Solutions

**Problem Set 5**, **Math 54-Lec 3, Linear Algebra, Fall 2017**

SEPTEMBER 13TH, 2017

**Problem 1.** Let $\mathbb{P}^n$ be the set of polynomials with real coefficients with degree less than or equal to $n$. That is, if $f \in \mathbb{P}^n$, then $f$ is of the form: $f(x) = c_0+c_1x+c_2x^2+\ldots+c_nx^n$, where the $c_i$'s are real numbers. For $f, g \in \mathbb{P}^n$ we define addition as $(f+g)(x) = f(x)+g(x)$ and scalar multiplication by a constant $k$ as $kf(x) = kc_0 + kc_1x \ldots kc_nx^n$. Verify that $\mathbb{P}^n$ is a vector space by showing it satisfies the vector space axioms.

Let $\vec{f} = a_0 + a_1x + \ldots + a_nx^n$, $\vec{g} = b_0 + \ldots + b_nx^n$, $\vec{h} = c_0 + \ldots + c_nx^n$

and let $r, s$ be constants. Then

1) $\vec{f}+\vec{g} = (a_0+b_0) + \ldots + (a_n+b_n)x^n = g+f$ ✓

2.) $(\vec{f}+\vec{g})+\vec{h} = (a_0+b_0+c_0) + \ldots + (a_n+b_n+c_n)x^n = f + (g+h)$ ✓

3) $r(\vec{f}+\vec{g}) = r(a_0+b_0) + \ldots + r(a_n+b_n) = r\vec{f} + r\vec{g}$ ✓

4.) $(r+s)\vec{f} = (r+s)a_0 + \ldots + (r+s)a_nx^n =$

$= ra_0 + sa_0 + \ldots + ra_nx^n + sa_nx^n = r\vec{f} + s\vec{f}$. ✓

5) $r(s\vec{f}) = rsa_0 + \ldots + rsa_nx^n = (rs)\vec{f}$ ✓

6.) $1\vec{f} = a_0 + \ldots + a_nx^n = \vec{f}$ ✓

7.) $\vec{f} + (-1)\vec{f} = (a_0-a_0) + \ldots + (a_n-a_n)x^n = \vec{0}$ ✓

8) $\vec{f} + (0) = \vec{f}$, so $\vec{0} = 0$. ✓

Finally, note sum of 2 polynomials of deg $\leq n$ is still a polynomial of deg $\leq n$, so the set is closed.

$\therefore$ $\mathbb{P}_n$ is a vector space.

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**Problem 2.** For $\mathbb{P}^n$ as in Problem 1, let $U$ be the set of all functions $f$ in $\mathbb{P}^n$ such that the coefficient of $x^n$ in $f$ is non-zero. That is, $f(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$ is in $U$ if and only if $c_n \neq 0$. Determine if $U$ is a subspace of $\mathbb{P}^n$ (you may assume that $\mathbb{P}^n$ is a vector space).

No, because the zero vector is not in U.