Quiz 3 **Math 54-Lec 3, Linear Algebra, Fall 2017**

SECTION: NAME: Solutions

You have 30 minutes to complete this quiz. To receive full credit, you must justify your answers.

Problem 1.(5 points.) Compute the inverse of the matrix A, below.

$$A = \begin{bmatrix} 2 & -4 \\ 4 & -6 \end{bmatrix}$$

$\det(A) = -12 + 16 = 4$

$$\therefore A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3/2 & 1 \\ -1 & 1/2 \end{bmatrix}$$

Problem 2.(5 points.) Compute the determinant of the following matrix.

$$\begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}$$

Expand over column 3,

$$\det(A) = 2 \det \begin{bmatrix} 0 & 3 & -4 \\ -5 & -8 & 3 \\ 0 & 5 & -6 \end{bmatrix} + 0 + 0 + 0$$

$$= 2 \cdot (-1)(-5) \det \begin{bmatrix} 3 & -4 \\ 5 & -6 \end{bmatrix}$$

$$= 10 \cdot (-18 + 20) = 20.$$

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Problem 3.(1 point each.) Label the following statements true or false. You do not need to justify your answers.

(a.) F For a square matrix A, if $\det(A) = 0$, then A is invertible.

(b.) T If two rows of a square matrix A are identical, then $\det(A) = 0$.

(c.) T If a matrix A is invertible, then the linear transformation T_A of A is one-to-one and onto.

(d.) T If A and B are invertible matrices, then their product AB is invertible.

(e.) F For a square matrix A, $\det(A^T) = -\det(A)$.

Explanations:

(a) See Theorem 4 in Section 4.2

(b) If 2 rows are identical, A will not have a pivot in every row so A is not invertible and $\det(A) = 0$.

(c) A invertible $\Rightarrow$ T is a bijection, so T is one-to-one and onto.

(d) True so long as the product is well defined, i.e. as long as A and B are both $n \times n$. I gave this question for free since I did not make that clear.

(e) See Theorem 5 in Section 3.2