AEE/CEE/EGM/MEE 546 Finite Element Analysis I
Fall 2022
Mid Term Exam – 5 Problems
Due Monday, Oct 31, 2022 at beginning of class
Late submissions will not be accepted without prior approval in extraordinary circumstances.

This is an open-book, open-notes take-home examination. You may use any resource at your disposal *except* the following:

You may not discuss the test nor seek help from another human being, including but not limited to students in the class, professors, your friends, and subject matter experts.

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1. Instead of the linear shape functions for a 1D bar element, the following shape functions have been proposed for an element with two nodes:

$$N_1 = \frac{-x(1-x)}{2} \qquad N_2 = \frac{x(1+x)}{2}$$

The resulting displacement field is $u = N_1 d_1 + N_2 d_2$

a) Develop the relation: $\varepsilon = [B]\{d\}$. That is, find the $[B]$ matrix in terms of x.
b) Develop the stiffness matrix, [K].
c) Are these valid shape functions? Why or why not?

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2. A one-dimensional, *second order* element is shown below:

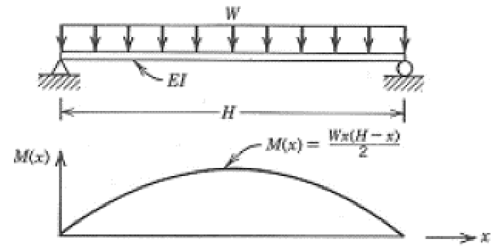
The physical nodal locations and nodal displacement values are shown in the table below:

Node 1		Node 2		Node 3	
x_1	d_1	x_2	d_2	x_3	d_3
2 in.	0.15 in.	4 in.	0.05 in	6 in.	–0.10 in

Find the physical location (x = ____) on the element where the displacement is zero.

OVER, PLEASE!

3. The potential energy for a simply supported beam under uniform distributed load (see the figure) is



$$\Pi = \int_0^H \left[\frac{EI}{2} \left(\frac{dy}{dx} \right)^2 + \left(\frac{Wx(H-x)}{2} y \right) \right] dx$$

in which:
y is the transverse deflection of the beam *W* = transverse distributed load

E, *I*, and *H* are constants independent of *x*

and the boundary conditions are $y(0) = 0$ and $y(H) = 0$

a) Use the Euler equation to solve for the deflection equation $y(x)$ of the beam
b) If we were to set $\delta\Pi = 0$, the result would be the Euler equation plus this boundary term:

$$y'\delta y|_0^H = 0$$

What does this boundary term tell us about the boundary conditions that must exist at the ends of the beam?

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4. Concerning FEA models with 3-node triangular and 4-node rectangular elements:

a) Using words and/or equations, explain why these elements perform poorly in bending.
b) Does "refining the mesh" (adding more of these elements) improve the performance of models with these elements? Explain.
c) What other approach can be taken to improve the performance of these models? Explain.

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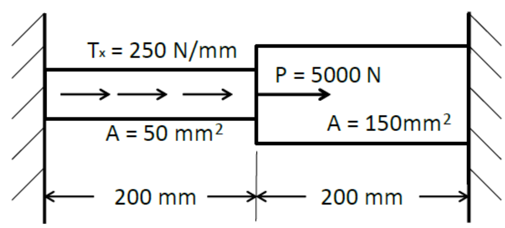
5. Consider the bar loaded as shown at right.



Assume *E* = 200 GPa and the bar is fixed at both ends.

a) Construct a 1D linear bar finite element model of the bar. *Use two elements in each section of the bar (4 elements in total).* Label all nodes and elements.
b) Write the global system of equations [K]{d} = {R}
c) Apply the boundary conditions to this global system of equations and solve for {d}
d) Plot the displacements $u(x)$ vs. *x* for the entire bar.
e) What are the reaction forces at the two ends?