Quiz 1 **Math 54-Lec 3, Linear Algebra, Fall 2017**

SECTION: NAME: Solutions

You have 30 minutes to complete this quiz. To receive full credit, you must justify your answers.

Problem 1.(5 points.) Solve the system of linear equations below.

$$\begin{aligned} 2x_1 \qquad - 6x_3 &= -8 \\ x_2 + 2x_3 &= 3 \\ 3x_1 + 6x_2 - 2x_3 &= -4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

Therefore $\boxed{x_3 = 2}$. And,

$x_2 + 2x_3 = 3$

$\Rightarrow x_2 + 2(2) = 3 \Rightarrow \boxed{x_2 = -1}$

And, $x_1 - 3x_3 = -4 \Rightarrow x_1 - 3(2) = -4 \Rightarrow \boxed{x_1 = 2.}$

Problem 2.(5 points.) For $\vec{v_1}, \vec{v_2}, \vec{v_3}$ below, find all constants c_1, c_2, c_3 that satisfy $c_1\vec{v_1}+c_2\vec{v_2}+c_3\vec{v_3} = \vec{0}$. Hence, are the vectors linearly independent or linearly dependent?

$$\vec{v_1} = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \ \vec{v_3} = \begin{bmatrix} 0 \\ 5 \\ -5 \end{bmatrix}$$

We want all c_1, c_2, c_3 such that $\begin{bmatrix} | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \vec{0}$.

So solve $\left[\begin{array}{ccc|c} -4 & -3 & 0 & 0 \\ 0 & -1 & 5 & 0 \\ 1 & 1 & -5 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ -4 & -3 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 1 & -20 & 0 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & -15 & 0 \end{array}\right].$$

Therefore $c_1 = c_2 = c_3 = 0$.

So $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$ has only the trivial solution so the vectors are independent.

Problem 3.(5 points.) For what values of c, a real number, does the following system of equations have infinitely many solutions?

$$\begin{aligned} 2x_1 + 5x_2 &= 6 \\ x_1 + cx_2 &= 3 \end{aligned}$$

First try to solve by putting it into a matrix.

$$\left[\begin{array}{cc|c} 2 & 5 & 6 \\ 1 & c & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 5 & 6 \\ 0 & 5-2c & 0 \end{array}\right].$$ Therefore we only

have a free variable (necessary for ∞-many solutions)

if $5-2c=0 \iff c=\frac{5}{2}$.