Solutions

Quiz 4 Math 54-Lec 3, Linear Algebra, Fall 2017

SECTION: NAME:

You have 30 minutes to complete this quiz. To receive full credit, you must justify your answers.

Problem 1.(5 Points) Let $W \subseteq \mathbb{R}^n$ be the subset of $\mathbb{R}^n$ containing all vectors whose entries sum to zero. Verify that W is a *subspace* of $\mathbb{R}^n$.

1.) Observe that $\vec{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ satisfies $0+0+\dots+0=0$ so $\vec{0} \in W$.

2.) Let $\vec{x}, \vec{y} \in W$ with $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, $\vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$. Then we have

$x_1 + \dots + x_n = 0$, $y_1 + \dots + y_n = 0$. Observe for $\vec{x}+\vec{y} = \begin{bmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{bmatrix}$

$(x_1+y_1)+\dots+(x_n+y_n) = (x_1+\dots+x_n)+(y_1+\dots+y_n) = 0+0=0$, so $\vec{x}+\vec{y} \in W$.

3) Similarly for $c \in \mathbb{R}$, $c\vec{x} = \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix} \Rightarrow cx_1+\dots+cx_n = c(x_1+\dots+x_n) = 0$ so $c\vec{x} \in W$.

Problem 2.(5 points) Determine whether the following set of vectors is a basis for $\mathbb{P}_2$, the vector space of polynomials of degree ≤ 2.

$$\{1+2x-x^2, 1+x^2, 2+x+x^2\}$$

Writing these polynomials as vectors in $\mathbb{R}^3$, and putting then as the columns of a matrix we get

$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ which we row reduce:

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -3 \\ 0 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

we only have 2 pivot columns so the $\text{rank}(A) = 2$ which is less then $\dim(\mathbb{P}_2) = 3$ so the vectors do not form a basis.

Problem 3.(1 point each.) Label the following statements true or false. You do not need to justify your answers.

(a.) F If $H = Span\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\}$, then $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\}$ is a basis for H.

(b.) T If A is an invertible $n \times n$ matrix, then the columns of A form a basis of $\mathbb{R}^n$

(c.) F $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

(d.) F If B is a row-echelon form of a matrix A, then the columns of B form a basis for $ColA$, the column space of A.

(e.) F If $A = A^T$, then the rows of A form a basis for $ColA$, the column space of A.

Explanation

(a.) $\{\vec{v}_1, \ldots, \vec{v}_n\}$ may be a linearly dependent set.

(b) A invertible $\Rightarrow$ columns of A are linearly independent, ~~so they~~ and any n linearly independent vectors form a basis for $\mathbb{R}^n$.

(c) False, $\mathbb{R}^2$ consists of vectors with 2 entries whereas $\mathbb{R}^3$ consists of vectors with 3 entries, so it doesn't make sense to say a vector in $\mathbb{R}^2$ is in $\mathbb{R}^3$.

(d) This is true for the pivot columns of A. Take for example $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ which has row echelon form $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = B$.

$Col\,A = Span\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ whereas Span{columns of B} = $Span\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$.

(e) True for the pivot rows, since rows may be linearly dependent.