UNIVERSIDAD NACIONAL DE COLOMBIA
SEDE BOGOTÁ

FACULTAD DE INGENIERÍA
Vicedecanatura Académica
POSGRADOS

PROPOSAL SUBMISSION

DOCTORAL THESIS:	x	MASTER THESIS:	
MASTER FINAL WORK:		SPECIALIZATION FINAL WORK:	

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2. **PROGRAM:** Phylosophy Doctoral in Computer Science and Systems Engineering
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4. **TITLE: Kernel Tensor Factorization**
5. **AREA:** Computer Science
6. **LINE OF RESEARCH:** Machine Learning
7. COMMENTARY WITH ADVISOR APROVAL
8. BIDDER SIGNATURE
9. SIGNATURE OF ADVISOR

Contents

1 Introduction

Tensors are multidimensional arraies. i.e. An N-way or N-order tensor is an element of tensor product of N vector spaces, each of which has its own coordinate system. A first order tensor is a vector, a second order tensor is a matrix, tensors of higher order are called high-order tensors.

Third-order tensor

1.1 Tensor decomposition

Tensor decomposition originated with Hitchcock in 1927 (Reference of Kolda), and the the multi-way model is attribuited to Cattel in 1944 (Reference of Kolda).

Tensor works had attention in 60s with Tucker, Carroll and Chang (Reference of Kolda) and Harshman in 1970 with applications in psychometrics. In 1981 Appellof and Davidson used tensor decomposition in chemometrics which have been an popular field of application of tensor decomposition since then.

In last twenty years tensor decomposition applications have expanded to many fields such as signal processing, numberical linear algebra, computer vision, numerical analysis, neuroscience, data mining, graph analysis.

We suggest to readers to refer to (Kolda, 2009) for an exhaustive a detailed review of fundamental decomposition methods.

1.1.1 Formulation for tensor completion

Following Ji Lu et. al notation

low rank matrix completion

$$\begin{aligned} &\min_X \operatorname{rank}(X) \\ &\text{s.t. } X_\Omega = M_\Omega \end{aligned} \tag{1}$$

where Ω is an index set, then X_Ω is coping entries of X in the indexes Ω and missed entries $\hat{\Omega}$ would be 0

The missing entries in X are determined in order to minimize the matrix X rank. i.e. a non convex optimization problem since rank is nonconvex.

Frequently, trace norm (or nuclear norm) $||\cdot||_*$ is used to approximate the rank of matrices. Trace norm is the tighest convex envelop for the matrices rank.

$$
\begin{aligned}
&\min_{X} ||X||_* \\
&\text{s.t. } X_\Omega = M_\Omega
\end{aligned}
\tag{2}
$$

Since tensor is a generalization of the matrix concept, we generalize the optimization problem as

$$
\begin{aligned}
&\min_{\mathcal{X}} ||\mathcal{X}||_* \\
&\text{s.t. } \mathcal{X}_\Omega = \mathcal{T}_\Omega
\end{aligned}
\tag{3}
$$

Where $\mathcal{X}$ and $\mathcal{T}$ are n-order tensors with identical size.

1.2 Tensor probability

Given a sample set

1.3 Kernel matrix

achievement of the project activities, including successful completion of the tasks and timely production of deliverables. resource accounting. It will also be in charge of managing the relations with collaborating institutions and administrative bodies within. stage before delivery hand over to ensure compliance and coherence. Also, it will follow up project progress anticipating corrective actions and assessing risk mitigation actions.