Homework 7

Problem 1 (90 pts)

In this problem, we will model an isentropic, CPG in the AFRL Mach-6 Ludwieg Tube as a long constant-area pipe to investigate unsteady shock waves within it. According to the AIAA paper on the facility, the inner diameter of the driver tube is 9.75 inches. The driver tube is 82 feet in length. A valve separates the driver tube from the converging-diverging nozzle. For this problem, we will assume the nozzle is also 9.75 inches in diameter and is 117 inches in length. Assume that the driver tube is sitting at $p_0 = 4000$ kPa and $T_0 = 500$ K before the valve opens. You may use the perfectly expanded back pressure from problem 2 in HW 6 for the back pressure. You may assume that the back air is at room temperature (295 K).

Normal Shock

The moment the valve opens, a normal shock wave propagates downstream through the "nozzle".

(a) Calculate the p_∞ and p_0 at the exit of the nozzle before the NS reaches it.

(b) Calculate the p_∞ and p_0 at the exit of the nozzle after the shock passes and before the contact surface arrives.

(c) If you had a blunt wind-tunnel model at the exit plane of the nozzle, what p_0 would it feel at the blunted tip surface in the state-2 flow? Briefly justify your answer.

(d) How long will it take for the NS to reach the probe from the time the valve opens? Generally speaking, what does this mean for your test-section data if your instrumentation within it is triggered to start collecting data right when the valve opens?

(e) How much time elapses between the passage of the NS and of the contact surface?

Expansion Wave

The moment the valve opens, a left-running centered expansion wave propagates upstream through the driver tube.

(f) In actuality, given the values in the Normal-Shock part of the problem, the expansion wave would propagate downstream initially. So, for this expansion-wave part of the problem re-do the necessary calculations from above for $p_0 = 1000$ kPa and $T_0 = 500$ K and the back air at $p = 101$ kPa and $T_0 = 295$ K.

(g) Use the method of characteristics to plot p/p_4 and T/T_4 as a function of time at the end wall of the driver tube. Do this for a few different number of characteristics. How many characteristics seems like enough?

(h) Plot the characteristics from the valve to the end wall and back.

(i) How long does it take for the head to get back to the location of the valve? If you approximated this time assuming a constant head speed equal to the local speed of sound (as done in the paper), would you expect it to be different than how you calculated if? If so would it be larger or smaller?

Problem 2 (80 pts)

As discussed in class, conical flows are extremely relevant in supersonic and hypersonic flow. In this problem, you will solve the exact, non-linear solution for an axisymmetric cone with a sharp nosetip, which is what engineers would have used when designing the axisymmetric spike for ramjets like on the SR-71. You will follow the analyses outlined in Chapter 10 in Anderson. Recall, we briefly discussed the differential form of the governing equations in HW 2. I would advise reviewing that in addition to Chapter 6 in Anderson for this problem.

(a - 22 pts.) You may either write a new code yourself, use the source code I've provided, or find a different source code from the internet so long as you are able to follow the code's methodology. Either way, you must separately write out a detailed methodology for the code indicating specific equations or equation numbers from Anderson's textbook (this will look something like a short technical report). If you write your own code, you may go about solving this in any fashion you see fit and utilizing any built-in ODE solver you like. You may assume $\gamma = 1.4$ throughout this problem.

As a warm-up to using your code, find the following:

(b - 10 pts.) Plot shock angle θ_s vs. cone half angle θ_c for $M_\infty = 1.25$, 2.0, 6.0, and 10.0. Hint: your code might only be able to handle guessed shock angles slightly less than the range of $0 - 90°$. Does the top or bottom of each curve align with the results from the Virginia-Tech Calculator? Why do you think that is?

(c - 10 pts.) Repeat part (b) but include results for a wedge with the same shock angles. Explain why you observe these differences between cones and wedges.

(d - 10 pts.) Plot the Mach number at the cone surface, M_c, vs. cone half angle θ_c for $M_\infty =$ 1.25, 2.0, 6.0, and 10.0. Hint: we are not talking about the stagnation point, where $M = 0$. Instead, we are talking about the Mach number along the ray of the cone. Does the top or bottom of each curve align with the results from the Virginia-Tech Calculator? Why is this?

Now that your warmed up, skim through the first 11 pages of the attached *Journal of Spacecraft and Rockets* article on a hypersonic flight vehicle similar to HiFIRE-1. As you can see, the researchers utilized the Taylor-Maccoll solutions to confirm the general pressure measurements at the cone wall.

(e - 20 pts.) Assuming an angle of attack of 0° and $\gamma = 1.4$, use the given flight data and your code to recreate the Taylor-Maccoll plot from left plot in Fig. 22. Note: you only need to recreate the green profile, not the others from this figure.

(f - 8 pts.) Continue reading and write a paragraph or two describing how you could implement your Taylor-Maccoll solution/code to estimate the angle of attack of the flight vehicle. This should be detailed such that somebody could carry it out.