Homework 6

Problem 1 (16 pts)

Starting with $\dot{m} = \rho u A$, prove that the mass flowrate through an isentropic choked nozzle can be written in the form:

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

Problem 2 (48 pts)

This problem will analyze the performance the the local Air Force Research Laboratory's (AFRL) Mach-6 Ludwieg Tube at the Wright Patterson Air Force Base. You might want to take a look at the attached conference paper again. The nozzle was designed using Method of Characteristics with supporting viscous CFD; however, our quasi-1D analyses are still quite helpful.

The nozzle is a traditional converging-diverging nozzle. For the first parts of this problem, let's assume the facility is operating at its maximum stagnation pressure ($p_0 = 4000$ kPa) and a stagnation temperature of $T_0 = 500$ K. The design exit Mach number is 6. The fluid is air. The throat diameter calculated inviscidly is 4.114 inches (note, the real (i.e. accounting for viscous effects) nozzle throat is 3.71 inch in diameter as discussed on page 3). For now, let us assume that the flow throughout the nozzle is isentropic everywhere except for through any shock waves that are present. Find:

(a) The mass flowrate through the nozzle under design conditions (i.e., fully isentropic flow).

(b) The exit area of the nozzle. How does this compare with the true exit area? We will use this exit area for the rest of the problem.

(c) The design back pressure and the temperature of the air leaving the nozzle with this back pressure.

(d) The lowest back pressure for which there is only subsonic flow in the nozzle. Recall, your area-Mach number relation plot has a subsonic section and supersonic section...

(e) The back pressure at which there is a normal shock wave on the exit plane of the nozzle

(f) The back pressure below which there are no shock waves in the nozzle. Hint: OS and EW are considered outside of the nozzle.

(g) The range of back pressures over which there are oblique shock waves in the exhaust from the nozzle.

(h) The range of back pressures over which there are expansion waves in the exhaust from the nozzle.

(i) The back pressure at which a normal shock wave occurs in the divergent section of the nozzle at a point where the nozzle area is a value equal to the average of the throat and the exit-plane areas.

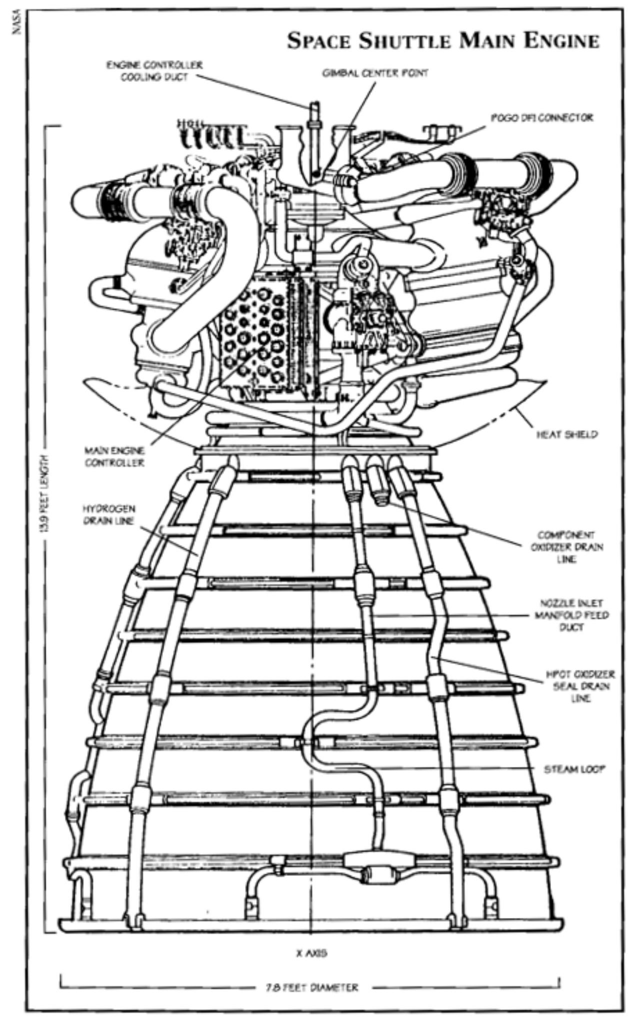
(j) The tunnel has two 2000-gallon receiving tanks (pg. 3). For the given stagnation conditions, calculate how long it will take to go from the design condition to the condition where there is a NS at the exit plane of the nozzle. You may neglect transient effects (i.e., you may assume that the entire volume of the receiving tanks is at the calculated back pressure(s)). You may also neglect thermal effects (i.e., you may assume that the air in the receiving tanks is always at 295 K). You may assume that the converging-diverging nozzle connects directly to the large receiving tanks. Briefly describe what effect the tunnel's diffuser would have on these calculations.

(k) From your calculations in part (j), determine the length (in feet) of the driver tube from which air is used over the course of this time frame. See pg. 2 for details of the driver tube. You may assume the pressure and temperature of the air in the driver tube are equal to the given stagnation conditions for the duration of this time frame.

Problem 3 (34 pts)

The Space Shuttle Main Engine (SSME) pictured below helped propel astronauts aboard the space shuttle into orbit since the 1970's. The three engines on the space shuttle burned approximately 500,000 gallons of liquid hydrogen fuel at a rate that could drain a swimming pool in under 30 seconds (WOW) — don't use this information to calculate the mass flowrate for this problem! The following equation, developed from conservation of mass and momentum can be used to calculate the thrust of the SSME:

$$Thrust = \dot{m}u_{\text{exit}} + (p_{\text{exit}} - p_{\text{ambient}})A_{\text{exit}} \qquad .$$



Homework 6

The combustion chamber at the top feeds burning compressible fluid into a small converging-nozzle section, through the throat, and out through a much larger diverging-nozzle section. Therefore, we can reasonably model this problem as a converging-diverging (CD) nozzle problem. The cross-sectional area of the entrance to the CD nozzle, throat, and exit are 0.21, 0.054, and 4.17 m^2, respectively. The pressure in the upstream combustion chamber (i.e., the stagnation pressure) is 20.408 MPa. The temperature of the fluid is 3600 K at the CD nozzle entrance (note, this is not the upstream stagnation temperature). We will assume $\gamma = 1.2$ and $R = 287$ J/kg/K for the fluid mixture. You may assume that the nozzle is choked and isentropic throughout this duration.

(a) Plot the thrust of the SSME as a function of altitude (from sea level to 20 km above sea level). Include markers where the nozzle flow is under-expanded, over-expanded, and at design conditions (aerodynamically speaking). The following equation can be used to calculate the ambient pressure:

$$p_{\text{ambient}} = 101325(1 - (2.25577 \cdot 10^{-5} \cdot h))^{5.25588} \qquad ,$$

where h is height (in meters) above sea level.

(b) Do you expect to see shock-diamonds or a plume when the space shuttle takes off?

(c) Does the "design condition" pertain to maximum thrust? Briefly explain.