

# Math 300 Homework 1

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This homework set corresponds to [S] 1.1-1.5 and [B] 1.1-1.3

1. ([B] 1.1) Let  $z = 1 + 2i$  and  $w = 2 - i$ . Compute the following:  
(a)  $z + 3w$ , (b)  $\bar{w} - z$ , (c)  $z^3$ ,  
(d)  $\operatorname{Re}(w^2 + w)$ , (e)  $z^2 + \bar{z} + i$
2. ([B] 1.2 (bc)) Find the real and imaginary parts of each of the following:  
(b)  $z = \frac{3 + 5i}{7i + 1}$ , (c)  $w = \left( \frac{-1 + i\sqrt{3}}{2} \right)^3$
3. ([B] 1.3 (ab)) Find the absolute value and conjugate of each of the following:  
(a)  $z = -2 + i$ , (b)  $w = (2 + i)(4 + 3i)$
4. ([B] 1.4 (abcd)) Write in polar form: (i.e., the form  $re^{i\theta}$ )  
(a)  $2i$ , (b)  $1 + i$ , (c)  $-3 + \sqrt{3}i$ , (d)  $-i$
5. ([B] 1.5 (ac)) Write in rectangular form: (i.e., the form  $a + bi$ )  
(a)  $\sqrt{2}e^{i\frac{3\pi}{4}}$ , (c)  $-e^{i250\pi}$
6. ([B] 1.10) Fix  $a \in \mathbb{C}$  and  $b \in \mathbb{R}$ . Show that the equation  $|z^2| + \operatorname{Re}(az) + b = 0$  has a solution if and only if  $|a^2| \geq 4b$ . When solutions exist, show the solution set is a circle.  
When does the circle degenerate to a point?
7. ([B] 1.11 (cd)) Find all solutions to the following equations:  
(c)  $z^6 = -9$  (d)  $z^6 - z^3 - 2 = 0$
8. ([B] 1.18) (a) Show that

$$z^5 - 1 = (z - 1)(z^2 + 2z \cos \frac{\pi}{5} + 1)(z^2 - 2z \cos \frac{2\pi}{5} + 1)$$

(b) (optional) Deduce from this closed formulas for  $\cos \frac{\pi}{5}$  and  $\cos \frac{2\pi}{5}$ .

9. ([B] 1.21) Given  $x, y \in \mathbb{R}$ , define the matrix  $M(x, y) := \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$ . Show that

$$M(x, y) + M(a, b) = M(x + a, y + b) \quad \text{and} \quad M(x, y)M(a, b) = M(xa - yb, xb + ya).$$

(This means that the set  $\{M(x, y) : x, y \in \mathbb{R}\}$ , equipped with the usual addition and multiplication of matrices, behaves exactly like  $\mathbb{C} = \{(x, y) : x, y \in \mathbb{R}\}$ .)