Solutions

Quiz 9 Math 54-Lec 3, Differential Equations, Fall 2017

SECTION: NAME:

You have 30 minutes to complete this quiz. To receive full credit, justify your answers.

Problem 1.(5 points) Solve the initial value problem: $y'' - 10y' + 25y = 0$, with initial conditions $y(0) = 1, y'(0) = 6$.

Aux. eqn: $r^2 - 10r + 25 = (r-5)^2$, so double root of 5.

$\therefore$ General solution is $y(t) = c_1 e^{5t} + c_2 t e^{5t}$

Now consider initial conditions, $y(0) = c_1(1) + c_2(0) = c_1$

$\therefore c_1 = 1$.

$y'(t) = 5c_1 e^{5t} + c_2 e^{5t} + 5te^{5t}$

$\Rightarrow y'(0) = 5c_1 + c_2 = c_2 + 5$, so $c_2 = 1$.

$\therefore y(t) = e^{5t} + te^{5t} = e^{5t}(1+t)$.

Problem 2.(5 points) Find a solution $y(t)$ to the differential equation $y'' + 4y' + 5y = 0$, that satisfies $y(0) = 2, y(\pi/2) = 0$.

Aux. eqn: $r^2 + 4r + 5$, roots: $\frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$

$\therefore$ gen soln: $c_1 e^{-2t}\cos(t) + c_2 e^{-2t}\sin(t)$

$y(0) = c_1(1)(1) + c_2(1)(0) = c_1 \Rightarrow c_1 = 2$

$y(\frac{\pi}{2}) = 0 + c_2 e^{-\pi}(1) \Rightarrow c_2 = 0$.

$\therefore y(t) = 2e^{-2t}\cos(t)$

Problem 3.(5 points) Let $y(t)$ be a non-trivial solution to the differential equation $y'' + cy = 0$, where c is a positive constant. As t goes to infinity what can we say about the behavior of $y(t)$? Specifically, as $t \to \infty$, does: $y(t) \to 0$? $y(t) \to \pm\infty$? Or does $y(t)$ diverge without going to $\pm\infty$?

Aux. eqn: $r^2 + c = 0$, since $c > 0$,

roots are $r = \pm i\sqrt{c}$ and general solution

is $y(t) = e^{0t}(c_1 \cos(\sqrt{c}) + c_2 \sin(\sqrt{c}))$

$= c_1 \cos(\sqrt{c}) + c_2 \sin(\sqrt{c})$

Since both the functions are bounded and periodic, $y(t)$ diverges w/ out going to infinity as t goes to infinity.