

# CS 372 Lecture #42

## Security

- encryption

**Note:** Many of the lecture slides are based on presentations that accompany *Computer Networking: A Top Down Approach*, 6<sup>th</sup> edition, by Jim Kurose & Keith Ross, Addison-Wesley, 2013.

# Password / Data encryption

- Messages are encoded by the sending protocol
- Encoded messages are decoded by the receiving protocol
- Many encryption algorithms (functions)
  - simple substitution to very complex computations
  - most use *mod* with large prime numbers
- *Private key* encryption
- *Public key* encryption

# Private key encryption

- Only sender and receiver have the key and the encrypt/decrypt algorithms
- (**sender**) For message  $M$ , with key  $K$ , the encrypted message  $E$  is
$$E = \text{encrypt}(K, M)$$
- (**receiver**) For encrypted message  $E$ , the original message is produced by the inverse of  $\text{encrypt}$ 
$$M = \text{decrypt}(K, E)$$
- Many algorithms
- Separate key for each correspondent
- Easy to change key
- Difficult to ensure confidentiality of key

# Private key encryption example

- Both sender and receiver have key

Example key (**K**):      abcdefghijklmnopqrstuvwxyz  
                             bdfhjlnprtvxzacegikmoqsuwy

- Sender:

**M** = **secret**    sends    **E** = *encrypt*(**K**, **M**) = **kjfijm**

- Receiver:

receives      **E** = **kjfijm**

decodes to get    **M** = *decrypt*(**K**, **E**) = **secret**

# Public key encryption

- Each user has two keys and the encrypt/decrypt algorithms
  - one *public* key, one *private* key
- (*sender*) For message *M*, with the destination user's public key *Kpublic*, the encrypted message *E* is
$$E = \text{encrypt}(K_{\text{public}}, M)$$
- (*receiver*) For a message *E* (encrypted with the destination user's *public* key) the original message can be produced only by the destination user's *private* key *Kprivate*
$$M = \text{decrypt}(K_{\text{private}}, E)$$
- Easy to change key
- Easy to ensure confidentiality of private key

# Public key encryption example (RSA\*)

- $K_{\text{public}} = \langle 3, 187 \rangle$
- $K_{\text{private}} = \langle 107, 187 \rangle$
- Message = 25
- $E = \text{encrypt}(K_{\text{public}}, \text{Message})$   
 $= \text{Message}^3 \bmod 187$   
 $= 25^3 \bmod 187 = 104$
- $M = \text{decrypt}(K_{\text{private}}, E)$   
 $= E^{107} \bmod 187$   
 $= 104^{107} \bmod 187 = 25 = \text{Message}$
- \*RSA: Rivest, Shamir, Adleman algorithm

# RSA: Choosing keys

1. Choose two large prime numbers  $p, q$ .  
(e.g., 1024 bits each)
2. Compute  $n = pq$ ,  $z = (p-1)(q-1)$
3. Choose  $e$  ( $e < n$ ) such that  $e$  has no common factors with  $z$ . ( $e, z$  are “relatively prime”).
4. Choose  $d$  such that  $ed-1$  is exactly divisible by  $z$ .  
(in other words:  $ed \bmod z = 1$ ).
5. Public key is  $(e, n)$ . Private key is  $(d, n)$ .  

$\underbrace{\hspace{1.5cm}}$   
 $K_B^+$

$\underbrace{\hspace{1.5cm}}$   
 $K_B^-$

# RSA: Encryption, decryption

Given  $(e,n)$  and  $(d,n)$  as computed above

1. To encrypt bit pattern,  $m$ , compute

$$c = m^e \bmod n$$

2. To decrypt received bit pattern,  $c$ , compute

$$m = c^d \bmod n$$

Magic  
happens!

$$m = \underbrace{(m^e \bmod n)}_c^d \bmod n$$



# Another RSA example:

Let  $p=5, q=7$  Then  $n=35, z=24$

Choose  $e=5$  (so  $e, z$  relatively prime)

$d=29$  (so  $ed-1 = 144$  is exactly divisible by  $z$ )

Suppose message  $m = 12$

encrypt:

$m$

12

$m^e$

248832

$c = m^e \bmod n$

17

decrypt:

$c$

17

$c^d$

481968572106750915091411825223071697

$m = c^d \bmod n$

12

# RSA: Why $m = (m^e \bmod n)^d \bmod n$ ?

Useful result from number theory

If  $p, q$  prime and  $n = pq$ , then:

$$x^y \bmod n = x^{y \bmod (p-1)(q-1)} \bmod n$$

$$(m^e \bmod n)^d \bmod n = m^{ed} \bmod n$$

$$= m^{ed \bmod (p-1)(q-1)} \bmod n$$

(using number theory result above)

$$= m^1 \bmod n$$

(since we chose  $ed$  to be divisible by  
 $(p-1)(q-1)$  with remainder 1 )

$$= m$$

# RSA: another important property

$$\underbrace{K_B^- (K_B^+ (m))}_{\text{apply public key first, then private key}} = m = \underbrace{K_B^+ (K_B^- (m))}_{\text{Apply private key first, then public key}}$$

apply **public** key  
first, then apply  
**private** key

Apply **private**  
key first, then  
apply **public** key

*Result is the same!*

- “Security” must be defined by an organization
  - Determine value of information and define a security policy
  - Aspects to consider include
    - privacy
    - data integrity
    - availability
    - confidentiality
- Mechanisms to provide aspects of security
  - Firewalls: packet filtering
  - Encryption: private and public key cryptosystems
  - Virtual private networks
  - etc.