MTH 547 — Design of Experiments

Department of Mathematics

Homework 1

Author:
Evan Burke

Instructor:
Dr. Maher Qumsiyeh

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Problem 2.7

Suppose that we are testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the P-value for the following observed values of the test statistic.

(a) $t_0 = 2.31$

(b) $t_0 = 3.60$

(c) $t_0 = 1.95$

(d) $t_0 = 2.19$

Givens:

$n_1 = n_2 = 10$
Sample variances are unknown but equal.

Assumptions:

For all values of the test statistic, we use a two-sample t-test because the *population variances* are unknown. This is a right-tailed t-test due to the alternative hypothesis, $H_1 : \mu_1 > \mu_2$. We use a pooled t-test because the sample variances are assumed equal. The t-test has $n_1 + n_2 - 2 = 18$ degrees of freedom.

Solution:

Claim: Population mean 1, μ_1, is equal to population mean 2, μ_2.

$$\mu_1 = \mu_2$$

Hypotheses:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

Because the sample variances are assumed equal, we do not perform an F-test, and move directly to a pooled t-test for our sample. The test statistic for a pooled t-test is given as

$$t_0 = \frac{\bar{y_1} - \bar{y_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where S_p is defined as

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}},$$

and S_1 and S_2 are the sample variances.

Given the values of the test statistic, t_o, we use the tcdf() function of the Ti-84 calculator with $n_1 + n_2 - 2$ degrees of freedom to generate a P-value. The tcdf() function's inputs are a lower bound, an upper bound, and the number of degrees of freedom. In our case, the lower bound will be our test statistic, t_o and the upper bound will be some very large number, in this case, $E99$ to approximate the infinite end of the t-distribution.

(a) $t_0 = 2.31$
tcdf(2.31,E99,18)
P-value $= 0.01647$

(b) $t_0 = 3.60$
tcdf(3.60,E99,18)
P-value $= 0.0010$

(c) $t_0 = 1.95$
tcdf(1.95,E99,18)
P-value $= 0.0335$

(d) $t_0 = 2.19$
tcdf(2.19,E99,18)
P-value $= 0.0210$

Given the observed test-statistics, we can bound the P-value on:

$$(0.0010 \leq P \leq 0.0335)$$

Problem 2.24

The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair times for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

Givens:

$n = 16$
Time is normally distributed and random.

Solution:

Claim: The mean repair time exceeds 225 hours.

$$\mu > 225$$

Hypotheses:

$$H_0 : \mu = 225$$
$$H_1 : \mu > 225$$

(b) Test the hypotheses formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

Givens:

$\alpha = 0.05$

Solution:

For a simple hypothesis test of one variable without a known population variance, we use a t-test. Because normality is assumed in the givens, we do not need to test for it. Our test statistic, t, is defined as:

$$t = \frac{\bar{y} - \mu}{S/\sqrt{n}},$$

where $\bar{y}$ is the sample mean, μ is the population mean, S is the sample variance, and n is the size of the sample.

The critical value is the value of the test statistic beyond which we reject the null hypothesis. The C.V. for a one-tailed test is given by

$$C.V. = t_{\alpha,n-1}$$

The C.V. can be found using the Ti-84's invT() function, which yields a t-value for a given area and number of degrees of freedom.

$$t_{0.95,15} = 1.7530$$

Using the T-Test function on the Ti-84 with the data from the table yields the value of the test statistic, t.

$$t = 0.6685$$

Our test statistic, t, is smaller than our critical value, therefore we fail to reject the null hypothesis H_0 at $\alpha = 0.05$.

$$\boxed{0.6685 < 1.7530 \rightarrow \text{Fail to reject } H_0}$$

This indicates that we cannot prove that the mean repair time is not 225 hours at $\alpha = 0.05$.

(c) Find the P-value for the test.

Solution:

Using the T-test function on the Ti-84 with the data from the table yields the P-value.

$$\boxed{P\text{-value} = 0.2570}$$

As before, this value is large compared to $\alpha = 0.05$, therefore we fail to reject H_0.

(d) Construct a 95 percent confidence interval on mean repair time.

Solution:

A confidence interval on a mean for a t-distribution is defined as

$$P\left(\bar{y} - t_{\alpha/2,n-1}S/\sqrt{n} \leq \mu \leq \bar{y} + t_{\alpha/2,n-1}S/\sqrt{n}\right) = 1 - \alpha$$

Using the Ti-84's TInterval function yields the 95 % confidence interval on mean repair time.

$$\boxed{(188.89 \leq \mu \leq 294.11)}$$

This result indicates we are 95% confident that the true value of mean repair time falls between 188.89 hours and 294.11 hours.

Problem 2.28

The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the mean and variance of the burning times.

Type 1		**Type 2**	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypothesis that the two variances are equal. Use $\alpha = 0.05$

Givens:

Data in table in minutes.
$\alpha = 0.05$

Assumptions:

The data is normally distributed.

Solution:

Claim: The variances of burning times of both formulations of chemical flares are equal.

$$\sigma_1 = \sigma_2$$

Hypotheses:

$$H_0' : \sigma_1 = \sigma_2$$
$$H_1' : \sigma_1 \neq \sigma_2$$

The test statistic for comparing variances of two different samples is F and is given by

$$F = \frac{S_1^2}{S_2^2}$$

Using the Ti-84's 2Samp-FTest function, we obtain the value of our test statistic F, as well as its P-value.

$$F = 0.9782$$

$$P\text{-value} = 0.9744$$

This value is sufficiently large to move forward assuming that $\sigma_1 = \sigma_2$. In other words, at $\alpha = 0.05$, we fail to reject $H_0' : \sigma_1 = \sigma_2$. We cannot prove that there is a statistically significant difference between the variances at $\alpha = 0.05$.

(b) Using the results from (a), test the hypothesis that the mean burning times are equal. Use $\alpha = 0.05$. What is the P-value for this test?

Givens:

$\alpha = 0.05$

Assumptions:

From part (a), we need to use a pooled t-test for comparison of means between the two samples. The data is normally distributed.

Solution:

Claim: The mean burning times are equal.

$$\mu_1 = \mu_2$$

Hypotheses:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Test Statistic:

$$t_0 = \frac{\bar{y_1} - \bar{y_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Using the Ti-84's 2-SampTTest function to perform a pooled two-sample t-test on the given data, we obtain the value of the test statistic and its P-value.

$$t_0 = 0.0480$$
$$P\text{-value} = 0.9622$$
$$0.9622 > 0.05 \rightarrow \text{Fail to reject } H_0$$

The P-value is large in comparison to $\alpha = 0.05$, therefore we fail to reject $H_0 : \mu_1 = \mu_2$. We cannot prove that there is a statistically meaningful difference between the mean burning times at $\alpha = 0.05$.

Problem 2.31

Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuitr pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kA) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

95 °C	**100 °C**
11.176	5.263
7.089	6.748
8.097	7.461
11.739	7.015
11.291	8.133
10.759	7.418
6.467	3.772
8.315	8.963

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use $\alpha = 0.05$.

Givens:

$\alpha = 0.05$

Assumptions:

Runs were made in random order.

Solution:

Claim: Higher baking temperatures result in wafers with a lower mean photoresist thickness.

$$\mu_1 > \mu_2$$

Hypotheses:

$$H_0 : \mu_1 \leq \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

In order to test these hypotheses, we will use a right-tailed two-sample t-test. Before doing so, we need to test if we need to use a pooled test or not.

Claim:

$$\sigma_1 = \sigma_2$$

Hypotheses:

$$H_0' : \sigma_1 = \sigma_2$$
$$H_1' : \sigma_1 \neq \sigma_2$$

The test statistic for comparing variances of two different samples is F and is given by

$$F = \frac{S_1^2}{S_2^2}$$

Using the Ti-84's 2Samp-FTest function, we obtain the value of our test statistic F, as well as its P-value.

$$F = 1.6381$$
$$P\text{-value} = 0.5306$$

This value is sufficiently large to move forward assuming that $\sigma_1 = \sigma_2$. In other words, at $\alpha = 0.05$, we fail to reject $H_0' : \sigma_1 = \sigma_2$. We cannot prove that there is a statistically significant difference between the variances at $\alpha = 0.05$.

We can now perform a pooled two-sample t-test.

Test Statistic:

$$t_0 = \frac{\bar{y_1} - \bar{y_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Using the Ti-84's 2-SampTTest function to perform a right-tailed pooled two-sample t-test on the given data, we obtain the value of the test statistic and its P-value.

$$t_0 = 2.6751$$
$$P\text{-value} = 0.0091$$
$$0.0091 < 0.05 \rightarrow \text{Reject } H_0$$

The P-value is small in comparison to $\alpha = 0.05$, therefore we reject $H_0 : \mu_1 \leq \mu_2$. At $\alpha = 0.05$ we have shown a statistically significant difference in means indicating that the mean photoresist thickness is lower when the baking temperature is higher.

(b) What is the P-value for the test conducted in part (a)?

Solution:

The P-value is 0.0091.

(c) Find a 95 percent confidence interval on the difference in means. Provide a practical interpretation of this interval.

Solution:

A (1-α)$\times$100% confidence interval on the difference in means for a t-distribution is given by

$$\left((\bar{Y}_1 - \bar{Y}_2) - t_{\alpha/2,n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{Y}_1 - \bar{Y}_2) + t_{\alpha/2,n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

Using the Ti-84's 2-SampTInterval function yields the 95 % confidence interval on the difference in mean photoresist thickness.

$$\boxed{(0.49957 \leq \mu_1 - \mu_2 \leq 4.5405)}$$

This result indicates we are 95% confident that the true value of the difference in mean thicnkess falls between 0.49957 and 4.5405 kA.