Problem Set 12, Math 54-Lec 3, Linear Algebra, Fall 2017

October 9th, 2017

Problem 1. (a.) Verify that for $\vec{x}, \vec{y} \in \mathbb{R}^n$ and c a constant, we have

$$||\vec{x} - c\vec{y}||^2 = ||\vec{x}||^2 - 2c(\vec{x} \cdot \vec{y}) + c^2||\vec{y}||^2$$

(b.) The Cauchey-Schwartz Inequality and is one of the most useful results in linear algebra. It states that for all vectors $\vec{x}, \vec{y}$ in $\mathbb{R}^n$ we have

$$(\vec{x} \cdot \vec{y}) \leq (||\vec{x}||)(||\vec{y}||).$$

Prove this statement. [*Hint:* use your result from part a with $c = \frac{\vec{x} \cdot \vec{y}}{\vec{y} \cdot \vec{y}}$]

Problem Set 12, Math 54-Lec 3, Linear Algebra, Fall 2017

October 9th, 2017

Problem 2. Another, perhaps even more powerful result from linear algebra is the Triangle Inequality. It states that for all vectors $\vec{x}, \vec{y}$ in $\mathbb{R}^n$ we have:

$$||\vec{x} + \vec{y}|| \leq ||\vec{x}|| + ||\vec{y}||.$$

Prove this statement. [*Hint:* use your result from problem 1]