

## EARTH AND OCEAN SCIENCES 453

### MATLAB homework assignment # 3: VOLCANIC ERUPTIONS AND CLIMATE CHANGE

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*Due* 25 October, 2024 by midnight

#### Preliminary Notes

This assignment involves developing a simple **radiative energy balance** model for the Earth and then investigating the effect of **volcanism** on this energy balance. Computationally, it is only slightly more involved than the carbon cycle model you did in the last project. However, the end product is far more ambitious: I want to see you do some interesting and solid science. This assignment asks explicitly for some creativity and for you to “follow your nose”. I am looking forward to the results of your curiosity. This project is based very loosely on the *Robock’s* [2000] review paper “**Volcanic eruptions and climate**”—the **results in Figure 4**. You might also explore topics related to the “**Faint Young Sun**” paradoxes on Earth with which you are now familiar. How, for example, does Earth’s response to volcanism change at 4 Ga compared to now? How might the longitudinal (meridional) heat transfer from the equator to the pole change the response if the Earth spins faster in the past? Feel free to explore similar problems that might emerge on Mars or Venus. You might explore the effect of volcanism on winter climate variability through its effect on the North Atlantic Oscillation (see: *Schleussner, C.F. and G. Feulner, A volcanically triggered regime shift in the subpolar North Atlantic Ocean as a possible origin of the Little Ice Age, Clim. Past, 9, 1321-1330, 2013*). Alternatively, you could investigate how the effect of volcanism on global climate might change under the “Faint Young Sun” conditions of Early Earth (see the review paper by *Feulner* you read first) or if the volcanic forcing occurs during a so-called “freshening” event, where freshwater from Lake Aggasiz, say, or from melting icebergs associated with Heinrich events during ice sheet retreat following the last glacial maximum (LGM) reduces the strength of Atlantic deep-water formation and meridional overturning circulation. To this end, see review **papers by Barreiro et al. and Hemming** from the reading list as well as recent papers by Ramstorff (e.g., **S. Rahmstorf: Thermohaline Ocean Circulation In: Encyclopedia of Quaternary Sciences, Edited by S. A. Elias. Elsevier, Amsterdam 2006**). For more information, borrow from me (and return) the excellent book ***Ocean Circulation* by Huang**.

The class of model that we will work with is sometimes referred to as a “**Budyko model**” in recognition of the work of the Russian scientist Mikhail I. Budyko who was among the first to develop such models. **Budyko is a bit of a legend**. Among other things he used this modeling approach to do the first theoretical investigation of the ice-albedo feedback mechanism for climate change, which he discovered by accident. That is, the more snow and ice, the more the solar radiation is reflected back into space, leading, in turn, to more snow and ice. You will have an opportunity to play with this feedback as well.

A usual starting point for modeling the radiative energy balance of a planet is the Stefan-Boltzmann radiation law

$$e = \sigma_B T^4 \quad (1)$$

where  $e$  is the flux of energy emitted ( $\text{W m}^{-2}$ ),  $\sigma_B = 5.6696 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$  is the Stefan-Boltzmann constant and  $T$  is absolute temperature. From the concepts summarized in Figure 1 an expression for

the radiation balance at the surface of a planet is

$$\rho c[Z] \frac{dT}{dt} = (1 - \alpha_S)S(t) - \varepsilon \sigma_B T^4 \quad (2)$$

where  $\rho$  is the density ( $\text{kg m}^{-3}$ ) of material comprising the surface,  $c$  is the specific heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ ),  $[Z]$  is a length scale that represents the effective thickness or “skin depth” of the irradiated surface,  $\alpha_S$  is the albedo,  $S(t)$  is the incident energy flux ( $\text{W m}^{-2}$ ) and  $\varepsilon$  is the total emissivity of the surface (for an ideal black body  $\varepsilon = 1$ ). Note that the units for equation 1 are ( $\text{W m}^{-2}$ ). In words, the LHS of equation 1 is the rate of change of thermal energy stored within the irradiated surface layer and the RHS gives the net flux of energy from this layer by radiative processes (i.e. [short-wavelength UV energy in from the sun] - [outgoing long-wavelength energy radiated from the top of the atmosphere (OLR)] ).

Some additional problems associated with describing a planetary radiation balance are that spinning planets have a day-side and a night-side and that the angle of incidence of light on the planetary surface depends on latitude. These considerations lead to

$$\rho c[Z] \frac{dT}{dt} = \gamma (1 - \alpha_S)S(t) - \varepsilon \sigma_B T^4 \quad (3)$$

where  $\gamma$  is a geometric factor accounting for the fact that, for a spinning spherical planet, radiation is intercepted by the disc area  $\pi R^2$  but reradiated to space by the spherical surface area  $4\pi R^2$ . It is easy to show that for this case  $\gamma = \frac{1}{4}$ , the ratio of the two areas. Finally, for planets with atmospheres including clouds there is an albedo associated with the top of the atmosphere  $\alpha_{\text{sky}}$  as well as an atmospheric transmissivity  $\tau$  for infrared radiation emitted back to space from the planetary surface. The combined results give

$$\rho c[Z] \frac{dT}{dt} = \frac{1}{4} (1 - \alpha_{\text{sky}})(1 - \alpha_S)S(t) - \varepsilon \tau \sigma_B T^4, \quad (4)$$

where the temperature  $T$  represents some kind of average surface temperature for the entire planet. Setting  $dT/dt = 0$  yields the steady-state condition (i.e., heat in = heat out)

$$\frac{1}{4} (1 - \alpha_{\text{sky}})(1 - \alpha_S)S_0 = \varepsilon \tau \sigma_B T^4 \quad (5)$$

where  $S_0$  is the solar constant (the radiative flux of solar energy at the Earth–Sun distance). Figure 1 summarizes the concepts leading to (5).

Note that Equation (5) provides no information about the distribution of temperature over the planetary surface and does not take into account such details as the distribution of land, sea and ice. We can introduce these possibilities by considering a *zonally-averaged model* in which the planet is divided into latitude-dependent zones or bands (Figure 2). Such a model can distinguish between equatorial and polar regions but has no longitudinal variation. Let us consider an Earth-like planet that is divided into six zones with the zone boundaries at 60°S, 30°S, 0°, 30°N and 60°N and suppose that each zone has its own temperature. Taking  $\varepsilon = 1$  and following the spirit of (4), we can write

$$A_k \rho_k c_k [Z_k] \frac{dT_k}{dt} = A_k \left\{ \gamma_k (1 - \alpha_k^{\text{sky}})(1 - \alpha_k)S(t) - \tau_k \sigma_B T_k^4 \right\} \quad (6)$$

where the  $k$  index identifies an individual zone so that, for our case,  $k = 1 \dots 6$ ;  $A_k$  is the surface area of zone  $k$ ,  $\gamma_k$  represent the geometric factor for the zone (no longer equal to 0.25 as for the whole

sphere) and the values of  $\rho_k$ ,  $c_k$ ,  $[Z_k]$ , etc. remain to be assigned. The atmosphere is mixed over time scales of weeks so I will take  $\tau$  to be constant for the entire atmosphere, rather than zonally dependent.

As written, the system of equations (6) would result in a complete thermal **decoupling** of the zones. This is obviously wrong: there are substantial heat fluxes between zones that are associated with atmospheric and oceanic circulation. The Gulf Stream, for example, transfers something like  $10^{15}$ W from Zone 4 to Zone 5. To take account of these exchanges we follow the layout of Figure 2 and modify (6) as follows:

$$A_1 \overline{\rho_1 c_1 [Z_1]} \frac{dT_1}{dt} = A_1 \left\{ \gamma_1 (1 - \alpha_1^{\text{sky}})(1 - \overline{\alpha_1})S_0 - \tau \sigma_B T_1^4 \right\} + L_{12} k_{12} (T_2 - T_1) \quad (7a)$$

$$A_2 \overline{\rho_2 c_2 [Z_2]} \frac{dT_2}{dt} = A_2 \left\{ \gamma_2 (1 - \alpha_2^{\text{sky}})(1 - \overline{\alpha_2})S_0 - \tau \sigma_B T_2^4 \right\} - L_{12} k_{12} (T_2 - T_1) + L_{32} k_{32} (T_3 - T_2) \quad (7b)$$

$$A_3 \overline{\rho_3 c_3 [Z_3]} \frac{dT_3}{dt} = A_3 \left\{ \gamma_3 (1 - \alpha_3^{\text{sky}})(1 - \overline{\alpha_3})S_0 - \tau \sigma_B T_3^4 \right\} - L_{23} k_{32} (T_2 - T_3) + L_{43} k_{43} (T_4 - T_3) \quad (7c)$$

$$A_4 \overline{\rho_4 c_4 [Z_4]} \frac{dT_4}{dt} = A_4 \left\{ \gamma_4 (1 - \alpha_4^{\text{sky}})(1 - \overline{\alpha_4})S_0 - \tau \sigma_B T_4^4 \right\} - L_{43} k_{43} (T_4 - T_3) - L_{45} k_{45} (T_4 - T_5) \quad (7d)$$

$$A_5 \overline{\rho_5 c_5 [Z_5]} \frac{dT_5}{dt} = A_5 \left\{ \gamma_5 (1 - \alpha_5^{\text{sky}})(1 - \overline{\alpha_5})S_0 - \tau \sigma_B T_5^4 \right\} + L_{45} k_{45} (T_4 - T_5) - L_{56} k_{56} (T_5 - T_6) \quad (7e)$$

$$A_6 \overline{\rho_6 c_6 [Z_6]} \frac{dT_6}{dt} = A_6 \left\{ \gamma_6 (1 - \alpha_6^{\text{sky}})(1 - \overline{\alpha_6})S_0 - \tau \sigma_B T_6^4 \right\} + L_{56} k_{56} (T_5 - T_6) \quad (7f)$$

(1)

where the overlined quantities represent zonally-averaged properties (see Appendix B). This tactic allows us to think of zones as containing a mix of land, water and ice surfaces. Dimensional analysis of the above equations confirms that the LHS and RHS now have dimensions of power (W) rather than flux (W/m<sup>2</sup>). Given that  $L_{ij}$  is a length, the transfer coefficients  $k_{ij}$  must have dimensions of W m<sup>-1</sup> K<sup>-1</sup>. That is, they are “effective” thermal conductivities that provide a parametric means to include the influence of longitudinal (i.e. meridional) heat transfer across the zones without adding a **fully dynamic model**.

For computational purposes we reorganize (7), noting that  $k_{ij} = k_{ji}$  and  $L_{ij} = L_{ji}$ , to obtain

$$\frac{dT_1}{dt} = \frac{1}{\overline{\rho_1 c_1 [Z_1]}} \left\{ \gamma_1 (1 - \alpha_1^{\text{sky}})(1 - \overline{\alpha_1})S_0 - \tau \sigma_B T_1^4 \right\} + \frac{L_{12} k_{12}}{A_1 \overline{\rho_1 c_1 [Z_1]}} (T_2 - T_1) \quad (8a)$$

$$\frac{dT_2}{dt} = \frac{1}{\overline{\rho_2 c_2 [Z_2]}} \left\{ \gamma_2 (1 - \alpha_2^{\text{sky}})(1 - \overline{\alpha_2})S_0 - \tau \sigma_B T_2^4 \right\} + \frac{1}{A_2 \overline{\rho_2 c_2 [Z_2]}} \left\{ -L_{12} k_{12} (T_2 - T_1) + L_{23} k_{23} (T_3 - T_2) \right\} \quad (8b)$$

$$\frac{dT_3}{dt} = \frac{1}{\overline{\rho_3 c_3 [Z_3]}} \left\{ \gamma_3 (1 - \alpha_3^{\text{sky}})(1 - \overline{\alpha_3})S_0 - \tau \sigma_B T_3^4 \right\} + \frac{1}{A_3 \overline{\rho_3 c_3 [Z_3]}} \left\{ -L_{23} k_{23} (T_3 - T_2) + L_{34} k_{34} (T_4 - T_3) \right\} \quad (8c)$$

$$\begin{aligned} \frac{dT_4}{dt} &= \frac{1}{\rho_4 c_4 [Z_4]} \left\{ \gamma_4 (1 - \alpha_4^{\text{sky}})(1 - \overline{\alpha}_4) S_0 - \tau \sigma_B T_4^4 \right\} \\ &\quad + \frac{1}{A_4 \rho_4 c_4 [Z_4]} \left\{ -L_{34} k_{34} (T_4 - T_3) + L_{45} k_{45} (T_5 - T_4) \right\} \quad (8d) \end{aligned}$$

$$\begin{aligned} \frac{dT_5}{dt} &= \frac{1}{\rho_5 c_5 [Z_5]} \left\{ \gamma_5 (1 - \alpha_5^{\text{sky}})(1 - \overline{\alpha}_5) S_0 - \tau \sigma_B T_5^4 \right\} \\ &\quad + \frac{1}{A_4 \rho_5 c_5 [Z_5]} \left\{ -L_{45} k_{45} (T_5 - T_4) + L_{56} k_{56} (T_6 - T_5) \right\} \quad (8e) \end{aligned}$$

$$\begin{aligned} \frac{dT_6}{dt} &= \frac{1}{\rho_6 c_6 [Z_6]} \left\{ \gamma_6 (1 - \alpha_6^{\text{sky}})(1 - \overline{\alpha}_6) S_0 - \tau_6 \sigma_B T_6^4 \right\} \\ &\quad - \frac{L_{56} k_{56}}{A_6 \rho_6 c_6 [Z_6]} (T_6 - T_5) \quad (8f) \end{aligned}$$

(2)

## APPENDIX A. SOME TRIGONOMETRY

### Area fraction of zones

For a sphere of radius  $R$ , the surface area of a zone in the latitude range  $[\theta_{k+1}, \theta_k]$  is given by

$$A_{k:k+1} = 2\pi R^2 \int_{\theta_k}^{\theta_{k+1}} d\theta = 2\pi R^2 [\sin \theta_{k+1} - \sin \theta_k] \quad (A.1)$$

where  $\theta$  is in radians. The total surface area of the sphere is  $A = 4\pi R^2$  so that the area fraction of the zone is

$$a_{k:k+1} = \frac{1}{2} [\sin \theta_{k+1} - \sin \theta_k]. \quad (A.2)$$

The length of the boundary separating zone  $k$  and zone  $k+1$  at latitude  $\theta_{k:k+1}$  (radians) is

$$L_{k:k+1} = 2\pi R \cos \theta_{k:k+1}. \quad (A.3)$$

Taking  $R = R_E$  these values are applied in Tables 2 and 3.

## APPENDIX B. AVERAGING

To calculate area-averaged properties such as  $\overline{\rho c[Z]}$  and  $\bar{\alpha}$  for each zone, use the area fractions of land, water and ice for each zone:

$$\overline{\rho c[Z]}_k = f_k^s \rho_s c_s [Z_s] + f_k^w \rho_w c_w [Z_w] + f_k^i \rho_i c_i [Z_i] \quad (B.1a)$$

$$\bar{\alpha}_k = f_k^s \alpha_s + f_k^w \alpha_w + f_k^i \alpha_i \quad (B.1b)$$

(3)

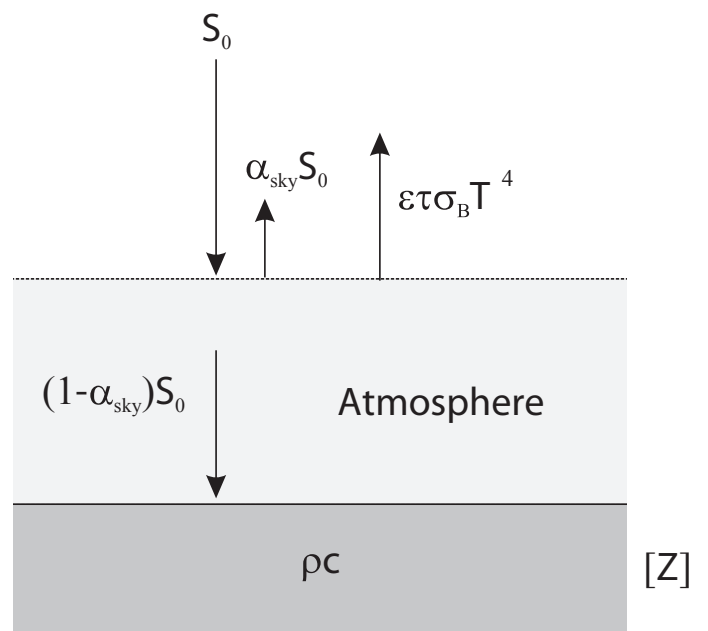


Figure 1. Basic radiative energy balance.

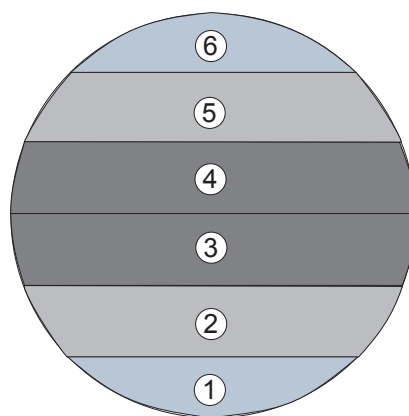


Figure 2. Zonally-averaged model.

## ASSIGNMENT

In this assignment you will first construct a radiative energy balance model for the Earth and then examine perturbations to this model related to volcanism. One goal is to learn about the effects of volcanism on climate change. Another is to learn how analyze this problem quantitatively using a simple model. The point is to **build some more understanding of how we use models to build understanding**. Once the basic volcano-climate exercise is complete, you can explore scenarios of your choice.

Write MATLAB or Python code to perform energy balance modeling as summarized in Equation 8. As a starting point use the parameter values summarized in Tables 1–3. Note that data for the fractional area of land, water and ice for the zones are not given. Estimate these for yourself using Google Earth or MODIS and SENTINEL 2 imagery. I strongly suggest that you write your program using consistent SI units. Thus you should be integrating in seconds even though you are probably interested in the month-to-year scale response. Following this advice I also strongly suggest that you use a Kelvin temperature scale and only after you have finished solving the system and are interested in graphic the results should you consider introducing Celsius temperatures. Note that you require initial conditions  $T_k(0)$  for each zone in order to launch the Runge–Kutta integration. You could, for example, assume that all zones were at 0°K at time  $t = 0$ . The only consequence of such a poor start is that it would take longer for the start-up transient to disappear and the solution to reach a steady state. If you are feeling confident and want to try method to simplify the numerical integration, AFTER you have a code that solves the dimensional model correctly, try non-dimensionalizing your model. Take some a reference temperature to non-dimensionalize time and a characteristic time scale of your choice to do the same for time. If you are interested in just volcanic effects, you might take a characteristic time  $t_{\text{pinatubo}} = 4$  years. If you are interested in effects of volcanism on a prescribed decadal climate oscillator like El Nino, you might take 10 years. If you are keen on snowball Earth, you might take longer.

1. Develop an algorithm for solving this problem on a computer. Please do it graphically and then add some text as needed. Please do this *before* you start writing your code and have this available in class OCTOBER 7
2. Write a code to examine the present, undisturbed (i.e. steady-state), energy balance of Earth. Start by assuming that all intra-zonal transfer is suppressed (i.e. set  $k_{ij}$  to zero). Next, change to the non-zero values suggested in Table 3. Note that  $k_{45}$  has been set to a larger value than other transfer coefficients, which emulates the effect of the Gulf Stream on Northern Hemisphere climate. You might wish to tinker with these coefficients (e.g. setting them to higher or lower values) to see what happens. That is, please play around with these values and see what you learn. Be aware of the questions you are asking when you do this. *Keep track of what you are trying to learn.*
3. Once you are satisfied that you have the program running and consider the results to be realistic, you can introduce the effect of volcanism. Refer to Figures 2 and Plate 5 of *Robock* [2000]. Note that Plate 5 is concerned with the latitudinal (i.e. zonal) spreading of volcanic aerosols. Note that Figure 2 is concerned with the change in solar radiation associated with the 1982 El Chichón and Pinatubo eruptions as observed at the Mauna Loa observatory. Examine the graph for “Direct Radiation” and use this to guide your choice of a magnitude, shape and timescale for a volcanic effect on incoming radiation. Use your examination of these graphs to propose a simple function  $\Phi_k(t)$  (where  $k$  is zone number and  $t$  is time (in seconds!!)) to represent the decrease in incoming



radiation related to the outgassing of aerosols. This is another place where you might wish to play with the mathematical form of this forcing to the model.

To implement this perturbation define a function  $\Phi_k(t)$  such that  $\Phi_k(t) = 1$  for the undisturbed case and, for example,  $\Phi_k(t) = 0.6$  for a 40% reduction in incoming radiation. Referring to Equation 8, modify the incoming radiation terms

$$\gamma_k (1 - \alpha_k^{\text{sky}})(1 - \overline{\alpha_k})S_0$$

to read

$$\gamma_k (1 - \alpha_k^{\text{sky}})(1 - \overline{\alpha_k})\Phi_k(t)S_0$$

and rerun your model.

4. Now torture the planet with volcanism in a way that you define. Here are some ideas (this is NOT a “to-do” list, just some ideas– please take as much latitude as you like):

- What happens if you include a CO<sub>2</sub> greenhouse effect in the problem in a way that is a little more realistic? The simplest way to do this is to rewrite the outgoing radiative flux in equation 2 using a fit for the outgoing long-wavelength radiation from a state-of-the-art model. That is:  $\epsilon\sigma_B T^4 \approx OLR(H_2O_v, CO_2)$ . For example the OLR heat flux (Pierrehumbert, 2011):

$$OLR(T, CO_2) = A(CO_2) + B(CO_2) * (T - T_{reference}),$$

with the climate sensitivity  $B = 2W/m^2$  K (independent of CO<sub>2</sub>) and  $A = A(CO_2(0)) - 4.*\log_2(CO_2/CO_2(0))$ , where you choose  $A(CO_2(0))$  to be where  $T = T_{reference}$ . Currently, a reference temperature for Earth might be the average temperature of 288 K. This form gives a climate sensitivity of 2 K per doubling of  $pCO_2$ . You will note that because changes in OLR are proportional to the log of the change in CO<sub>2</sub> that it takes a really big change in  $pCO_2$  to have a large effect on the surface temperature.

- What happens if the transmissivity  $\tau$  depends on optical wavelength because of clouds (that may vary in structure or altitude or be distributed only in certain latitudes) and greenhouse gases? (Please find some data on this to justify your approach. Google sky might be a place to start for estimating cloud cover).
- Do your results change if you add more zones with appropriate meridional (i.e., longitudinal) transport coefficients?
- What if the residence time of volcanic aerosols is a few times longer than might be estimated from the Robock measurements or varies according to eruption intensity (i.e., the more energetic the eruption, the higher the eruption column and the longer the residence time– look, for example, in the *Encyclopedia of Volcanoes* to learn about eruption intensity and column height (the measure of energy release is the “Volcano Explosivity Index ” or VEI index with 8 as the highest value).
- What happens if the “diffuse radiation” in Figure 2 of the Robock paper is also considered?
- What if the continents were distributed differently– how might the effect of volcanism on climate change during different epochs in Earth’s history? Be specific about which epoch and justify your distribution of continental landmass.
- What is the effect of the monotonic increase in the solar constant over Earth’s history?

- What if you include effects related to the differing albedos of fresh snow, sea ice, glacier ice, dirty ice, forest, grassland, desert, tundra, shallow water, deep water, ... etc?
- (This is a little more ambitious and interesting). The review paper by *Barreiro et al* [2008, sections 3 and 4] in the course reading list show that a critically large freshening of the North Atlantic will drastically reduce the strength of the thermohaline circulation (Figure 8) and reduce the meridional (longitudinal) heat transfer in the Atlantic. To parameterize this effect in terms of the rate of freshening, you might use Figure 9 in: S. Rahmstorf: Thermohaline Ocean Circulation. In: Encyclopedia of Quaternary Sciences, Edited by S. A. Elias. Elsevier, Amsterdam 2006. Yes... for a start, just use the stability diagram in Figure 9 to define approximately bounds for when the gulf stream is on or off and for the effect of freshening rate. See the last bullet on the ice-albedo feedback for a way in which such a bifurcation diagram might be implemented computationally.
- (This is also pretty interesting and builds on the Atlantic freshening idea above) An emerging potential cause of the widespread famines that defined the Medieval Little Ice Age (LIA) is the collective effect of multiple explosive eruptions on the dynamics of the North Atlantic Oscillation. The North Atlantic Oscillation (NAO) is the main driver of winter climate variability in the northern hemisphere (NH) and is related to a “seesaw” driven by variations in the atmospheric masses above the Azores (a permanent high-pressure region) and Iceland (a permanent low-pressure region). A “positive” mode corresponds to a large pressure difference, strong westerly winds, warm and wet winters in Europe and an enhanced production of sea ice near Labrador and Greenland. Reduced Atlantic deep-water formation and a weakened meridional (equator to pole) heat transfer related to this enhanced sea ice production can drive NH cooling. In contrast, a “negative” mode corresponds to weak winds and relatively cold, dry winters in northern Europe. A provocative correlation between explosive volcanism within or near the tropics and the persistence of a positive mode suggests a link between stratospheric eruptions and this class of multidecadal climate variability. Climate models suggest that a temporal correlation between the LIA with a number of explosive eruptions is explained by a volcanically-stabilized positive mode. Using figure 1 of the paper by Schleussner et al. as a guide for the forcing (Schleussner, C.F. and G. Feulner, A volcanically triggered regime shift in the subpolar North Atlantic Ocean as a possible origin of the Little Ice Age, *Clim. Past*, 9, 1321-1330, 2013), together with Figure 9 from the Ramstorff paper above, see if you can parameterize effects of explosive volcanism on the NAO and build some understanding of the sensitivity to, say, where volcanism occurs, how intense are the eruptions in terms of their radiative forcing, etc..
- From Archer’s 2000 review paper in the reading list, it is clear that glacial and interglacial oceans are different beasts. In particular, the longitudinal temperature difference driving ocean stirring is different during glacial times. Among other things the whole planet is colder (there are smaller equator - pole temperature gradients). This might lead intuitively to you hypothesizing proportionally less “meridional” heat transfer. However, one effect of a colder atmosphere and floating ice is to reduce the density gradient in the ocean that otherwise retards the descent of cold, salty water at high latitudes—deep water formation. Furthermore, the advance of floating ice sheets involves the removal of freshwater from the surface ocean and the ejection of salt— a mechanism called “brine rejection”. This additional production of salty water might provide a greater driving force for this meridional overturning circulation. How does your model change if you implement a weaker gulf stream? How about a stronger one? What is the right picture for a glacial ocean? Qualitatively, varying the meridional temperature gradient has an effect similar to the

freshening discussed in the Barriero paper but for different reasons. Which effect is larger?

- (This is also a little more ambitious and interesting). What if the reduction in solar radiation is particularly extreme? One way to explore this is to investigate what happens if the zonally-averaged albedo varies nonlinearly with temperature— this is a proxy for the so-called “ice-albedo feedback” that may have given rise to so-called “snowball Earth” events (see the Hoffmann and Schrag paper at: [http://www-eps.harvard.edu/people/faculty/hoffman/snowball\\_paper.html](http://www-eps.harvard.edu/people/faculty/hoffman/snowball_paper.html)). Should you choose to investigate this effect note that the temperature-dependent albedo in each zone will vary between the zonally-averaged initial value  $\alpha_o$ , say, used for part 2 above and the value for ice  $\alpha_i$ . In addition, it will obviously take more cooling (a greater temperature change) in an equatorial zone to change its average albedo than a zone in the high latitudes.

One parameterization for the variation of albedo with surface temperature that is not inconsistent with results from fully-dynamic climate models is:

$$\alpha(T) = \begin{cases} \alpha_i & T \leq T_i \\ \alpha_o + (\alpha_i - \alpha_o) \frac{(T - T_o)^2}{(T_i - T_o)^2} & T_i < T < T_o \\ \alpha_o & T \geq T_o \end{cases},$$

where I take  $T_i = 260$  K and  $T_o = 290$  K is a threshold temperature below which  $\alpha_o$  will increase quadratically with declining temperature (to a minimum value of  $\alpha_i$ ). Can you find a snowball solution related to volcanism? Can you recover from a snowball solution (we are not living on a frozen planet currently)?

5. Submit a **concise** and **precisely-written** report with a **maximum of 2000 words** on your modelling efforts. Please follow the structure on the website. Your paper must indicate the precise questions that you address with your model(s). Please email me and Luke the MATLAB or Python scripts separately. Please submit the report as a PDF.

**Table 1. Model parameters**

Property	Value	Units
Stefan-Boltzmann constant, $\sigma_B$	$5.6696 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
Solar constant, $S_0$	1368	$\text{W m}^{-2}$
Radius of Earth, $R_E$	$6371 \times 10^3$	m
Surface area of Earth, $A_E$	$\pi R_E^2$	$\text{m}^2$
Total emissivity of Earth, $\varepsilon$	1	
Atmospheric transmissivity, $\tau$	0.63	
Atmospheric albedo, $\alpha_{\text{sky}}$	0.2	
Albedo of land surface, $\alpha_s$	0.4	
Albedo of ocean surface, $\alpha_w$	0.1	
Albedo of ice, $\alpha_i$	0.6	
Density of land surface, $\rho_s$	2500	$\text{kg m}^{-3}$
Density of ocean water, $\rho_w$	1028	$\text{kg m}^{-3}$
Density of ice, $\rho_i$	900	$\text{kg m}^{-3}$
Thermal scale depth for land, $[Z_s]$	1.0	m
Thermal scale depth for ocean, $[Z_w]$	70.0	m
Thermal scale depth for ice, $[Z_i]$	1.0	m
Specific heat capacity for land, $c_s$	790	$\text{J kg K}^{-1}$
Specific heat capacity for water, $c_w$	4187	$\text{J kg K}^{-1}$
Specific heat capacity for ice, $c_i$	2060	$\text{J kg K}^{-1}$

**Table 2. Zone properties**

Property	Value	Units
<i>Zone 1</i> (90°S–60°S)		
Area fraction, $a_1$	0.0670	
Geometric factor, $\gamma_1$	0.1076	
Land fraction, $f_1^s$	–	
Ocean fraction, $f_1^w$	–	
Ice fraction, $f_1^i$	–	
<i>Zone 2</i> (60°S–30°S)		
Area fraction, $a_2$	0.1830	
Geometric factor, $\gamma_2$	0.2277	
Land fraction, $f_2^s$	–	
Water fraction, $f_2^w$	–	
Ice fraction, $f_2^i$	–	
<i>Zone 3</i> (30°S–0°)		
Area fraction, $a_3$	0.2500	
Geometric factor, $\gamma_3$	0.3045	
Land fraction, $f_3^s$	–	
Water fraction, $f_3^w$	–	
Ice fraction, $f_3^i$	–	
<i>Zone 4</i> (0°–30°N)		
Area fraction, $a_4$	0.2500	
Geometric factor, $\gamma_4$	0.3045	
Land fraction, $f_4^s$	–	
Water fraction, $f_4^w$	–	
Ice fraction, $f_4^i$	–	
<i>Zone 5</i> (30°N–60°N)		
Area fraction, $a_5$	0.1830	
Geometric factor, $\gamma_5$	0.2277	
Land fraction, $f_5^s$	–	
Water fraction, $f_5^w$	–	
Ice fraction, $f_5^i$	–	
<i>Zone 6</i> (60°N–90°N)		
Area fraction, $a_6$	0.0670	
Geometric factor, $\gamma_6$	0.1076	
Land fraction, $f_6^s$	–	
Water fraction, $f_6^w$	–	
Ice fraction, $f_6^i$	–	

**Table 3. Intra-zonal exchanges**

Property	Value	Units
<i>Boundary 12</i> (60°S)		
Boundary length, $L_{12}$	$2.0015 \times 10^7$	m
Thermal exchange coefficient, $k_{12}$	$1.0 \times 10^7$	$\text{W m}^{-1} \text{K}^{-1}$
<i>Boundary 23</i> (30°S)		
Boundary length, $L_{23}$	$3.4667 \times 10^7$	m
Thermal exchange coefficient, $k_{23}$	$1.0 \times 10^7$	$\text{W m}^{-1} \text{K}^{-1}$
<i>Boundary 34</i> (0°)		
Boundary length, $L_{34}$	$4.0030 \times 10^7$	m
Thermal exchange coefficient, $k_{34}$	$1.0 \times 10^7$	$\text{W m}^{-1} \text{K}^{-1}$
<i>Boundary 45</i> (30°N)		
Boundary length, $L_{45}$	$3.4667 \times 10^7$	m
Thermal exchange coefficient, $k_{45}$	$5.0 \times 10^7$	$\text{W m}^{-1} \text{K}^{-1}$
<i>Boundary 56</i> (60°N)		
Boundary length, $L_{56}$	$2.0015 \times 10^7$	m
Thermal exchange coefficient, $k_{56}$	$1.0 \times 10^7$	$\text{W m}^{-1} \text{K}^{-1}$