Homework 1

Evan Burke
8 September 2022

Problem 1

The final Reynolds Transport Theorem we derived in class looked like:

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}(mb)_{sys}}{\mathrm{d}t} = \frac{\partial}{\partial t}\int_{CV} \rho b \mathrm{d}V + \int_{CS,out} b\rho|\vec{V_n}|\mathrm{d}A - \int_{CS,out} b\rho|\vec{V_n}|\mathrm{d}A$$

(a) In your own words, describe what each of the three terms on the right-hand-side of the equation mean related to an arbitrary fluid extensive property, B.

(b) If our problem was in the $x-y-z$ space, how would you represent the integrals $\int_{CV}\mathrm{d}V$ and $\int_{CS}\mathrm{d}A$ in terms of triple and double integrals, respectively?

(c) Why are the last two terms integral terms?

(d) What does the subscript "n" mean for the last two terms? Why do we need that there?

(e) Why do we need the absolute magnitude signs around the $\vec{V_n}$ terms?

(f) Why is the derivative with-respect-to t a partial derivative?

(g) Explain to a classmate how our

$$\int_{CS,out} b\rho|\vec{V_n}|\mathrm{d}A - \int_{CS,out} b\rho|\vec{V_n}|\mathrm{d}A$$

term is equivalent to

$$\int_{CS} b\rho\boldsymbol{V}\cdot\hat{\boldsymbol{n}}\mathrm{d}A\,,$$

which is equivalent to

$$\int_{CS} b\rho\vec{V}\cdot\mathrm{d}\vec{A}\,.$$

Be sure to explain the different math concepts. You may find it easier to "explain" by using a simple control-volume problem as an illustration.