MATH 302 Assignment 1

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1[1]

Let's consider all the sequences obtained after rolling a die with 6 faces n times.

1. How many possible sequences are there?
2. How many sequences do not contain ⚀ ? How many sequences do not contain neither ⚀, or ⚁?
3. Using the exclusion inclusion principle show that the number of sequences that contain at least one ⚀ and one ⚁ is $6^n - 2 \times 5^n + 4^n$.

Problem 2

1. A player receives 13 cards from a deck of 52. How many possible hands can the player receive?
2. Two other players join and they receive 13 cards each from the remaining deck. How many ways are there to deal the cards to the three players?
3. Find the answer directly as a multinomial coefficient and verify it is equal to the value found in the previous question.

Problem 3

Assuming a poker deal of 5 cards distributed from a deck of 52, how many ways are there to get[2]

1. royal flush

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[1] Abraham de Moivre used the inclusion exclusion principle to solve a generalized version of this problem in Problems 24 and 25 from De Moivre A., *The Doctrine of Chances: or, a Method of Calculating the Probability of Events in Play*, 1718

[2] if needed, see `https://en.wikipedia.org/wiki/List_of_poker_hands` for the definition of these poker hands)

2. straight flush

3. flush

4. straight

5. two pairs

Problem 4

1. We toss a fair die three times. What is the probability that all tosses produce different outcomes?

2. You own n colors, and want to use them to color 6 objects. For each object, you randomly choose one of the colors. What is the probability that there exists a pair of objects that shares the same colour?

Problem 5 (Bell numbers)

We call a *partition* of a set E, a set of non-empty subsets of E such that every element x in E is in exactly one of these subsets (i.e., the subsets are nonempty mutually disjoint sets). For example, if $E = \{a, b, c\}$, then $\{\{a\}, \{b, c\}\}$ is a partition of E.

1. What are the other partitions of $\{a, b, c\}$?

2. We denote $\mathcal{B}_n$ (n-th Bell's number) the number of possible partitions from a set of cardinal n, and we also set $\mathcal{B}_0 = 1$. Show that

$$\mathcal{B}_{n+1} = \sum_{k=0}^{n} \binom{n}{k} \mathcal{B}_k.$$

Problem 6 (Chu-Vandermonde's formula)

Show that

$$\binom{p+q}{n} = \sum_{i+j=n} \binom{p}{i} \binom{q}{j},$$

where the sum goes over all the values $(i, j) \in \mathbb{N}^2$, such that $i + j = n$.

Recommended practice exercises (not to be handed in)

Textbook exercices 1.4 to 1.8, and C.1 to C.17