MATH 302 Assignment 6

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

1. Let X be a random variable that takes only non-negative integer values. Then show that

$$\mathbb{E}(X) = \sum_{i=1}^{+\infty} P(X \geq i)$$

2. Use this result to find the expectation of a Geometric r.v.

Problem 2

Let X and Y be 2 independent r.v.'s that each follow a Geometric law of parameter p.

1. Find the distribution of $\min(X, Y)$ (hint: use the cdf).
2. Find $P(\min(X, Y) = X)$ (i.e. $P(Y \geq X)$).
3. Show that $P(X + Y = z) = (z + 1)p^2(1 - p)^2$.
4. Find $P(Y = y | X + Y = z)$, for $y = 0, 1, \ldots, z$

Problem 3 (Coupon's collector problem)

We are trying to collect a set of n different cards, that are put in boxes of cereal: each box contains exactly one card, and it is equally likely to be any of the n cards. Let T_n be the number of boxes that are bought until we have collected at least one copy of every card. What is $\mathbb{E}(T_n)$? (hint: think about the geometric distribution)

Problem 4

Use the Poisson approximation to estimate the probability that at most 2 out of 50 given people will have invalid driver's license if normally 5% of the people do.

Problem 5

To get to LSK 201, students can take two different doors, one located down the room and the other located upstairs. Assume that every day the number of students who come to class in LSK 201 follows a Poisson law of parameter λ, and that every student chooses independently which door to get in, with probability to choose the upper door p.

1. Show that the number of students that come using the upper door X_u follows a Poisson distribution of parameter $p\lambda$ (hint: Find $P(X_u = k)$ by conditioning on the total number of students that come to class)

2. Are X_u and X_l (number of students who take the lower door) independent? (hint: use the fact that $P(X_l = k, X_u = k) = P(X_l = k, X_l + X_u = k + l)$)

Recommended practice exercises (not to be handed in)

Textbook exercices 3.51-57, 4.30-36