Week 1 Assignment

Wednesday, April 3, 2019 8:34 PM

Solve the following problems from Chapter 1:

5 Use the analytical technique, $dJ/d\mathrm{DV}|_{\mathrm{DV}^*} = 0$, to determine the value of x that maximizes $y = xe^{-x}$.

7 Choose three (x, y) data pairs, and generate a linear model that best fits the data in a least squares sense. For each of the $i = 1,2,3$ pairs, the equation $y_i = a + bx_i$ is true. Use the analytical technique $dJ/d\mathrm{DV}|_{\mathrm{DV}^*} = 0$ to solve for the $\{a, b\}$ coefficient values. Here, the OF is the sum of squared deviations between model and data. Because there are two decision variables, there are two $dJ/d\mathrm{DV}|_{\mathrm{DV}^*} = 0$ equations, termed the normal equations. They will be linear in coefficients $\{a, b\}$. Solve them simultaneously. Show each stage of your process. To verify that you have gotten the right answer, compare your answer to any automated method of curve fitting (such as the trend line function in Excel charts).

8 A person goes to bed at midnight and has to be on the job at 8 : 00 am the next day. Before going to bed she makes a decision about the time, T (hour), for the alarm clock to ring. It takes 30 min to travel to work. The longer she sleeps, the more sleep benefit she gets. Benefit $= 1 - e^{-T/3}$. However, the longer she sleeps the greater is the anxiety about, and potential cost associated with, being late to work. Badness $= 1/(7.5 - T)$. Graph these to show a qualitative relation of benefit versus hours of sleep and the cost of wake-up time. Additively combining these multiple and competing objectives as a single OF, $J = \left(1 - e^{-T/3}\right) - 1/(8 - T)$. Use the analytical technique, $dJ/d\mathrm{DV}|_{\mathrm{DV}^*} = 0$, to solve for the best wake-up time.

10 A right circular open tank of height, h, and radius, r, has volume $\pi r^2 h$ and surface area $\pi r^2 + 2\pi rh$. The cost of a tank is related to the amount of material in the surface of thickness, t, density, ρ, and cost per unit mass, c. Cost $= ct\rho(\pi r^2 + 2\pi rh)$. What dimensions, r and h, minimize cost while meeting a specified volume, V? State the optimization in standard form. Explain simplifications that you are using. Use the analytical technique, $dJ/d\mathrm{DV}|_{\mathrm{DV}^*} = 0$, to solve the simplified application for $\{r^*, h^*\}$.

And finally, try to set up that one:

27 Return benefit from effort has a diminishing returns response. There is also a threshold effort before there is any return. Consider building a piece of furniture. Effort may start with buying the wood, stain and varnish, and sharpening and aligning the cutting tools. This effort is necessary, but so far there is nothing to sell; there is no benefit to the carpenter. After assembly, light sanding and one coat of finish may make a salable item; but more sanding and another coat of finish makes it an excellent item, increases perceived value to the buyer, and willingness to pay a higher price. Additional carpenter effort beyond excellence moves the item toward perfection, but doesn't increase value over something already excellent. Often an S-shaped relation is used to relate value to effort. The logistic relation is one, $V = a/\left[1 + e^{-b(E-c)}\right]$, where "$E$" represents the time or effort invested and "V" the value. You probably have 20 things that you are working on and only 100% effort to give. If you work totally, you don't eat or sleep, which are necessary functions to be able to invest energy in any project. As you personally seek to optimize life outcome, what are DVs, OF, and constraints? For simplicity and clarity, only consider three projects.