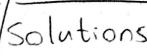
Problem Set 8, Math 54-Lec 3, Linear Algebra, Fall 2017

September 20th, 2017



This problem was originally assigned on the homework but I felt it deserved extra emphasis.

Problem 1. Let $M_{2\times 2}$ be the vector space of all two by two matrices, and define $T: M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$.

Part a. Show that T is a linear transformation.

Part b. Let B be any two by two matrix such that $B = B^T$. Find an $A \in M_{2\times 2}$ such that T(A) = B.

Part c. Describe the kernel of T.

The Kernel of Tirall matrices Ac Mexz such that T(A)= 0.

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Part d. Show that the range of T is the set of B in $M_{2\times 2}$ with the property that $B = B^T$.

We went to show Ronge (T)= { all 2x2 mutities B = uch }

Call this set W. To show the sets one equal we must show Range (T) CW and Range (T) ZW because 2 sets are subsets of each other iff they are equal.

1) Let $\vec{x} \in Range(T)$, thu $\vec{x} = T(A)$ for some $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Therefore $\vec{x} = A + A^T = \begin{bmatrix} 2a & btc \\ c+b & 2d \end{bmatrix} \in W$. Thus $Range(T) \subseteq W$.

2) Let BEW. We want to show Be Range (T) which ITS
equivalent to showing there exists an AE Mexic such that T(A) = B. We know B = BT, so from part b, we have $T(\frac{1}{2}B) = B$ so Bt Range (T). Thus $W \subseteq Range$ T.

Thus, Ronge (T) = W.