Quiz 7

Math 54-Lec 3, Linear Algebra, Fall 2017

SECTION: NAME:

You have 40 minutes to complete this quiz. To receive full credit, justify your answers.

Problem 1.(10 points)

(a.)(3 points) Let $\vec{v_1} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Find an **orthonormal** basis $\beta = \{\vec{b_1}, \vec{b_2}\}$ for the plane spanned by $\vec{v_1}$ and $\vec{v_2}$.

(b.)(4 points) Let $\vec{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Using whichever method you prefer, find the least square solution(s) to the inconsistent system $A\vec{x} = \vec{y}$, where A is the matrix with columns $\vec{v_1}, \vec{v_2}$.

(c.)(3 points) Finally, find a third vector $\vec{b_3}$ such that the matrix B with columns $\vec{b_1}, \vec{b_2}, \vec{b_3}$ is orthogonal.

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Problem 2.(5 points) Consider the inner product space $C[0, 2\pi]$, that is the vector space of all continuous functions on the interval $[0, 2\pi]$, with inner product:

$$\langle f, g\rangle = \int_0^{2\pi} f(t)g(t)dt$$

(a.)(2 points) Verify that $sin(x)$ is orthogonal to $cos(x)$.

(b.)(3 points) Using part a and given the fact that $\langle sin(x), sin(x)\rangle = \langle cos(x), cos(x)\rangle = \pi$, find the projection of $f(x) = 1$ onto the span of $sin(x), cos(x)$.