MATH 302 Assignment 4

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

Consider the experiment of tossing a coin three times where the probability of a head on an individual toss is p. Suppose that for each toss that comes up heads we win \$ 1, but for each toss that comes up tails we lose \$1. Let X denote the total winnings.

1. List the possible elementary events ω associated with the random experiment, the associated value of $X(\omega)$, and the probability $P(\omega)$.
2. Calculate the expectation and variance of X.

Problem 2

Let X take values $\{1, 2, 3, 4, 5\}$, with p.m.f. given by

Table 1: The p.m.f. of X

k	1	2	3	4	5
$\mathbb{P}(X = k)$	1/7	1/14	3/14	2/7	2/7

1. Calculate $\mathbb{P}(X \leq 3)$
2. Calculate $\mathbb{P}(X < 3)$
3. Calculate $\mathbb{P}(X < 4.12 | X > 1.6)$
4. Calculate $\mathbb{E}\, X$
5. Calculate $\mathbb{E}|X - 2|$

Problem 3

Consider the following lottery: There are a total of 10 tickets, of which 5 are "win" and 5 are "lose". You draw tickets until you draw the first "win". Drawing one ticket costs \$2, 2 tickets \$4, 3 tickets \$8, and so on. A winning ticket pays out \$8.

1. Let X be the number of tickets you draw in the lottery (i.e. the number of tickets until the first win, including the winning ticket). Calculate the p.m.f. of X.
2. Calculate the expectation $\mathbb{E}\, X$.
3. Calculate the variance $\sigma^2(X)$.
4. What are your expected winnings in this game?

Problem 4

Prove the following claims. Here, X, Y are random variables on the same finite sample space, and $a, b \in \mathbb{R}$.

1. $\mathbb{E}\,(aX + b) = a\mathbb{E}\, X + b$
2. $\sigma^2(aX + b) = a^2\sigma^2(X)$
3. $\sigma^2(X) = \mathbb{E}(X^2) - (\mathbb{E}\, X)^2$

Problem 5

An urn contains n red balls and n white balls. We simultaneously pick n balls and record X the number of red balls that are picked.

1. Assume that one can distinguish the balls by labeling them (from 1 to $2n$). How many ways are there to get $X = k$? Deduce the law of X
2. Find its expectation

Problem 6 (Linear regression)

Let X and Y be 2 real random variables, with $Var(X) > 0$. Find $a, b \in \mathbb{R}$ that minimize $E([y - (aX + b)]^2)$ (*hint:* Decompose $E([y - (aX + b)]^2)$ into a sum of two terms using a formula from class, and minimize each term separately).

Recommended practice exercises (not to be handed in)

Textbook exercices 1.16-1.19, 3.1, 3.2, 3.8, 3.10, 3.15, 3.19, 3.21-24, 3.28, 3.29, 3.75