# Homework 1

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## Problem 2.7

Suppose that we are testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the $P$-value for the following observed values of the test statistic.

(a) $t_0 = 2.31$

(b) $t_0 = 3.60$

(c) $t_0 = 1.95$

(d) $t_0 = 2.19$

**Givens:**

$n_1 = n_2 = 10$
Sample variances are unknown but equal.

**Assumptions:**

For all values of the test statistic, we use a two-sample t-test because the *population variances* are unknown. This is a right-tailed t-test due to the alternative hypothesis, $H_1 : \mu_1 > \mu_2$. We use a pooled t-test because the sample variances are assumed equal. The t-test has $n_1 + n_2 - 2 = 18$ degrees of freedom.

**Solution:**

Claim: Population mean 1, $\mu_1$, is equal to population mean 2, $\mu_2$.

$$\mu_1 = \mu_2$$

Hypotheses:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

Because the sample variances are assumed equal, we do not perform an F-test, and move directly to a pooled t-test for our sample. The test statistic for a pooled t-test is given as

$$t_0 = \frac{\bar{y_1} - \bar{y_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where $S_p$ is defined as

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}},$$

and $S_1$ and $S_2$ are the sample variances.

Given the values of the test statistic, $t_o$, we use the tcdf() function of the Ti-84 calculator with $n_1 + n_2 - 2$ degrees of freedom to generate a $P$-value. The tcdf() function's inputs are a lower bound, an upper bound, and the number of degrees of freedom. In our case, the lower bound will be our test statistic, $t_o$ and the upper bound will be some very large number, in this case, $E99$ to approximate the infinite end of the t-distribution.

(a) $t_0 = 2.31$
tcdf(2.31,E99,18)
$P$-value $= 0.01647$

(b) $t_0 = 3.60$
tcdf(3.60,E99,18)
$P$-value $= 0.0010$

(c) $t_0 = 1.95$
tcdf(1.95,E99,18)
$P$-value $= 0.0335$

(d) $t_0 = 2.19$
tcdf(2.19,E99,18)
$P$-value $= 0.0210$

Given the observed test-statistics, we can bound the $P$-value on:

$$(0.0010 \leq P \leq 0.0335)$$