MATH 302 Assignment 5

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

You have two dice, one with three sides labeled $0, 1, 2$ and one with 4 sides, labeled $0, 1, 2, 3$. Let X_1 be the outcome of rolling the first die, and X_2 the outcome of rolling the second. The rolls are independent.

1. What is the joint p.m.f. of (X_1, X_2)? You can write your answer as a table (see lecture notes).
2. Let $Y_1 = X_1 \cdot X_2$ and $Y_2 = \max\{X_1, X_2\}$. Make a table for the joint p.m.f. of (Y_1, Y_2).
3. Are Y_1, Y_2 independent?

Problem 2

Prove that $Corr(X, Y) = 1$ if and only if there exist $a > 0$ and $b \in \mathbb{R}$ such that $Y = aX + b$ (third statement of Proposition 15 from lecture notes chapter 4). Hint: see Thursday's tutorial.

Problem 3

An urn contains a white ball and a red ball. One draws a ball in this urn, checks its color and replace it with 2 more balls of the same color. One repeats the operation indefinitely.

1. What is the probability that the first n drawn balls are red? (*hint:* define the events A_n = "the first n balls drawn are red" and use the law of total probability after evaluating $P(A_k|A_{k-1})$)
2. What is the probability to draw red balls indefinitely? We recall the Stirling formula $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.
3. Does the result change if we replace the ball with three more balls of the same color instead of two? We admit that $\lim_{n\to+\infty} \sum_{k=1}^{n} ln\left(1 - \frac{1}{3k-1}\right)) = -\infty$

Recommended practice exercises (not to be handed in)

Textbook exercises 8.14, 8.16, 8.17,