Homework 4

Problem 1 (63 pts)

The compressor fan in a turbojet engine is used to compress the incoming air (i.e., increase its pressure and temperature) before combustion. Scientists recognized that naturally-occurring shock waves could be used for similar purposes. For example, this is a major component of how ramjets/scramjets work. Ramjets/scramjets compress incoming supersonic air via a shock (or series of shocks) before entering fuel combustion chamber. We will use our knowledge of **normal-shock-wave** behavior to compare how effective normal shocks are at compressing air compared to traditional mechanical sources (e.g., compressor fans) modeled as isentropic compression. Note, the same theory is utilized for explosive/detonation engineering, which is pretty cool!

(a) Our normal-shock relations derived in class were functions of Mach number; however, when viewing shock waves as compressors of air, it is more useful to obtain relations in terms of strictly thermodynamic variables. This process results in the "Hugoniot Equation", which represents the effectiveness of normal-shock-wave compression in terms of pressure and density (or mass-specific volume depending on your preference). Parts of this derivation are outlined in section 3.7 from Modern Compressible Flow (Anderson, 3rd edition). Starting with the steady one-dimensional continuity equation, following the steps laid out in section 3.7, derive the following equation (i.e., the Hugoniot equation) for the effective pressure ratio (i.e., compression) across a given normal-shock wave in terms of the density ratio (i.e., decrease in specific volume):

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_2}{\rho_1}} \quad .$$

Make sure you list all steps and assumptions throughout the derivation. Hint: it might be helpful to utilize a control volume such as that drawn in HW 2, Problem 3, understanding that properties at points 1 and 2 may now be different because of the normal shock separating them.

(b) Assume that a turbojet compressor fan isentropically compresses air for a range $1 < \frac{\rho_2}{\rho_1} < 5$. Plot the compression (i.e., $\frac{p_2}{p_1}$) for this range of $\frac{\rho_2}{\rho_1}$ for both traditional isentropic compression (e.g., a turbojet compression fan) and normal-shock-wave compression. These curves should be on the same plot with a clearly labeled legend.

(c) If you were tasked with deciding which mechanism to use to compress air, which would you choose? For relatively large $\frac{\rho_2}{\rho_1}$, what other real-world considerations are there for choosing between isentropic versus normal-shock-wave compression? Think of "efficiency".

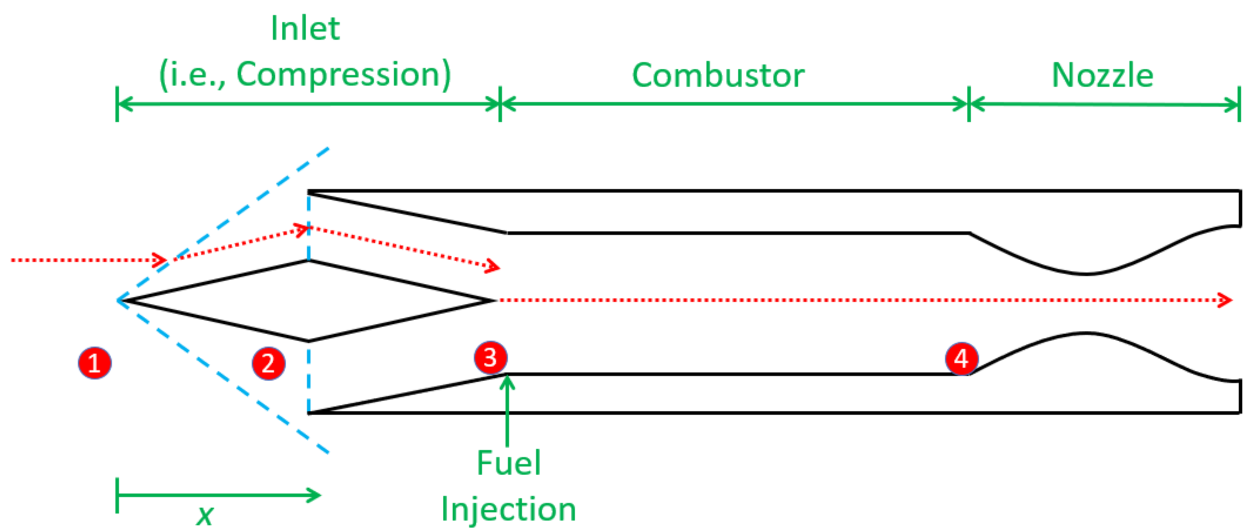
(d) Calculate the change in entropy ($s_2 - s_1$) from the given range of $\frac{\rho_2}{\rho_1}$ for both the isentropic and normal-shock compression. Plot $s_2 - s_1$ versus $\frac{\rho_2}{\rho_1}$, and comment on what you find. Does this affect your thoughts on part (c)? Research, and then briefly explain, the physical explanation for the $s_2 - s_1$ behavior across the normal shock.

(e) Using your figure from part (b), for what range of $\frac{\rho_2}{\rho_1}$, are the two processes comparable? Explain why this makes sense. For what Mach-number (i.e., M_1) range would this correspond to regarding the normal-shock-wave case? Does this help you explain?

Problem 2 (46 pts)

Ramjet

Now that we have gained some understanding of the compressibility effects of shock waves, we will focus in on one particular application — Ramjets. Ramjets are an incredible utilization of the basic compressible-flow theories we have been discussing in class. This problem will design a (slightly) simplified ramjet to optimize its efficiency. Consider the two-dimensional ramjet schematic below. We will indeed solve the problem in a two-dimensional sense (i.e., the leading-edge "spike" will be treated as a two-dimensional wedge, not a cone). The blue dashed lines represent the shock waves we will account for in the problem. The red dashed lines represent a streamline of the flow. Supersonic horizontal flow over the spike creates an oblique shock. Downstream of the oblique shock, the flow encounters a normal shock. This is labeled as the inlet (i.e., compression) region. Fuel is injected in the combustor (i.e., burner). The fluid then flows through a converging-diverging nozzle before exiting to the atmosphere.



We will use the SR-71 as our motivation for obvious reasons (i.e., it's awesome). Therefore, we will use a cruise Mach number of 3.0. The freestream temperature and pressure at the altitude of the aircraft are 217 K and 20 kPa, respectively. We will use: $\gamma = 1.4$, $R = 287$ J/kg/K, $c_p = 1000$ J/kg/K. We will assume a constant $q = 500$ kJ/kg in the combustor.

The following assumptions will be made in order to solve this problem:

i Steady

ii Inviscid

iii CPG

iv Uniform velocity, pressure, temperature, density, enthalpy, and energy at each x location

v The oblique shock from the spike is an attached weak shock from a two-dimensional wedge

vi We will neglect the angularity of the streamline through the inlet. For example, we will assume that the fluid is travelling horizontally between the oblique and normal shocks and between the normal shock and fuel injection.

vii Our general isentropic, oblique-shock, and normal-shock equations apply (where appropriate)

viii We will assume that the static Mach number at the inlet of the combustor is the same value throughout the entire combustor.

ix You may use an isentropic relation to calculate the static temperature at the exit of the combustor.

x We will utilize one-dimensional flow with heat transfer to obtain the difference in stagnation temperature between the inlet and exit of the combustor from the given constant value of q.

xi We will neglect the fuel mixture in the air in the combustor.

Homework 4

(a) Spend a small amount of time researching how a ramjet works and briefly describe it in a paragraph or two here.

(b) A (slightly) simplified ramjet cycle efficiency is given by:

$$\eta = 1 - \left(\left(\frac{p_{1,\infty}}{p_{3,\infty}} \right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_{4,\infty} - \left(\frac{p_{3,0}}{p_{1,0}} \right)^{\frac{\gamma-1}{\gamma}} \cdot T_{3,\infty} \right)}{(T_{4,\infty} - T_{3,\infty})} \right) \quad .$$

Use this equation to come up with the optimal spike half angle (to the nearest degree) for the given cruise conditions. You must write out your general methodology for the grader. Include a plot of the efficiency versus spike half angle.

(c) What is the efficiency of the ramjet if we get rid of the spike altogether?

(d) Write up a description of your observations of the ramjet with and without the spike. Be sure to include relevant compressible-flow theories/jargon. Be sure to include a conversation of the effect of inlet freestream pressure ratio, inlet total pressure ratio, and combustor freestream temperature difference.

(e) For the optimal spike half angle solved for in part (b), let's now explore the effect of cruise Mach number on efficiency. Plot efficiency versus cruise Mach number.

(f) Write up a description of your observations of the Mach-number effect on ramjets. Be sure to include the phenomena that limit the ramjet's performance at higher Mach numbers. Be sure to include relevant compressible-flow theories/jargon. Be sure to include a conversation of the effect of inlet freestream pressure ratio, inlet total pressure ratio, and combustor freestream temperature difference.

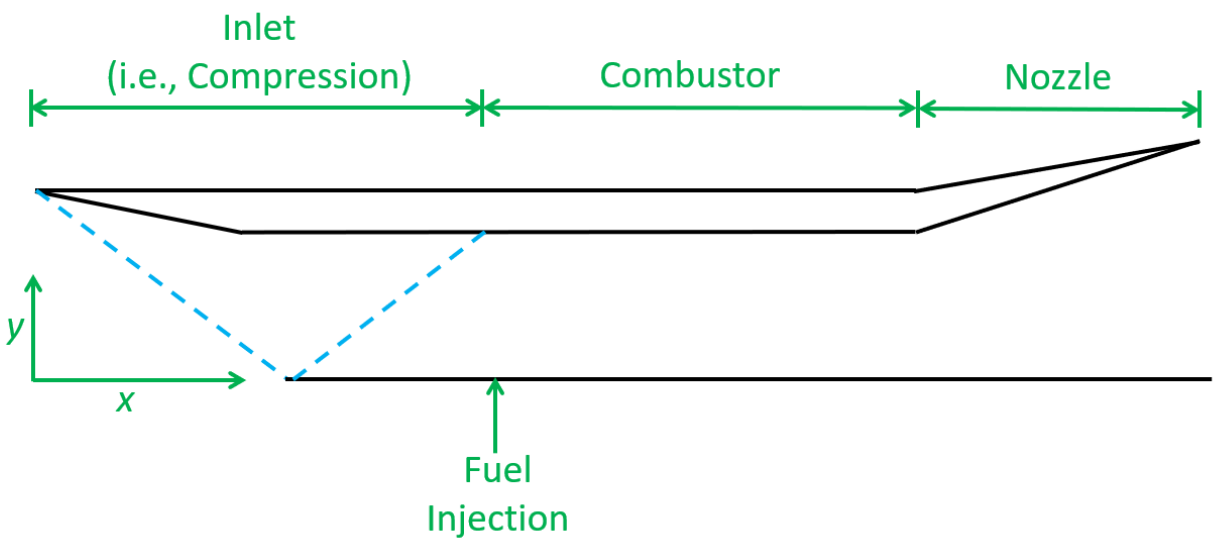
(g) What do you expect to happen if we instead treat the spike as an axisymmetric cone? Briefly justify your answer with words.

Problem 3 (30 pts)

Scramjet

(a) After what you've learned from problems 1 and 2, do some more research and briefly describe why scramjets (theoretically) solve some of the specific issues that ramjets encounter at higher Mach numbers.

Consider the (very) simplified two-dimensional scramjet schematic below. We will indeed solve the problem in a two-dimensional sense. The blue dashed lines represent the shock waves we will account for in the problem. The leading edge creates an angle of $-7°$ with-respect-to the horizontal plane.



We will use the X-51 Waverider as our motivation for obvious reasons (i.e., it's awesome). Therefore, we will use a cruise Mach number of 5.0. Other than Mach number, we will use the same freestream conditions as the ramjet example. We will not deal with the combustor in this problem.

(b) How do the inlet freestream pressure ratio, inlet total pressure ratio, and combustor inlet freestream temperature compare for this scramjet design compared to the ramjet (with spike) design? You must actually calculate the numbers. Does this support what you said in part (a)?

(c) Under these conditions, what is the Mach number of the flow at the inlet of the combustor? What combustion challenges do we face in efficiently burning fuel at this Mach number?

(d) Let's now investigate the qualitative effect of viscosity on shock reflections. We will focus our attention to the last reflected shock before the fuel-injection site. An effect of viscosity is to decelerate the flow in the vicinity of the wall such at $u_{\text{wall}} = 0$. We will assume that the scramjet is a height of H from the bottom wall to the top wall. We will also assume that u is equal to its local freestream value a distance δ away from the walls. Sketch y versus β for $0 < y < H$. In addition, sketch the more realistic shape of the shock.