Problem Set 5, Math 54-Lec 3, Linear Algebra, Fall 2017

September 13th, 2017

Problem 1. Let $\mathbb{P}^n$ be the set of polynomials with real coefficients with degree less than or equal to n. That is, if $f \in \mathbb{P}^n$, then f is of the form: $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$, where the c_i's are real numbers. For $f, g \in \mathbb{P}^n$ we define addition as $(f+g)(x) = f(x) + g(x)$ and scalar multiplication by a constant k as $kf(x) = kc_0 + kc_1 x \ldots kc_n x^n$. Verify that $\mathbb{P}^n$ is a vector space by showing it satisfies the vector space axioms.

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Problem 2. For $\mathbb{P}^n$ as in Problem 1, let U be the set of all functions f in $\mathbb{P}^n$ such that the coefficient of x^n in f is non-zero. That is, $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$ is in U if and only if $c_n \neq 0$. Determine if U is a subspace of $\mathbb{P}^n$ (you may assume that $\mathbb{P}^n$ is a vector space).