EARTH AND OCEAN SCIENCES 453

MATLAB homework assignment # 1:
FOSSIL FUEL EFFECTS ON THE GLOBAL CARBON CYCLE

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Due 6 pm 7 September, 2024

Preliminary Notes

Figure 1, depicting the present global carbon cycle, is an example of a so-called "box model" to which you have been roughly introduced through a thermal history calculation for Earth. In this case the boxes are numbered sequentially (1,2,3. . . 9). Each box contains a mass of carbon, *i.e.*, the "burden", and is connected to other boxes by arrows indicating fluxes into or out of the box. The values of the fluxes have been adjusted so as to produce a steady-state situation (that is, all the fluxes add up to zero or the "net flux" of carbon is 0 and the input = the output). In general the rate of change of the contents of boxes can be represented by the sum of all the influxes and outfluxes so that, for example, a three-box system could be represented by the set of ordinary differential equations (ODEs)

$$dM_1/dt = -F_{1:2} + F_{2:1} - F_{1:3} + F_{3:1} \tag{1}$$

$$dM_2/dt = +F_{1:2} - F_{2:1} - F_{2:3} + F_{3:2} \tag{2}$$

$$dM_3/dt = +F_{1:3} - F_{3:1} + F_{2:3} - F_{3:2} \tag{3}$$

where the flux denoted by $F_{i:j}$ is read as "from box i to box j".

In non-steady state situations the box fluxes are expected to be related to the size of the box. Assuming a linear flux law gives

$$F_{1:2} = k_{1:2}M_1 \tag{4}$$

$$F_{2:1} = k_{2:1}M_2 \tag{5}$$

where the $k_{i:j}$ are rate coefficients associated with the flow of material from box i to box j. Real systems are not necessarily linear but this is a good place from which to start. From the above relations it is apparent that the rate coefficients for a system near steady-state can be approximated

$$k_{i:j} = \frac{F_{i:j}}{M_i}.$$

Thus Figure 1 provides sufficient information to construct a 9-box coupled system and estimate the various rate coefficients.

The initial conditions are $M_i(0) = (M_0)_i$ where $(M_0)_i$ is the initial mass of a box i. If one solves for the response of this unforced system which is already in steady state, not surprisingly, there will be no change in $M_i(t)$ with time. Referring to Figure 1. The dashed arrows represent the addition of CO_2 to the atmosphere by fossil fuel burning and the effects of deforestation. For now, ignore the effect of deforestation (which is currently around $1\,\mathrm{Gt\,yr^{-1}}$) and consider the system response to adding CO_2 emissions according to the scenarios considered in the following assignment.

Box Model of Global Carbon Cycle

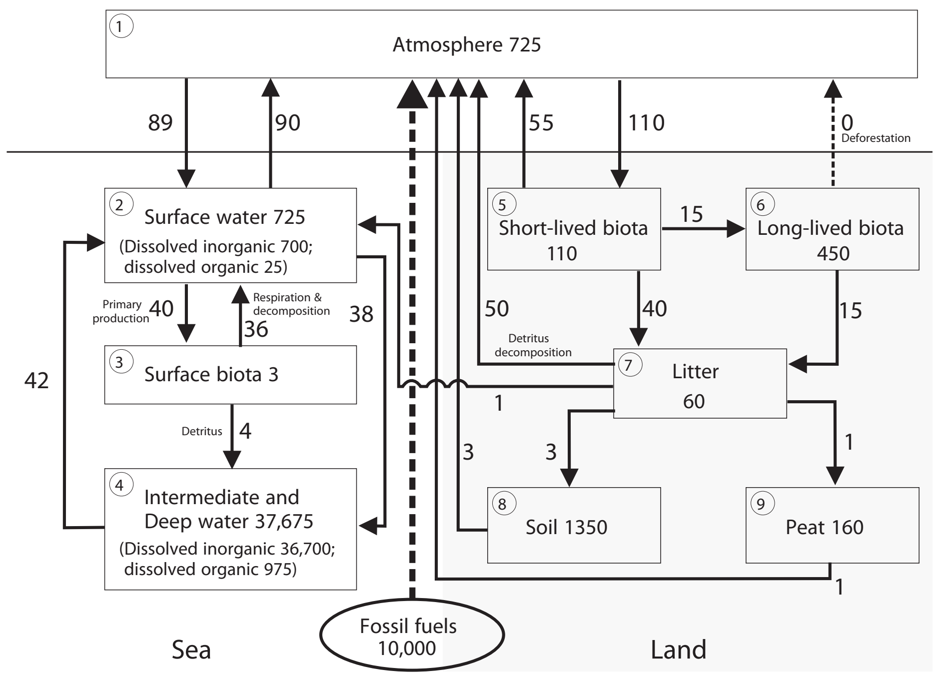

Figure 1. Principal reservoirs and fluxes in the carbon cycle. Units are Gt (10^9 metric tons = 10^{12} kg) for "burdens" and Gt/yr for fluxes. After Bolin (1986) with some modifications for this assignment.

ASSIGNMENT

- Construct a graphical algorithm indicating how you will solve this problem computationally. Include a 1 paragraph discussion or bulleted list of your plan with this figure if you wish.

  Some Preliminaries...

- Step 0. Complete assignment 0 or revisit it: Make sure you really understand the examples on the web from global variables to the setup of the main scripts to the handling of the odes.

- Step 0.1. Once you are comfortable with the examples on the web, use *either* the Euler or Runge Kutta (RK4) methods to solve a simplified box model for the C-cycle. To do this, use only boxes 1, 2, 5 and 7. For an initial steady state use the following adjusted fluxes. Make the flux from 1:2 and 2:1 90 Gt/yr; 7:1 55 Gt/yr; 5:7 55 Gt/yr; 1:5 110 Gt/yr; 5:1 55 Gt/yr and 7:2 0 Gt/yr. Make sure that the net flux is 0 Gt/yr. Now, add an arbitrary new flux of C to the atmosphere and see what happens. Alternatively, read a little further in the assignment and apply an IPCC A2 scenario to your simplified box model (you will find this forcing in the MATLAB script `emissions.m`. Play around (you will hear this a lot). What happens if you make the forcing fast or slow relative to the response time for the surface ocean, atmosphere, litter or short-lived biota boxes? The response time is given by the inverse of the rate constant. What do you have to do to your Euler method to keep it from exploding? Anything? Does the Runge kutta method suffer the same fate?

  The business of assignment 1

- Write a MATLAB or Python script that will calculate the time-evolving content of the nine boxes in Figure 1 when forced by a prescribed rate of CO_2 emissions. See the Notes on the numerical method below for explicit instructions. Try to solve this model using either the Euler method or RK4 from Assignment 0. Next for MATLAB, the MATLAB integrator ode45 or ode115s (great for stiff problems). For Python use odeint. What, if any, are some of the differences between the speed or accuracy of the integrators? How did you choose a time step with the Euler method when you have 4 boxes with 4 rate constants? Why do differences among the integrators emerge? Please be prepared to discuss what you learn through this exercise.

  1. Download and read the summary of the 2012 IPCC report from the course website. Note that this has been superceded (go to the web and download the current IPCC report if you wish) but this does not matter for this exercise. Apply this script to solve for the response to the modified A2 scenario over the time interval 1800–2200. Plot the content of boxes 1 (atmosphere) and 2 (ocean surface water) over time. (MATLAB files `emissionplotmain.m` and `emissions.m` which define and plot the modified A2 scenario can be downloaded from the course web site.)

  2. Modify the function `emissions.m` so that instead of dropping to zero in 2101 AD it maintains the 2000 AD value indefinitely. Plot the content of boxes 1 and 2 over time.

  3. Modify the function `emissions.m` in order to investigate periodic and decaying periodic CO_2 forcing to the atmosphere. Plot the content of boxes 1 and 2 over time.

  4. Try an experiment of your own design. What about other emissions scenarios (you might check the most recent IPCC report or create one of your own)? What about different flux laws (please justify any changes)?

5. Please read the paper by Byrne et al. (2024) in Nature entitled "Carbon emissions from the 2023 Canadian wildfires", which reports a truly astonishing result related to the magnitude of carbon emissions from the 2023 Canadian forest fires in comparison to anthropogenic emissions. Using the time series from Figure 2 and whatever other data in the paper implement the emissions from 2023. Forest fire intensity and frequency are both increasing in North America, Australia and even Europe. How might implement these changes into your model and learn about possible effects of the shifting behavior of this carbon source? Over what time scales?
6. Write a report on what you have learned that includes an explicit comparison of the solution methods you tried as well as a discussion of your scientific results. The format should follow the guidelines on the website

- Please submit your scripts electronically to me by email. Use comments in your code to clarify your programming intentions (recall that % denotes a comment). Please note: NO COMMENTS MEANS NO CREDIT.
- Please submit a hard copy and an electronic version of your paper in PDF format. PLEASE LIMIT YOUR REPORT TO 1500 WORDS. PLEASE INDICATE THE WORD COUNT ON YOUR PAPER AT THE TOP OF PAGE 1. This is not very much space so you will have to be clear, precise and concise: Build your figures to do much of the work for you.

Notes on Numerics

To solve this problem you will have to define a `function`, say `dYbydt(Y,t)`, where `Y` is a vector of length 9 and the individual terms might resemble, $dM_1/dt = -F_{1:2} + F_{2:1} - F_{1:3} + F_{3:1}$ (This function defines the system of ODEs for the box model). Note that you can pass information such as the definitions of $k_{i:j}$ from the main program to the function by using the MATLAB `global` variable specification. See the script `odeRKexamplemain.m` on the course website for examples of the use of global variables and other constructs. Assignment 0 had such goals...