MATH 392 Power Calculations

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Given $\mu = 100, \sigma^2 = 15^2, \bar{X} = 105, \alpha = .05$, for what sizes n will power be .7? .75? .9?

$$H_0 : \mu = 100$$

$$H_A : \mu > 100$$

Power = .7

Under each hypothesis, the sampling distributions will follow

$$X_{H_0} \sim N(\mu, \sigma/\sqrt{n})$$

$$X_{H_A} \sim N(\bar{X}_{obs}, \sigma/\sqrt{n})$$

With $\alpha = .05$, the critical value x_{crit}at which we reject the null hypothesis is μ $+1.645(\sigma/\sqrt{n})$. β is the area under the alternative hypothesis distribution to the left of this critical value. With $\beta = .3$, we can refer to the standard normal distribution to find how many standard errors away from the mean our critical value is:

```
qnorm(.3,0,1)
```

```
## [1] -0.5244005
```

Thus,

$$x_{crit} = 100 + 1.645\frac{\sigma}{\sqrt{n}} = 105 - .5244\frac{\sigma}{\sqrt{n}}$$

Now solve for n:

$$(1.645 + .5244)\frac{\sigma}{\sqrt{n}} = 5$$
$$(\frac{(2.189)15}{5})^2 = n$$
$$n = 42.35$$

Thus for a power of at least .7, the sample size must be at least 43.

Power = .75

Generalizing the equations for x_{crit},

$$100 + 1.645\frac{\sigma}{\sqrt{n}} = 105 - q_{z_\beta}\frac{\sigma}{\sqrt{n}}$$

where q_{z_β} is the quantile for the standard normal distribution under which the area to the left of the quantile is β. Thus for $\beta = .25$, q_{z_β} is

```r
qnorm(.25,0,1)
```

```
## [1] -0.6744898
```

Thus

$$\begin{aligned}(1.645 + .6745)\frac{\sigma}{\sqrt{n}} &= 5\\ (\frac{(2.319)15}{5})^2 &= n\\ 48.4 &= n\end{aligned}$$

A sample size of at least 49 is needed for a power of .75

Power = .9

```r
qnorm(.1,0,1)
```

```
## [1] -1.281552
```

$$\begin{aligned}(1.645 + 1.282)\frac{\sigma}{\sqrt{n}} &= 5\\ (\frac{(2.927)15}{5})^2 &= n\\ 77.11 &= n\end{aligned}$$

A sample size of at least 78 is needed for a power of .9.