Problem 2

Note: All calculations performed in Python using custom module designed to analyze isentropic flow and normal shocks. See appendix **??**.

(a) Plot the Mach number of the F-16 during its 20-minute flight.

Givens:

Time series for p_t and p, the total and static pressure, respectively, measured by the F-16's pitot-probe. The flight data given covers a simple trajectory including a take-off, acceleration to max-speed, and deceleration to landing.

Assumptions:

Let region 1 be the area upstream of the bow shock. Let region 2 be the area *immediately behind the shock* but not at the probe's stagnation point. The static pressure measurement taken by the probe is valid for the static pressure *upstream* of the shock. The flow before and after the bow shock is isentropic, calorically perfect air. $\gamma_{air} = 1.4, R_{air} = 287\,\mathrm{J/kg \cdot K}$. The shock can be evaluated as a normal shock in front of the pitot probe. A shock is only present in front of the pitot probe when the F-16 is traveling supersonically, $M > 1$. All flow is isentropically brought to rest at the tip of the pitot-probe.

Solution:

During the subsonic portion of flight, the F-16's Mach number is found using isentropic equations relating p_t, p, and M.

$$\frac{p_{t,1}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

Rearranging to isolate M:

$$M_1 = \sqrt{\left[\left(\frac{p_{t,1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \frac{2}{\gamma - 1}}$$

During the subsonic portion of flight, we have data for the quantities $p_{t,1}$ and p_1, and can easily solve for M_1. However, once the F-16 reaches supersonic velocities, this relation is no longer valid for our measurements. The total pressure measurement becomes the total pressure experienced by the probe *behind the shock*, $p_{t,2}$. We must now find a new relationship using $p_{t,2}$, p_1, and M_2. The ratio of total pressure behind a shock to static pressure in front of a shock can be expressed via the multiplication of other ratios we have isentropic/normal shock relations for.

$$\frac{p_{t,2}}{p_1} = \frac{p_{t,2}}{p_2} \frac{p_2}{p_1}$$

The isentropic relationship for total and static pressure has already been shown for the subsonic Mach caculations.

$$\frac{p_{t,2}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

The relationship connecting static pressure across a normal shock is given below:

$$\frac{p_2}{p_1} = \left(\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}\right)$$

Combining these two equations yields an expression for $p_{t,2}/p_1$ in terms of γ, M_1, and M_2.

$$\frac{p_{t,2}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}\right)$$

We do not know M_1 or M_2, but we do have a normal shock relationship relating the two of them:

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1}$$

Substituting this relation into our previous defined ratio:

$$\frac{p_{t,2}}{p_1} = \left(1 + \frac{\gamma - 1}{2} \left[\frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1}\right]\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}\right)$$

This equation cannot be solved by hand, so a numerical solver will be used in Python to determine the Mach number based on $p_{t,2}/p_1$ while the F-16 is supersonic. The final question that must be examined before calculating the F-16's Mach number during its flight is how to decide whether the F-16 is subsonic or supersonic without knowing the Mach number. The answer comes by examining the limiting case of exactly sonic velocity, i.e. $M = 1$. The isentropic total-to-static pressure ratio for $M = 1$ is given below, where p^* represents static pressure at sonic conditions.

$$\frac{p_t}{p^*} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

Inserting our known value of γ yields the critical pressure ratio for sonic flight:

$$\frac{p_t}{p^*} \approx 1.89$$

From this value we can determine when the F-16 reaches sonic velocities. By definition, a normal shock at sonic conditions is an infinitely weak shock and is isentropic. Examining the relationship for $p_{t,2}/p^*$ at sonic conditions:

$$\frac{p_{t,2}}{p_1} = \left(1 + \frac{\gamma-1}{2}\left[\frac{1+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}-1}\right]\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\right)$$

$$\frac{p_{t,2}}{p_1} = \left(1 + \frac{\gamma-1}{2}\left[\cancelto{1}{\frac{\gamma+1}{\gamma+1}}\right]\right)^{\frac{\gamma}{\gamma-1}} \left(\cancelto{1}{\frac{\gamma+1}{\gamma+1}}\right)$$

$$\frac{p_{t,2}}{p_1} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{t,2}}{p_1} \approx 1.89$$

We have now proven that both the subsonic and supersonic relationships we have found converge onto the same value at the sonic condition. Any time the pressure ratio from the pitot-probe exceeds this critical value, the F-16 is supersonic, and any time it is below, it is subsonic.

Discussion:

Figure 1 shows the time history of the pitot-probe's total to static pressure ratio, highlighting the critical threshold beyond which flight is supersonic.

Figure 2 shows the F-16's Mach number (Mach in region 1) over the time of its flight using the relationships we have defined.

Figure 3 shows a comparison of the F-16's Mach number calculated correctly, taking shocks into account, and calculated as if there were no shocks in front of the probe.

Figure 4 shows the Mach number in region 2, which is immediately after the normal shock during supersonic flight.

Figure 5 shows a comparison of the F-16's flight Mach number in region 1 and that of the post-shock flow in region 2. Note that the region just upstream of the pitot probe experiences a local minimum Mach number when the F-16 is flying at its largest Mach number. This is indicative of the fact that shock strength increases with incoming Mach number.

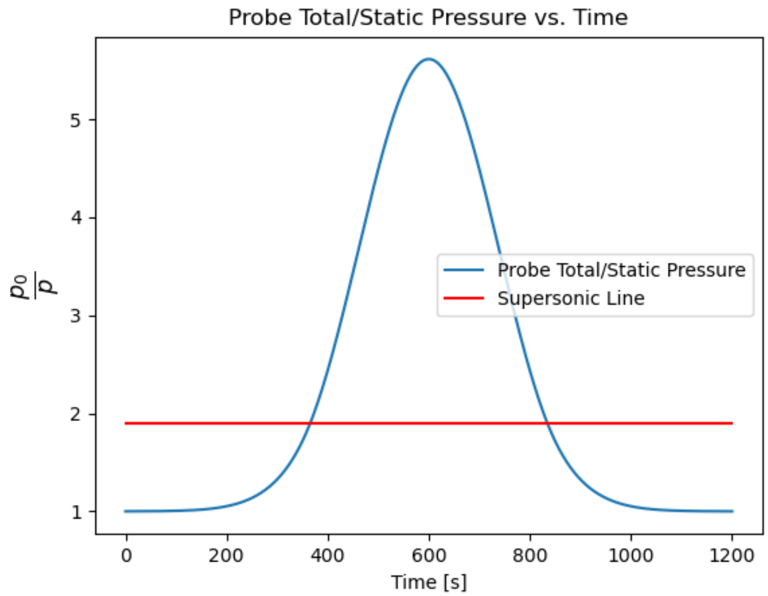

Figure 1: Probe total-to-static pressure ratio and supersonic threshold

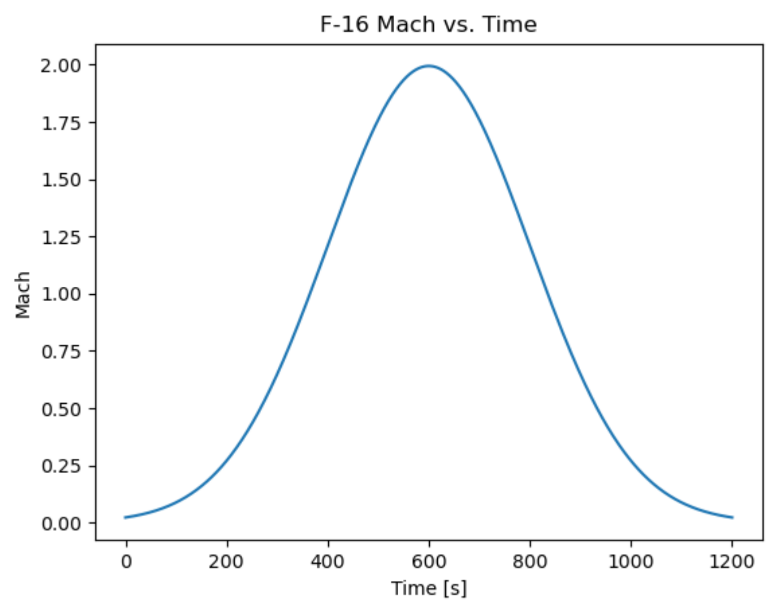

Figure 2: F-16 Mach vs. Time during flight, calculated using isentropic and normal shock relations

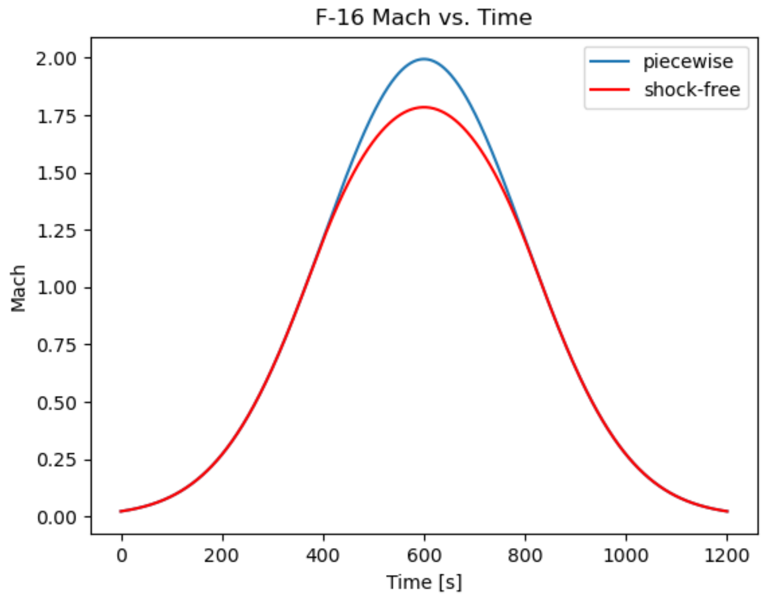

Figure 3: F-16 Mach vs. Time during flight, calculated correctly and as if there were no shocks

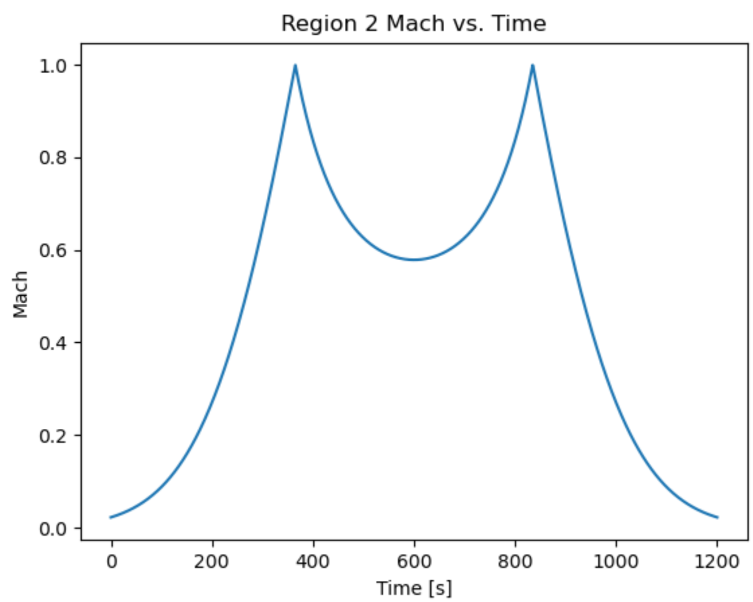

Figure 4: Mach in region 2, upstream of pitot probe vs. Time during flight

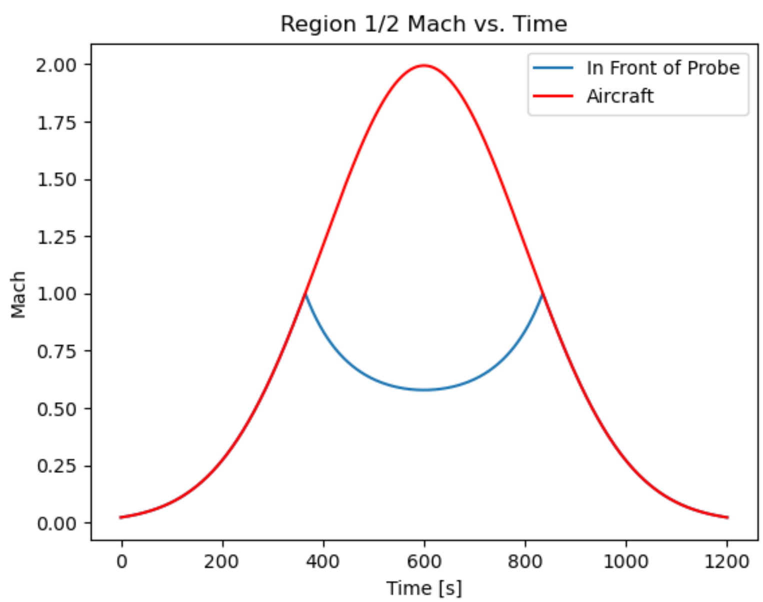

Figure 5: Mach vs Time of flow in regions 1 (pre-shock/vehicle Mach) and 2 (post shock/before pitot) vs. Time during flight

(b) Plot the air temperature experienced at the tip of the probe for this 20-minute flight.

Givens:

Time series for p_t and p, the total and static pressure, respectively, measured by the F-16's pitot-probe. The flight data given covers a simple trajectory including a take-off, acceleration to max-speed, and deceleration to landing. There is a constant atmospheric temperature of $T = 298\,K$ at all altitudes in the trajectory.

Assumptions:

Let region 1 be the area upstream of the bow shock. Let region 2 be the area *immediately behind the shock* but not at the probe's stagnation point. The static pressure measurement taken by the probe is valid for the static pressure *upstream* of the shock. The flow before and after the bow shock is isentropic, calorically perfect air. $\gamma_{air} = 1.4, R_{air} = 287\,\mathrm{J/kg \cdot K}$. The shock can be evaluated as a normal shock in front of the pitot probe. A shock is only present in front of the pitot probe when the F-16 is traveling supersonically, $M > 1$. All flow is isentropically brought to rest at the tip of the pitot-probe. Total temperature is constant across a shock because shocks are adiabatic.

Solution:

Given a constant freestream air temperature of $T_\infty = 298K$ and the previously calculated vehicle Mach number, we can determine the total temperature experienced by the F-16 over the duration of its flight using the following isentropic relationship:

$$\frac{T_{t,1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$T_{t,1} = 298\,[K] * \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

We know that shocks are adiabatic and therefore total temperature across a shock is constant.

$$T_{t,1} = T_{t,2}$$

Because we are assuming that the tip of the probe is a stagnation point ($M = 0$), total temperature is equal to static temperature for the probe.

$$T_{t,2} = T_{t,probe} = T_{probe}$$

Figure 6 shows the time history of temperature experienced at the tip of the pitot probe.

These temperatures are safely below the melting point of aluminum.

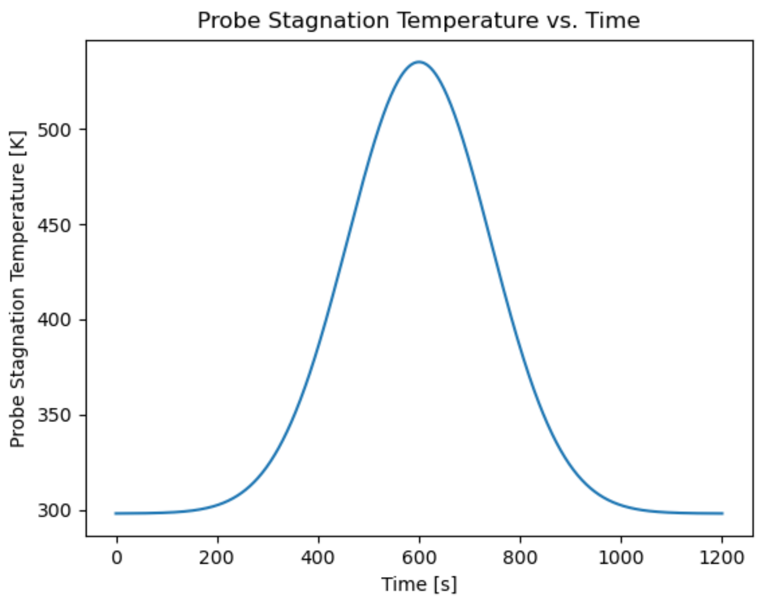

Figure 6: Pitot Probe Total Temperature vs. Time