Syllabus

Tuesday, April 2, 2019 10:31 AM

ENGR 4620 Course Syllabus Spring 2019

Winter 2016 University of Denver

ENGR 4620 – Optimization
Course Syllabus

Instructor Information:
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Class Time: 05:00 pm – 08:30 pm, Tuesdays
Class Location: TBD
Office hours: 9:00 am – 11:00 am, Mondays and Wednesdays

Catalog Description:

Engineering problems will be formulated as different programming problems to show the wide applicability and generality of optimization methods. The development, application, and computational aspects of various optimization techniques will be discussed with engineering examples. The application of nonlinear programming techniques will be emphasized. A design project will be assigned. 4.0 Lecture hours.

Textbooks: R. Rhinehart, *Engineering Optimization: Applications, Methods and Analysis*, John Wiley and Sons: New York, 2018, ISBN: 978-1-118-93631-3|
(Optional) S.S. Rao, Engineering Optimization: Theory and Practice, 4th Ed., John Wiley and Sons: New York, 2009.
(Optional) G. Vanderplaats, Numerical Optimization Techniques for Engineering Design, Vanderplaats Research & Development: Colorado Springs, CO, 2005.
(Optional) P. Venkataraman, Applied Optimization with MATLAB Programming, 2nd Ed., John Wiley and Sons: New York, 2009.

Software: MATLAB Student Edition, available from Mathworks.
Codes can be found at:
www.r3eda.com
www.wiley.com/go/venkat2e

Topical Outline:
1. Optimization of continuous functions using linear programming
2. Nonlinear programming
3. Numerical techniques for constrained one dimensional optimization
4. Unconstrained and constrained optimization
5. Advanced optimization techniques including global optimization and stochastic optimization techniques.

Grading:
The grade will be determined in the following way:

- 30% - Homework
- 10% - Class Participation
- 25% - Midterm Project
- 35% - Final Project

We will use the following grading scale: A ≥ 92.5%, A- ≥ 89.5%, B+ ≥ 87.5%, B ≥ 82.5%, B- ≥ 79.5%, C+ ≥ 77.5%, C ≥ 72.5%, C- ≥ 79.5%, D+ ≥ 67.5%, D ≥ 62.5%, D- ≥ 59.5%, F otherwise.

Course Objectives:

1. Students will develop an understanding of basic optimization algorithms as well as the pros and cons of each method.
2. Students will be able to implement basic optimization algorithms in a programming language such as MATLAB or C++.
3. Students will be able to complete a project involving the application of optimization methods to model and assist in the design of a physical system.
4. Students will be able to write clearly and concisely in a formal technical report.

Homework: Weekly assignments will be posted on Canvas and will consist of problems from the text, in-class worksheets, and other sources. It will be due **at the next class session each Tuesday**. Late homework be accepted up to one week late, with the 10% penalty. Exceptions to the late policy are at the instructor's discretion. Students are encouraged to study in groups and to discuss problems. However, every student is responsible for the work submitted.

Attendance:
Students are expected to attend the class. The student who does not attend the class is responsible for the materials covered and for obtaining all other information given that day in class, e.g., notes, homework problems, due date, etc. – preferably from another student - and should return to class caught up.

Other:
We will try and utilize technology in the classroom this quarter, however, please be mindful to use it for classroom purposes, not personal use. If you must use an electronic device for a personal reason, please first leave the classroom. You do not need to ask permission to leave class for personal reasons, just do your best to avoid causing any disruption on your way out.

Students with Disabilities/Medical Issues
If you qualify for academic accommodations because of a disability or medical issue please submit a Faculty Letter to me from Disability Services Program (DSP) in a timely manner so that your needs may be addressed. DSP is located on the 4 floor of Ruffatto Hall; 1999 E. Evans Ave.303.871. / 2372 / 2278/ 7432. Information is also available on line at http://www.du.edu/disability/dsp (http://www.du.edu/disability/dsp)

Inclusive Learning Environment

In this class, we will work together to develop a learning community that is inclusive and respectful. The goal of inclusiveness, in a diverse community, encourages and appreciates expressions of different ideas, opinions, and beliefs, so that conversations and interactions that could potentially be divisive turn instead into opportunities for intellectual and personal enrichment. A dedication to inclusiveness requires respecting what others say, their right to say it, and the thoughtful consideration of others' communication. Both speaking up and listening are valuable tools for furthering thoughtful, enlightening dialogue. Respecting one another's individual differences is critical in transforming a collection of diverse individuals into an inclusive, collaborative and excellent learning community. Our core commitment shapes our core expectation for behavior inside and outside of the classroom.

Honor Code/Academic Integrity

All work submitted in this course must be your own and produced exclusively for this course. The use of sources (ideas, quotations, paraphrases) must be properly acknowledged and documented. For the consequences of violating the Academic Misconduct policy, refer to the University of Denver website on the Honor Code (www.du.edu/honorcode). See also http://www.du.edu/studentconduct for general information about conduct expectations from the Office of Student Conduct.

*Note: Information given in this document may be subject to adjustment.

Introduction

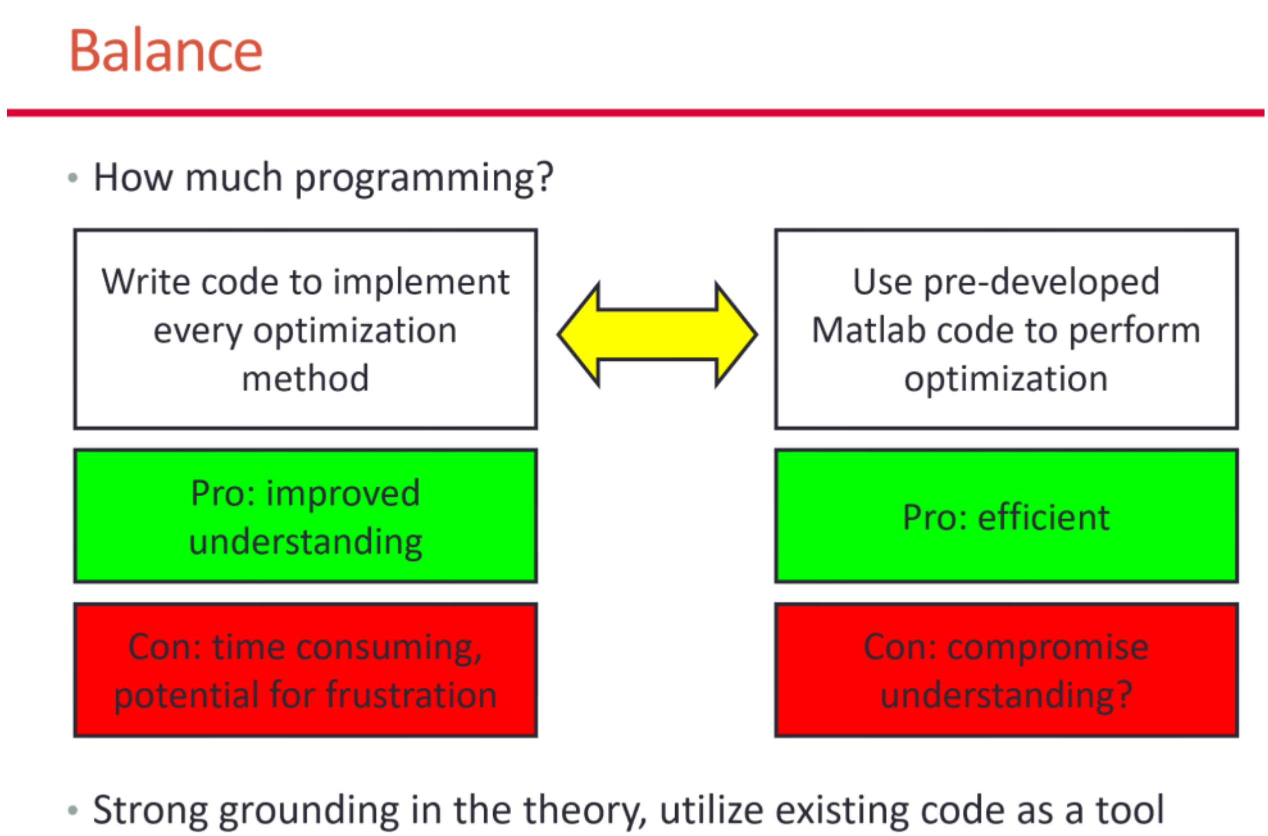
Tuesday, April 2, 2019 11:07 AM

Topical Outline:

- Optimization of continuous functions using linear programming
- Nonlinear programming
- Numerical techniques for constrained one dimensional optimization
- Unconstrained and constrained optimization
- Advanced optimization techniques including global optimization

Students will

- develop an understanding of basic optimization algorithms, as well as the pros and cons of each method
- implement basic optimization algorithms in a programming language such as MATLAB or C++.
- complete a project involving the application of optimization methods to model and assist in the design of a physical system
- write clearly and concisely in a formal technical report



Optimization

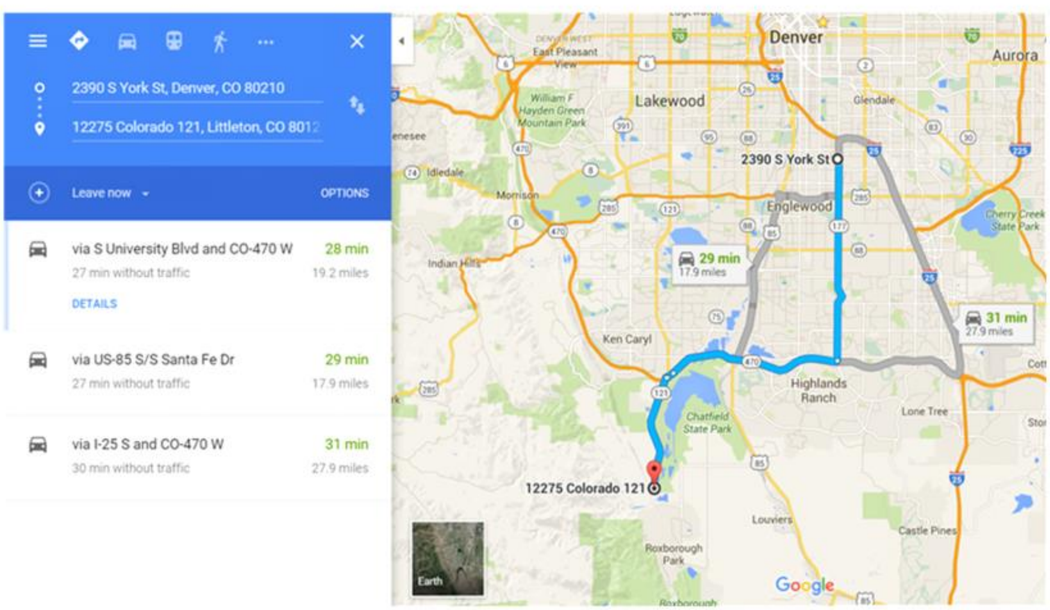
- The mathematical study of problems in which one seeks to maximize or minimize a real function by systematically searching for and finding the values of real or integer variables from within an allowed set that accomplishes the maximization or minimization.

- i.e.,

minimize/maximize	Objective function
subject to	Set of Constraints

Finding the shortest/fastest route

- Objective: To minimize the travel time
  To minimize the toll cost
  To avoid traffic



Buying a car

- Objective: To minimize the total payment
  To minimize the interest

- Constraints: Budget
  Convenience
  Looks

Optimization in Aerospace

- Objective: Minimize weight
- Constraints: Payload weighs xx lbs
  Maintaining structural integrity with a positive margin of safety
  Costs within budget

www.ulalaunch.com

Optimization in Energy Industry

- Objective: Minimize fuel cost
- Constraints: Generation-load balance
  Physical constraints of each generation unit
  Physical constraints of the power grid
  Fuel and emission limits

More Examples:

1. Design of aircraft and aerospace structures for minimum weight
2. Finding the optimal trajectories of space vehicles
3. Design of civil engineering structures such as frames, foundations, bridges, towers, chimneys, and dams for minimum cost
4. Minimum-weight design of structures for earthquake, wind, and other types of random loading
5. Design of water resources systems for maximum benefit
6. Optimal plastic design of structures
7. Optimum design of linkages, cams, gears, machine tools, and other mechanical components
8. Selection of machining conditions in metal-cutting processes for minimum production cost
9. Design of material handling equipment, such as conveyors, trucks, and cranes, for minimum cost
10. Design of pumps, turbines, and heat transfer equipment for maximum efficiency
11. Optimum design of electrical machinery such as motors, generators, and transformers
12. Optimum design of electrical networks
13. Shortest route taken by a salesperson visiting various cities during one tour
14. Optimal production planning, controlling, and scheduling
15. Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of the physical phenomenon
16. Optimum design of chemical processing equipment and plants
17. Design of optimum pipeline networks for process industries
18. Selection of a site for an industry
19. Planning of maintenance and replacement of equipment to reduce operating costs
20. Inventory control

21. Allocation of resources or services among several activities to maximize the benefit
22. Controlling the waiting and idle times and queueing in production lines to reduce the costs
23. Planning the best strategy to obtain maximum profit in the presence of a competitor
24. Optimum design of control systems

Table 1.1 Methods of Operations Research

Mathematical programming or optimization techniques	Stochastic process techniques	Statistical methods
Calculus methods	Statistical decision theory	Regression analysis
Calculus of variations	Markov processes	Cluster analysis, pattern recognition
Nonlinear programming	Queueing theory	Design of experiments
Geometric programming	Renewal theory	Discriminate analysis (factor analysis)
Quadratic programming	Simulation methods	
Linear programming	Reliability theory	
Dynamic programming		
Integer programming		
Stochastic programming		
Separable programming		
Multiobjective programming		
Network methods: CPM and PERT		
Game theory		
Modern or nontraditional optimization techniques		
Genetic algorithms		
Simulated annealing		
Ant colony optimization		
Particle swarm optimization		
Neural networks		
Fuzzy optimization		

Here are a few essential aspects of optimization:

Point 1: Although optimization offers the joys of solving an intellectual puzzle, it is not just a stimulating mathematical game. Optimization applications are complicated, and the major challenges are the clear and complete statement of:

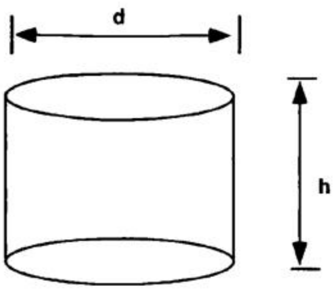
1) The objective function (OF—the outcome you wish to minimize or maximize)
2) Constraints (what cannot or should not be violated, or exceeded)
3) The decision variables (DV—what you are free to change to seek a minimum)
4) The model (how DVs relate to OF and constraints)
5) The convergence criterion (the indicator of whether the algorithm has found a close enough proximity to the minimum or maximum and can stop or needs to continue)
6) The DV initialization values
7) The number of starts from randomized locations to be confident that the global optimum has been found
8) The appropriate optimization algorithm (for the function aberrations, for utility, for precision, for efficiency)
9) Computer implementation in code Oh yes,
10) The mathematics of the optimization algorithm (understanding this is also important)

not only that!

Point 2: Do not study. Learning is most effective if you integrate the techniques into your daily life. You will forget the material that you memorized in order to pass a test. Since this book provides skills that are essential for both personal and career life, I want you to take the techniques with you. I want this book to be useful in your future. Although memorization and high-level mathematical analysis are both elements of the book, understanding the examples and doing of the exercises is more important. To maximize the impact of this material, you need to integrate it into your daily life. You need to practice it.

Example: Oil Can

- Consider the design of an oil can from aluminum.
- The can must be of standard size, 1 quart.
- Assume the can will be a cylinder of height h, and diameter d
- Design variables: d, h



30

Example: Oil Can

- Consider that the aluminum has a cost of \$0.02 per cm^2
- For effective pouring, the height of the can must be larger than the diameter, $h > d$
- Design parameter: C (cost per cm^2)
- Design functions: volume, surface area
  - $V = (\pi d^2/4)\,h$ $= \pi r^2 \cdot h$
  - $A = 2\,(\pi d^2/4) + \pi dh$

- Minimize total cost of material to construct the can

Example: Oil Can

31

OF

Minimize $f(d,h)$: $C\,[2\,(\pi d^2/4) + \pi dh]$

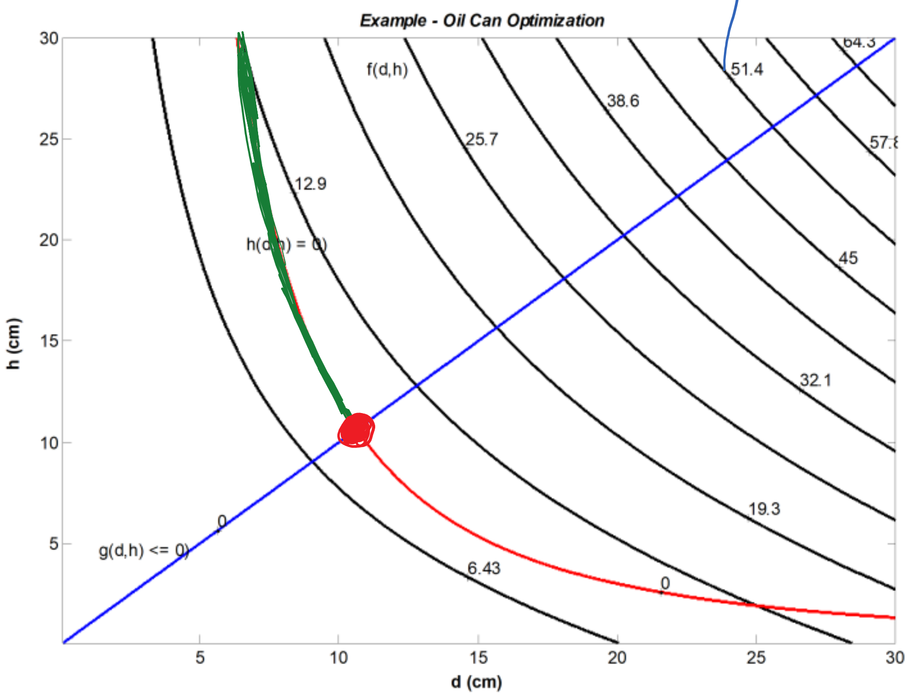
level curves

Subject to: $h_1(d,h)$: $(\pi d^2/4)\,h - 1 = 0$

$g_1(d,h)$: $d - h \leq 0$

$g_1(d,h): \quad d - h \leq 0$

Example: Oil Can



33

Terminology

Tuesday, April 2, 2019 11:22 AM

Mathematical formulation

Minimize $f(x_1, x_2, \ldots, x_n)$ — Objective Function

Subject to:

$h_1(x_1, x_2, \ldots, x_n) = 0$
$h_2(x_1, x_2, \ldots, x_n) = 0$
$h_l(x_1, x_2, \ldots, x_n) = 0$ — Equality Constraints

$g_1(x_1, x_2, \ldots, x_n) \le 0$
$g_2(x_1, x_2, \ldots, x_n) \le 0$
$g_m(x_1, x_2, \ldots, x_n) \le 0$ — Inequality Constraints

$x_i^l \le x_i \le x_i^u,\ i = 1, \ldots, n$ — Side Constraints

Standard Form

12

Objective Funcion (OF)

Decision Variables (DV)

You might want to minimize (costs, expenses, risk, etc.) or maximize (profit, reliability, probability of success, etc.). Optimization will seek DV values that lead to the optimum, either minimum or maximum.

A trial solution (TS) is a particular DV choice. It might not be the optimum. The optimum is denoted as DV*. For simple, idealized applications, one can determine the exact value of DV*; but, for most applications, this is an ideal concept, like the value of π. You'll get close enough to the true value for a particular need and call the close enough TS the DV*.

Type of Problems

- The problem definition often dictates the solution method

- Linear versus nonlinear
  - Are design functions (objective or constraints) nonlinear?
- Constrained versus unconstrained

Solutions Methods

- Small number of design parameters (e.g. n=2)
  → Graphical solutions (Ch. 2)

- If all design functions are linear
  → Linear programming problem (Ch. 3)

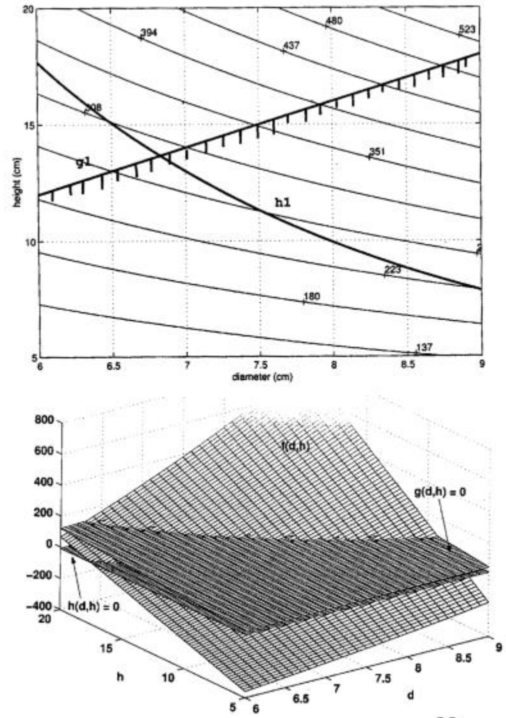
- If any 1 design function is nonlinear
  → Nonlinear programming problems (Ch. 4-6)
    - Non-gradient versus gradient
  → Constrained problems (Ch.7)

37

Graphical solutions

- Objective function is represented as scaled contours
- Feasible region
- Which constraints are active?

- Non-linearity (in objective and constraints) complicates solution procedures



38

Figure 1.4 Graphical representation of Example 1.1.

Linear Programming Problems

- If objective function and all constraints are linear functions

- Common in business, commerce

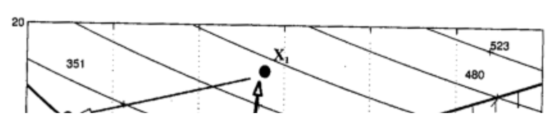
- Solutions lie on boundaries of feasible region

Nonlinear Programming Problems

- If any of the design functions are nonlinear

- Solutions obtained via numerical analysis

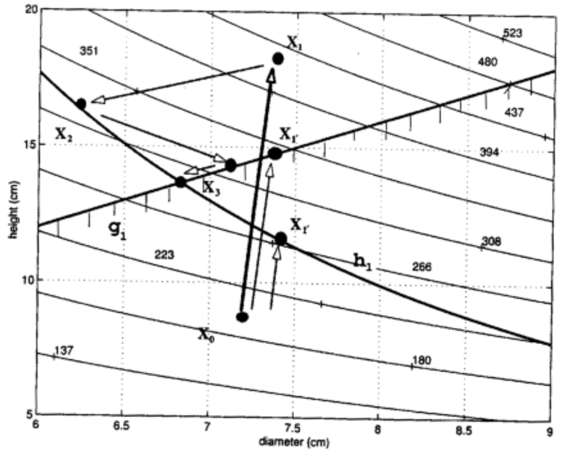
- "Search methods" involve



Nonlinear Programming Problems

- If any of the design functions are nonlinear
- Solutions obtained via numerical analysis
- "Search methods" involve iterative techniques

Step 0:	Choose $\mathbf{X_0}$
Step 1:	Identify $\mathbf{S}$
	Determine α
	$\Delta\mathbf{X} = \alpha\mathbf{S}$
	$\mathbf{X_{new}} = \mathbf{X_0} + \Delta\mathbf{X}$
	Set $\mathbf{X_0} \leftarrow \mathbf{X_{new}}$
	Go To Step 0



40

Examples of Constraints:

- Constraint on transition: When visiting major league ballparks, don't go to NYC immediately after Boston or vice versa because of fan loyalty. When operating a mixer, you must fill and blend material in the mixer before dumping.
- Constraint on rate of change: Don't immediately "floor" the automobile accelerator pedal position, because it startles passengers. Gradually move it from one position to another, no faster than 5° of arc per second.
- Constraint on DV: A composition must be between 0 and 100%. A flow rate must be nonnegative.
- Constraint on secondary variables (auxiliary variables): Don't let a downstream tank overflow or run dry. The sum of all compositions must be equal to unity. Keep the fire temperature below the melting point of the nozzle material. An automobile engine temperature cannot exceed 180°F.
- Constraints may relate to natural laws: Human bodies cannot float in the Earth's atmosphere because of the density difference between mostly water and air. Heat does not naturally flow from cold to hot, because entropy must increase. As close as cellulose is to sugar, and even though grazing animals can digest cellulose, humans cannot live on cellulose.
- Constraints may relate to human laws or procedures: You cannot build your house on your neighbor's property. You cannot freely travel from any one country to any other. Wear your uniform and follow this procedure to clock your work hours.
- Constraints may occur in the future: Today you can make the purchases on your credit card; but the constraint happens in the future when all of your income is paying accumulated interest. You can drive the race car now and pass a pit stop; but the worn tires will fail in half a lap.
- Constraints may be soft or hard: It is OK to violate soft constraints. The speed limit is 60 mph, but enforcement permits you to drive 4 mph higher. Other soft constraint examples, in which mild violation is permissible might include, "Don't yell at your children." Wear white hats from Memorial Day to Labor Day. By contrast, hard constraints may not be violated. "Don't let the mixture composition get into the explosive limits." "Pay your taxes." Don't try to take the logarithm of a negative number.

Minimize or maximize?
We focus on minimization.

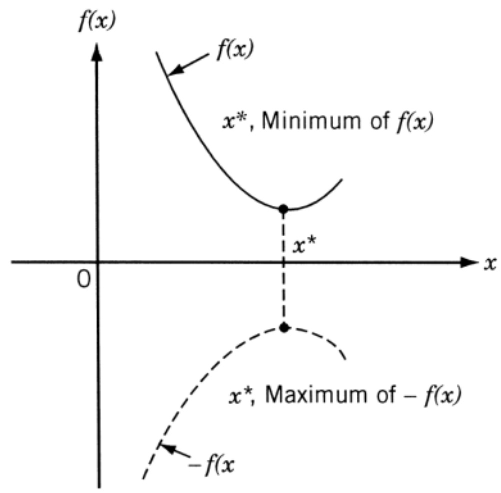


Figure 1.1 Minimum of $f(x)$ is same as maximum of $-f(x)$.

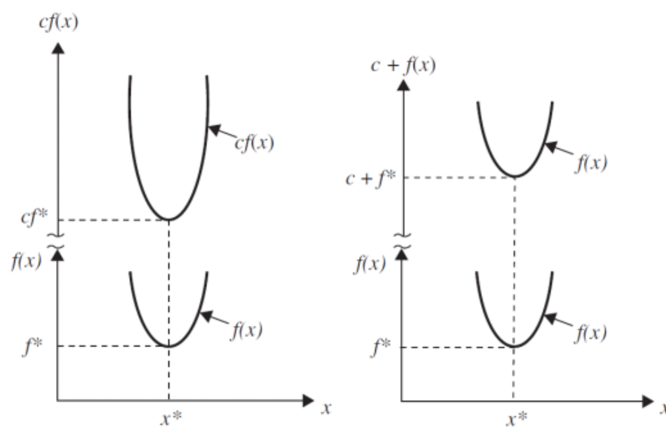


Figure 1.2 Optimum solution of $cf(x)$ or $c + f(x)$ same as that of $f(x)$.

"Canonical" Form of the Optimization Statement

$$
\begin{aligned}
&\min_{\{DV\}} J = \text{you define this relation} \\
&\text{S.T.:} \quad P_1 = f(DV) \le P_{10} \\
&\qquad a \le DV_1 \le b \\
&\qquad \text{rate} = \frac{dP_2}{dt} \le \text{rate}_0 \\
&\qquad\qquad \vdots
\end{aligned}
$$

Or perhaps a bit less formally:

$$
\text{Find } \mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \text{ which minimizes } f(\mathbf{X})
$$

subject to the constraints

$$
\begin{aligned}
g_j(\mathbf{X}) &\le 0, \qquad j = 1, 2, \ldots, m \\
l_j(\mathbf{X}) &= 0, \qquad j = 1, 2, \ldots, p
\end{aligned} \tag{1.1}
$$

Optimization Procedure:

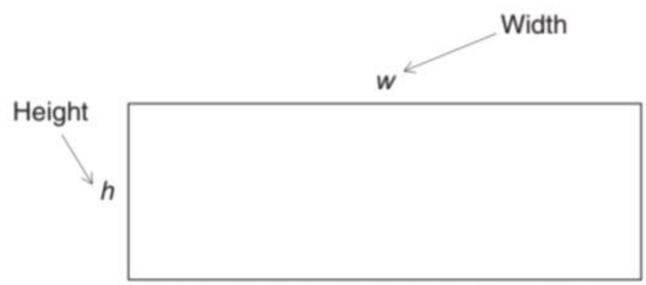
There are eight basic stages in an optimization procedure:

1) *State the OF, or objective function.*
2) *State whether you wish to minimize or maximize.*
3) *State the DVs, or decision variables, and what you are free to change to effect the OF value.*
4) *Use model relationships to relate the DV to all parts of the OF.*
5) *Define constraints.*
6) *State the method used for solving the optimization application. This is the optimization approach, algorithm, procedure, or logic.*
7) *Execute the procedure.*
8) *Reflect on it all.*

Example 5 Minimize the perimeter of a rectangle that provides a desired area by choosing the height and width. Figure 1.4 represents a mental construct of a rectangle. We need the mental construct to derive models that represent the interaction of DVs and OF. The model of the mental construct is not the reality, but it is a mathematical approximation of reality.

1) State the OF, or objective function: J = perimeter = $2h + 2w$.
2) State whether you wish to minimize or maximize: Minimize $J = 2h + 2w$.
3) State the DVs, or decision variables, and what you are free to change to effect the OF value. In this case it is just one item, either the height or the width. Given one value, the other is

Figure 1.4 Analysis of a rectangle.



calculated from the desired area. $A = hw$. I'll choose height as the DV. The basic optimization statement is

$$\min_{\{h\}} J = 2h + 2w \tag{1.15}$$

$\{h, w\} \Leftarrow$ at first

4) Use model relationships to relate the DV to all parts of the OF. In general we need a model of the device or process to relate state variables to influences. In this case h is explicit in the OF. However, h also affects w, which is not explicitly revealed in the OF. We need the complete relation. These models are often termed constitutive relations. From the definition of area in a rectangle,

$$w = \frac{A}{h} \tag{1.16}$$

Inserting it into the OF completely indicates how the choice of h-value affects the OF value:

$$\min_{\{h\}} J = 2h + \frac{2A}{h} \tag{1.17}$$

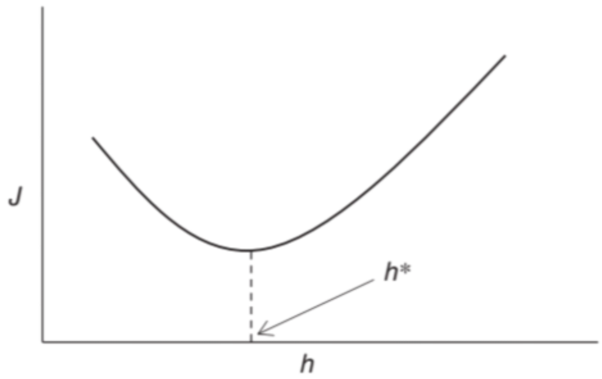
5) Define constraints

$$h > 0 \tag{1.18}$$

6) State the method used for solving the optimization application and for adjusting h to find the value h^* that minimizes the OF. This is the optimization approach, algorithm, method, or logic. The analytical method is one algorithm. Since the OF permits its use, I'll use it. Assume J is a function with one minimum, as illustrated in Figure 1.5.

Figure 1.5 A function with one minimum.

Figure 1.5 A function with one minimum.



At the minimum, $dJ/dh|_{h^*} = 0$. Elsewhere, the slope is not zero. So, get the analytical derivative of J w.r.t. h, equate it to zero, and solve for the value of h^*.

7) Execute the procedure:

$$\left.\frac{dJ}{dh}\right|_{h^*} = 0 = 2 - \frac{2A}{(h^*)^2} \tag{1.19}$$

The solution is

$$h^* = \sqrt{A} \tag{1.20}$$

which also means that $w^* = h^*$ and the rectangle is a square.

8) Reflect on it all (the model, the assumptions, the answer, the method, the acceptance of your solution by others in the enterprise). Evaluate your procedure and results. The first application and solution provides progressive insight. What would you do differently in OF, DV, constraints, model, and method? What idealizations limit validity of result? When others look at your analysis, what might they find incomplete or overlooked? Redo the work with this added depth of insight. This was an ideal initial analysis.

   For instance: Is the item being modeled an ideal rectangle of lines of zero width? Other than mental constructs and intellectual exercises, where do you find such things? An exercise like this might be the idealization of an application to choose a window pane shape to minimize the consumption of frame material. But the frame has thickness, and it must both overlap the window a bit and extend beyond. So, perhaps the volume of frame material, not the perimeter, is the real OF. Also, is it the window pane area or the open area inside the frame that is important? The answer to the analysis above was a square, but how does this fit with your personal experience? Are window panes square? What considerations seem to override the analysis to give an alternate (not $h = w$) solution?

Example 6 Maximize power delivered to a resistor from a battery with voltage V. Figure 1.6 represents a mental construct of a battery within the dashed lines as an ideal battery followed by an internal

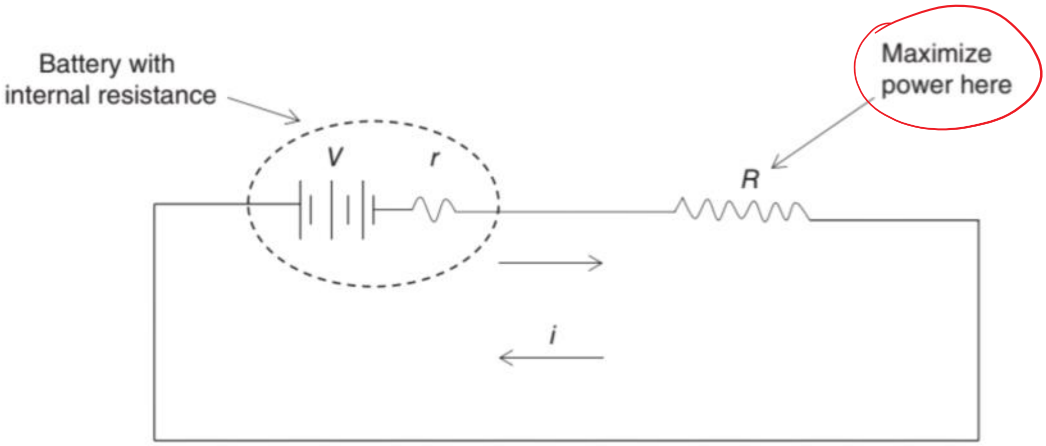


Figure 1.6 Mental construct of a battery in a resistance circuit.

1) State the OF, or objective function: $J = \text{Power} = i^2 R$.
2) State whether you wish to minimize or maximize: Maximize $J = i^2 R$.
3) State the DVs, or decision variables, and what you are free to change to effect the OF value. In this case it is just one item, the external resistor resistance, DV = R. The basic optimization statement is

$$\max_{\{R\}} J = i^2 R \tag{1.21}$$

4) Use model relationships to relate the DV to all parts of the OF. In general we need a model of the device or process to relate state variables to influences. In this case R is explicit in the OF. However, R also affects i, and the model is not explicitly revealed in the OF. We need the complete relation. These models are often termed constitutive relations. From elementary circuit analysis there is a relation between current and resistance:

$$V = i(R + r) \rightarrow i = \frac{V}{R + r} \tag{1.22}$$

Inserting it into the OF completely indicates how the choice of R value affects the OF value:

$$\max_{\{R\}} J = \frac{V^2 R}{(R + r)^2} = V^2 R (R+r)^{-2} \tag{1.23}$$

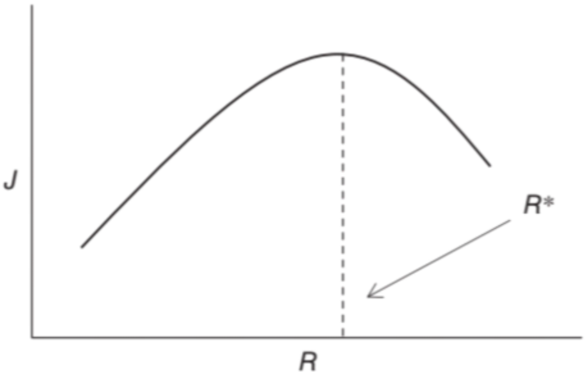
5) Define constraints

$$R > 0 \tag{1.24}$$

6) State the method used for solving the optimization application and for adjusting R to find the value R^* that maximizes OF. This is the optimization approach, algorithm, method, or logic. The analytical method is one algorithm and applicable to this example. Assume J is a function with one maximum as illustrated in Figure 1.7.

At the maximum, $dJ/dR|_{R^*} = 0$. Elsewhere, the slope is not zero. So, get the analytical derivative of J w.r.t. R, equate it to zero, and solve for the value of R.

Figure 1.7 Illustration of a function with a single maximum.



7) Execute the procedure:

$$\left.\frac{dJ}{dR}\right|_{R^*} = 0 = \frac{\cancel{V^2}}{(R+r)^2} - 2\frac{\cancel{V^2} R}{(R+r)^3} = 0 \tag{1.25}$$

$R + r - 2R = 0$

$r - R = 0$

$R = r$

The solution is

$$R^* = r \tag{1.26}$$

8) Reflect on it all (the model, the assumptions, the answer, the method, the acceptance of your solution by others in the enterprise). Evaluate your procedure and results. The first application and solution provides progressive insight. What would you do differently in OF, DV, constraints, model, and method? What idealizations limit validity of result? When others look at your analysis, what might they find incomplete or overlooked? Redo the work with this added depth of insight. This was an ideal initial analysis.

For instance, is the model of the battery correct (is the internal resistance in series with the ideal battery or is it more complicated)? Is the circuit model correct (zero resistance in the wire)? Is there an impedance effect on start-up? Does the battery voltage remain at the initial "V" while it is discharging? If you buy a resistor of $R^* = r$ what value will R actually be? Are the "given" V and r values the absolute truth for all circuits to be made? This OF looked at maximizing delivered power, but are there other measures of performance or cost that have been overlooked?

Problem statement

- In July, Venkat's Fruit Orchard usually sees a scheduling problem. Both cherries and blueberries ripen around the same time.
  - A cartload of cherries sells for $2750 while a cartload of blue berries fetches $1750.
  - The farm must deliver at least a half cartload of cherries and one cartload of blue-berries to the local supermarket.
  - Three persons can pick 2 cartloads of cherries in a day while one person will pick a cartload of blue-berries in the same time.
  - Only 46 people show up for work during the day.
  - One cartload of cherry is contained in 4 pallets while the same amount of blueberries occupies 6 pallets. There are 251 pallets.
  - Each picker is paid $45 per day.
  - The packaging and storage can handle at most 30 cartload per day.

- Set up the LP problem for the Orchard to maximize profits.

42

Problem formulation

- Design parameters:
  - All of the numbers in the problem statement
- Design variables:
  - x: cartload of cherries picked per day
  - y: cartload of blueberries picked per day
- Objective function:
  - Profit (total sales minus total labor cost)
- Constraints:
  - Labor constraint
  - Storage constraint
  - Pallet constraint
  - Side constraints

43

Problem formulation – objective

- A cartload of cherries sells for $2750 while a cartload of blue berries fetches $1750.
- Each picker is paid $45 per day.
- Three persons can pick 2 cartloads of cherries in a day while one person will pick a cartload of blue berries in the same time.

- Total sales: $2750x + 1750y$
- Total labor cost: $45*(1.5x + y)$
- Objective function: f(x,y): $2750x + 1750y – (45*(1.5x + y))$

  $= 2682.5x + 1705y$

44

Problem formulation – inequality constraints

- Only 46 people show up for work during the day.
- Labor constraint: $g_1(x,y)$: $1.5x + y <= 46$

- The packaging and storage can handle at most 30 cartload per day.
- Storage constraint: $g_2(x,y)$: $x + y <= 30$

- One cartload of cherry is contained in 4 pallets while the same amount of blueberries occupies 6 pallets. There are 251 pallets.
- Pallet constraint: $g_3(x,y)$: $4x + 6y <= 251$

Problem formulation – side constraints

- The farm must deliver at least a half cartload of cherries and one cartload of blue berries to the local supermarket.
- Side constraints: $x >= 0.5$

  $y >= 1$

Problem formulation

- Total sales: $2750x + 1750y$
- Total labor cost: $45*(1.5x + y)$
- Objective function: $f(x,y)$: $2750x + 1750y - (45*(1.5x + y))$
  $= 2682.5x + 1705y$

- Labor constraint: $g_1(x,y)$: $1.5x + y <= 46$
- Storage constraint: $g_2(x,y)$: $x + y <= 30$
- Pallet constraint: $g_3(x,y)$: $4x + 6y <= 251$
- Side constraints: $x >= 0.5$
  $y >= 1$

47

Example Problems

Tuesday, April 2, 2019 1:15 PM

Example 1 A one-line function or equation

$$y = 8x - 2x^2 + 4 = -2(x-2)^2 + 12 \tag{1.3}$$

The objective is to find the value of x that maximizes the value of y. This can be graphically illustrated in Figure 1.1.

y = OF (often called dependent response)
x = DV (often called independent input)

Using the analytical procedure of setting the derivative of the function to zero and then solving for the DV* value, we easily obtain

$$x_{\text{optimum}} = 2 = x^* \tag{1.4}$$

$$y(x_{\text{optimum}}) = 12 = y^* \tag{1.5}$$

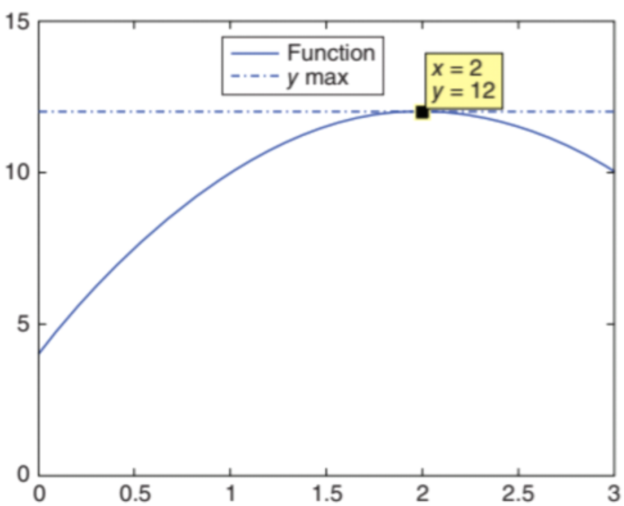


Figure 1.1 Illustrating a function with a maximum.

Example 2 Economic Optimization

Find the optimal thickness of insulation in an attic. The initial capital cost is the cost of insulation as price per volume times area times thickness, $C = pAt$, and thickness must be nonnegative, so there is a constraint $t \geq 0$. There may be alternate constraints related to total weight of insulation that the ceiling can support, $\rho At < W$. The annual value of energy lost, an expense, might be simply modeled as $E = eA(T_{\text{house}} - T_{\text{outside}})/(r + kt)$, where k is the specific factor for the insulation, r is the insulation capacity of the walls, and e is the unit cost of energy. Ignoring the time value of money and looking at an N-year horizon, the total cost is the initial capital and the N annual expenses. $J = C + NE = pAt + NeA(T_{\text{house}} - T_{\text{outside}})/(r + kt)$. The optimization statement then becomes

$$\begin{aligned} &\min_{\{t\}} J = pAt + \frac{NeA(T_{\text{house}} - T_{\text{outside}})}{r + kt} \\ &\text{S.T.}: \quad \begin{matrix} t \geq 0 \\ \rho At < W \end{matrix} \end{aligned} \tag{1.6}$$

Analytically, this admits a solution. Set $dJ/dt = 0$, and solve for insulation thickness.

$$t^* = \frac{1}{k}\left[\sqrt{\frac{p}{kNe(T_{\text{house}} - T_{\text{outside}})}} - r\right] \tag{1.7}$$

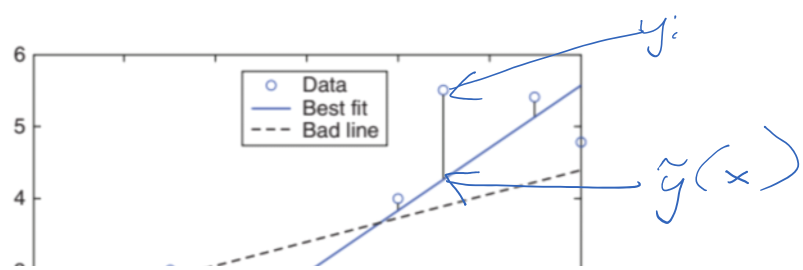
However: are the other parameters truly given?
Can we use a different material?
Finally, what about setting a different temperature?

The equations reveal that the value of energy lost is related to the house temperature setting. Is an option for minimizing costs to change the house temperature setting? If yes, then perhaps there needs to be a comfort penalty for deviations from the nominally ideal 72°F (22°C). Characteristically, the penalty scales with the square of the deviation from desired, but there needs to be a weighting factor that makes the temperature deviation equivalent to the cost. With such, the optimization statement has evolved to

$$\begin{aligned} &\min_{\{t,\, T_{\text{house}}\}} J = pAt + \frac{NeA(T_{\text{house}} - T_{\text{outside}})}{r + kt} + \lambda(T_{\text{house}} - 72)^2 \\ &\text{S.T.}: \quad \begin{matrix} t \geq 0 \\ \rho At < W \end{matrix} \end{aligned} \tag{1.8}$$

up to me.

Example 3: Least squares regression



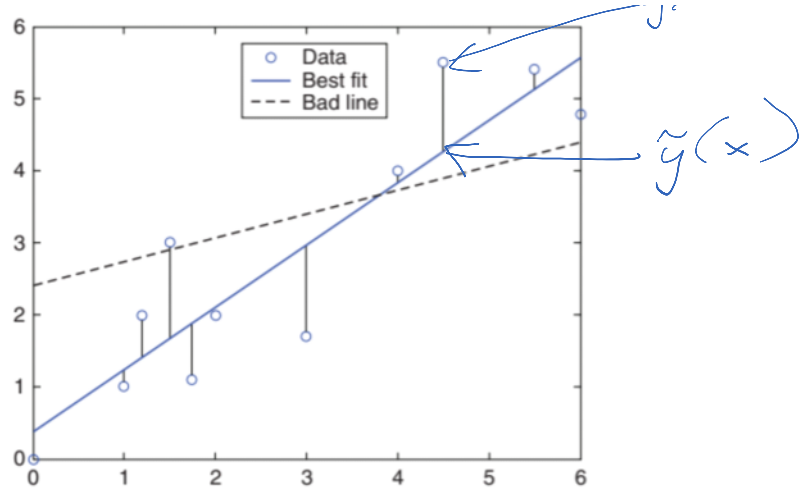


Figure 1.2 Regression illustration.

In classic regression, we evaluate best as minimizing sum of squared deviation:

$$\text{OF} = \text{SSD} = \sum_{i=1}^{N} (y_i - \tilde{y}(x_i))^2 \tag{1.9}$$

where y_i and x_i are data pairs, N is the number of data points, and $\tilde{y}$ is a model value.

The equation of a line is $\tilde{y} = a + bx$, which is linear in both of the coefficients. The objective is to find the values for a and b that minimize the OF. Here, the decision variables are the model coefficients $\{a, b\}$, and the optimization application can be stated as

$$\begin{aligned} &\min_{\{a,b\}} J = \sum_{i=1}^{N} (y_i - \tilde{y}(x_i))^2 \\ &\text{S.T.:} \quad \tilde{y}_i = \tilde{y}(x_i) = a + bx_i \end{aligned} \tag{1.10}$$

Substituting the model into the OF, $J = \sum_{i=1}^{N} (y_i - (a + bx_i))^2$, taking the derivatives, and setting them to zero,

$$\left.\begin{aligned} \frac{\partial J}{\partial a} &= 0 = -2\sum_{i=1}^{N} (y_i - a - bx_i) && (1.11a) \\ \frac{\partial J}{\partial b} &= 0 = -2\sum_{i=1}^{N} x_i(y_i - a - bx_i) && (1.11b) \end{aligned}\right\} \text{linear eq. in } a \text{ and } b$$

Rearranging, the normal equations are

$$(N)a + \left(\sum x_i\right)b = \left(\sum y_i\right) \tag{1.12a}$$

$$\left(\sum x_i\right)a + \left(\sum x_i^2\right)b = \left(\sum x_i y_i\right) \tag{1.12b}$$

Since each term in parentheses has known values from the data, the normal equations result in two linear equations in two unknowns, variables a and b.

Example 4 Best Path or Sequence

Find the best path to visit major baseball parks in the Northeastern United States. Should this be least miles, least time, lowest cost, or most scenic? Start at home, and return to home. One possible sequence is to go to Chicago (C), then Boston (Bo), then New York (N), and so on.

Give a number to each city in sequence. Assign home as points 1 and 8, start and finish. For example, here is one plan, one sequence, one path:

H	C	Bo	N	Pit	Phi	Bal	H
1	2	3	4	5	6	7	8

But this sequence might have fewer total miles:

H	C	Bo	N	Pit	Phi	Bal	H
1	7	6	5	2	3	4	8

If minimizing distance for the total trip is the objective, using l_{ij} = distance between city i and j, the optimization application can be stated as

$$\min_{\text{sequence}\{2,3,\ldots,7\}} J = \sum_{i=1}^{N} l_{ij} \tag{1.13}$$

This is a classic example of the traveling salesman problem (TSP), which is a key element in scheduling planning, sequencing, and logistics. The example reveals several issues. One is that the decision variables are nominal (or category variables), indicated here as integers for the sequence. Accordingly, there is no concept for a derivative, revealing that many applications are not amenable to the classic analytical approach to determining DV*.

The other issue is an additional concept, the constraint. Here the sequence is constrained so that (i) each city number is used, (ii) each is used only once in the sequence, and (iii) the initial and final cities are constrained to H.

Issues that Shape Optimization Procedure

Tuesday, April 2, 2019 4:41 PM

Examples:

- Nonlinearities—The objective function may not have an analytically tractable derivative. If it does, the derivative may require iterative nonlinear root-finding procedures to solve an implicit nonlinear relation.
- Discontinuities—The OF or its derivatives may have discontinuities.
- Discretization—The decision variable might represent integer values (number of queuing lines, number of parallel devices, number of samplings for a delay) or discretized sizes (pipes, resistors, and shirts come in discrete sizes). Further, the numerical time or space discretization in the model used for the OF will generate steps or ridges (striations) on what might be considered as a smooth OF. Convergence criterion on root-finding techniques used within the model can also cause striations. For these, analytical optimization techniques, which were developed for continuum-valued OFs and DVs, will be confounded.
- Multiple optima—Many objective functions have local minima that would trap the optimization procedure within a local, not the global optimum.
- Flat spots—Some OFs have zero, or effectively zero, response to the DV in regions of saturation or inconsequence, and if the derivative is zero, there is no guidance as to how to improve the DV solution.
- Stochastic response—When OF data is being generated experimentally, there is noise (uncertainty, experimental error) on the OF value. A replicate experiment (an attempt to implement the same DV values) will not produce exactly the same OF value. Here, because of experimental vagaries, moving the TS toward the true DV^* value might return a worse, not better OF value, which would indicate that the optimum is in the false direction. When Monte Carlo simulations are used as a surrogate for experiments, the impact is the same.
- Uncertainty—There is uncertainty in the givens. Air pressure, temperature, humidity, and wind velocity continually change. If you design an airplane wing for one set of conditions, what is the outcome at other realizations of the givens? Should the design be to minimize the worst possible outcome over all possible realizations?
- Constraints—Types of constraints were described previously. When constraints are hard (cannot be violated), the optimization procedure needs to be modified to choose an alternate path. If you encounter a high wall blocking your downhill walk, but want to get to the other side, perhaps walk along the wall, not directly downhill.

Over- and Under-specification

Tuesday, April 2, 2019 6:01 PM

DOF - Degrees of Freedom (linear systems)

Over-specified means that there are more independent equations than independent variables, more constraints than variables—in this case the degrees of freedom <0. A straight line can go through any two points, but if you add a third point that is not on the line, the single line cannot simultaneously go through all three points.

Under-specified means that there are fewer equations (constraints) than variables, degrees of freedom >0. For example, if there is only one point, then you could choose the slope of the line and still find a line to go through the point. You are free to choose a variable value. You have a degree of freedom.

Balanced means that there are the same number of equations as variables: DoF = 0.

degrees of freedom = number of DVs − number of specifications (active constraints)

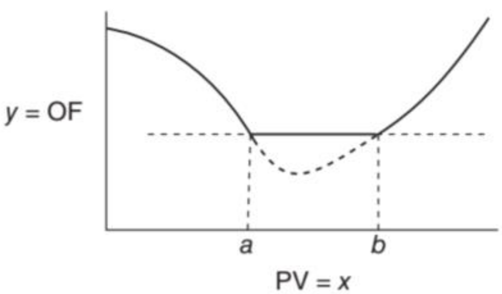


Figure 1.12 Illustrating an under-specified application.

There are parallels in optimization. Figure 1.12 illustrates an under-specified application, for example:

$$\min_{\{x\}} J = y \begin{cases} = c & \text{if } a \le x \le b \\ = 1 - e^{-(x-d)^2} & \text{otherwise} \end{cases} \qquad (1.35)$$

Any x in the $[a, b]$ range is equivalent to any other x-value. There is flexibility in the solution. There are extra choices you can make. This may also arise if there are distinct but identical solutions. For instance,

$$\min_{\{x\}} J = \left(x^2 - 9\right)^2 \qquad (1.36)$$

Here there are two identical solutions: $x = (3, -3)$. If you can choose one, which is preferred? In this case there are probably reasons other than that expressed in the OF as to why one x-value is better than another. Consider resource conservation, future flexibility, reliability, political capital, etc., and include this additional concept into the OF.

Iterative Procedures

Tuesday, April 2, 2019 7:07 PM

There are many that will be revealed in subsequent chapters, but here, I'll introduce a heuristic direct search. The term direct search contrasts procedures that use gradient (derivative) information. A direct search only uses the function evaluation. The term heuristic means that it uses intuitive or tried-and-true human experience, as opposed to a mathematically formulated rule. To understand the heuristic direct search, imagine how a blindfolded tightrope walker would find the bottom of the rope span if he were placed randomly along the rope. The rule is take a step in one direction. If you go downhill, the step was the right direction; so, from this best-so-far spot, make the next step in the same direction. If you go uphill, the step is probably the wrong direction; so, return to the best-so-far spot, and make the next step in the other direction. If the reverse step had the same size, then you return to a prior spot, so when reversing direction, also cut the step size in half. By contrast, if going in the right direction, make the next step a bit larger. Near the minimum the alternating steps are progressively cut in half. Stop, and claim success when the step distance is small enough.

Here is an algorithm outline for the heuristic direct search

1) Initialize
   - Choose the initial feasible TS, the base case DV
   - Evaluate the OF-value, OF_{base} at the DV_{base}
   - Choose a step size, DV_{delta}
   - Choose a convergence threshold for DV_{delta}
2) Test a new TS
   - Set the new $TS = DV_{base} + DV_{delta}$
   - Evaluate the OF-value, OF_{TS} at the TS
   - IF OF_{TS} is better than OF_{base} THEN
     - Set $OF_{base} = OF_{TS}$
     - Set $DV_{base} = TS$
     - Set $DV_{delta} = 1.2*DV_{delta}$
   - ELSE (meaning that the OF is worse or a constraint is hit)
     - Set $DV_{delta} = -0.5*DV_{delta}$
   - ENDIF
   - IF $|DV_{delta}| <$ convergence threshold EXIT

   Return to a new TS (Go to Point 2)