

MATH 302 Assignment 7

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this [one](#)).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a “we trust you” policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

Suppose that X has generating function

$$G_X(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^3.$$

1. Find the mean and variance of X .
2. Find the p.m.f. of X . Use your expression for the p.m.f. to check your answers from part (a).

Problem 2

1. Prove proposition 6 (chapter 5) of the lecture using generating functions.
2. Apply proposition 10 to find the expectation and variance of the binomial, geometric and Poisson distributions

Problem 3 (Negative binomial distribution)

We want to find the pmf of the first time T_m to get the m -th success in a sequence of Bernoulli trials with probability of success p .

1. Justify why T_1 and $T_m - T_{m-1}$ (for $m > 1$) follow a geometric distribution.
2. Find the generating function of T_m .
3. By differentiating $\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k$ for $x \in (-1, 1)$, express $\frac{1}{(1-x)^m}$ as a power series and determine the law of T_m .

Problem 4

Let $c > 0$ and $X \sim \text{Unif}[0, c]$. Show that the RV $Y = c - X$ has the same c.d.f. and therefore also the same p.d.f. as X .

Problem 5

Let X be a random variable with p.d.f.

$$f(x) = \begin{cases} cx^{-3} & x > 2 \\ 0 & \text{otherwise} \end{cases}$$

1. Find c so that f is a p.d.f.
2. Compute the c.d.f. of X .
3. Find $\mathbb{P}(X > 3 | X < 5)$.
4. Find the median of X , i.e. the value m such that $\mathbb{P}(X > m) = \mathbb{P}(X \leq m)$.
5. Calculate $\mathbb{E}\sqrt{X}$.

Recommended practice exercises (not to be handed in)

Textbook exercises 3.3, 3.4, 3.7, 3.16, 3.31-33, 3.39, 3.43, 3.44, 3.56, 3.57, 4.9-11