MATH 302 Assignment 3

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

Let Ω be a sample space and $\mathbb{P}$ be a probability. Prove that there can't exist events E, F that satisfy

$$\mathbb{P}(E \setminus F) = \frac{2}{5},\ \mathbb{P}(E \cup F) = \frac{1}{2},\ \text{and}\ \mathbb{P}\left((E \cap F)^c\right) = \frac{3}{4}.$$

Problem 2

Assume that the events E_1, E_2 are independent.

1. Prove that the events E_1^c, E_2^c are also independent.
2. If, in addition, $\mathbb{P}(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{3}$. Prove that

$$\mathbb{P}(E_1 \cup E_2) = \frac{2}{3}$$

3. If, in addition, E_3 is a third event that is independent of E_1 and of E_2, and such that $\mathbb{P}(E_3) = \frac{1}{4}$. Prove that

$$\frac{17}{24} \leq \mathbb{P}(A \cup B \cup C) \leq \frac{19}{24}.$$

Problem 3

In a small town, there are three bakeries. Each of the bakeries bakes twelve cakes per day. Bakery 1 has two different types of cake, bakery 2 three different types, and bakery 3 four different types. Every bakery bakes equal amounts of cakes of each type. You randomly walk into one of the bakeries, and then randomly buy two cakes.

1. What is the probability that you will buy two cakes of the same type?

2. Suppose you have bought two different types of cake. Given this, what is the probability that you went to bakery 2?

Problem 4

An airplane manufacturer has three factories A B and C which produce 50%, 25%, and 25%, respectively, of a particular airplane. Seventy percent of the airplanes produced in factory A are passenger airplanes, 25% of those produced in factory B are passenger airplanes, and 25% of the airplanes produced in factory C are passenger airplanes. If an airplane produced by the manufacturer is selected at random, use a probability tree diagram to calculate the probability the airplane will be a passenger plane.

Problem 5 (Laplace's rule of succession)

There are $N+1$ urns labeled from 0 to N. Urn k contains k white balls and $n-k$ red balls. We randomly pick an urn, and once the urn is chosen, pick balls from there with replacement.

1. Let A_k be the event "urn k is chosen" and B_n the n-th drawn ball is white. Write $\mathbb{P}(A_k)$ and $\mathbb{P}(B_n|A_k)$

2. What is the probability that the $n+1$th drawn ball is white given that the n previous balls were also all white?

3. What does this probability become as the number of urns N goes to infinity? We also give $\sum_{k=1}^{N} k^n \sim \frac{N^{n+1}}{n+1}$ as N goes to infinity.

Problem 6 (Pólya's urn)

Consider an urn with b white balls and r red balls. We draw a ball, note its color, and put it back with d more balls of the same color, and repeat the experiment indefinitely. Let's denote B_n the event "the n-th drawn ball is white"

1. Find B_1 and B_2

2. Find a general formula for B_n

Recommended practice exercises (not to be handed in)

Textbook exercices 2.1-2.19