Problem Set 1, Math 54-Lec 3, Linear Algebra, Fall 2017

September 1st, 2017

Problem 1. Let $A \in M_{3\times 3}$, that is a 3×3 matrix, such that $A\vec{x} = \vec{b}$ is consistent for all $\vec{b} \in \mathbb{R}$. Show that $A\vec{x} = \vec{0}$ has only the trivial solution.

Problem 2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a function with

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}.$$

Determine if T a linear transformation.

Problem 3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ be linearly dependent in $\mathbb{R}^n$. Determine whether $\{T(\vec{v_1}), T(\vec{v_2}), T(\vec{v_3})\}$ is linearly dependent or independent in $\mathbb{R}^m$.