Homework 3

Problem 1 (82 pts)

For this problem, you are going to take on the role of a hypersonic researcher dealing with matters of flight and wind-tunnel tests.

The attached AIAA-2011-3413 article gives an overview of a hypersonic flight test conducted by the U.S. Air Force in 2010. Please read pages 1–7. Figure 3 gives the altitude of the vehicle for the duration of its flight.

(a) Your boss/advisor wants to know whether you should take the time/effort to use accurate values for the local air temperature during the duration of the flight or simply use a constant room-temperature estimate in order to calculate the vehicle's freestream Mach number.

Knowing the vehicle's altitude trajectory (i.e., Fig. 3 in the article), you consult with atmospheric scientists to come up with best estimates of the air temperature at every altitude using the Committee on Space Reserach International Reference Atmosphere (CIRA)-86 model. You are able to look up with the freestream temperature T_∞, pressure p_∞, and density ρ_∞ during the ascent portion of the flight. "Freestream" in this case refers to the air in front of any shock waves associated with the vehicle. The freestream velocity of the vehicle u_∞ was measured with its instrumentation. All of these data are included in the attached Excel sheet for the ascent portion of the flight in addition to the flight time t. Use this data to plot the vehicle's freestream Mach number M_∞ versus flight time t using these realistic temperature values.

What would happen if you instead assume that the freestream for the duration of the flight was simply constant at room temperature (i.e., $T_\infty = 298$ K)? Provide a plot showing the absolute magnitude of the difference in calculated M_∞ between the two approaches versus t. Use this to make a case to your boss/advisor.

(b) Your boss/advisor, who's never taken a compressible flow course, thinks the isentropic relations are a bunch of bologna and that incompressible fluid-mechanics equations such as the Bernoulli equation are sufficient to calculate things like the stagnation pressure p_0 of the flow. You need to provide a good argument for your choice of using your compressible-flow equations.

The ratio $\frac{\rho_\infty}{\rho_0}$ represents the variation of flow density due to flow velocity and is therefore directly related to the flow's compressibility. Assuming isentropic flow, at what M_∞, are ρ_∞ and ρ_0 equal? As M_∞ increases, this ratio gets smaller and smaller. Using the flight data from above, provide your boss/advisor with the time during the flight for which the flow first becomes compressible. Show this plot of $\frac{\rho_\infty}{\rho_0}$ versus t as support to your argument.

Provide your boss/advisor with a figure of p_0 versus t for the duration of the flight test using compressible flow, isentropic relations and his beloved incompressible flow, Bernoulli equation (i.e., $p_0 = p_\infty + \frac{1}{2}\rho_\infty u_\infty^2$). Make sure these are on the same plot to highlight the differences. You may use the Excel data for p_∞, ρ_∞, etc. Explain what you see to your boss/advisor. Who's right?

Based on your plot, are there any times during the flight where the p_0 agree? If so, when and why?

Mathematically prove to your boss/advisor that for $M << 1$ (i.e., incompressible flows), the isentropic equation for $\frac{p_0}{p_\infty}$ becomes exactly the Bernoulli equation. Hint: The binomial approximation $(1+x)^\alpha \approx 1 + \alpha x$ holds for $x << 1$.

(c) You are now tasked with conducting some wind-tunnel experiments to see how they compare with your analyzed flight data. You decide to test in the local AFRL Mach-6 Ludwieg Tube (which is only able to produce one Mach number, $M_\infty = 6$. Your boss/advisor wants to you find a test condition in that facility where both M_∞ and u_∞ exactly match the flight M_∞ and u_∞ at a particular time during the trajectory. How many flight points do you have as options? Choose one (if multiple options exist), and indicate the flight time t. What are the flight M_∞ and u_∞ at that t?

Recall that the air in the AFRL Mach-6 Ludwieg Tube is heated using heated blankets. These heating blankets set the flow's stagnation temperature T_0 for the duration of a test. Assuming isentropic flow, what temperature do these blankets need to be set to for your test?

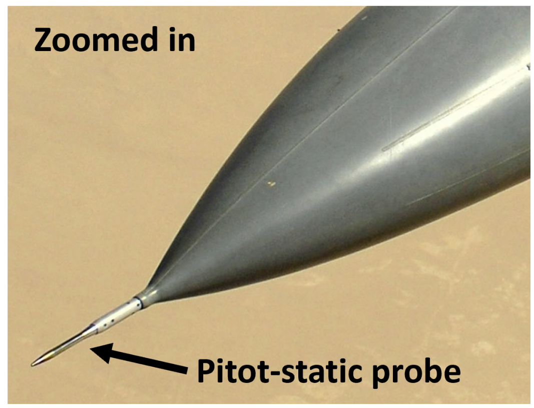
Does this seem like a reasonable value of T_0 to achieve in the facility? Indicate to your boss/advisor whether or not you think it's possible to match M_∞ and u_∞ simultaneously in this facility. If not, what other ideas can you think of?

We will get to this later in the course, but the Mach number is always equal to one in the "throat" of supersonic or hypersonic nozzles. The nozzle "throat" is simply the location of smallest cross-sectional area. Assuming that you could achieve the value of T_0 you calculated above, what would be the freestream value of temperature in the throat of the AFRL nozzle?

(d) Now that you have indicated to your boss/advisor that you have matching M_∞ and u_∞ conditions he/she thinks you should measure the same surface temperature on the model. Is this true? Explain yourself to him/her. You may assume that the vehicle was the same size and at the same Reynolds number here.

Problem 2 (76 pts)

For this problem you are going to help write the internal computer software for the F-16 fighter jet pictured on the left below. A zoomed-in picture of the nose of the aircraft is provided on the right. As one can see, sticking out front of the aircraft's nose is a long pitot-static probe, which measures stagnation pressure at the front tip and static pressure along the side.



Below is a visualization of the flow around the pitot-static probe under supersonic conditions. One can see that a shock wave can form upstream of the port. This shock wave is curved and is called a bow shock as we will see in the next chapter; however, the shock structure immediately upstream of the probe is more-or-less vertical and is therefore often modeled as a normal shock, as will be done for this problem. Note, even under supersonic conditions, the *static* measurement is for the static air *upstream* of the bow shock (we will learn why this is valid only for the static measurement in HW 5). You may assume calorically perfect, isentropic air.

You are provided flight data for a short flight in the attached Excel spreadsheet. Note, this data is tabulated from take-off, to max speed, and back down to landing over a 20-minute interval.

(a) Plot the Mach number of the aircraft for this 20-minute flight.

(b) Plot the air temperature experienced at the tip of the probe for this 20-minute flight. You may assume a constant atmospheric air temperature of 298 K over the entirety of altitudes experienced during the flight. If the probe were, made of aluminum, could it withstand these temperatures?