

## Problem 2

A one-dimensional, *second order* element is shown below:



The physical node locations and nodal displacement values are shown in table 1:

Node 1		Node 2		Node 3	
$x_1$	$d_1$	$x_2$	$d_2$	$x_3$	$d_3$
2 in.	0.15 in.	4 in.	0.05 in.	6 in.	-0.10 in.

Table 1: 1D element coordinates and displacements.

Find the physical location ( $x =$ ) on the element where the displacement is zero.

**Solution:**

$$u = a_1 + a_2x + a_3x^2$$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = [A] \begin{Bmatrix} 1 \\ x \\ x^2 \end{Bmatrix}$$

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}$$

$$[N] = [1 \quad x \quad x^2][A]^{-1}$$

$$[A]^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -\frac{5}{4} & 2 & \frac{3}{4} \\ \frac{1}{8} & -\frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

$$[N_1 \quad N_2 \quad N_3] = [1 \quad x \quad x^2] \begin{bmatrix} 3 & -3 & 1 \\ -\frac{5}{4} & 2 & \frac{3}{4} \\ \frac{1}{8} & -\frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

$$[N_1 \quad N_2 \quad N_3] = \left[ \left( \frac{x^2}{8} - \frac{5x}{4} + 3 \right) \quad \left( -\frac{x^2}{4} + 2x - 3 \right) \quad \left( \frac{x^2}{8} - \frac{3x}{4} + 1 \right) \right]$$

$$\{u\} = [N]\{d\}$$

$$u = \left[ \left( \frac{x^2}{8} - \frac{5x}{4} + 3 \right) \quad \left( -\frac{x^2}{4} + 2x - 3 \right) \quad \left( \frac{x^2}{8} - \frac{3x}{4} + 1 \right) \right] \begin{Bmatrix} 0.15 \\ 0.05 \\ -0.10 \end{Bmatrix}$$

$$u = -\frac{x^2}{160} - \frac{x}{80} + \frac{1}{5}$$

$$u(x = 4.7445) = 0$$

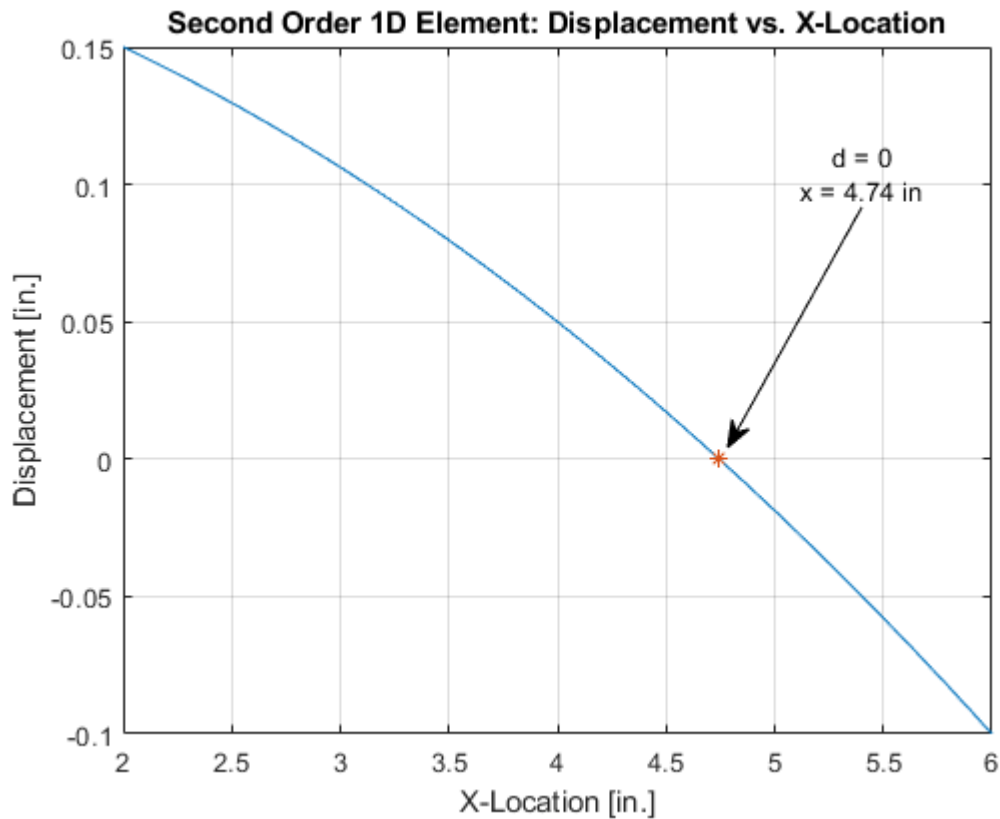


Figure 1: Deflection vs. X-location for second order 1D element. Zero deflection point starred in red.