

Problem 1 (24 pts)

In an inviscid flow, a small change in pressure dp , is related to a small change in velocity, du , by

$$dp = -\rho u du \quad ,$$

which is referred to as Euler's equation and is derived from the conservation of momentum.

(a) Using this relation, derive a differential relation for the fractional density change $d\rho/\rho$ as a function of the fractional change in velocity du/u , with the fluid's compressibility τ as a coefficient.

(b) Recall from lecture that the general definition for τ is,

$$\tau = \frac{1}{\rho} \frac{d\rho}{dp} \quad .$$

Using our results for isentropic flows, show that τ for isentropic flows simplifies to,

$$\tau = \frac{1}{\gamma p} \quad .$$

(c) The velocity at a point in an **isentropic** flow of air is $u = 63$ m/s travelling by a Cessna 172 prop aircraft. The density and pressure are 1.23 kg/m³ and 1.01×10^5 Pa, respectively. If the fractional velocity change is 0.01 , what is the fractional density change? Do not look up a value for τ_{air} . Instead, use your result from part (b). $\gamma = 1.4$ for air.

(d) Repeat part (c) except for a local velocity of $u = 980$ m/s travelling by an SR-71 Blackbird. Allow the fractional velocity change to still be 0.01 . You may still assume isentropic flow.

(e) Comment on the order-of-magnitude differences in the fractional density change between parts (c) and (d). What causes this large difference? Are both compressible flows?

(f) Use this finding to explain to a classmate why high-speed (i.e., supersonic, hypersonic) flows are inherently compressible.

Problem 2 (10 pts)

(a) Derive the following equations for c_p and c_v (recall, $\gamma = c_p/c_v$):

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$c_v = \frac{R}{\gamma - 1}$$

(b) For what assumptions are these equations valid?

Problem 3 (49 pts)

This problem will introduce you to the Air Force Research Laboratory's (AFRL) Mach-6 Ludwig Tube (i.e., wind tunnel) located just down the road from us at the Wright Patterson Air Force Base. We will use this facility as a basis for some of our homework problems throughout the semester.

Skim through the first ten pages of the attached article on the AFRL Mach-6 Ludwig Tube. You do not have to digest it all, but I want you to focus on the basic working principles of the facility. I also want you to focus on the measured pressure of the air as it moves through different parts of the facility. Does Eq. 6 look familiar??!

(a) As you now know from reading, the researchers utilize the isentropic relations to compare pressure, density, and temperature at various locations in the tunnel. They do this by experimentally measuring the air pressure at discrete points on the walls of the tunnel/nozzle. Make your case for why this is valid. In addition, make the case of a reviewer of this paper who might try to argue why this is not valid.

(b) Whether it is valid or not, we will move forward with this problem utilizing the isentropic relations. Experimentally-measured air pressure at the point "Noz Exit" are provided in the attached Excel document along with the related test time t . The "Noz Exit" air properties are those experienced by a model placed in the tunnel's test section (see Fig. 4 in the article). As discussed in the paper, researchers using this facility have some control over the upstream starting air pressure and temperature. Assume that these are set to $p = 1380$ kPa and $T = 500$ K and can be assumed constant throughout the duration of the test because the air there is more-or-less stagnate and does not move. Write a computational script to plot the pressure, density, and temperature of the flow in the test section versus the test time t . Note, $t = 0$ ms is when air starts flowing through the wind tunnel. Include your figures and text code in your single submitted pdf. Label all axes and include units/legends where appropriate. Include your code in your submission.

(c) Generally useful wind-tunnel data is collected in the test section between $25 < t < 75$ ms. How "steady" is the pressure during that period (i.e., what is the standard deviation about the mean value during that time)?

(d) At $t = 50$ ms, what is the entropy difference between the upstream and test-section air? Recall,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad .$$

You may use $c_p = 1000$ J/kg K and $R = 287$ J/kg K. Does this make sense? Why or why not?

(e) What do the isentropic relations show happens to the temperature of the flow as it expands to Mach-6? Look up the liquefaction temperature of air. What is it in Kelvin? With this knowledge, why do you think the air accelerated through high-speed wind tunnels is heated before every test?

With an upstream starting pressure of $p = 1380$ kPa, calculate the upstream starting temperature for which the air temperature in the test section is equal to the liquefaction temperature at $t = 50$ ms.

(f) A wind tunnel is used to simulate a flight vehicle flying through stagnate air. The difference is clearly the reference frame. If a hypersonic boost-glide vehicle is flying at Mach-6 at an altitude of 40 km, where the air temperature is ≈ 250 K, do you think the air liquefies as it travels over the vehicle? Explain your reasoning.