Problem Set 9, Math 54-Lec 3, Linear Algebra, Fall 2017

September 22th, 2017

This problem was taken from Professor Nadler's Fall 2015 Math 54 midterm.

Problem 1

(a) State the rank theorem for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.

(b) Compute the rank of

$$A = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & -1 & 1 & 2 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

(c) Is the linear transformation defined by A injective? Justify your answer. [*Hint:* It may be helpful to use the previous parts]

(d) Use the rank theorem to show that any linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ cannot be one-to-one (injective) whenever $n > m$.