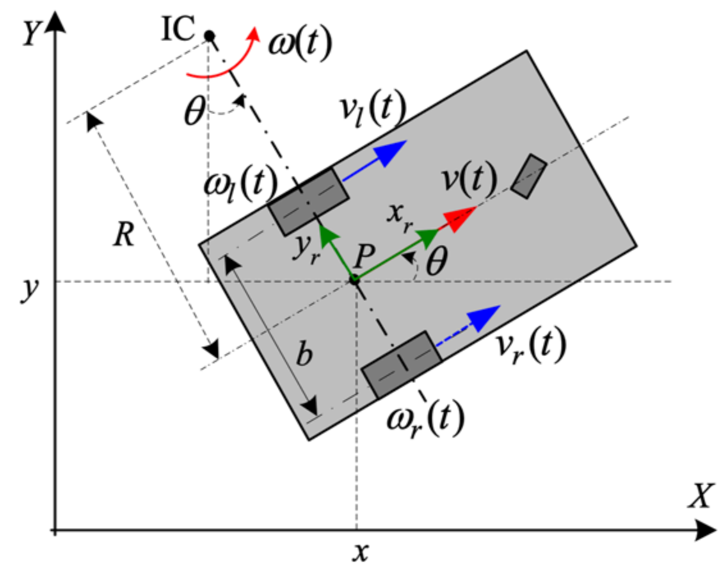
Mobile Robot Kinematics

- Differential drive robots
  - Kinematics
    - Notations
      - $\omega_l(t)$: angular velocity of left wheel
      - $\omega_r(t)$: angular velocity of right wheel
      - $v_l(t)$: linear velocity of left wheel ($\leftarrow$ $v_l(t) = r\,\omega_l(t)$)
      - $v_r(t)$: linear velocity of right wheel ($\leftarrow$ $v_r(t) = r\,\omega_r(t)$)
    - IC: Instantaneous center of rotation
      - R: Instantaneous curvature radius of the robot trajectory

$$\mathrm{IC} = (\, x - R\sin\vartheta \,,\; y + R\cos\vartheta)$$



Mobile Robot Kinematics



- Differential drive robots
  - Kinematics
    - Control Input

$$
\begin{cases}
\omega(t) = \dfrac{v_r(t)}{R + b/2} \\
\omega(t) = \dfrac{v_l(t)}{R - b/2}
\end{cases}
\Rightarrow
\begin{cases}
\omega(t) = \dfrac{v_r(t) - v_l(t)}{b} \\
R = \dfrac{b}{2}\dfrac{v_r(t) + v_l(t)}{v_r(t) - v_l(t)}
\end{cases}
$$

$$
\Rightarrow \; v(t) = R\,\omega(t) = \frac{1}{2}[v_r(t) + v_l(t)]
$$

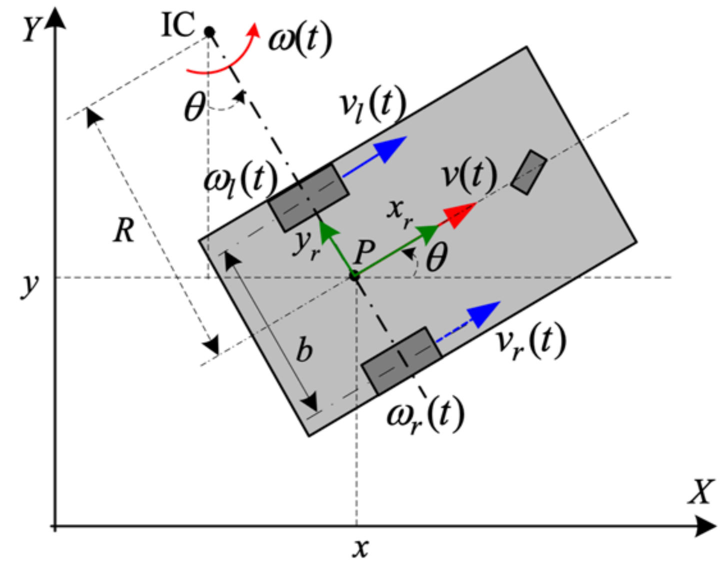
    - Linear and Angular velocity of a mobile robot

$$
\begin{cases}
v(t) = R\,\omega(t) = \dfrac{1}{2}[v_r(t) + v_l(t)] \\
\omega(t) = \dfrac{v_r(t) - v_l(t)}{b}
\end{cases}
$$

Mobile Robot Kinematics

- Differential drive robots
  - Kinematics

$$\begin{cases} v(t) = R\,\omega(t) = \dfrac{1}{2}[v_r(t) + v_l(t)] \\ \omega(t) = \dfrac{v_r(t) - v_l(t)}{b} \end{cases}$$



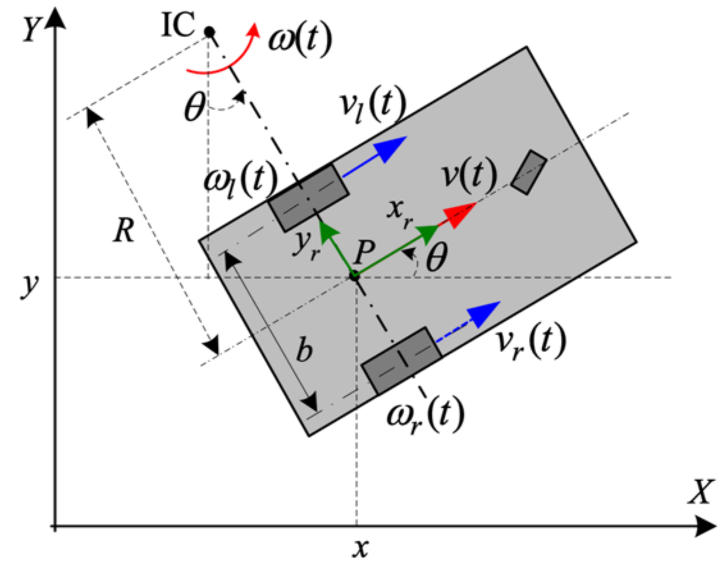
- Special case 1: $v_l = v_r$
  - $v(t) = v_l(t) = v_r(t)$ & $\omega(t) = 0$ → Moving in a straight-line.
- Special case 2: $v_l = -v_r$
  - $v(t) = 0$ & $\omega(t) = 2v_r(t)/b$ → Pure rotation about the robot center.
- General case: R = finite nonzero
  - Following a curved path
- Very sensitive to the relative velocity of two wheels.
  - Small errors in the velocity provided to each wheel result in different trajectories.

Mobile Robot Kinematics

- Differential drive robots
  - Kinematics model in the robot frame

$$\begin{Bmatrix} v_x(t) \\ v_y(t) \\ \omega(t) \end{Bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/b & r/b \end{bmatrix} \begin{Bmatrix} \omega_l(t) \\ \omega_r(t) \end{Bmatrix}$$

    - Useful for velocity control



  - Kinematics model in the robot frame
    - Robot pose $[x(t), y(t), \theta(t)]$

$$\begin{cases} \dot{x}(t) = v(t)\cos\theta(t) \\ \dot{y}(t) = v(t)\sin\theta(t) \\ \dot{\theta}(t) = \omega(t) \end{cases} \Rightarrow \begin{Bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{Bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} v(t) \\ \omega(t) \end{Bmatrix}$$

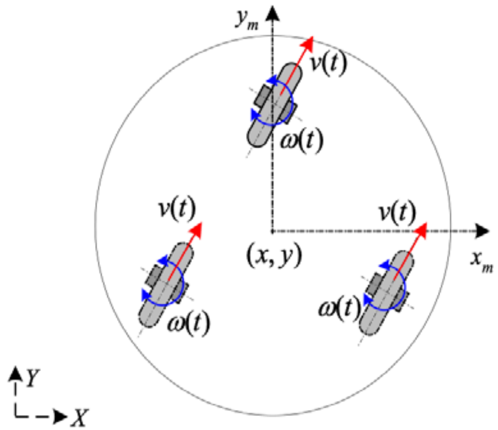
$$\begin{cases} x(t) = \int_0^t v(\tau)\cdot\cos\theta(\tau)\,d\tau \\ y(t) = \int_0^t v(\tau)\cdot\sin\theta(\tau)\,d\tau \\ \theta(t) = \int_0^t \omega(\tau)\,d\tau \end{cases}$$

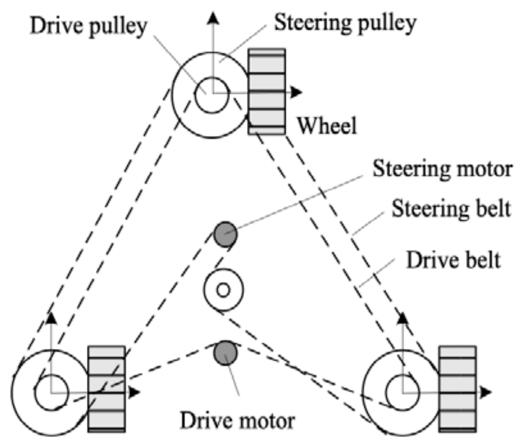
Mobile Robot Kinematics

- Synchronous drive robots
  - Synchronous drive mobile robots
    - Each wheel is capable of being driven and steered.
    - Typical configuration: Three steered wheels are arranged at the vertices of an equilateral triangle
    - All of the wheels steer and drive in unison.
      - One motor rotates all of the wheels at the same speed.
      - Another motor steers all of the wheels so that they always point in the same direction.
    - The IC is always at infinity. The orientation of a robot cannot be changed.
    - Often used with turret.
    - Mechanical chain might result in misalignment of wheels

Mobile Robot Kinematics

- Synchronous drive robots
  - Synchronous drive mobile robots



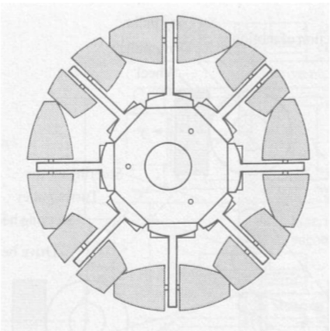


- Forward kinematics
  - Control variables: translational speed $v(t)$ and rotational velocity $\omega(t)$

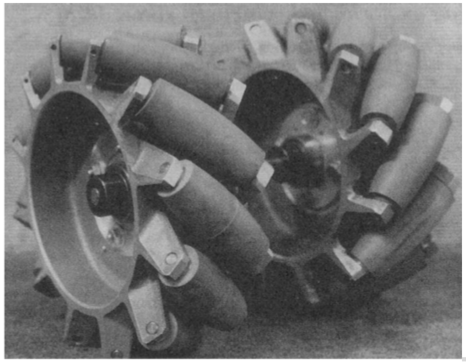
$$
\begin{cases}
x(t) = \int_0^t v(\tau) \cdot \cos\theta(\tau)\, d\tau \\
y(t) = \int_0^t v(\tau) \cdot \sin\theta(\tau)\, d\tau \\
\theta(t) = \int_0^t \omega(\tau)\; d\tau
\end{cases}
$$

Mobile Robot Kinematics

- Omnidirectional mobile robots
  - Omnidirectional mobile robots
    - Capable of 3 DOF motion
    - Inverse kinematics is significant.
    - Design problem is closely related to solving nonholonomic constraints.
  - Roller wheels
    - Composed of a circular hub surrounded by passive rollers.
    - A hub is driven and the rollers are idle (i.e., passive)
  - Types of roller wheels
    - Universal wheels, Mecanum wheels (Swedish wheels), etc



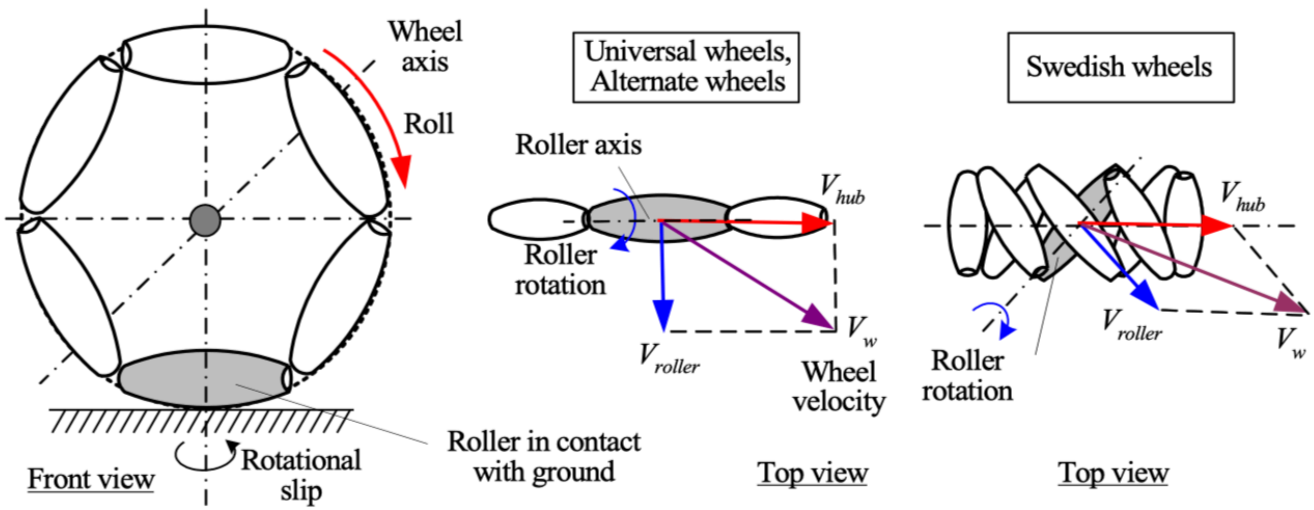
Universal wheels



Mecanum wheels (or Swedish wheel)

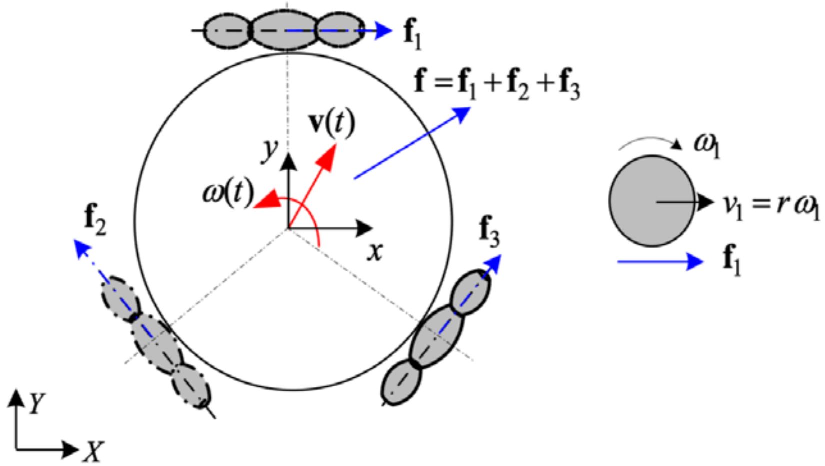
Mobile Robot Kinematics

- Omnidirectional mobile robots
  - Kinematics of roller wheels
    - Hub rotation: rotation (or roll) about the hub axis with the rollers remaining still.
    - Roller rotation: translation in the direction of the hub axis with the roller in contact with the ground spinning and the hub fixed.
    - Motion in other directions involves a combination of hub rotation and roller rotation.



Mobile Robot Kinematics

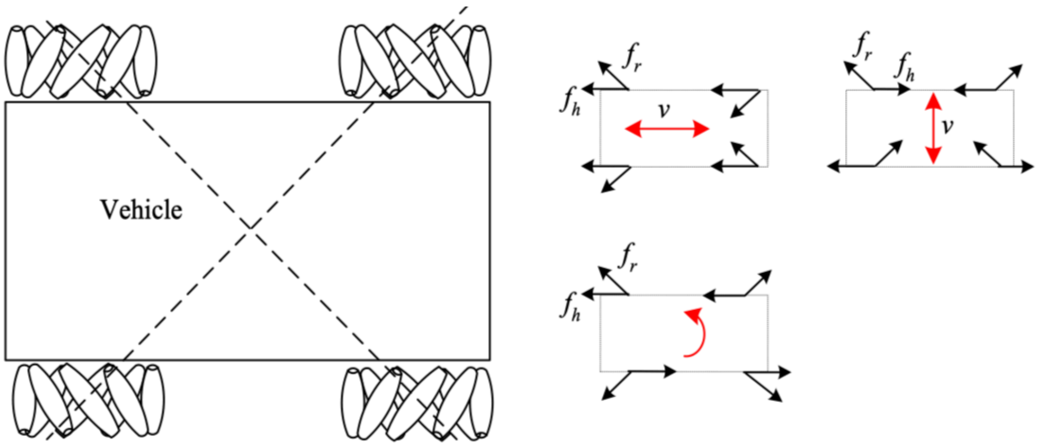
- Omnidirectional mobile robots
  - Three-wheeled omnidirectional mobile robot with universal wheels



  - A resultant force vector **f** from three wheel forces determines the motion of a robot.
  - The motion is decomposed into a translation of the robot center and a rotation about the robot center.
    - Pure rotation: **f** = 0

Mobile Robot Kinematics

- Omnidirectional mobile robots
  - Four-wheeled mobile robot with Swedish wheels



  - Drawbacks
    - Vertical vibration due to discontinuous contact
    - Reliability problem
    - Complicated design