# Homework 4

Evan Burke
1 November 2022

Develop and integrate the stiffness matrix of the Physical Element described below. This element is a four-node plane stress quadrilateral element with two degrees of freedom per node. This stiffness matrix is given by:

$$\left[K\right] = \int_{-1}^{1}\int_{-1}^{1} \left[B\right]^T \left[D\right] \left[B\right] \det \left[J\right] t_e \mathrm{d}\xi \mathrm{d}\eta$$

in which $\det \left[J\right]$ is the determinant of the Jacobian Matrix:

$$\left[J\right] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

$\left[B\right]$ is the strain-displacement matrix with $\left\{\varepsilon\right\} = \left[B\right]\left\{d\right\}$ and $\left[D\right]$ is the linear isotropic stiffness matrix:

$$\left[D\right] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Use the following material properties: $E = 70\,\mathrm{GPa}$, $\nu = 0.33$.

Assume the element is 2.5 mm thick.

The nodal coordinates $(x, y)$ of the physical element are:

$$\begin{aligned} \text{Node } 1 &= (-0.25, -1.0) \\ \text{Node } 2 &= (1.0, -0.50) \\ \text{Node } 3 &= (0.75, 0.25) \\ \text{Node } 4 &= (0.75, 0.50) \end{aligned}$$

with shape functions:

$$\begin{aligned} N_1 &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2 &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned}$$

**Solution:**

The stiffness matrix $\left[K\right]$ is defined by the following integral:

$$\left[K\right] = t_e \int_{-1}^{1}\int_{-1}^{1} \left[B\right]^T \left[D\right] \left[B\right] \det\left[J\right] \mathrm{d}\xi \mathrm{d}\eta$$

with integrand $\Phi$:

$$\Phi_i = \left[B\right]^T \left[D\right] \left[B\right] \det\left[J\right]$$

Evaluation of this integral for a four-node element as described in the problem statement is complex or impossible to do analytically. To approximate the stiffness matrix of this element we will utilize Gaussian Quadrature. Using preexisting knowledge regarding the formation of a 4-point quadrature, we can bypass several complex steps. The quadrature for our stiffness matrix takes the following form:

$$[K] = ([K]_1 + [K]_2 + [K]_3 + [K]_4) * t_e$$

where the $[K]_i$ are the integrand of the stiffness matrix equation evaluated at some pre-selected quadrature points.

$$\begin{aligned}
[K]_1 &= \Phi(\xi_1, \eta_1) \\
[K]_2 &= \Phi(\xi_2, \eta_2) \\
[K]_3 &= \Phi(\xi_3, \eta_3) \\
[K]_4 &= \Phi(\xi_4, \eta_4)
\end{aligned}$$

For this quadrature the points are as follows:

$$\begin{aligned}
\Phi(\xi_1, \eta_1) &= \frac{1}{\sqrt{3}}(-1,-1) \\
\Phi(\xi_2, \eta_2) &= \frac{1}{\sqrt{3}}(1,-1) \\
\Phi(\xi_3, \eta_3) &= \frac{1}{\sqrt{3}}(1,1) \\
\Phi(\xi_4, \eta_4) &= \frac{1}{\sqrt{3}}(-1,1)
\end{aligned}$$

$$\left[J\right] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix}$$

From MATLAB, $\left[J\right]$ for an arbitrary $\xi$ and $\eta$:

$$J = \left[\begin{array}{cc} \frac{\eta}{8}+\frac{3}{4} & \frac{1}{16}-\frac{3\,\eta}{16} \\ \frac{\xi}{8}-\frac{1}{4} & \frac{9}{16}-\frac{3\,\xi}{16} \end{array}\right]$$

From MATLAB, $\det\left[J\right]$ for an arbitrary $\xi$ and $\eta$:

$$\det\left[J\right] = \frac{3\,\eta}{128} - \frac{19\,\xi}{128} + \frac{7}{16}$$

$\left[B\right]$ is given by:

$$\left[B\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \left[J\right]^{-1} & 0 \\ 0 & \left[J\right]^{-1} \end{bmatrix} \begin{bmatrix} N_{1,\xi} & 0 & N_{2,\xi} & 0 & N_{3,\xi} & 0 & N_{4,\xi} & 0 \\ N_{1,\eta} & 0 & N_{2,\eta} & 0 & N_{3,\eta} & 0 & N_{4,\eta} & 0 \\ 0 & N_{1,\xi} & 0 & N_{2,\xi} & 0 & N_{3,\xi} & 0 & N_{4,\xi} \\ 0 & N_{1,\eta} & 0 & N_{2,\eta} & 0 & N_{3,\eta} & 0 & N_{4,\eta} \end{bmatrix}$$

From MATLAB, $\left[B\right]$ for an arbitrary $\xi$ and $\eta$ (denoted by k and e respectively):

$$B =$$

```
/ #1,  0, #3,  0, #2,  0, #4,  0 \
|                                |
|  0, #5,  0, #7,  0, #6,  0, #8 |
|                                |
\ #5, #1, #7, #3, #6, #2, #8, #4 /

where

                 / k   1 \      / e   1 \
       (3 e - 1) | - - - | 8    | - - - | (k - 3) 24
                 \ 4   4 /      \ 4   4 /
  #1 == --------------------- - --------------------
          3 e - 19 k + 56         3 e - 19 k + 56

                 / k   1 \      / e   1 \
       (3 e - 1) | - + - | 8    | - + - | (k - 3) 24
                 \ 4   4 /      \ 4   4 /
  #2 == --------------------- - --------------------
          3 e - 19 k + 56         3 e - 19 k + 56
```

```
      / e   1 \                          / k   1 \
      | - - - | (k - 3) 24   (3 e - 1) | - + - | 8
      \ 4   4 /                          \ 4   4 /
#3 == -------------------- - ---------------------
         3 e - 19 k + 56        3 e - 19 k + 56

      / e   1 \                          / k   1 \
      | - + - | (k - 3) 24   (3 e - 1) | - - - | 8
      \ 4   4 /                          \ 4   4 /
#4 == -------------------- - ---------------------
         3 e - 19 k + 56        3 e - 19 k + 56

      / k   1 \              / e   1 \
      | - - - | (e + 6) 16   | - - - | (k - 2) 16
      \ 4   4 /              \ 4   4 /
#5 == -------------------- - --------------------
         3 e - 19 k + 56        3 e - 19 k + 56

      / k   1 \              / e   1 \
      | - + - | (e + 6) 16   | - + - | (k - 2) 16
      \ 4   4 /              \ 4   4 /
#6 == -------------------- - --------------------
         3 e - 19 k + 56        3 e - 19 k + 56

      / e   1 \              / k   1 \
      | - - - | (k - 2) 16   | - + - | (e + 6) 16
      \ 4   4 /              \ 4   4 /
#7 == -------------------- - --------------------
         3 e - 19 k + 56        3 e - 19 k + 56

      / e   1 \              / k   1 \
      | - + - | (k - 2) 16   | - - - | (e + 6) 16
      \ 4   4 /              \ 4   4 /
#8 == -------------------- - --------------------
         3 e - 19 k + 56        3 e - 19 k + 56
```

From MATLAB, the integrand $\Phi$ for an arbitrary $\xi$ and $\eta$ (denoted by k and e respectively):

```
[[((#23 - #25) (#31 - #33) + (#17 - #7) (#29 - #13)) #1,
```

```
  ((#27 - #5) (#31 - #33) + (#23 - #25) (#29 - #13)) #1,
-((#23 - #25) (#31 - #32) + (#17 - #7) (#29 - #12)) #1,
-((#27 - #5) (#31 - #32) + (#23 - #25) (#29 - #12)) #1,
((#23 - #25) (#30 - #32) + (#17 - #7) (#28 - #12)) #1,
((#27 - #5) (#30 - #32) + (#23 - #25) (#28 - #12)) #1,
-((#23 - #25) (#30 - #33) + (#17 - #7) (#28 - #13)) #1,
-((#27 - #5) (#30 - #33) + (#23 - #25) (#28 - #13)) #1],
[((#9 - #11) (#29 - #13) + (#15 - #3) (#31 - #33)) #1,
((#15 - #3) (#29 - #13) + (#31 - #33) (#19 - #21)) #1,
-((#9 - #11) (#29 - #12) + (#15 - #3) (#31 - #32)) #1,
-((#15 - #3) (#29 - #12) + (#31 - #32) (#19 - #21)) #1,
((#9 - #11) (#28 - #12) + (#15 - #3) (#30 - #32)) #1,
((#15 - #3) (#28 - #12) + (#30 - #32) (#19 - #21)) #1,
-((#9 - #11) (#28 - #13) + (#15 - #3) (#30 - #33)) #1,
-((#15 - #3) (#28 - #13) + (#30 - #33) (#19 - #21)) #1],
[-((#23 - #24) (#31 - #33) + (#17 - #6) (#29 - #13)) #1,
-((#27 - #4) (#31 - #33) + (#23 - #24) (#29 - #13)) #1,
((#23 - #24) (#31 - #32) + (#17 - #6) (#29 - #12)) #1,
((#27 - #4) (#31 - #32) + (#23 - #24) (#29 - #12)) #1,
-((#23 - #24) (#30 - #32) + (#17 - #6) (#28 - #12)) #1,
-((#27 - #4) (#30 - #32) + (#23 - #24) (#28 - #12)) #1,
```

```
((#23 - #24) (#30 - #33) + (#17 - #6) (#28 - #13)) #1,
((#27 - #4) (#30 - #33) + (#23 - #24) (#28 - #13)) #1],
[-((#9 - #10) (#29 - #13) + (#15 - #2) (#31 - #33)) #1,
-((#15 - #2) (#29 - #13) + (#31 - #33) (#19 - #20)) #1,
((#9 - #10) (#29 - #12) + (#15 - #2) (#31 - #32)) #1,
((#15 - #2) (#29 - #12) + (#31 - #32) (#19 - #20)) #1,
-((#9 - #10) (#28 - #12) + (#15 - #2) (#30 - #32)) #1,
-((#15 - #2) (#28 - #12) + (#30 - #32) (#19 - #20)) #1,
((#9 - #10) (#28 - #13) + (#15 - #2) (#30 - #33)) #1,
((#15 - #2) (#28 - #13) + (#30 - #33) (#19 - #20)) #1],
[((#22 - #24) (#31 - #33) + (#16 - #6) (#29 - #13)) #1,
((#26 - #4) (#31 - #33) + (#22 - #24) (#29 - #13)) #1,
-((#22 - #24) (#31 - #32) + (#16 - #6) (#29 - #12)) #1,
-((#26 - #4) (#31 - #32) + (#22 - #24) (#29 - #12)) #1,
((#22 - #24) (#30 - #32) + (#16 - #6) (#28 - #12)) #1,
((#26 - #4) (#30 - #32) + (#22 - #24) (#28 - #12)) #1,
-((#22 - #24) (#30 - #33) + (#16 - #6) (#28 - #13)) #1,
-((#26 - #4) (#30 - #33) + (#22 - #24) (#28 - #13)) #1],
[((#8 - #10) (#29 - #13) + (#14 - #2) (#31 - #33)) #1,
((#14 - #2) (#29 - #13) + (#31 - #33) (#18 - #20)) #1,
-((#8 - #10) (#29 - #12) + (#14 - #2) (#31 - #32)) #1,
```

```
-((#14 - #2) (#29 - #12) + (#31 - #32) (#18 - #20)) #1,
((#8 - #10) (#28 - #12) + (#14 - #2) (#30 - #32)) #1,
((#14 - #2) (#28 - #12) + (#30 - #32) (#18 - #20)) #1,
-((#8 - #10) (#28 - #13) + (#14 - #2) (#30 - #33)) #1,
-((#14 - #2) (#28 - #13) + (#30 - #33) (#18 - #20)) #1],
[-((#22 - #25) (#31 - #33) + (#16 - #7) (#29 - #13)) #1,
-((#26 - #5) (#31 - #33) + (#22 - #25) (#29 - #13)) #1,
((#22 - #25) (#31 - #32) + (#16 - #7) (#29 - #12)) #1,
((#26 - #5) (#31 - #32) + (#22 - #25) (#29 - #12)) #1,
-((#22 - #25) (#30 - #32) + (#16 - #7) (#28 - #12)) #1,
-((#26 - #5) (#30 - #32) + (#22 - #25) (#28 - #12)) #1,
((#22 - #25) (#30 - #33) + (#16 - #7) (#28 - #13)) #1,
((#26 - #5) (#30 - #33) + (#22 - #25) (#28 - #13)) #1],
[-((#8 - #11) (#29 - #13) + (#14 - #3) (#31 - #33)) #1,
-((#14 - #3) (#29 - #13) + (#31 - #33) (#18 - #21)) #1,
((#8 - #11) (#29 - #12) + (#14 - #3) (#31 - #32)) #1,
((#14 - #3) (#29 - #12) + (#31 - #32) (#18 - #21)) #1,
-((#8 - #11) (#28 - #12) + (#14 - #3) (#30 - #32)) #1,
-((#14 - #3) (#28 - #12) + (#30 - #32) (#18 - #21)) #1,
((#8 - #11) (#28 - #13) + (#14 - #3) (#30 - #33)) #1,
((#14 - #3) (#28 - #13) + (#30 - #33) (#18 - #21)) #1]]
```

```
where

        3 e   19 k    7
   #1 == --- - ---- + --
        128    128   16

        3616814565052631 (3 e - 1) #36
   #2 == ------------------------------
                17179869184 #37

        3616814565052631 (3 e - 1) #38
   #3 == ------------------------------
                17179869184 #37

        7125664516223095 (3 e - 1) #36
   #4 == ------------------------------
                34359738368 #37

        7125664516223095 (3 e - 1) #38
   #5 == ------------------------------
                34359738368 #37

        5398230694108405 (3 e - 1) #36
   #6 == ------------------------------
                8589934592 #37

        5398230694108405 (3 e - 1) #38
   #7 == ------------------------------
                8589934592 #37

        7125664516223095 #34 (k - 2)
   #8 == ----------------------------
               17179869184 #37

        7125664516223095 #35 (k - 2)
   #9 == ----------------------------
               17179869184 #37

         7125664516223095 #36 (e + 6)
   #10 == ----------------------------
                17179869184 #37
```

```
       7125664516223095 #38 (e + 6)
#11 == ----------------------------
              17179869184 #37

       8 (3 e - 1) #36
#12 == ---------------
             #37

       8 (3 e - 1) #38
#13 == ---------------
             #37

       10850443695157893 #34 (k - 3)
#14 == -----------------------------
              17179869184 #37

       10850443695157893 #35 (k - 3)
#15 == -----------------------------
              17179869184 #37

       16194692082325215 #34 (k - 3)
#16 == -----------------------------
               8589934592 #37

       16194692082325215 #35 (k - 3)
#17 == -----------------------------
               8589934592 #37

       5398230694108405 #34 (k - 2)
#18 == ----------------------------
              4294967296 #37

       5398230694108405 #35 (k - 2)
#19 == ----------------------------
              4294967296 #37

       5398230694108405 #36 (e + 6)
#20 == ----------------------------
              4294967296 #37

       5398230694108405 #38 (e + 6)
```

```
#21 == ----------------------------
              4294967296 #37

       3616814565052631 #34 (k - 2)
#22 == ----------------------------
              8589934592 #37

       3616814565052631 #35 (k - 2)
#23 == ----------------------------
              8589934592 #37

       3616814565052631 #36 (e + 6)
#24 == ----------------------------
              8589934592 #37

       3616814565052631 #38 (e + 6)
#25 == ----------------------------
              8589934592 #37

       21376993548669285 #34 (k - 3)
#26 == -----------------------------
              34359738368 #37

       21376993548669285 #35 (k - 3)
#27 == -----------------------------
              34359738368 #37

       24 #34 (k - 3)
#28 == --------------
             #37

       24 #35 (k - 3)
#29 == --------------
             #37

       16 #34 (k - 2)
#30 == --------------
             #37

       16 #35 (k - 2)
#31 == --------------
             #37
```

```
        16 #36 (e + 6)
#32 == --------------
             #37

        16 #38 (e + 6)
#33 == --------------
             #37

        e   1
#34 == - + -
        4   4

        e   1
#35 == - - -
        4   4

        k   1
#36 == - + -
        4   4

#37 == 3 e - 19 k + 56

        k   1
#38 == - - -
        4   4
```

From MATLAB, $\left[K\right]_1$:

$$\left[K\right]_1 =$$

```
  Columns 1 through 2

  14020.8948929524         7969.14111437039
  7969.14111437039          25974.606564508
 -9714.81607093324        -6231.22823878262
 -6308.67080895101        -6244.91306623978
 -3756.88746372825         -2135.3249259652
  -2135.3249259652        -6959.87485267606
 -549.191358290926         397.412050377436
  474.854620545818        -12769.8186455922
```

```
Columns 3 through 4

   -9714.81607093324          -6308.67080895101
   -6231.22823878262          -6244.91306623978
    12462.9822177267           1605.00089114581
    1605.00089114581           4595.39016861992
    2603.07712082345           1690.40324857222
    1669.65257443581           1673.31941290152
   -5351.24326761689           3013.26666923298
    2956.57477320101          -23.796515281649

Columns 5 through 6

   -3756.88746372825           -2135.3249259652
    -2135.3249259652          -6959.87485267606
    2603.07712082345           1669.65257443581
    1690.40324857222           1673.31941290152
    1006.65496196059           572.158589490423
    572.158589490423           1864.89284619623
    147.155380944205          -106.486237961031
   -127.236912097439           3421.66259357831

Columns 7 through 8

   -549.191358290926           474.854620545818
    397.412050377436          -12769.8186455922
   -5351.24326761689           2956.57477320101
    3013.26666923298          -23.796515281649
    147.155380944205          -127.236912097439
   -106.486237961031           3421.66259357831
    5753.27924496361          -3304.19248164938
   -3304.19248164938            9371.9525672955
```

From MATLAB, $[K]_2$:

$$[K]_2 =$$

```
Columns 1 through 2

7588.9253023072          3526.13735617572
```

```
   3526.13735617572           6684.57871393278
  -7045.36906314338          -206.378670591415
  -283.821240759797           3401.69433500741
  -2431.35719001231          -3375.05768370428
  -3318.36578767231          -9174.79179897743
   1887.80095084848           55.2989981199782
   76.0496722563868          -911.481249962762

Columns 3 through 4

  -7045.36906314338          -283.821240759797
  -206.378670591415           3401.69433500741
   17130.4542465048          -7504.44800653865
  -7504.44800653865           13753.4526435624
  -5494.99380203214           5777.45846430507
   5700.01589413668          -13469.9204495876
  -4590.09138132925           2010.81078299338
   2010.81078299338          -3685.22652898223

Columns 5 through 6

  -2431.35719001231          -3318.36578767231
  -3375.05768370428          -9174.79179897743
  -5494.99380203214           5700.01589413668
   5777.45846430507          -13469.9204495876
   6453.97184037591          -854.335450785886
  -854.335450785886           19035.4579419865
   1472.37915166854          -1527.31465567849
   -1548.0653298149           3609.25430657845

Columns 7 through 8

   1887.80095084848           76.0496722563868
   55.2989981199782          -911.481249962762
  -4590.09138132925           2010.81078299338
   2010.81078299338          -3685.22652898223
   1472.37915166854           -1548.0653298149
  -1527.31465567849           3609.25430657845
   1229.91127881223           -538.79512543487
   -538.79512543487           987.453472366537
```

From MATLAB, $[K]_3$:

$$[K]_3 =$$

```
  Columns 1 through 2

   1404.37112147306          798.210936563748
   798.210936563748          2601.68752631868
   1984.32974657417          937.719704897028
    916.96903076062          6825.14587349537
  -5241.18437801994         -2978.96377041304
  -2978.96377041304         -9709.63003363951
   1852.48350997271          1243.03312895227
   1263.78380308868          282.796633825457

Columns 3 through 4

   1984.32974657417           916.96903076062
   937.719704897028          6825.14587349537
   6912.33021532338         -1308.18949358814
  -1308.18949358814          20122.5274204681
  -7405.61943318509         -3422.17501176582
   -3499.6175819342         -25471.7911689538
  -1491.04052871246          3813.39547459334
   3870.08737062532         -1475.88212500972

Columns 5 through 6

  -5241.18437801994         -2978.96377041304
  -2978.96377041304         -9709.63003363951
  -7405.61943318509          -3499.6175819342
  -3422.17501176582         -25471.7911689538
   19560.3663906067          11117.6441450884
   11117.6441450884          36236.8326082394
  -6913.56257940168         -4639.06279274118
  -4716.50536290956         -1055.41140564606

Columns 7 through 8

   1852.48350997271          1263.78380308868
   1243.03312895227          282.796633825457
  -1491.04052871246          3870.08737062532
   3813.39547459334         -1475.88212500972
```

```
   -6913.56257940168          -4716.50536290956
   -4639.06279274118          -1055.41140564606
    6552.11959814143           -417.36581080443
    -417.36581080443           2248.49689683032
```

From MATLAB, $\left[K\right]_4$:

$$\left[K\right]_4 =$$

```
    Columns 1 through 2

    7449.82559864112           3100.30015594989
    3100.30015594989           19171.8899629945
    68.5872100647694           -912.34651261755
   -933.097186753958           2482.19802935211
   -7262.44185599477          -5592.87718252932
   -5649.56907856129          -12390.3988323572
   -255.970952711119           3404.92353919698
    3482.36610936536          -9263.68915998941

Columns 3 through 4

    68.5872100647694          -933.097186753958
    -912.34651261755           2482.19802935211
    775.134467548404          -339.568129723115
   -339.568129723115           622.328809173003
    2049.11953797532           5.37980371926812
   -15.3708704171405          -781.964103677629
   -2892.84121558849           1267.28551275781
    1267.28551275781          -2322.56273484748

Columns 5 through 6

   -7262.44185599477          -5649.56907856129
   -5592.87718252932          -12390.3988323572
    2049.11953797532          -15.3708704171405
    5.37980371926812          -781.964103677629
    12860.7405445254           5607.5750796251
     5607.5750796251           10254.0331714148
   -7647.41822650596           57.3648693533256
   -20.0777008150566           2918.32976461997
```

```
Columns 7 through 8

   -255.970952711119          3482.36610936536
    3404.92353919698         -9263.68915998941
   -2892.84121558849          1267.28551275781
    1267.28551275781         -2322.56273484748
   -7647.41822650596         -20.0777008150566
    57.3648693533256          2918.32976461997
    10796.2303948056         -4729.57392130811
   -4729.57392130811          8667.92213021693
```

Finally, from MATLAB, $\big[K\big]$:

$$\big[K\big] = (\big[K\big]_1 + \big[K\big]_2 + \big[K\big]_3 + \big[K\big]_4) * t_e =$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7.62E+04 | 3.85E+04 | -3.68E+04 | -1.65E+04 | -4.67E+04 | -3.52E+04 | 7.34E+03 | 1.32E+04 |
| 3.85E+04 | 1.36E+05 | -1.60E+04 | 1.62E+04 | -3.52E+04 | -9.56E+04 | 1.28E+04 | -5.67E+04 |
| -3.68E+04 | -1.60E+04 | 9.32E+04 | -1.89E+04 | -2.06E+04 | 9.64E+03 | -3.58E+04 | 2.53E+04 |
| -1.65E+04 | 1.62E+04 | -1.89E+04 | 9.77E+04 | 1.01E+04 | -9.51E+04 | 2.53E+04 | -1.88E+04 |
| -4.67E+04 | -3.52E+04 | -2.06E+04 | 1.01E+04 | 9.97E+04 | 4.11E+04 | -3.24E+04 | -1.60E+04 |
| -3.52E+04 | -9.56E+04 | 9.64E+03 | -9.51E+04 | 4.11E+04 | 1.68E+05 | -1.55E+04 | 2.22E+04 |
| 7.34E+03 | 1.28E+04 | -3.58E+04 | 2.53E+04 | -3.24E+04 | -1.55E+04 | 6.08E+04 | -2.25E+04 |
| 1.32E+04 | -5.67E+04 | 2.53E+04 | -1.88E+04 | -1.60E+04 | 2.22E+04 | -2.25E+04 | 5.32E+04 |