Problem 2

A one-dimensional, *second order* element is shown below:

The physical node locations and nodal displacement values are shown in table 1:

Node 1		**Node 2**		**Node 3**	
x_1	d_1	x_2	d_2	x_3	d_3
2 in.	0.15 in.	4 in.	0.05 in.	6 in.	-0.10 in.

Table 1: 1D element coordinates and displacements.

Find the physical location ($x =$) on the element where the displacement is zero.

Solution:

Note: Matrix calculations and plotting performed in MATLAB, see appendix **??**.

The displacement of the element as a function of x is given by the following second-order equation:

$$u = a_1 + a_2 x + a_3 x^2$$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = [A] \begin{Bmatrix} 1 \\ x \\ x^2 \end{Bmatrix}$$

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}$$

Shape functions for the element can be found using the relation below:

$$[N] = \begin{bmatrix} 1 & x & x^2 \end{bmatrix} [A]^{-1}$$

$$[A]^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -\frac{5}{4} & 2 & \frac{3}{4} \\ \frac{1}{8} & -\frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

$$\begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} = \begin{bmatrix} 1 & x & x^2 \end{bmatrix} \begin{bmatrix} 3 & -3 & 1 \\ -\frac{5}{4} & 2 & \frac{3}{4} \\ \frac{1}{8} & -\frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

The final shape functions for the element:

$$\begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{x^2}{8} - \frac{5x}{4} + 3\right) & \left(-\frac{x^2}{4} + 2x - 3\right) & \left(\frac{x^2}{8} - \frac{3x}{4} + 1\right) \end{bmatrix}$$

The displacement field for the element can be found using the following relation between shape functions and known nodal degrees of freedom:

$$\{u\} = [N]\{d\}$$

$$u = \begin{bmatrix} \left(\frac{x^2}{8} - \frac{5x}{4} + 3\right) & \left(-\frac{x^2}{4} + 2x - 3\right) & \left(\frac{x^2}{8} - \frac{3x}{4} + 1\right) \end{bmatrix} \begin{Bmatrix} 0.15 \\ 0.05 \\ -0.10 \end{Bmatrix}$$

The following equation defines displacement along the element as a function of x location:

$$u = -\frac{x^2}{160} - \frac{x}{80} + \frac{1}{5}$$

The coordinate of the element where there is no displacement is $x = 4.7445\,\text{in}$.

$$\boxed{u(x = 4.7445) = 0}$$

Figure 1 shows the element displacement versus x-location with the point of zero-displacement annotated.

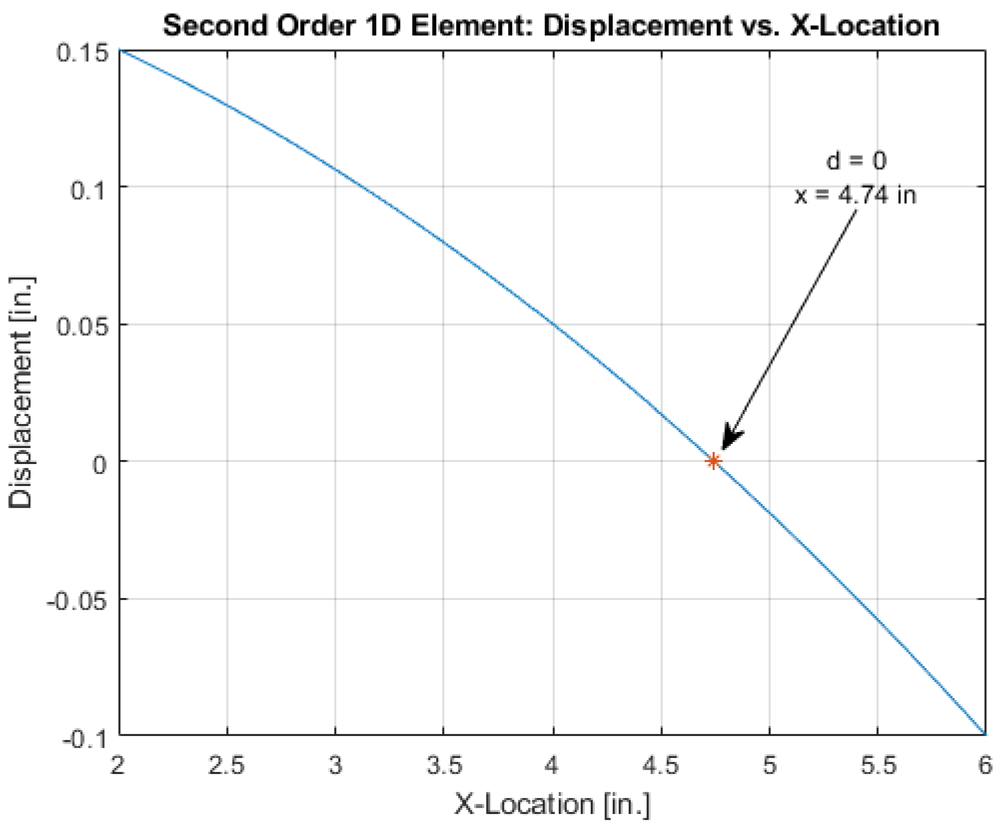

Figure 1: Deflection vs. X-location for second order 1D element. Zero deflection point starred in red.