Efficient object classification using Euler Characteristic

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General methodology

CONSIDER a n-dimensional object $X = (V_0, V_1, \ldots, V_n)$, with V_k the set of all its k-cells. We define its Euler Characteristic (EC) χ as:

$$\chi = \sum_{k=0}^{n} (-1)^k \, |V_k|.$$

Fix a **function** $g : V_0 \to [a, b]$, with $[a, b]$ a closed interval, and a fixed **number** T of thresholds. From it, auxiliary functions $g_k : V_k \to [a, b]$ are constructed as follows: for each k-cell $\{v_0, v_1, \ldots, v_k\}$ with $v_0, v_1, \ldots, v_k \in V_0$, define

$$g_k(\{v_0, v_1, \ldots, v_k\}) = \min_{0 \leq i \leq k} \{g(v_i)\}.$$

Then the interval $[a, b]$ is divided into T equally-spaced thresholds $a = t_0 < t_1 < t_2 < \ldots < t_T = b$. For each of them, the cardinality of the vertices subset $V_0^{(i)} \subset V_0$ defined as $V_0^{(i)} = \{x \in V_0 : g(x) > t_i\}$ might be computed using a similar method as in *bucket-sort*. Proceed analogously with the subsets $V_k^{(i)}$ of k-cells. Thus, **the EC at i-th threshold** is defined as:

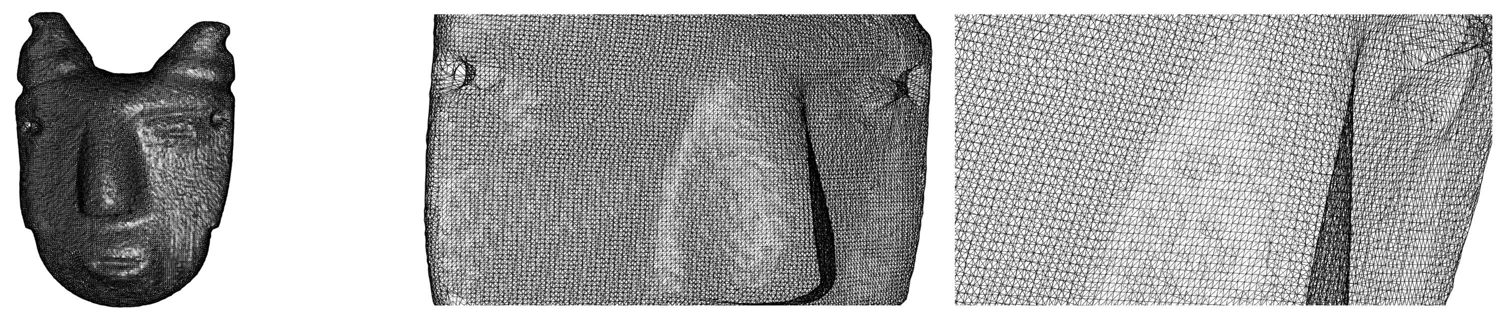
$$\chi_i = \sum_{k=0}^{n} (-1)^k \, |V_k^{(i)}|.$$

Assuming that the numerical values of function g have already been computed, assigning numerical values to every k-cell is an $O(V_0)$ complexity algorithm; computing and storing every $|V_k^{(i)}|$ value has $O(V_0 + T)$ complexity.

Once the ECG are computed, a machine-learning algorithm can be invoked to accomplish the assortment task. SVM with linear kernel was heavily used in this particular case.

One particular problem

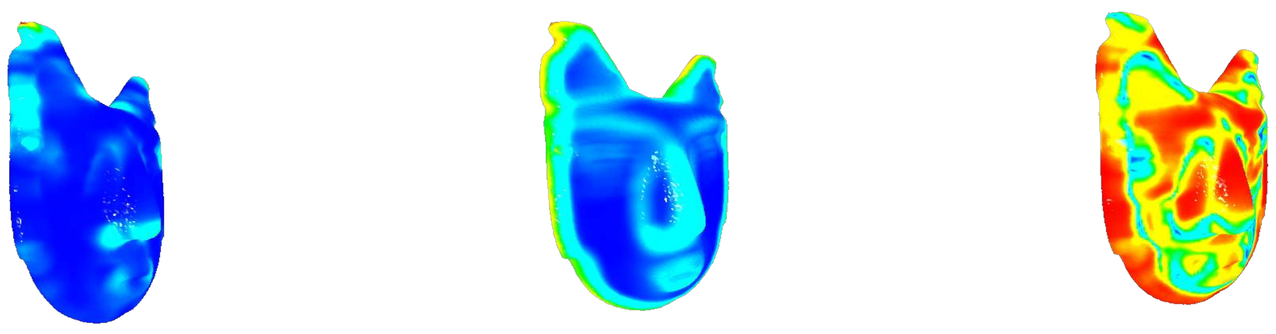
The goal was to specifically assort 128 masks into 9 different groups.



(a) Sample of a mesh **(b)** Details **(c)** More details

Figure 1: *Meshes are very detailed. Approximately 60,000 vertices each.*

Different ECGs, using different filtration functions, for each mask were computed. For example, using their principal curvature values:



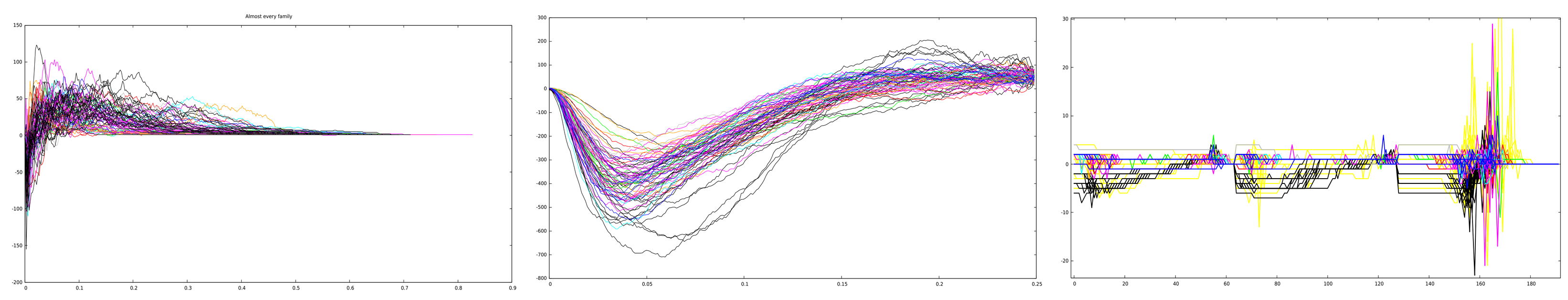
(a) Minimum curvature **(b)** Mean Curvature **(c)** Shape Index

Figure 2: *Heatmaps under different filtration functions*

The fact that each mask is embedded in the $[-1, 1]^3$ cube, with its mass centered at origin, can be exploited. The projections of each vertex to each $x = 0, y = 0, z = 0$ planes were considered as well as potential filtrations.

Figure 3: *Heatmap based on projections of each vertex to a $[-1, 1]^3$ cube's face.*

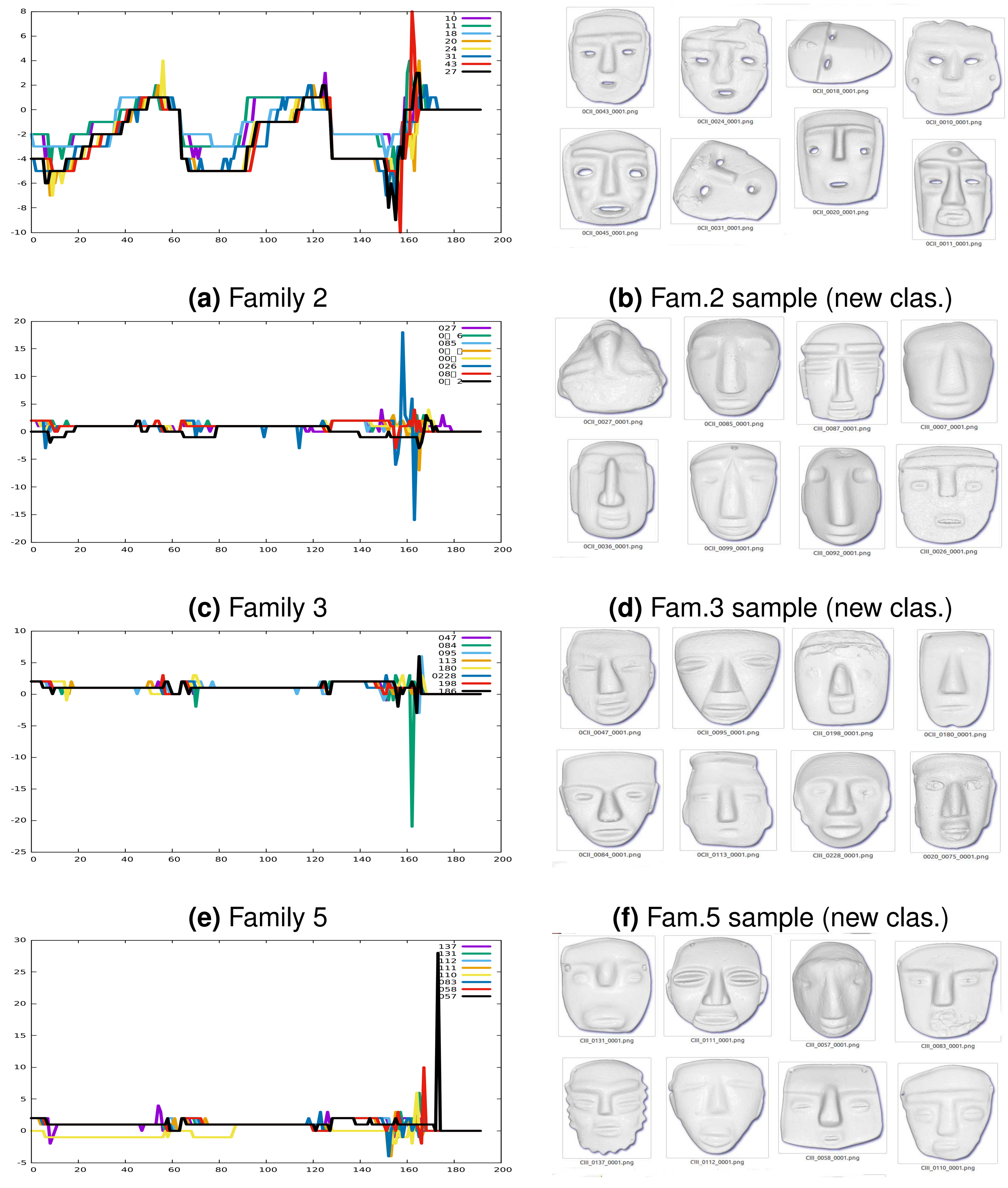
All the ECGs for all the masks were computed and plotted on the same plotspace, from which we can poor performance based on curvature values.



(a) Mean curvature **(b)** Shape Index **(c)** Main projections

Figure 4: *ECGs with $T = 256$ and $T = 64$ thresholds. Different color corresponds to different original classification. Fig. 4(c) looks promising while Figs 4(a) and 4(b) do not.*

After running the training phase, a new classification was obtained. The first striking difference between this assortment and the original one is the homogenization of number of specimens per family, since only two out of nine families possess less than 10 items. Out of the remaining seven families, 8 items were chosen randomly and each family was plotted. Examples below.



(a) Family 2 **(b)** Fam.2 sample (new clas.)

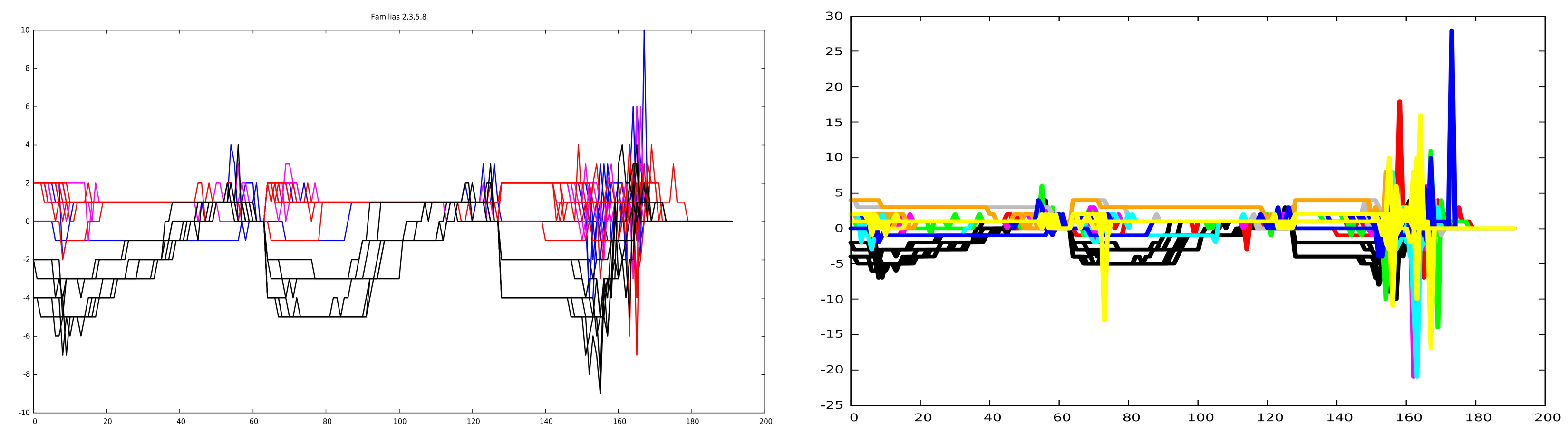
(c) Family 3 **(d)** Fam.3 sample (new clas.)

(e) Family 5 **(f)** Fam.5 sample (new clas.)

(g) Family 9 **(h)** Fam.9 sample (new clas.)

Figure 5: *ECGs of different masks within the same family according to the new classification. Different colors refer to different items.*

Removing the outlying masks and plotting the same four families simultaneously, as in figure 6(a), it is reasonable to claim a different pattern for each different family.



(a) Families 2,3,5,9 **(b)** New classification

Figure 6: *ECGs based on the three main projections, now colored according to the new classification*

Conclusions

The computation of the ECG associated to a given object, especially if it is based on projections, is a simple algorithm of linear complexity and memory.

The ECG algorithm is quite general and is open for further experimentation with other functions $g : V \to [a, b]$.

A larger database, with more specimens per family might solve the lack of characterization problem faced throughout the project.

Outliers such as the one presented in Figures 5(e) and 5(g) suggest that there might not be just nine families in total but more. It would be an interesting problem to determine the appropriate number of families of masks in first place.

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