MATH 392 Problem Set 4

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6.2

Let $x_1, ..., x_n \sim$ Poisson(λ). Show the maximum likelihood estimate of λ is $\hat{\lambda} = \bar{x}$:

$$L(\lambda|x_1, ..., x_n) = P(X_1 = x_1, ..., X_n = x_n)$$

$$L(\lambda|x_1, ..., x_n) = P(X = x_1)...P(X = x_n)$$

$$L(\lambda|x_1, ..., x_n) = \prod_i^n \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$L(\lambda|x_1, ..., x_n) = \frac{e^{-\lambda n} \lambda^{\sum_i^n x_i}}{\prod_i^n x_i!}$$

Take the natural log of both sides to make the derivation easier:

$$ln(L(\lambda|x_1, ..., x_n)) = ln(e^{-n\lambda}) + ln(\lambda^{\sum_i^n x_i}) - ln(\prod_i^n x_i!)$$

$$ln(L(\lambda|x_1, ..., x_n)) = -n\lambda + ln(\lambda) \sum_i^n x_i - ln(\prod_i^n x_i!)$$

Derive with respect to λ and set equal to 0:

$$\frac{\partial ln(L(\lambda|x_1, ..., x_n))}{\partial \lambda} = -n + \frac{\sum_i^n x_i}{\lambda} = 0$$

$$\frac{\partial ln(L(\lambda|x_1, ..., x_n))}{\partial \lambda} = \lambda = \frac{\sum_i^n x_i}{n} = \bar{x}$$

Thus $\lambda_{MLE} = \bar{x}$

6.8

$$f(x; \theta) = \frac{\sqrt{2/\pi} x^2 e^{-x^2/2\theta^2}}{\theta^3}$$

$$L(\theta|x_1, ..., x_n) = \prod_i^n f(x_i; \theta)$$

$$L(\theta|x_1,...,x_n) = \prod_i^n \frac{\sqrt{2/\pi}x_i^2 e^{-x_i^2/2\theta^2}}{\theta^3}$$

$$L(\theta|x_1,...,x_n) = (\frac{\sqrt{2/\pi}}{\theta^3})^n \prod_i^n x_i^2 e^{-x_i^2/2\theta^2}$$

$$ln(L(\theta|x_1,...,x_n)) = ln(\frac{\sqrt{2/\pi}}{\theta^3})^n + \sum_i^n ln(x_i^2) + \sum_i^n ln(e^{-x_i^2/2\theta^2})$$

$$ln(L(\theta|x_1,...,x_n)) = ln\sqrt{2/\pi} - ln\theta^3 + \sum_i^n ln(x_i^2) + \sum_i^n -x_i^2/2\theta^2$$

Derive with respect to θ and set to 0:

$$\frac{\partial(L(\theta|x_1,...,x_n))}{\partial\theta} = -\frac{n}{\theta^3}3\theta^2 + \sum_i^n x_i^2/\theta^3 = 0$$

$$\frac{3n}{\theta} = \frac{\sum_i^n x_i^2}{\theta^3}$$

$$\theta = \frac{\bar{x}\sqrt{n}}{\sqrt{3}}$$

Thus $\theta_{MLE} = \frac{\bar{x}\sqrt{n}}{\sqrt{3}}$.

6.11

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6.13