MATH 392 Problem Set 5

EJ Arce

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$\{\hat{\beta}_n\}$ is consistent if and only if $lim_{n\to\infty}P(|\hat{\beta}_n - \beta| < \varepsilon) = 1\ \forall\ \varepsilon > 0$.

It has been shown before that $\forall$ n ϵ N, $E[\hat{\beta}_n] = \frac{n}{n+1}\beta$. Thus,

$$\begin{aligned} lim_{n\to\infty}P(|\hat{\beta}_n - \beta| < \varepsilon) = 1 &\Leftrightarrow lim_{n\to\infty}P(|E[\hat{\beta}_n] - \beta| < \varepsilon) = 1 \\ &\Leftrightarrow lim_{n\to\infty}|E[\hat{\beta}_n] - \beta| < \varepsilon \\ &\Leftrightarrow lim_{n\to\infty}|\frac{n}{n+1}\beta - \beta| < \varepsilon \\ &\Leftrightarrow lim_{n\to\infty}|(\frac{n}{n+1} - 1)\beta| < \varepsilon \end{aligned}$$

It suffices to show that $lim_{n\to\infty}|\frac{n}{n+1} - 1| < \varepsilon$:

Let $\varepsilon > 0$. Then $\exists$ m ϵ $\mathbb{N}$ st $\forall n \geq m$,

$$\frac{1}{n} < \varepsilon \Rightarrow \frac{1}{n+1} < \varepsilon \Rightarrow |\frac{-1}{n+1}| < \varepsilon \Rightarrow |\frac{n-n-1}{n+1}| < \varepsilon \Rightarrow |\frac{n}{n+1} - \frac{n+1}{n+1}| < \varepsilon \Rightarrow |\frac{n}{n+1} - 1| < \varepsilon$$

Thus $lim_{n\to\infty}|\frac{n}{n+1} - 1| < \varepsilon$, so $lim_{n\to\infty}P(|\hat{\beta}_n - \beta| < \varepsilon) = 1$. Thus $\{\hat{\beta}_n\}$ is consistent.

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