MATH 302 Assignment 8

Instructions

- Submit a pdf file of written work on Canvas. Be careful of the size of your file. If it exceeds 5Mb, use a compression tool to reduce it (like this one).
- Each homework assignment is worth 0.5% of your final course mark. They are not graded by the TA; instead they are (randomly) checked for appropriate content. Students who submit significant attempts at solving at least half of the problems in each assignment will receive full mark (*do not submit work otherwise*).
- We implement a "we trust you" policy and assume that all students will try hard to solve the problems in the homework assignments, and will receive full credit for trying hard. However, *students who submit garbage files, work that is not their own or that contains attempted solutions for less than half of the problems will receive a penalty of - 10 points on their final course mark.*

Problem 1

Let $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}} dx$ denote the cdf of $\mathcal{N}(0,1)$. We report in the next page different values that it can take.

1. Suppose that the weight of a person selected at random from some population is normally distributed with parameters μ and σ. Suppose also that $P(X \leq 160) = 0.5$ and $P(X \leq 140 = 0.25)$. Find μ and σ and find $P(X \geq 200)$.
2. Of all the people in the population weighing at least 200 pounds what percentage will weigh over 220 pounds?

Problem 2

1. Let $X \sim \mathcal{N}(0, \sigma^2)$. Find the density of $Y = |X|$.
2. Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Find the density of $Y = e^X$. *remark*: this density is called a *lognormal density.*

Problem 3

Let X be an exponential variable with unknown parameter λ. We know that $\mathbb{P}(X \leq 10) = \frac{5}{9}$.

1. Compute $\mathbb{P}(X \geq 15)$.
2. Compute $\mathbb{P}(X \geq 15 | X \geq 10)$ (use the memoryless property).
3. Compute $\mathbb{P}(X \geq 15 | 10 \leq X \leq 20)$.
4. Find the number $y > 0$ for which $\mathbb{P}(y \leq X \leq 2y)$ is the largest.

Problem 4 (Minimum of independent exponential random variables)

1. Let X_1, X_2 be independent exponential r.v.'s with respective rates λ_1, λ_2 and λ_3 (not necessarily equal). Find the probability that $X_1 = \min(X_i,\ 1 \leq i \leq 3)$ (*hint*: Use a result from tutorial and compute $P(X_1 \leq \min(X_2, X_3))$.

2. Find the distribution of the minimum of n independent exponential r.v.'s.

3. 8 runners R_i $(i = 1, \ldots, 8)$ enter a race, with a time to complete respectively $\sim Exp(i)$ (and independent from the other runners). Suppose the winner of the race (with the shortest time) earns 10 dollars and other runners lose 1. What is the expected gain of runner 1?

Recommended practice exercises (not to be handed in)

Textbook exercices 3.17, 3.18, 3.66, 3.70, 3.71, 4.12-14, 4.48, 4.52, 4.53

Standard Normal CDF Phi(z)

z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)	z	Phi(z)
0.00	0.5000	**0.50**	0.6915	**1.00**	0.8413	**1.50**	0.9332	**2.00**	0.9772	**2.50**	0.9938
0.01	0.5040	**0.51**	0.6950	**1.01**	0.8438	**1.51**	0.9345	**2.01**	0.9778	**2.51**	0.9940
0.02	0.5080	**0.52**	0.6985	**1.02**	0.8461	**1.52**	0.9357	**2.02**	0.9783	**2.52**	0.9941
0.03	0.5120	**0.53**	0.7019	**1.03**	0.8485	**1.53**	0.9370	**2.03**	0.9788	**2.53**	0.9943
0.04	0.5160	**0.54**	0.7054	**1.04**	0.8508	**1.54**	0.9382	**2.04**	0.9793	**2.54**	0.9945
0.05	0.5199	**0.55**	0.7088	**1.05**	0.8531	**1.55**	0.9394	**2.05**	0.9798	**2.55**	0.9946
0.06	0.5239	**0.56**	0.7123	**1.06**	0.8554	**1.56**	0.9406	**2.06**	0.9803	**2.56**	0.9948
0.07	0.5279	**0.57**	0.7157	**1.07**	0.8577	**1.57**	0.9418	**2.07**	0.9808	**2.57**	0.9949
0.08	0.5319	**0.58**	0.7190	**1.08**	0.8599	**1.58**	0.9429	**2.08**	0.9812	**2.58**	0.9951
0.09	0.5359	**0.59**	0.7224	**1.09**	0.8621	**1.59**	0.9441	**2.09**	0.9817	**2.59**	0.9952
0.10	0.5398	**0.60**	0.7257	**1.10**	0.8643	**1.60**	0.9452	**2.10**	0.9821	**2.60**	0.9953
0.11	0.5438	**0.61**	0.7291	**1.11**	0.8665	**1.61**	0.9463	**2.11**	0.9826	**2.61**	0.9955
0.12	0.5478	**0.62**	0.7324	**1.12**	0.8686	**1.62**	0.9474	**2.12**	0.9830	**2.62**	0.9956
0.13	0.5517	**0.63**	0.7357	**1.13**	0.8708	**1.63**	0.9484	**2.13**	0.9834	**2.63**	0.9957
0.14	0.5557	**0.64**	0.7389	**1.14**	0.8729	**1.64**	0.9495	**2.14**	0.9838	**2.64**	0.9959
0.15	0.5596	**0.65**	0.7422	**1.15**	0.8749	**1.65**	0.9505	**2.15**	0.9842	**2.65**	0.9960
0.16	0.5636	**0.66**	0.7454	**1.16**	0.8770	**1.66**	0.9515	**2.16**	0.9846	**2.66**	0.9961
0.17	0.5675	**0.67**	0.7486	**1.17**	0.8790	**1.67**	0.9525	**2.17**	0.9850	**2.67**	0.9962
0.18	0.5714	**0.68**	0.7517	**1.18**	0.8810	**1.68**	0.9535	**2.18**	0.9854	**2.68**	0.9963
0.19	0.5753	**0.69**	0.7549	**1.19**	0.8830	**1.69**	0.9545	**2.19**	0.9857	**2.69**	0.9964
0.20	0.5793	**0.70**	0.7580	**1.20**	0.8849	**1.70**	0.9554	**2.20**	0.9861	**2.70**	0.9965
0.21	0.5832	**0.71**	0.7611	**1.21**	0.8869	**1.71**	0.9564	**2.21**	0.9864	**2.71**	0.9966
0.22	0.5871	**0.72**	0.7642	**1.22**	0.8888	**1.72**	0.9573	**2.22**	0.9868	**2.72**	0.9967
0.23	0.5910	**0.73**	0.7673	**1.23**	0.8907	**1.73**	0.9582	**2.23**	0.9871	**2.73**	0.9968
0.24	0.5948	**0.74**	0.7704	**1.24**	0.8925	**1.74**	0.9591	**2.24**	0.9875	**2.74**	0.9969
0.25	0.5987	**0.75**	0.7734	**1.25**	0.8944	**1.75**	0.9599	**2.25**	0.9878	**2.75**	0.9970
0.26	0.6026	**0.76**	0.7764	**1.26**	0.8962	**1.76**	0.9608	**2.26**	0.9881	**2.76**	0.9971
0.27	0.6064	**0.77**	0.7794	**1.27**	0.8980	**1.77**	0.9616	**2.27**	0.9884	**2.77**	0.9972