Efficient classification using Euler Characteristic

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Question

Can topology tell us something about the origins of pre-Columbian masks?

General methodology

- The classification algorithm relies on the **Euler Characteristic Graph** [1].
- We introduced the **extended ECG**, concatenating ECGs from the same artifact.
- Consider a 3D model $X = (V, E, F)$ and its **Euler Characteristic** (EC):

$$\chi = |V| - |E| + |F|$$

# of vertices — # of edges — # of faces

- Then fix a **vertex filtration** function $g : V \to [a, b]$ and extend it to the rest of **faces**:

$$g_F(\{v_0, v_1, v_2\}) = \min\{g(v_0), g(v_1), g(v_2)\}$$

$g_F : F \to [a, b]$ — Some face — Each vertex is assigned some # in interval $[a, b]$

- Same idea to extend the filtration to the rest of **edges**.
- The interval $[a, b]$ is split into T equally-spaced **thresholds**
- Say $a = t_0 < t_1 < t_2 < \ldots < t_T = b$. Consider **the EC at i-th threshold**:

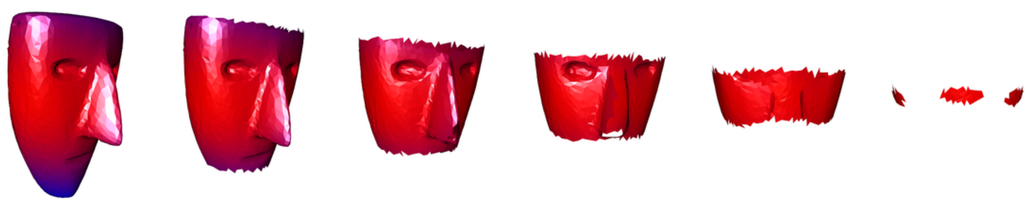
$$\chi_i = |V_i| - |E_i| + |F_i|$$

# of vertices v such that $g(v) > t_i$ — # of faces f such that $g_F(f) > t_i$

- The **EC Graph (ECG)** is obtained by comparing χ_i vs. t_i.

Pre-Columbian morphologies

- 3D models from pre-Columbian masks found in Templo Mayor, Mexico City.
- Need to assort **128 masks** into **8 different groups**.
- 70 masks already classified (**ground truth**) with 58 masks to decide [2].

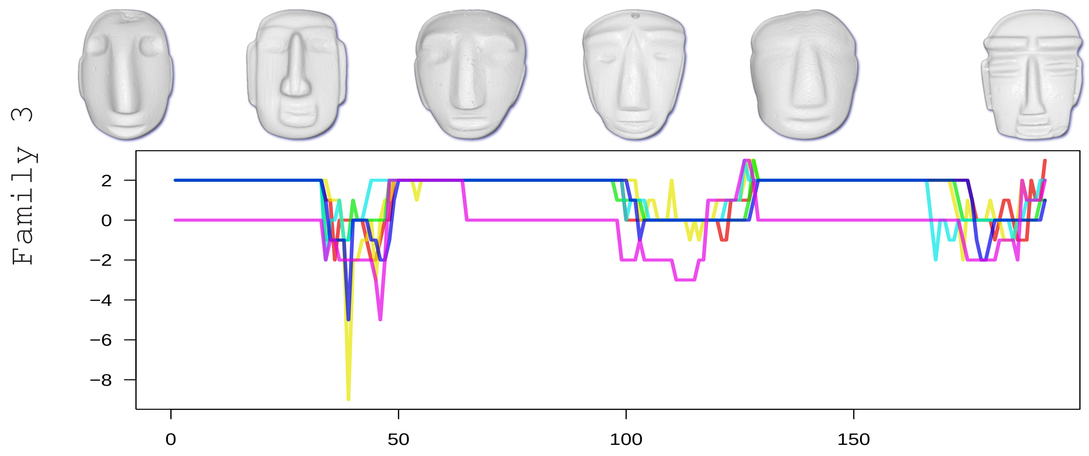


- Each mask is embedded in the $[-1, 1]^3$ cube, with its mass centered at origin.
- The **cylindrical distances** of each vertex, $\sqrt{x^2 + y^2}$, were considered as filtrations.
- These were **concatenated** to form an extended ECG.

- **Spherical** filters $\sqrt{x^2 + y^2 + z^2}$ were also considered.
- This defined training and testing sets to classify via **polynomial-kernel SVM**.

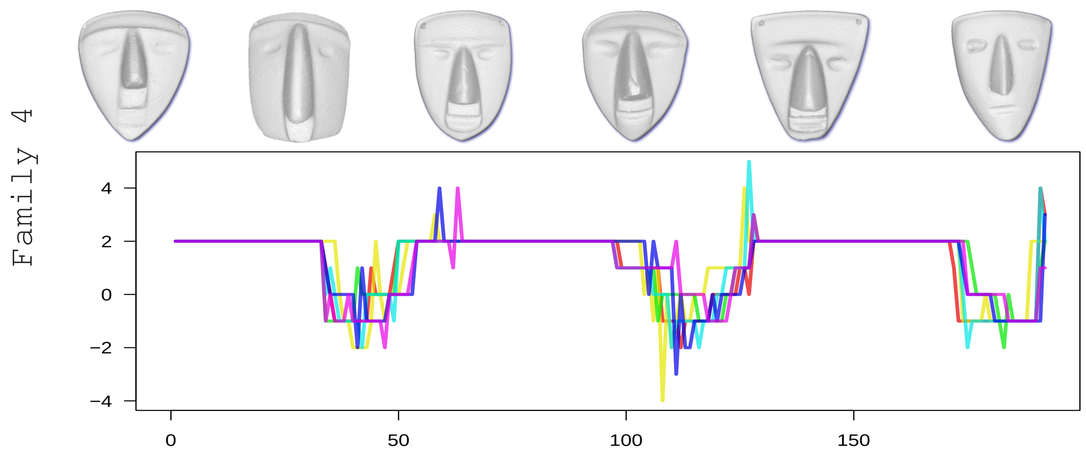
Results

- Masks with holes in their eyes or mouth were grouped in one unique family
- $T = 32$ produced similar results than $T = 256$
- Different colors refer to different items.

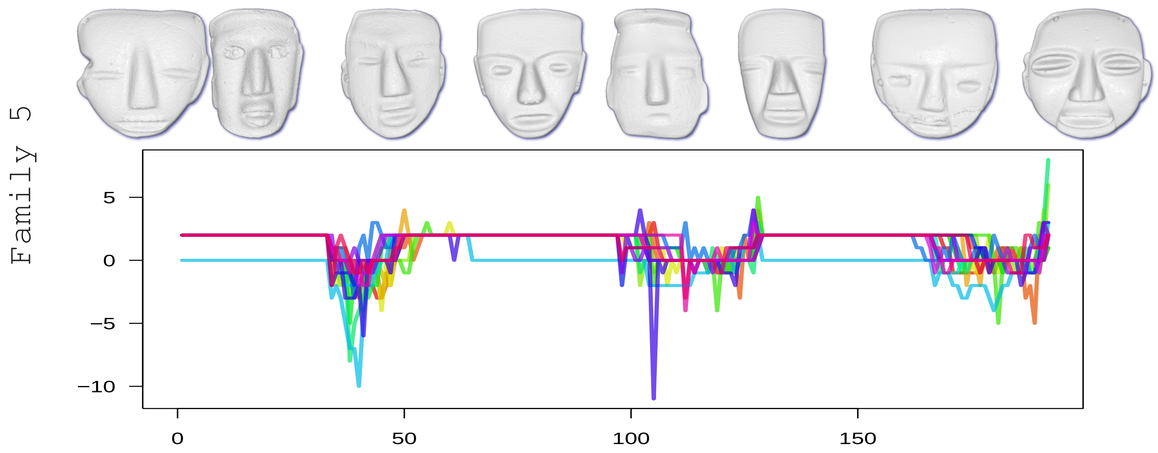


- The cylindrical filters managed to identify additional similar masks

- None of the unknown masks were assigned to set `04`



- The cylindrical filters also identified patterns in `Fam 05`.



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Conclusions

- Answer to the original question: **Yes it can!**
- Archaeologists have manifested approval towards some proposed groupings.
- There is a large number of variables to tune.
- Outliers suggest that there might not be just eight families in total but more. We need to determine the appropriate number of families of masks in first place.
- The computation of the ECG is a simple algorithm of linear complexity.
- This technique can be used to **analyze 3D plant models** and their **morphology**.

Acknowledgments

The authors would like to thank both Instituto Nacional de Antropología e Historia through its project *Desarrollo de Aplicaciones de Computación en Arqueología* and Red Temática CONACYT de Tecnologías Digitales para la Difusión del Patrimonio Cultural for providing the 3D scanned pre-Columbian masks which made possible this project.

References

[1] E. Richardson, M. Weirman Efficient classification using the Euler Characteristic In *Pattern Recognition Letters* Vol.49 pp.99-106, 2014.

[2] D. Jiménez Badillo, S. Ruíz Correa, O. Mendoza Montoya *Analyzing formal features of archaeological artefacts through the application of Spectral Clustering*. Conferencia del Digital Classicist Seminar (06/11/2012). Deutsches Archäologisches Institut, Berlin. `http://hdl.handle.net/11858/00-1780-0000-000B-216A-E`