Discrete-Time Neoclassical Growth Model

1. Model

We consider a standard discrete-time neoclassical growth model with a representative agent who maximizes lifetime utility over consumption:

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} \beta^t \ln(c_t) \tag{1}$$

subject to the capital accumulation equation:

$$k_{t+1} - (1-\delta)k_t = Ak_t^\alpha - c_t, \quad t = 0, 1, \ldots, T-1, \tag{2}$$

with parameters and variables:

- $c_t \geq 0$: consumption at time t
- k_0: given initial capital stock
- k_T: terminal capital (free or fixed)
- $\alpha \in (0,1)$: capital share in output
- $\beta \in (0,1)$: discount factor
- $\delta \in [0,1]$: depreciation rate
- $A > 0$: productivity

The resource constraint can also be written as:

$$c_t + k_{t+1} = Ak_t^\alpha + (1-\delta)k_t. \tag{3}$$

2. First-Order Conditions

The first-order conditions (FOCs) of the representative agent are:

Euler Equation

For $t = 0, \dots, T-2$:

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \left[\alpha A k_{t+1}^{\alpha-1} + 1 - \delta \right], \tag{4}$$

or equivalently:

$$c_{t+1} = \beta c_t \left[\alpha A k_{t+1}^{\alpha-1} + 1 - \delta \right]. \tag{5}$$

Capital Accumulation / Resource Constraint

For $t = 0, \dots, T-1$:

$$c_t + k_{t+1} = A k_t^{\alpha} + (1-\delta) k_t. \tag{6}$$

Boundary Conditions

$$k_1 = k_0, \tag{7}$$

$$k_T = k_{T-1} \quad \text{(or free terminal capital).} \tag{8}$$

These FOCs correspond directly to the equations implemented in the MATLAB function *func_cpo.m*.