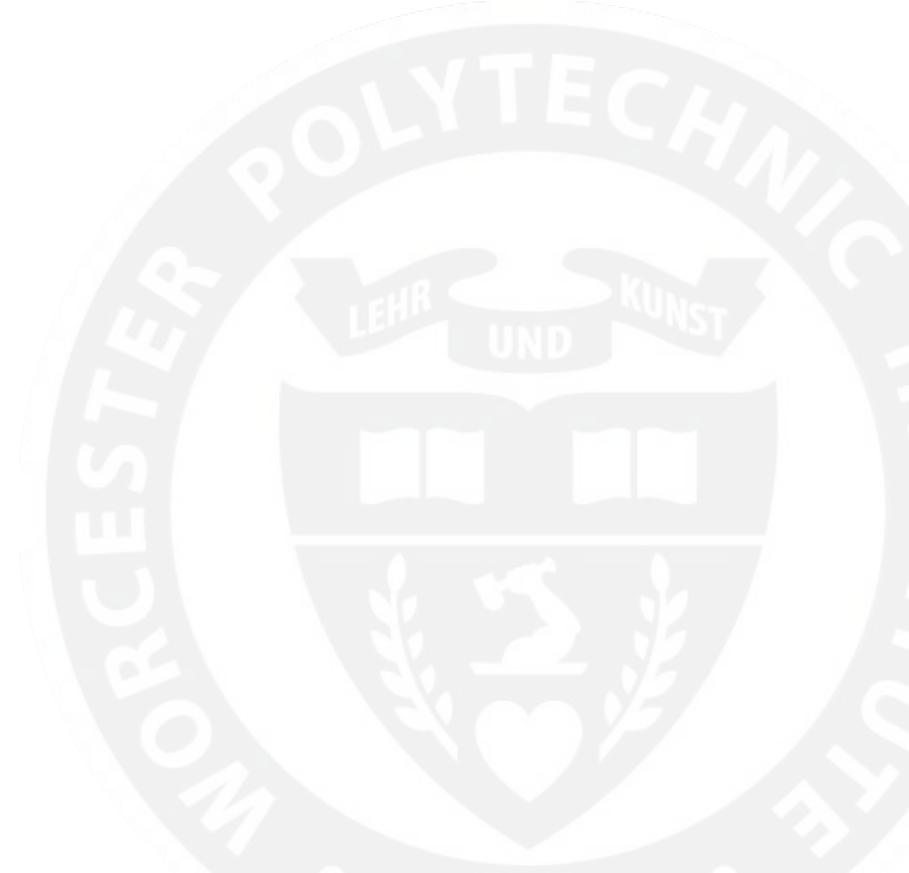


WPI

Hybrid Holistic Mobile Manipulator

Edward Jackson

Adviser: Professor Berk Calli

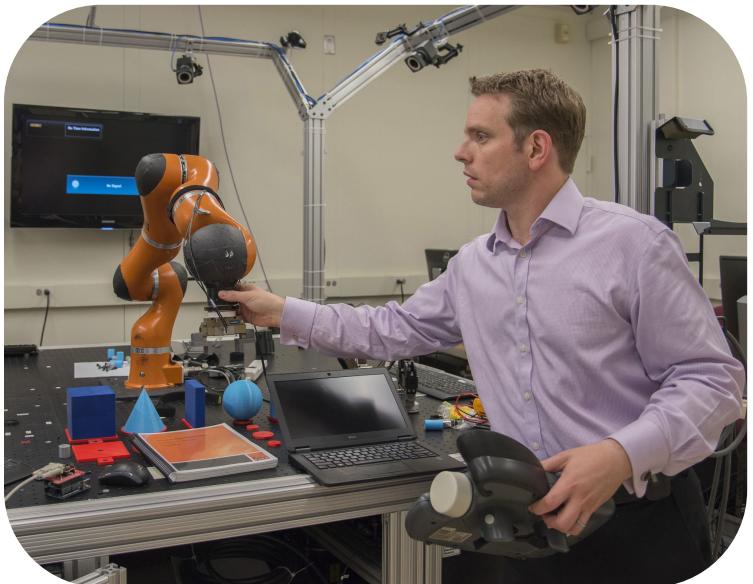


Project Overview

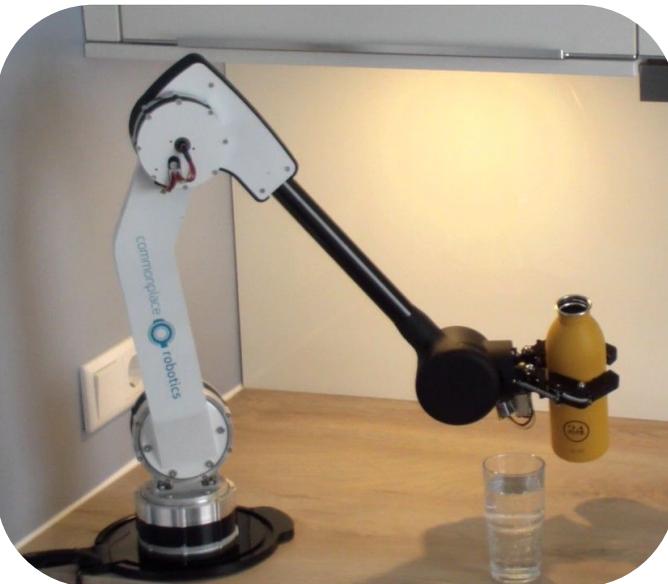
- Continuation of *Design and Implementation of a Modular Mobile Manipulator*
- Implemented a controller to govern the mobile robot in one cost function
 - Moves the system gracefully to a desired Pose
 - Maximizes Manipulability while traversing
 - Controller is reactive to changes in environment and conditions



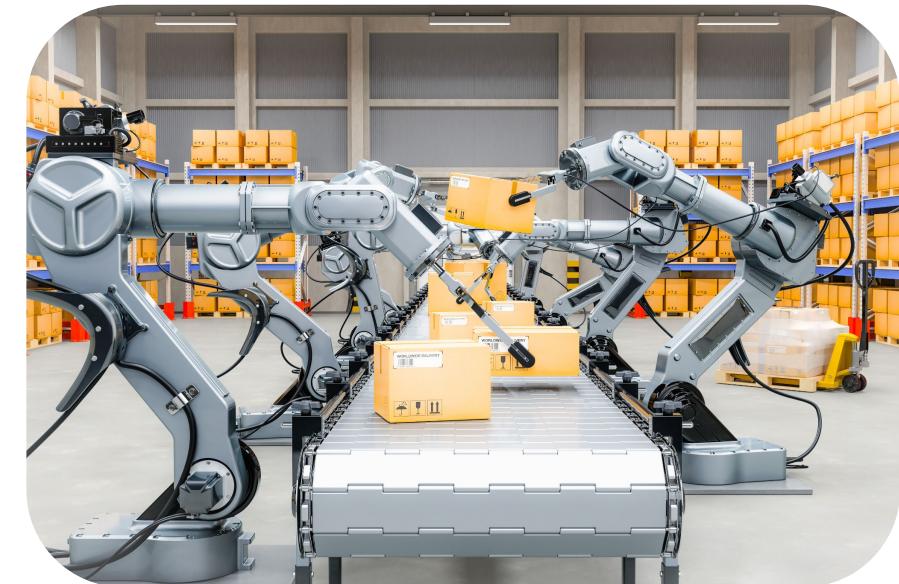
Robotic Arms and Their Applications



Research



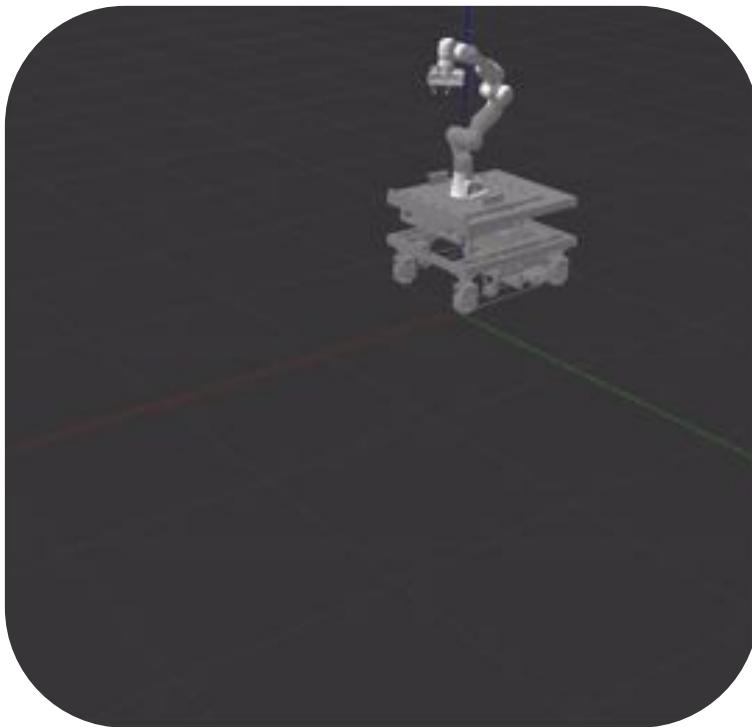
Service



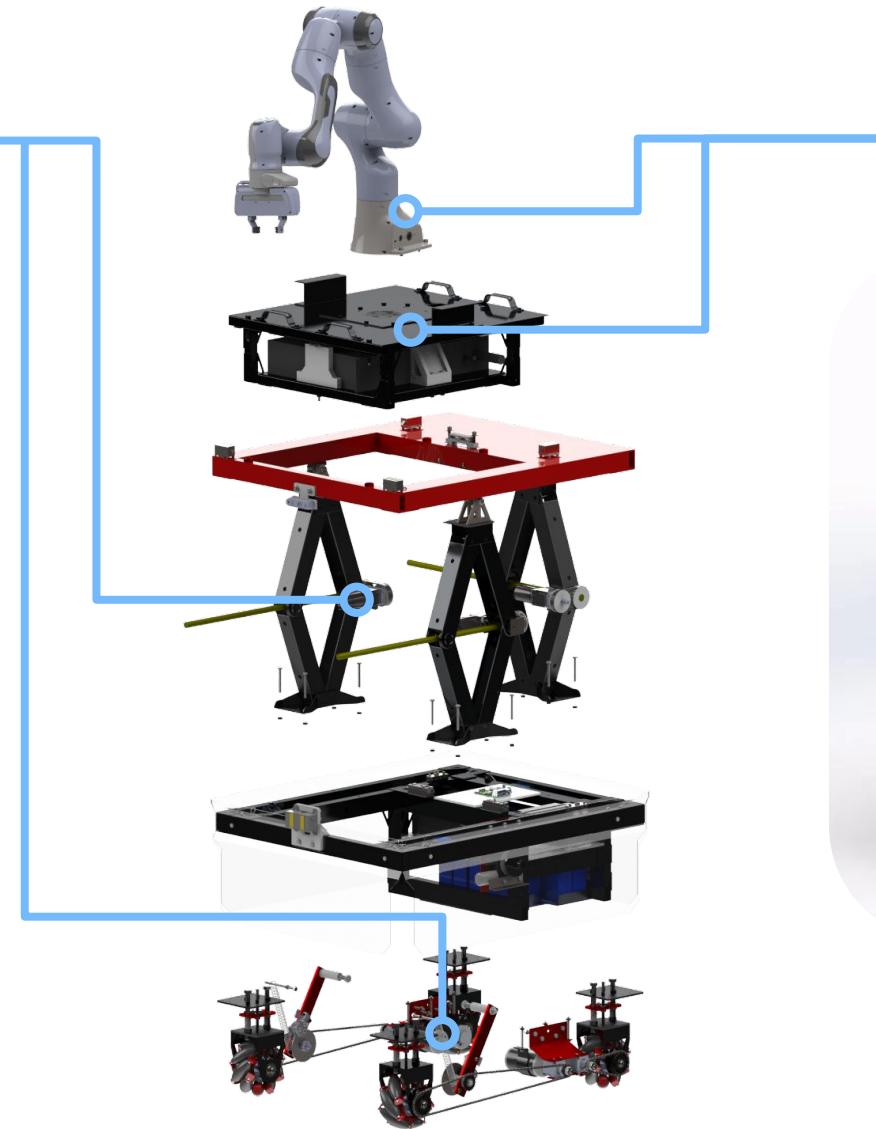
Industrial & Manufacturing

Proposed MQP Design

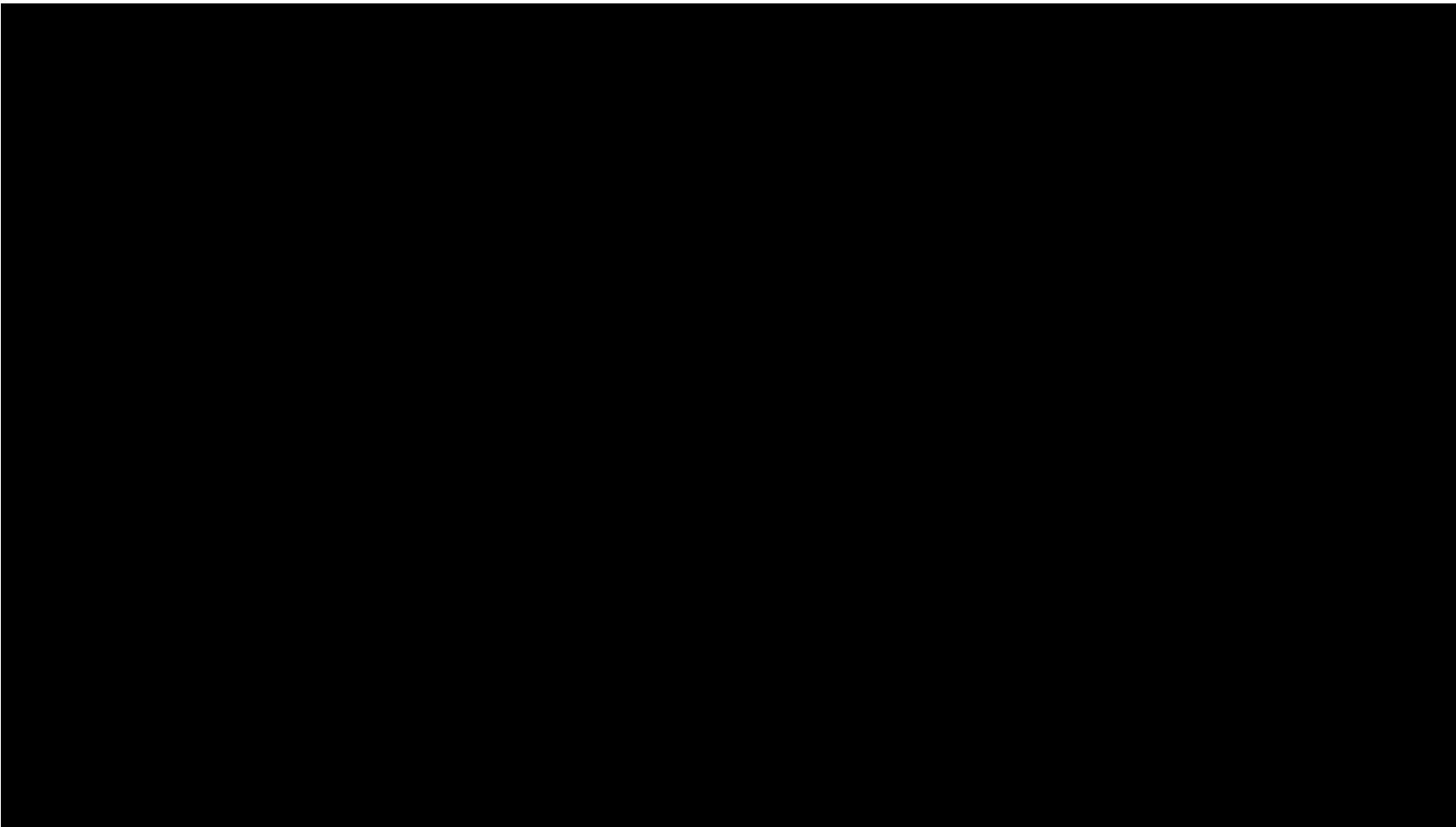
Mobility



Modularity



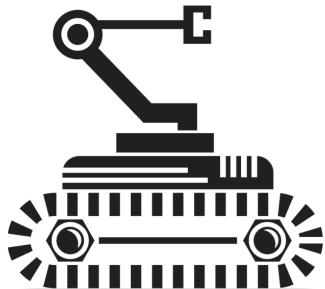
Robot Performing a Task





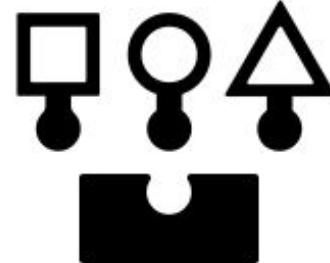
DR Objectives

Holistic



- Considers all aspects of system functionality together

Solves Joint Mismatch



- Various joint types are irrelevant

Robust & Efficient



- Maximizes Manipulability during operation
- Faster solve time than existing Motion Planners

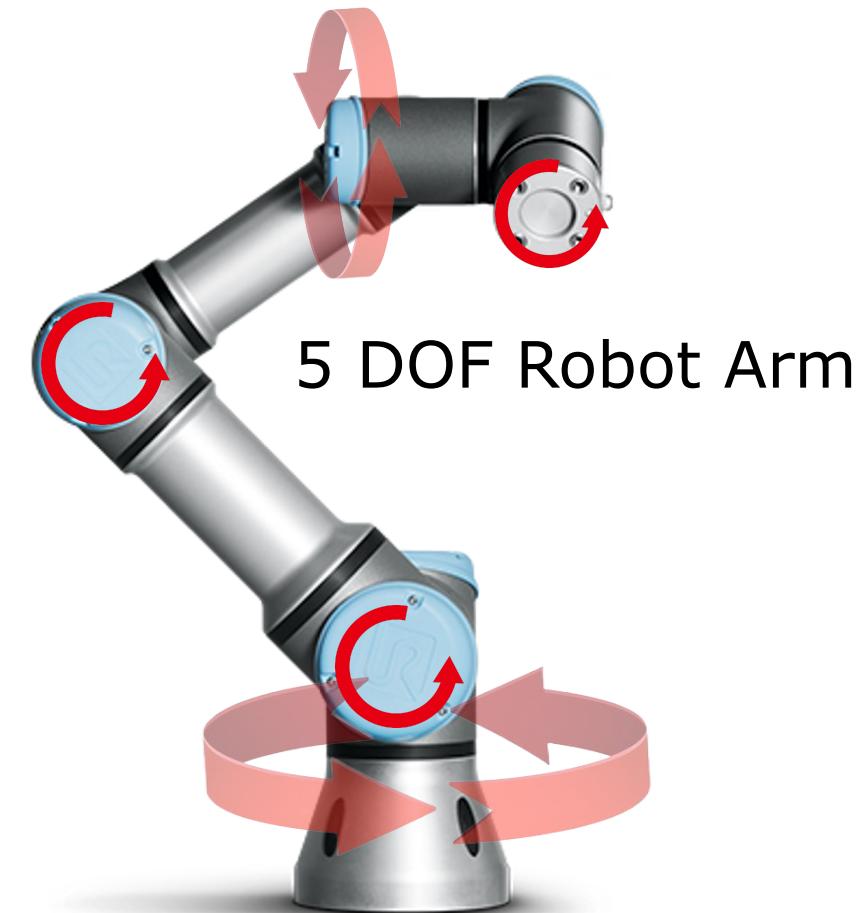
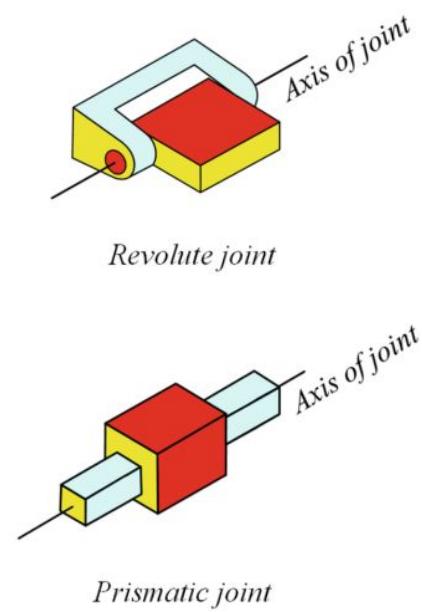
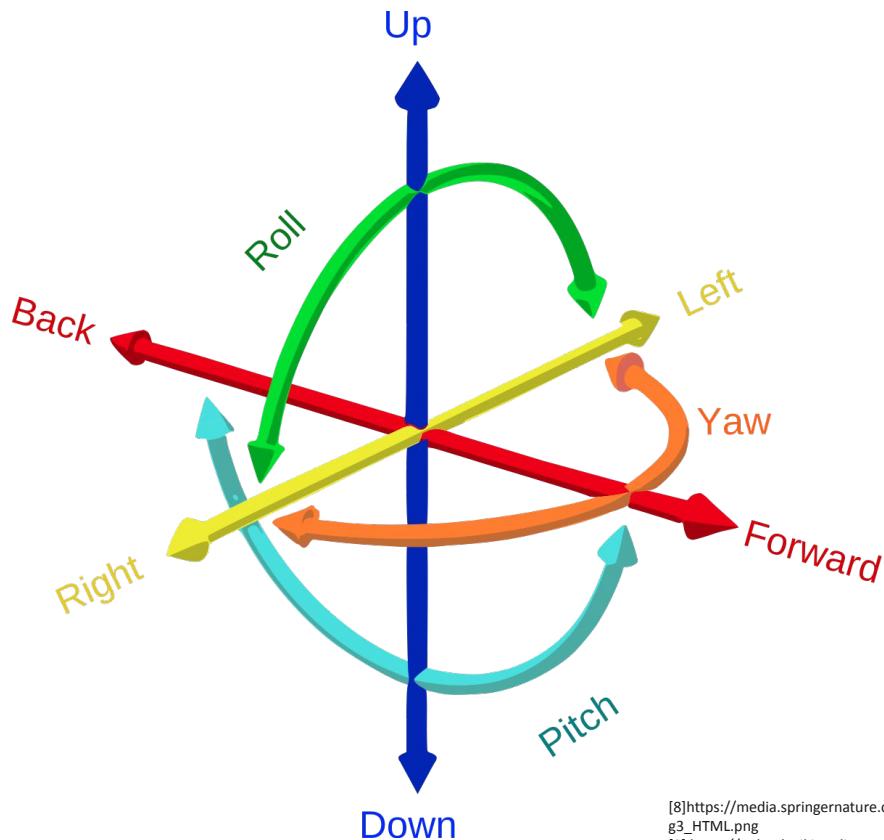


WPI

Background Terminology & Info

What is a Degree of Freedom (DOF)?

- Ability to translate or rotate about an axis
- DOF of a Robot Arm = # of joints



[8] https://media.springernature.com/lw685/springer-static/image/chp%3A10.1007%2F978-3-030-93220-6_1/MediaObjects/133289_3_En_1_Fig3_HTML.png
[9] <https://upload.wikimedia.org/wikipedia/commons/thumb/2/2a/6DOF.svg/1200px-6DOF.svg.png>
[10] https://cdn.shopify.com/s/files/1/1750/5061/products/ur5_300x300.png?v=1541621897

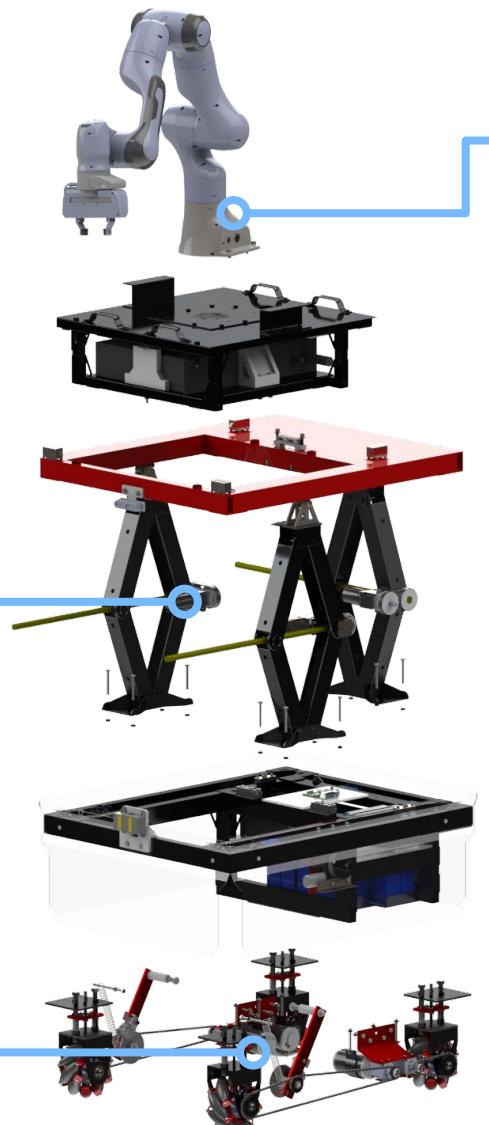
Integrated Redundancy

3 DOF Parallel
Manipulator
Elevator

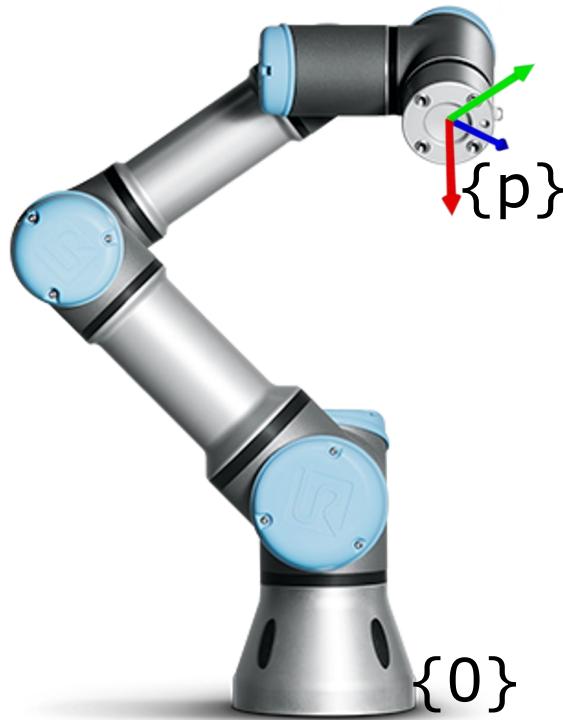
2/3 DOF
Driving System

7 DOF Robot
Arm

=12/13 DOF
System



Robot Arm Forward Kinematics



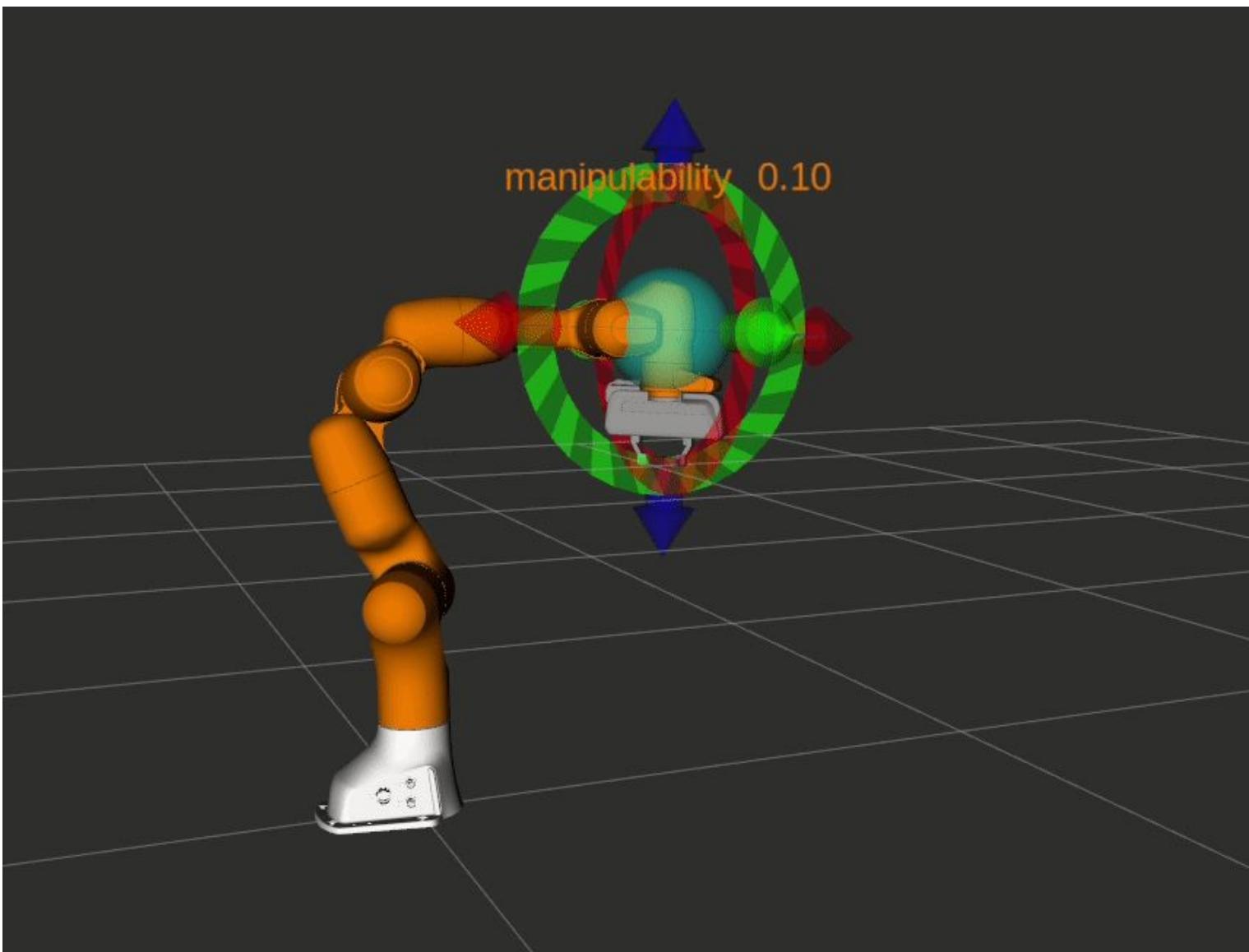
$$p = F(q)$$

$$\frac{d}{dt}(p) = \frac{d}{dt}(F(q)) \Rightarrow v = J(q)\dot{q}$$

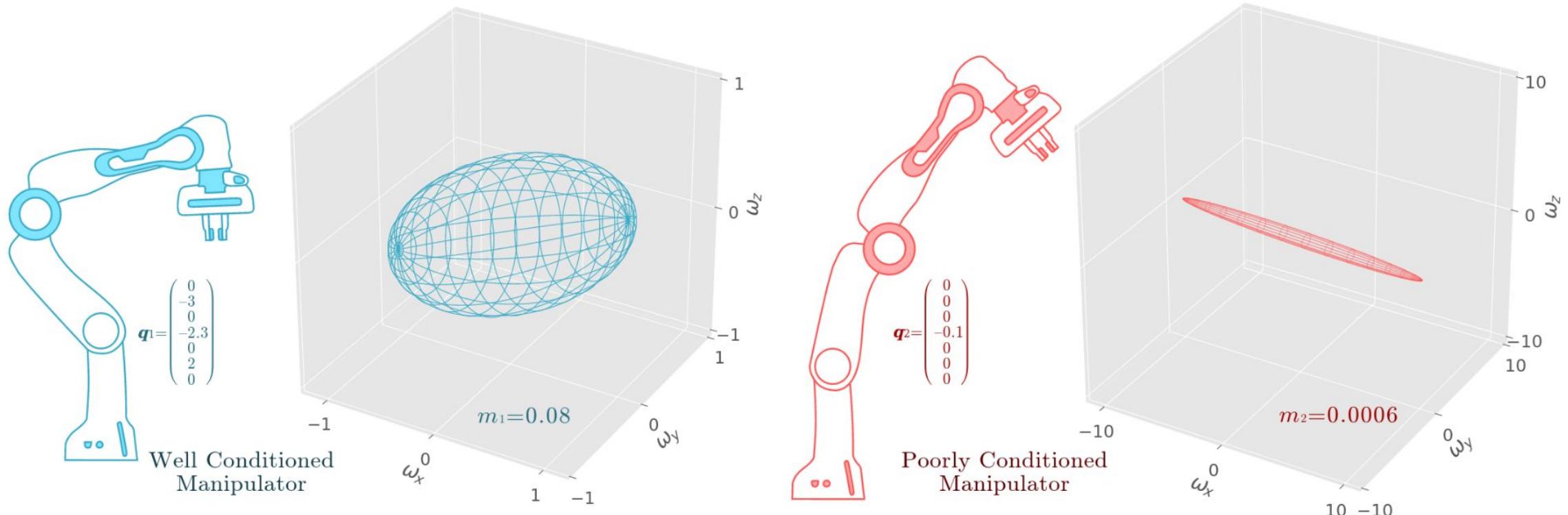
$$p = \begin{bmatrix} [x] \\ [y] \\ [z] \\ [a] \\ [b] \\ [c] \end{bmatrix}, \quad v = \begin{bmatrix} [x/s] \\ [y/s] \\ [z/s] \\ [a/s] \\ [b/s] \\ [c/s] \end{bmatrix}$$

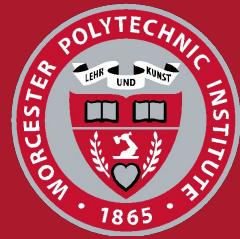
Kinematically Redundant Pose Exploration

- End Effector **Pose** remains *constant*
- Arm Configuration and Manipulability index can *change*



What is Manipulability?



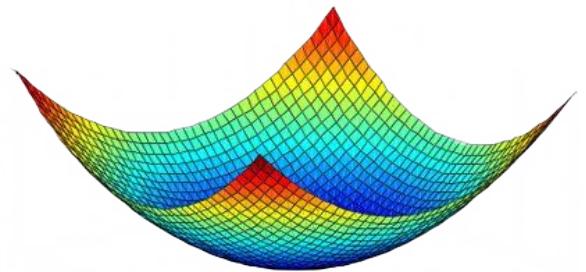


WPI

Proposed Holistic Controller

Holistic Controller Basics

Quadratic Program



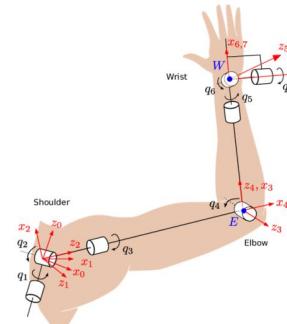
- Model system as multivariate Quadratic Program

Maximize Manipulability



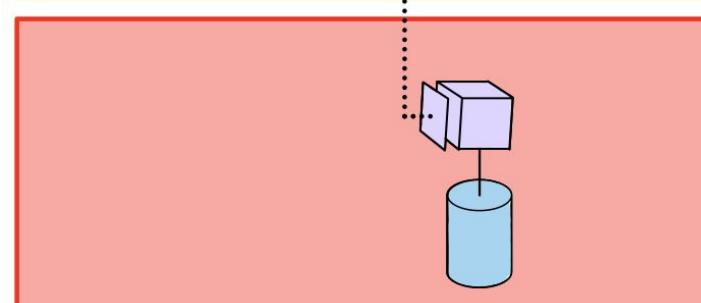
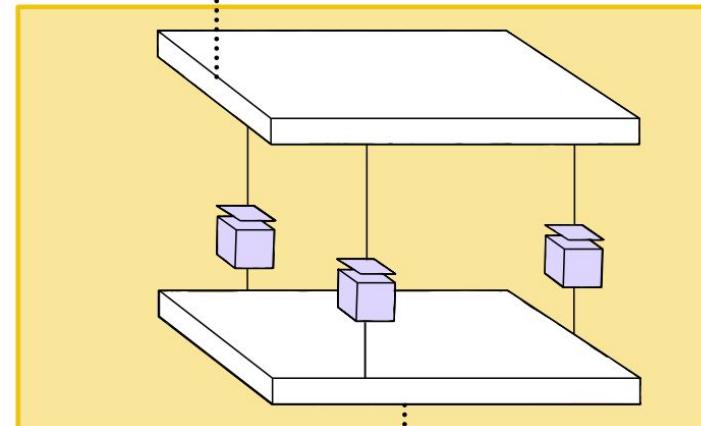
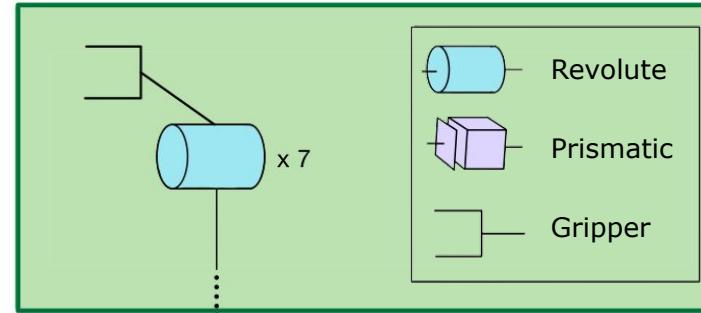
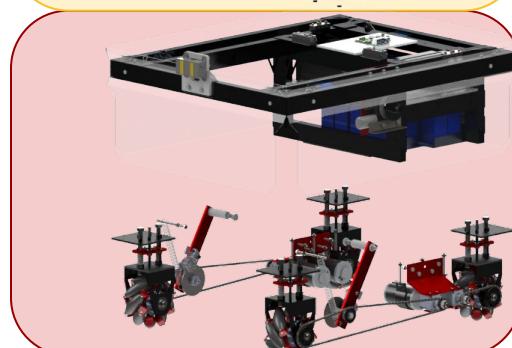
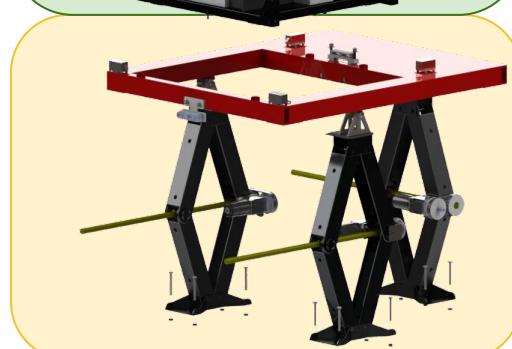
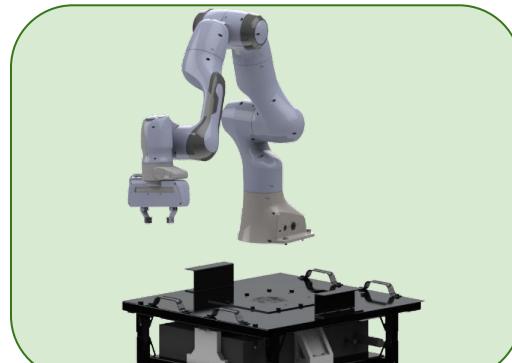
- Set additional sub objectives to maximize “good” robot configurations

Simplify System



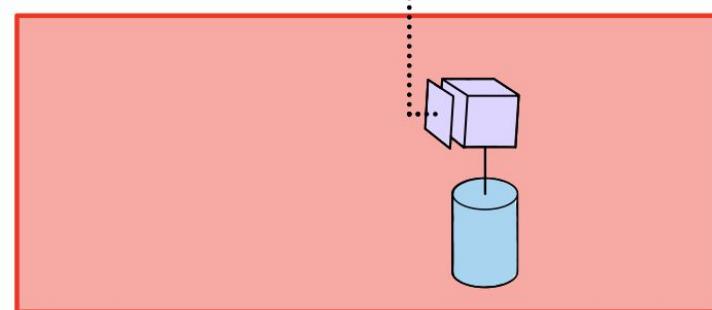
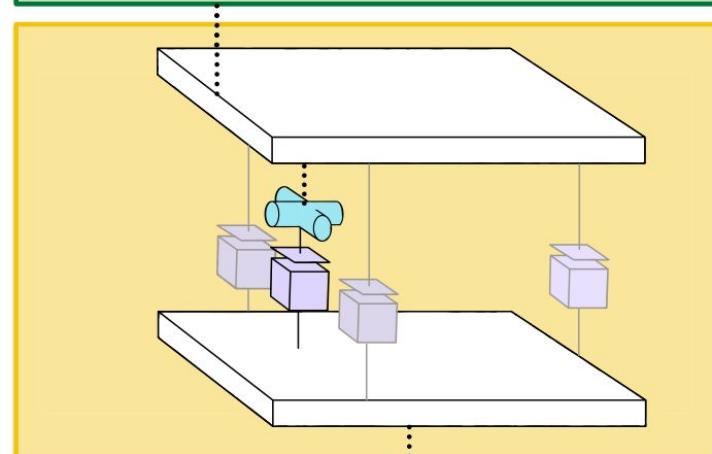
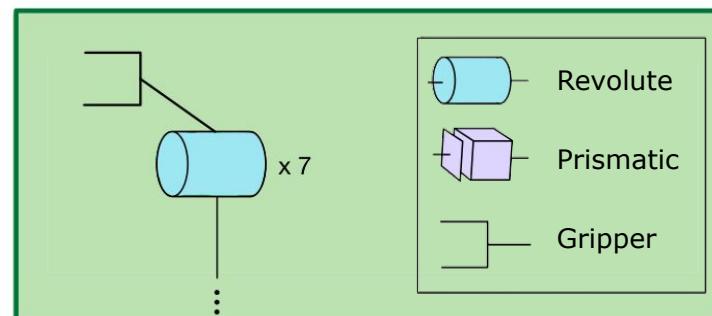
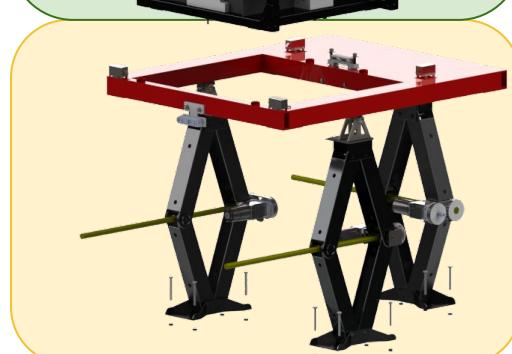
- Decompose System into Linkages and Joints

Simplifying System into Prismatic/Revolute Joints & Linkages



- 7 DOF *Serial* Linkage
- 3 DOF *Parallel* Linkage
- 2 DOF *Serial* Linkage

Simplify Elevator into Serial Manipulator



Pivoting



Vertical
Translation

Quadratic Program Background

$$\begin{array}{ll}\text{minimize} & (1/2)\mathbf{x}^T \mathbf{P} \mathbf{x} + \mathbf{q}^T \mathbf{x} \\ \text{subject to} & \mathbf{G} \mathbf{x} \leq \mathbf{h} \\ & \mathbf{A} \mathbf{x} = \mathbf{b}\end{array}$$

\mathbf{x} : Optimization Variables

\mathbf{P}, \mathbf{q} : Define general QP

\mathbf{G}, \mathbf{h} : Inequality Constraints

\mathbf{A}, \mathbf{b} : Equality Constraints

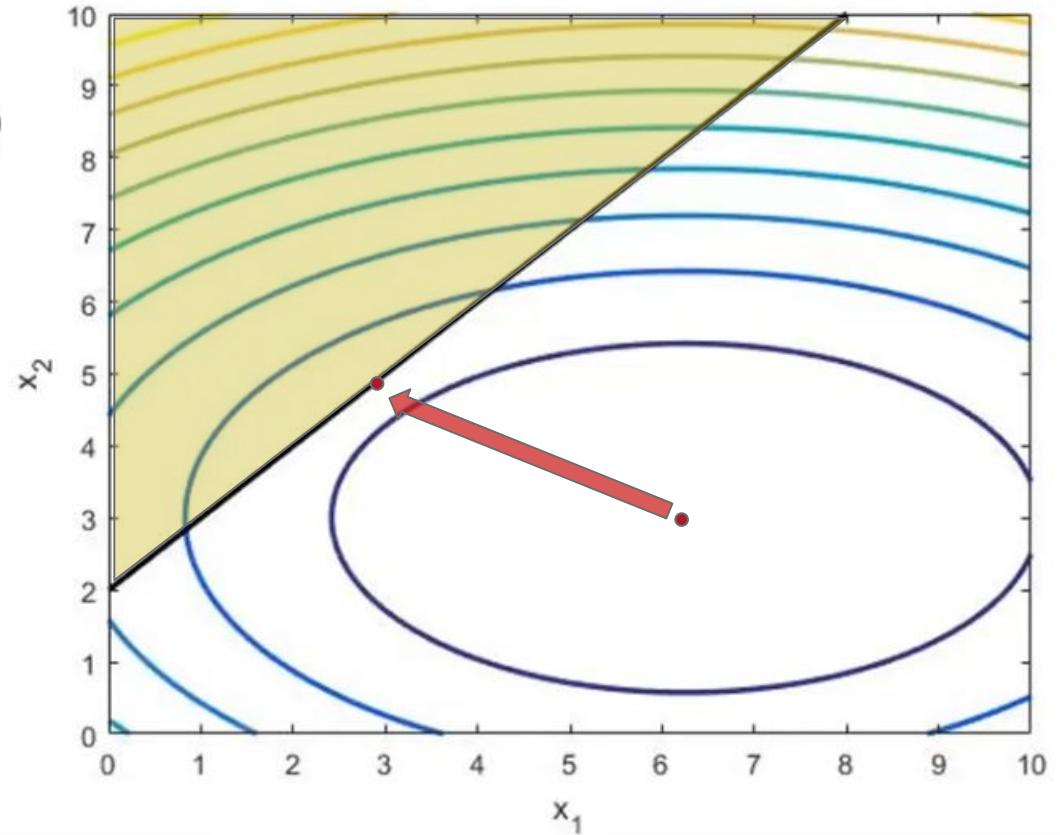
Simple QP Example

$$\begin{aligned} \min_{x_1, x_2} : & \text{ cost} = 0.4x_1^2 - 5x_1 + x_2^2 - 6x_2 + 50 \\ \text{subject to: } & x_2 - x_1 \geqslant 2 \end{aligned}$$

$$x_1 = 2.5$$

$$x_2 = 4.5$$

$$\text{cost} = 33.25$$



Robot Arm RRMC Controller with QP

Recall:

$$\begin{array}{ll}\text{minimize} & (1/2)\mathbf{x}^T \mathbf{P}\mathbf{x} + \mathbf{q}^T \mathbf{x} \\ \text{subject to} & \mathbf{G}\mathbf{x} \leq \mathbf{h} \\ & \mathbf{A}\mathbf{x} = \mathbf{b}\end{array}$$

$$v_{[6x1]} = J_{[6xn]} \dot{q}_{[nx1]}$$

$$\dot{q} = J^+ v$$

$$v \propto e$$

$$e = p^* - p$$



$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} \dot{q}^T \mathbf{1}_n \dot{q}$$

$$\begin{array}{ll}\text{subject to} : & J(q)\dot{q} = v \\ & \dot{q}^- \leq \dot{q} \leq \dot{q}^+\end{array}$$

Resolved Rate Motion
Controller

RRMC as QP
Worcester Polytechnic Institute

QP of Mobile Manipulator

$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} \dot{x}^T \boxed{Q} \dot{x} + \boxed{C^T} \dot{x}$$

Objective Function

subject to: $\mathcal{J}(q)x = {}^b v_e$

RRMC with Tolerance

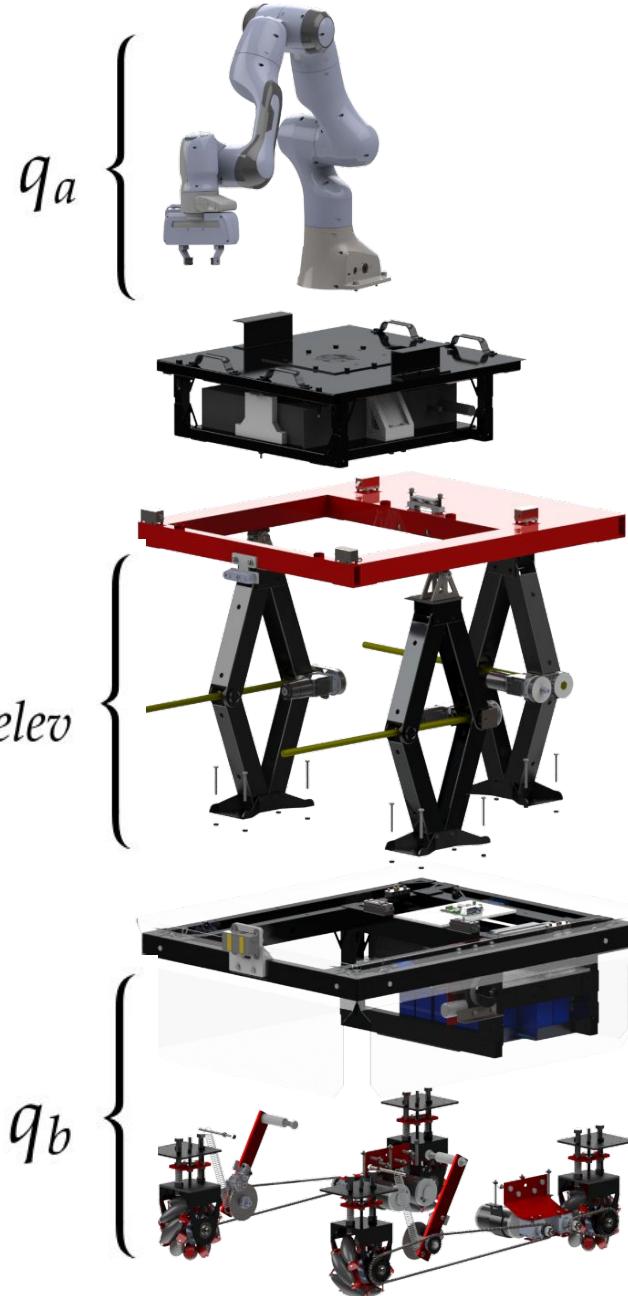
$$Ax \leq B$$

Joint Position Limits

$$X^- \leq x \leq X^+$$

Joint Velocity Limits

- $x = \begin{bmatrix} \dot{q} \\ \delta \end{bmatrix}$
 - Joint Velocities
 - Cartesian Tolerance
- QP evaluated at every instance



Subject To Conditions

$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} x^T Q x + C^T x$$

Objective Function

subject to: $\mathcal{J}(q)x = {}^b v_e$

RRMC with Tolerance

$$Ax \leq B$$

Joint Position Limits

$$X^- \leq x \leq X^+$$

Joint Velocity Limits

RRMC with Tolerance

$$\mathcal{J}(q)x = {}^b v_e$$

$$\mathcal{J}(q) = [{}^b J_e(q) \quad 1_{6x6}]$$

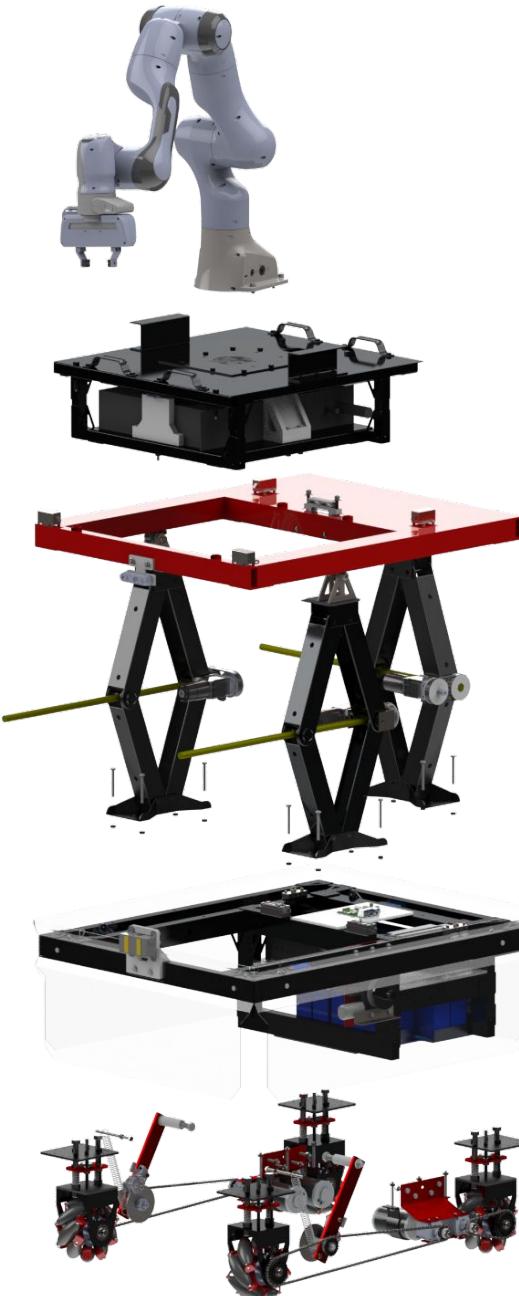
$${}^b v_e(t) = {}^b v_e^*(t) - \delta(t)$$

Joint Velocity Limits

$$X^- \leq x \leq X^+$$

$$X^- = \begin{bmatrix} \dot{q}^- \\ \delta^- \end{bmatrix}, \quad X^+ = \begin{bmatrix} \dot{q}^+ \\ \delta^+ \end{bmatrix}$$

$$x = \begin{bmatrix} \dot{q} \\ \delta \end{bmatrix}$$



Subject To Conditions

$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} x^T Q x + C^T x \quad \text{Objective Function}$$

subject to: $\mathcal{J}(q)x = {}^b v_e$ RRMC with Tolerance

$Ax \leq B$ Joint Position Limits

$X^- \leq x \leq X^+$ Joint Velocity Limits

Joint Position Limits

$$Ax \leq B$$

$$A = \begin{bmatrix} A_b \\ A_{elev_p} \\ A_{elev_r} \\ A_a \\ 1_6 \end{bmatrix} = [\pm 1_{nxn+6}] \quad B = \begin{bmatrix} 0_b \\ B_{elev_p} \\ B_{elev_r} \\ B_a \end{bmatrix}$$

$$B_{elev_{p,r}}, B_a = \begin{bmatrix} \frac{q_{elev_{p,r,a}} - p_{s_{elev_{p,r,a}}}}{p_{i_{elev_{p,r,a}}} - p_{s_{elev_{p,r,a}}}} \\ \vdots \\ \frac{q_n - p_{s_{elev_{p,r,a}}}}{p_{i_{elev_{p,r,a}}} - p_{s_{elev_{p,r,a}}}} \end{bmatrix}$$



Objective Function Analysis: Q

$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} x^T \boxed{Q} x + \boxed{C^T} x$$

Objective Function

$$\text{subject to: } \boxed{\mathcal{J}(q)x = {}^b v_e}$$

RRMC with Tolerance

$$Ax \leq B$$

Joint Position Limits

$$X^- \leq x \leq X^+$$

Joint Velocity Limits

Objective Function: Q

$$Q = \begin{bmatrix} diag(\lambda_b) & & & \\ & diag(\lambda_{elev}) & & \\ & & diag(k_a) & \\ 0_{(n+6)x(n+6)} & & & diag(\lambda_\delta) \end{bmatrix}$$

$$k_a = \text{constant}$$

$$\lambda_{elev} = \left(\frac{1}{\|e\|}, k_{elev_r} \right)$$

$$\lambda_b, \lambda_\delta = \frac{1}{\|e\|}$$



Objective Function Analysis: C^T

$$\min_{\dot{q}} : f_0(\dot{q}) = \frac{1}{2} \dot{x}^T Q \dot{x} + C^T \dot{x}$$

Objective Function

$$\text{subject to: } J(q)x = {}^b v_e$$

RRMC with Tolerance

$$Ax \leq B$$

Joint Position Limits

$$X^- \leq x \leq X^+$$

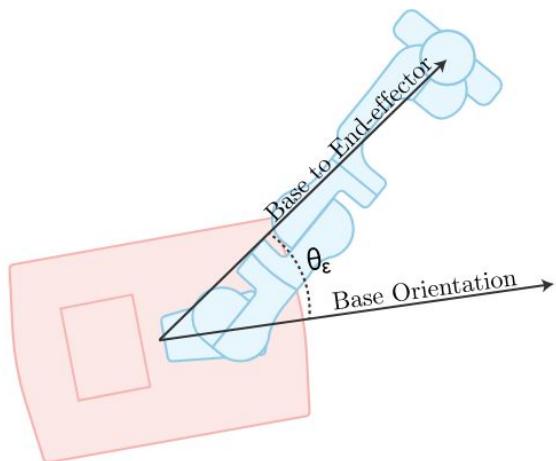
Joint Velocity Limits

Objective Function: C^T

$$C = \begin{bmatrix} J_m + \epsilon \\ 0_{6 \times 1} \end{bmatrix}$$

$$J_m = \begin{bmatrix} 0_b \\ J_{m,a}^T \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} -k_\epsilon \theta_\epsilon \\ 0_{n-1} \end{bmatrix}$$



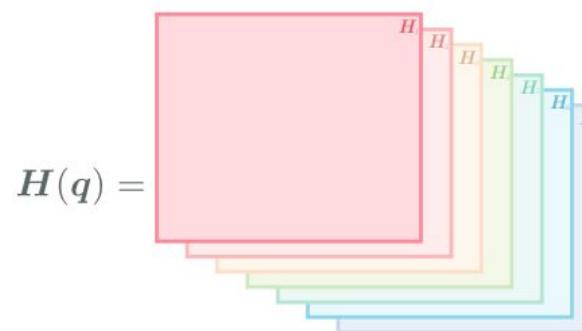
The Manipulability Jacobian \mathbf{J}_m^T

$$m(q) = \sqrt{\det(\hat{J}(q)\hat{J}(q)^T)}$$

$$\dot{m} = J_m^T(q)\dot{q} \quad \longleftrightarrow \quad v = J(q)\dot{q}$$

$$J_{m,a}^T = m \begin{bmatrix} \text{vec}(J(q)H_1(q)^T)^T \text{vec}(J(q)J(q)^T)^{-1} \\ \text{vec}(J(q)H_2(q)^T)^T \text{vec}(J(q)J(q)^T)^{-1} \\ \vdots \\ \text{vec}(J(q)H_n(q)^T)^T \text{vec}(J(q)J(q)^T)^{-1} \end{bmatrix}$$

Hessian
Tensor
Review



$$\begin{aligned} \mathbf{j} &= \frac{d\mathbf{J}(\mathbf{q})}{dt} \\ &= \frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_1} \dot{q}_1 + \frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_2} \dot{q}_2 + \cdots + \frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_n} \dot{q}_n \\ &= \left(\frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_1} \quad \frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_2} \quad \dots \quad \frac{\partial \mathbf{J}(\mathbf{q})}{\partial q_n} \right) \dot{\mathbf{q}} \\ &= \mathbf{H}(\mathbf{q}) \dot{\mathbf{q}} \\ &= \begin{pmatrix} \mathbf{H}_a(\mathbf{q}) \\ \mathbf{H}_{\alpha}(\mathbf{q}) \end{pmatrix} \dot{\mathbf{q}} \end{aligned}$$

$$\mathbf{H}_i(\mathbf{q}) = \begin{cases} \mathbf{H}_{a_i} \\ \mathbf{H}_{\alpha_i} \end{cases}$$

Diagram illustrating the structure of the Hessian tensor $\mathbf{H}_i(\mathbf{q})$. It shows a stack of colored rectangular blocks, each labeled with a small H , representing the components of the tensor. The top row of labels shows the indices a_1, a_2, \dots, a_n and $\alpha_1, \alpha_2, \dots, \alpha_n$ corresponding to the columns of the blocks.

Final Robot Pictures



Modular Arm Swap



Reaching Low



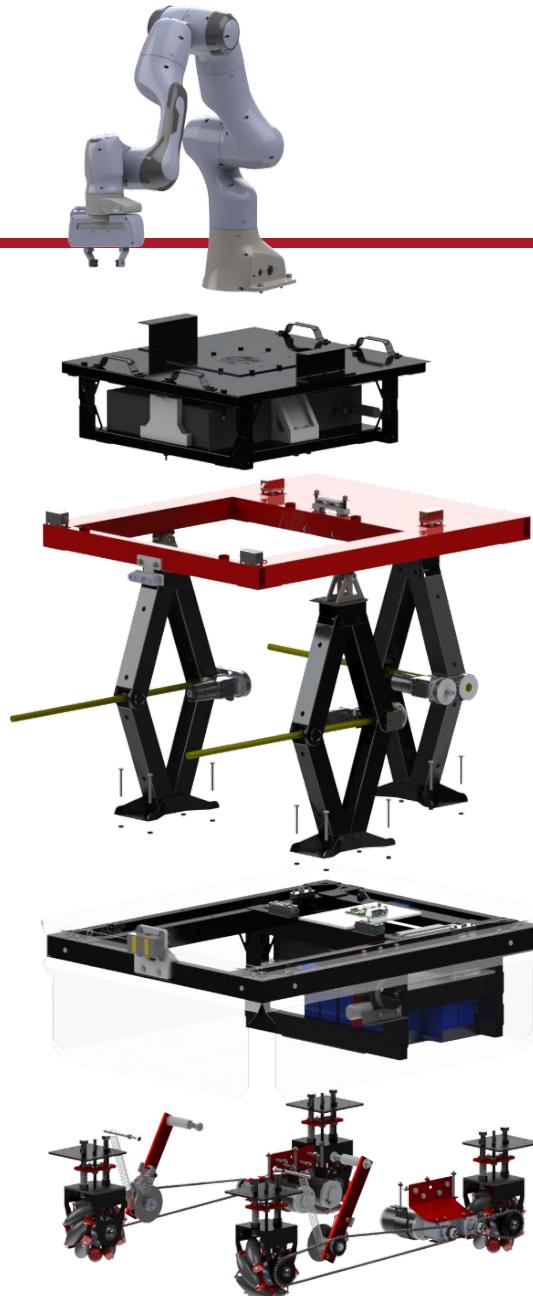
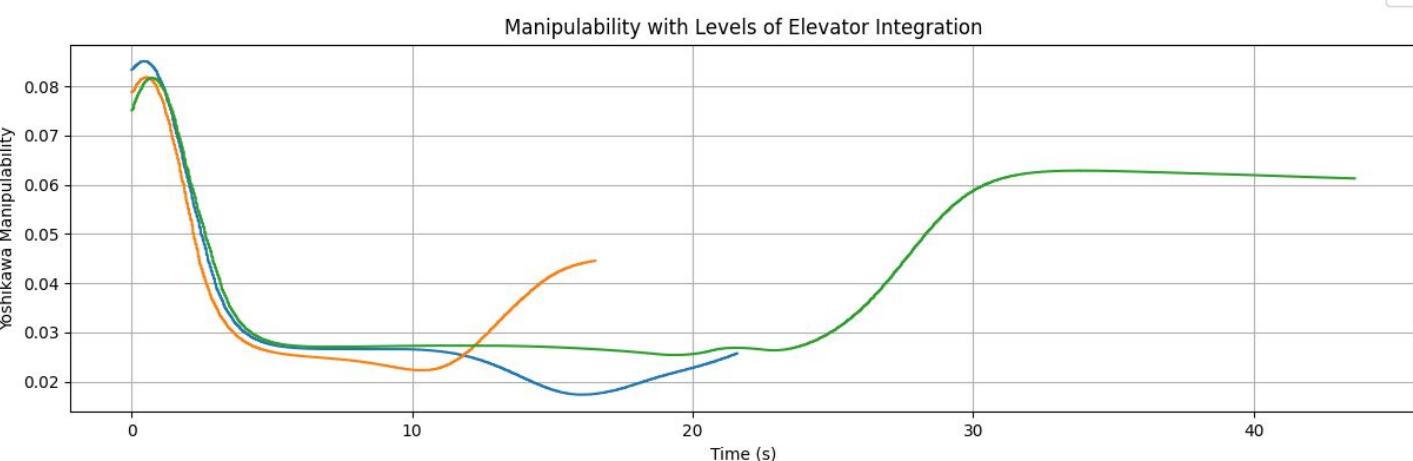
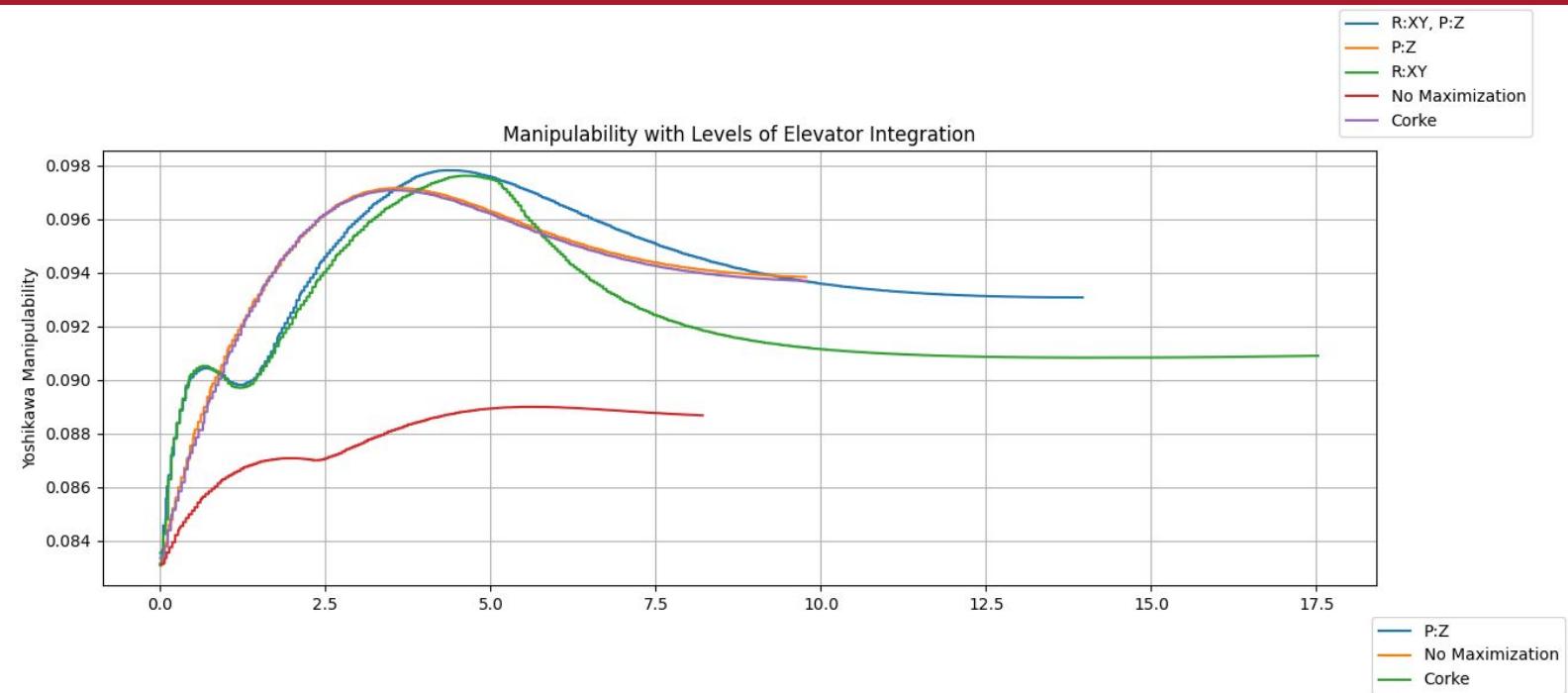
Reaching High

Worcester Polytechnic Institute

Video Demonstrations



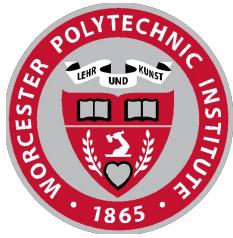
Different Manipulability Approaches



Ongoing Research

1. Manipulability Weighting / Prioritizing
2. Power Manipulability Representation
3. Visualizing QP “Pitfalls”





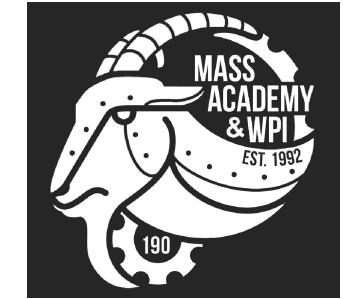
WPI

Thank you for listening!

Questions?



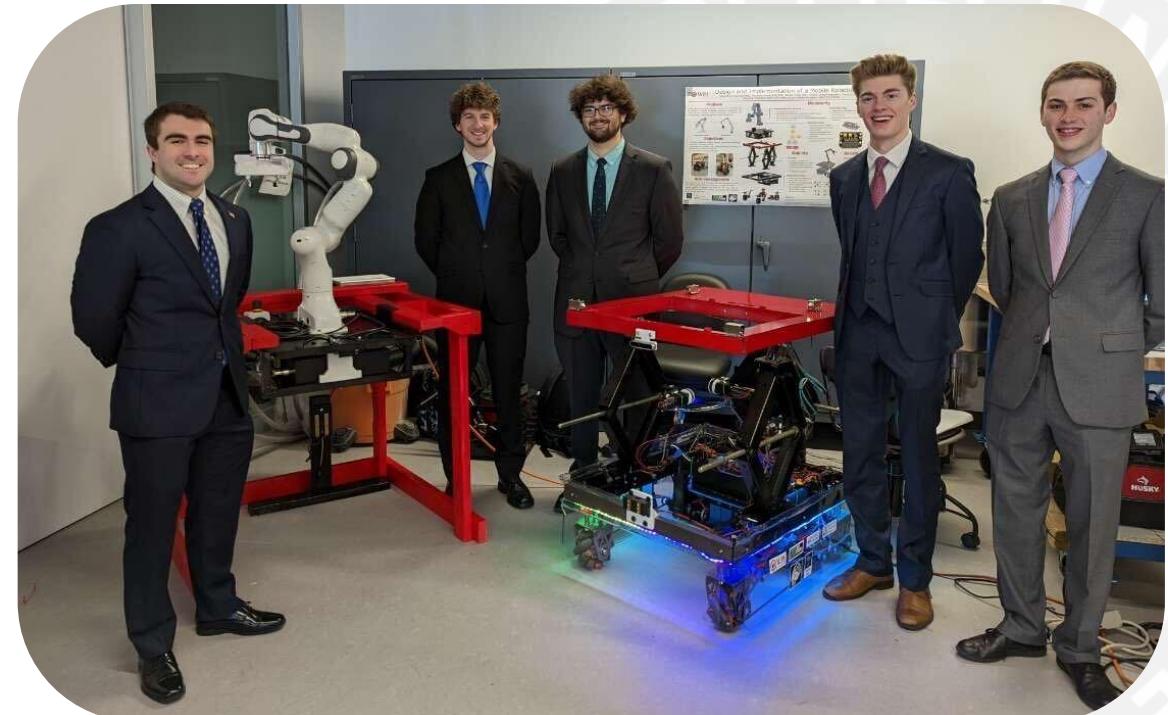
TINKER
BOX



METAL
RECYCLING
REIMAGINED



Manipulation and
Environmental
Robotics lab.



-
- A project overview/problem definition
 - System-level diagrams, as needed, to explain how your system operates and how it solves the problem
 - More detailed design slides for key sub-systems
 - Demonstrations (videos, data analysis, etc.) of system performance