Problem Set 3, Math 54-Lec 3, Linear Algebra, Fall 2017

September 8th, 2017

Problem 1. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Prove that T is one-to-one iff $T(\vec{x}) = \vec{0}$ implies $\vec{x} = \vec{0}$, that is to say the **only** vector in $\mathbb{R}^n$ who's image under T is the zero vector is the zero vector itself.

Problem 2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Let $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_k}\}$ be a linearly independent set in $\mathbb{R}^n$. Prove that if T one-to-one then $\{T(\vec{v_1}), T(\vec{v_2}), \ldots, T(\vec{v_k})\}$ is also linearly independent. [*Hint:* use the result from Problem 1, above.]