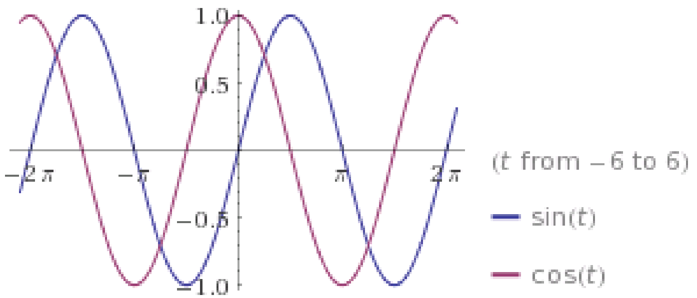
Computing phase difference



1. Instantaneous Phase Difference (IPD): phase difference at t.

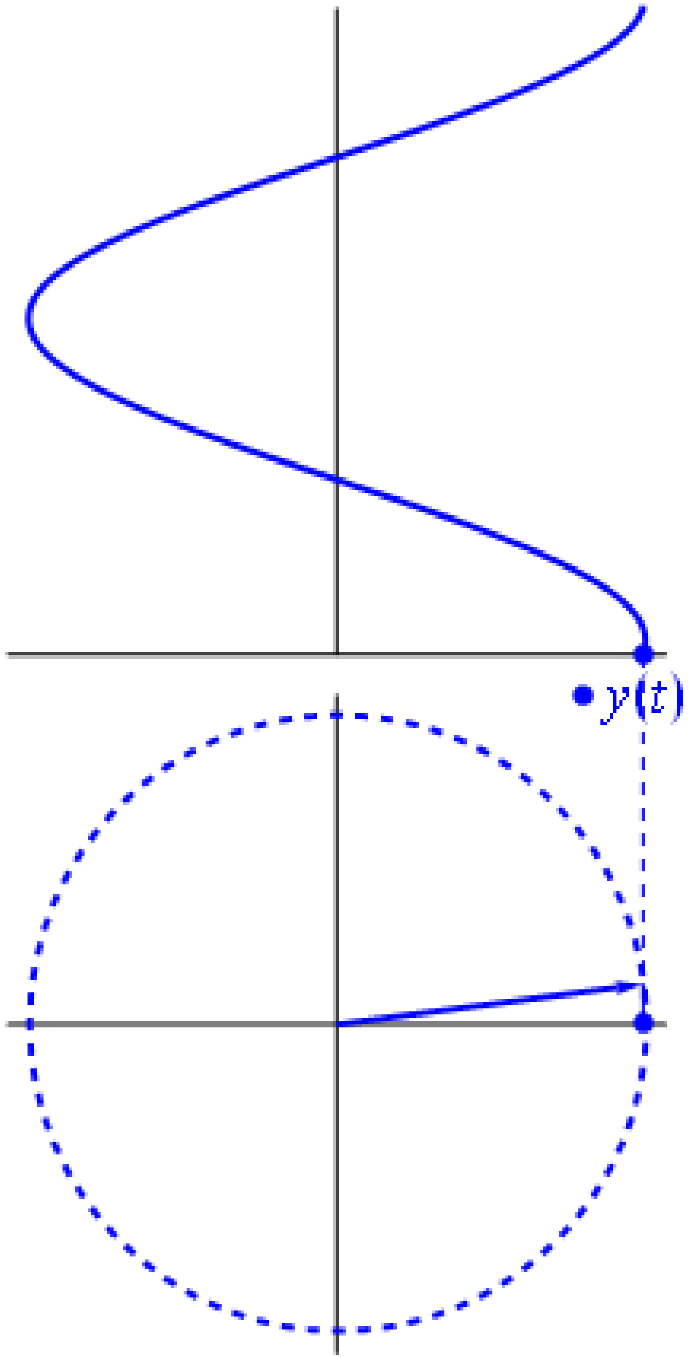
$$\Delta\varphi_t = \varphi_{i,t} - \varphi_{j,t}$$

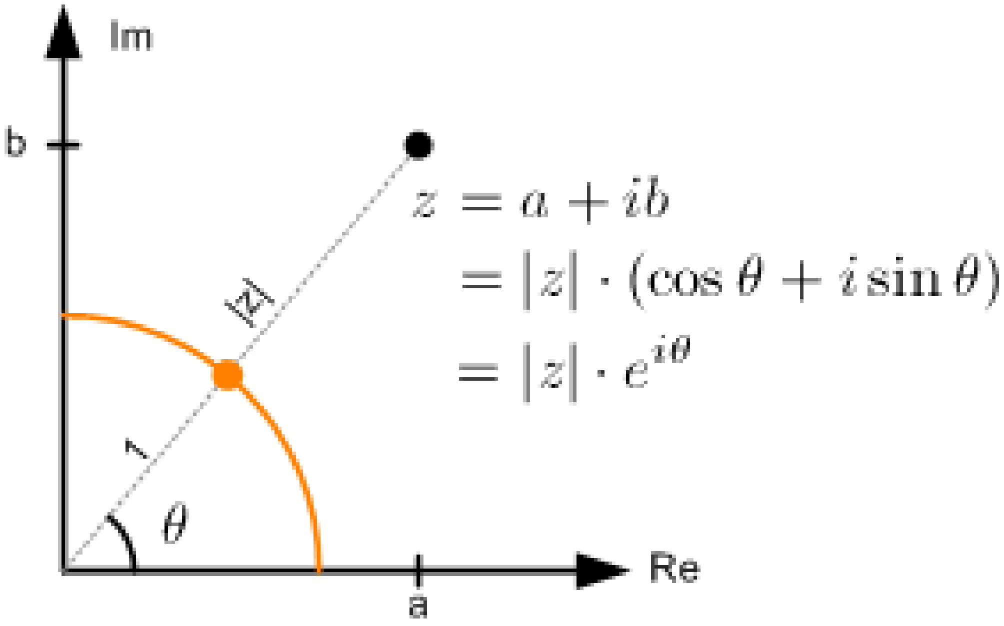
2. We will use exponential form:

$$S = re^{i\varphi} = r(cos\varphi + i\ sin\varphi)$$

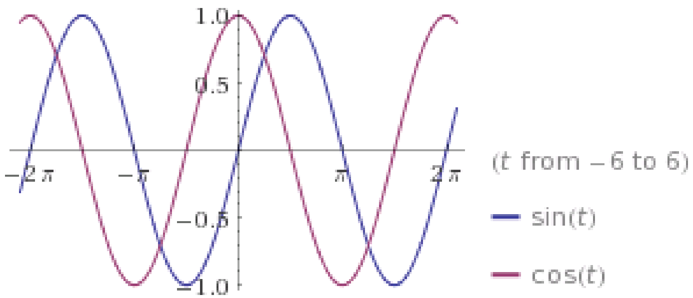
3. We can take the angle of the exponent form:

$$\Delta\varphi = \arg(re^{i\Delta\varphi})$$


$y(t)$


Im
b
Re
a
|z|
1
θ
$z = a + ib$
$= |z| \cdot (\cos\theta + i\sin\theta)$
$= |z| \cdot e^{i\theta}$

Directionality measure: dPLI



1. Instantaneous Phase Difference (IPD): phase difference at t.

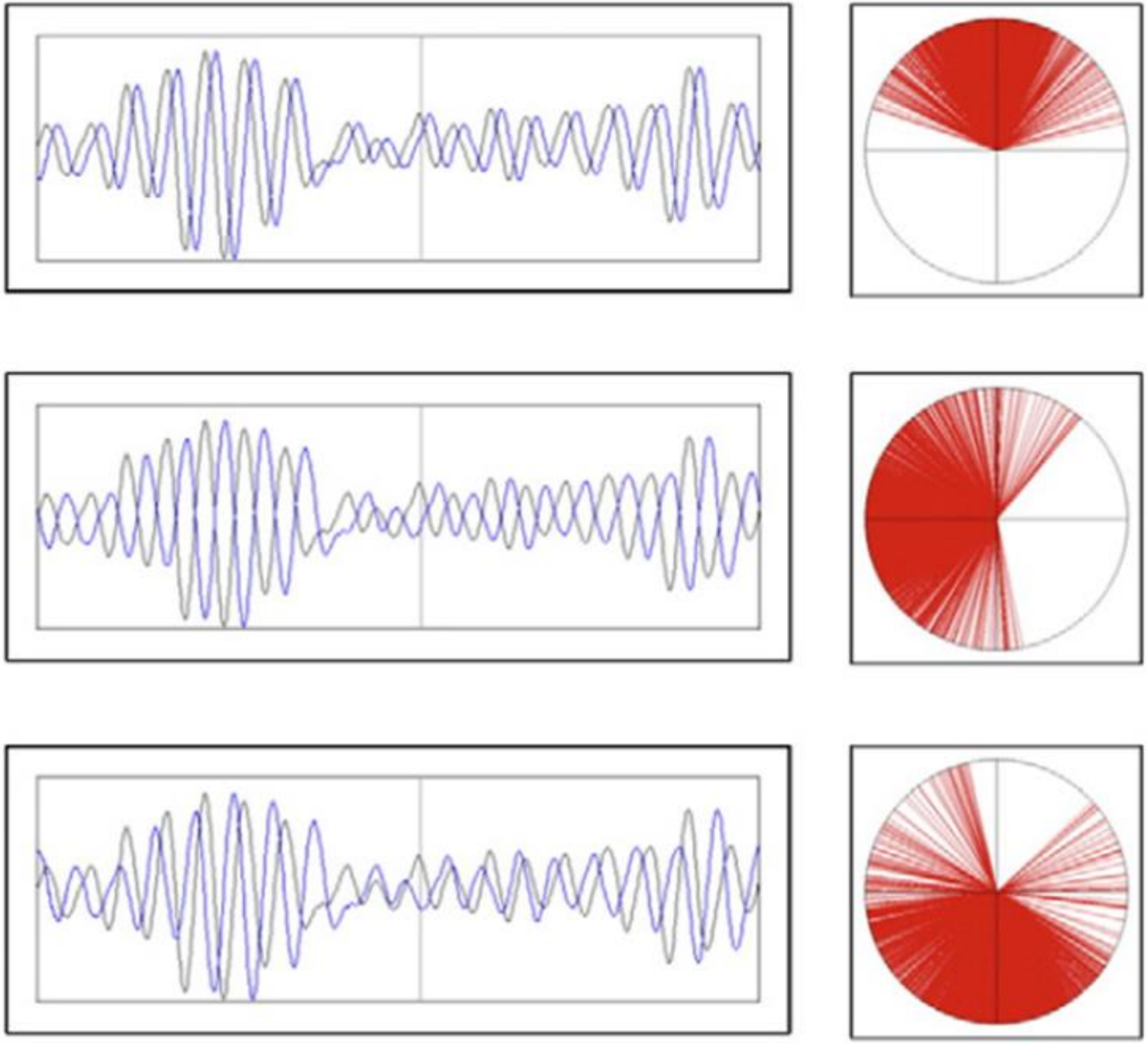
$$\Delta\varphi_t = \varphi_{i,t} - \varphi_{j,t}$$

2. directional Phase Lag Index (dPLI): time average of sign of IPD
   ➡ captures the phase lead and lag relationship.

$$\text{dPLI} = < sign(\Delta\varphi_t) >$$

If 0<dPLI≤1, *i* lead *j*.
If -1≤dPL<0, *i* lag *j*.
In dPLI=0, neither *i* or *j* lead/lag.

CJ Stam, et al. NeuroImage 2012



dPLI = 1.0

dPLI = -0.08

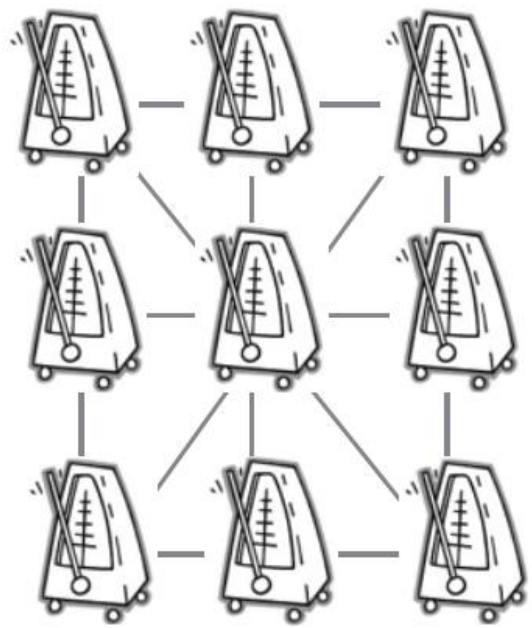
dPLI = -0.95

CCNSS 2018 Module 5
Tutorial 3:

Introduction to Kuramoto Model

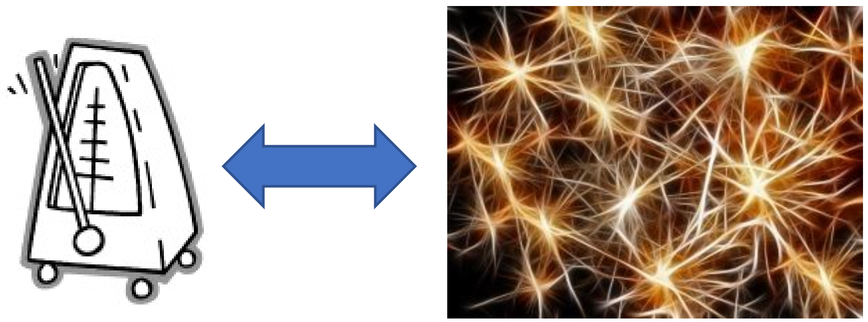
Collective Oscillators

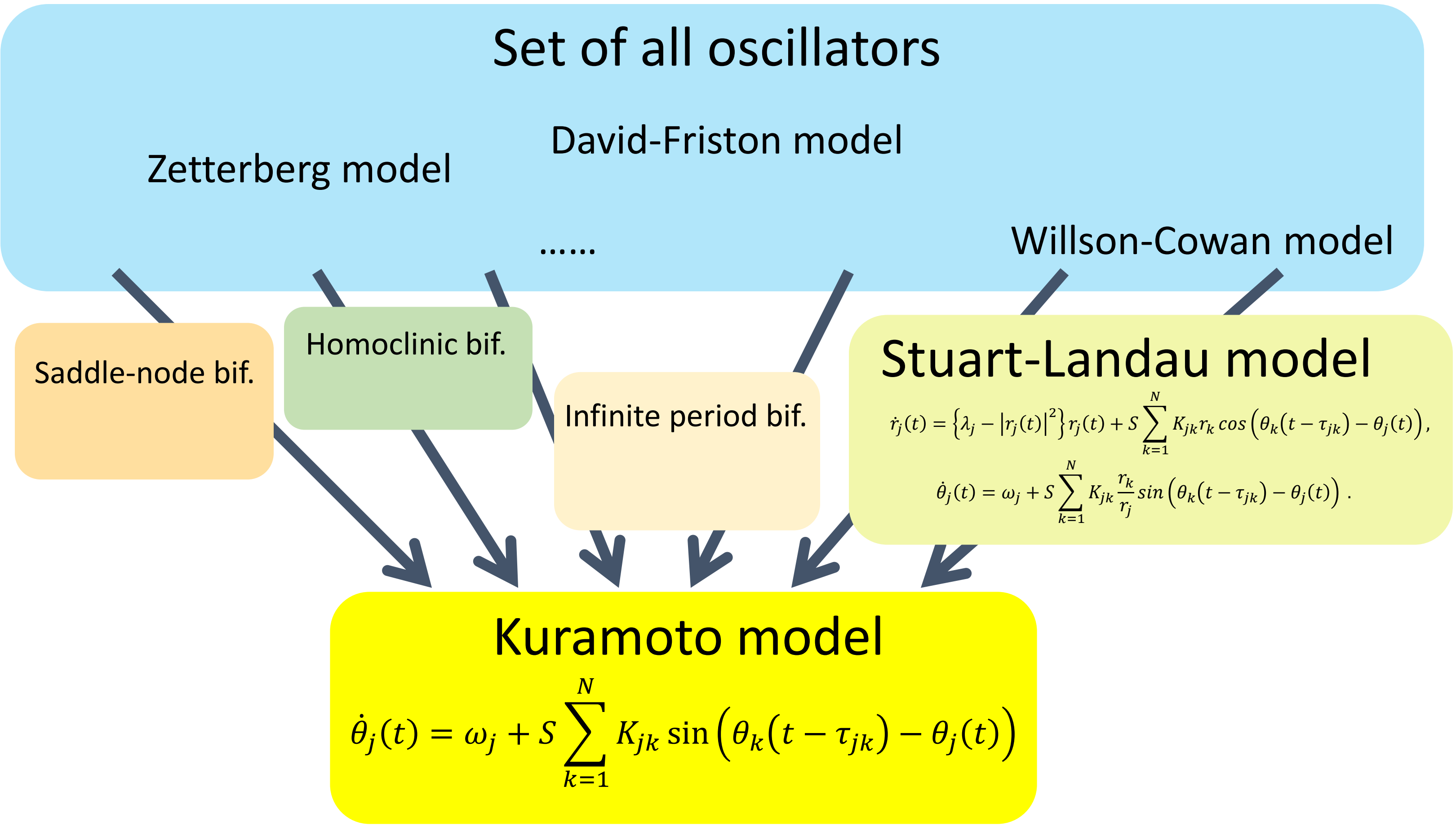
Ensemble of Coupled Oscillators



Arthur T. Winfree, 1967
Yoshiki Kuramoto, 1975

We consider neural masses as oscillators

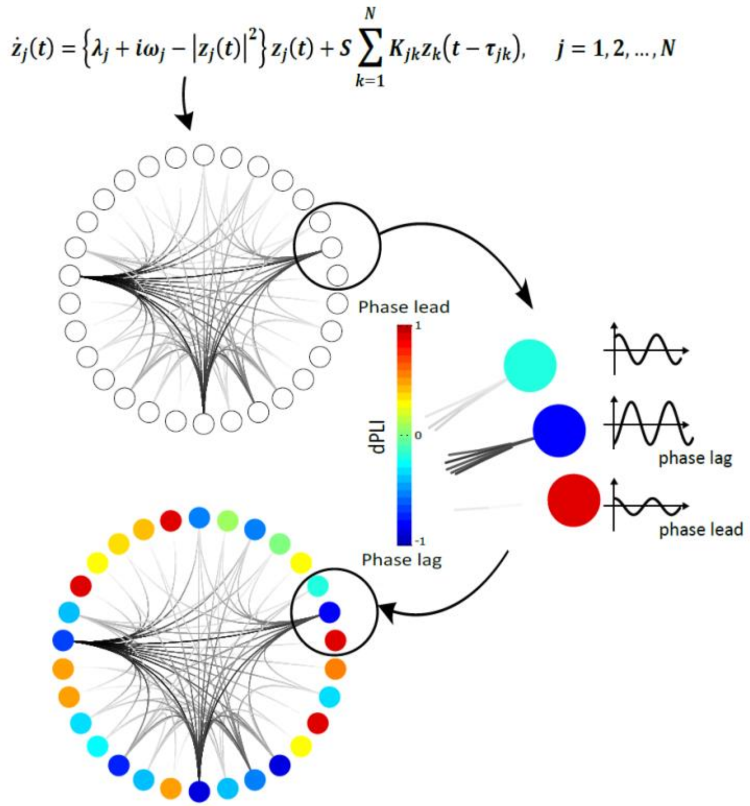




There exists mappings from all oscillators to the Kuramoto model, as a first-order approximation.
There exists mappings from some oscillators to the Stuart-Landau model, as the next-order appx.

Kuramoto/ Stuart-Landau model are the canonical models of oscillators.
If we can show that the K. model and S.-L. model yield *a specific property*,
it suggests that other oscillators can possibly yield *that property*.

Oscillator Models



Kuramoto Model

$$\dot{\theta}_j(t) = \omega_j + S \sum_{k=1}^{N} K_{jk} \sin\left(\theta_k\left(t - \tau_{jk}\right) - \theta_j(t)\right), \qquad j = 1,2, \ldots, N$$

Stuart-Landau Model

$$\dot{Z}_j(t) = \left\{\lambda_j + i\omega_j - \left|Z_j(t)\right|^2\right\} Z_j(t) + S \sum_{k=1}^{N} K_{jk} Z_k\left(t - \tau_{jk}\right), \qquad j = 1,2, \ldots, N$$

Wilson-Cowan Model

$$\dot{E}_j(t) = -E_j + F\left[C_{EE} E_j - C_{IE} I_j + P_j + S \sum_{k=1}^{N} K_{jk} E_k\right]$$

$$\dot{I}_j(t) = -I_j + F\left[C_{EI} E_j - C_{II} I_j + Q_j\right], \qquad F[x] = \left(1 - e^{-x}\right), \qquad j = 1,2, \ldots, N.$$

Kuramoto/Stuart-Landau models are general/canonical models of the oscillators, and have general properties of which most complex models also have.

Moon, et al. PLOS Comp. Biol. 2015

Kuramoto model

$$\dot{\theta}_j(t) = \omega_j + \frac{K}{N}\sum_{k=1}^{N} A_{jk} \sin\Big(\theta_k(t-\tau) - \theta_j(t)\Big), \qquad j = 1,2,\dots,N.$$

$$Re^{\Theta} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

Correlation and Covariance

Covariance is sum of volume of rectangles (+ if in 1 or 3 quadrants, - if in 2 or 4 quadrants.
Correlation is covariance normalized by it

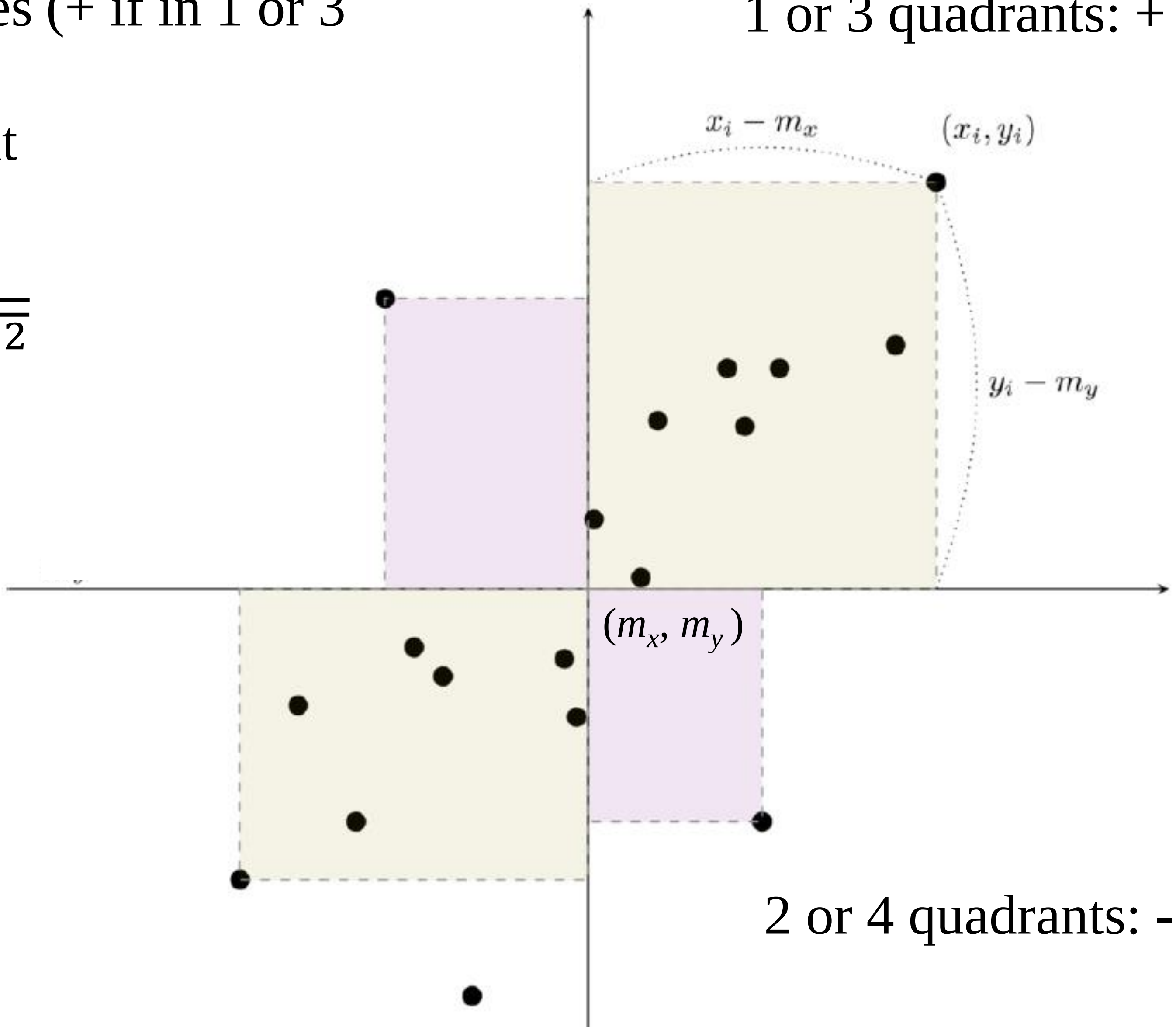
$$C_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

$$C_{xy} = \frac{cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}}$$

$$cov(x,y) = \langle (x_i - \bar{x})(y_i - \bar{y}) \rangle$$

$$var(x) = \langle x_i - \bar{x}^2 \rangle_i$$
$$var(y) = \langle y_i - \bar{y}^2 \rangle_i$$

1 or 3 quadrants: +

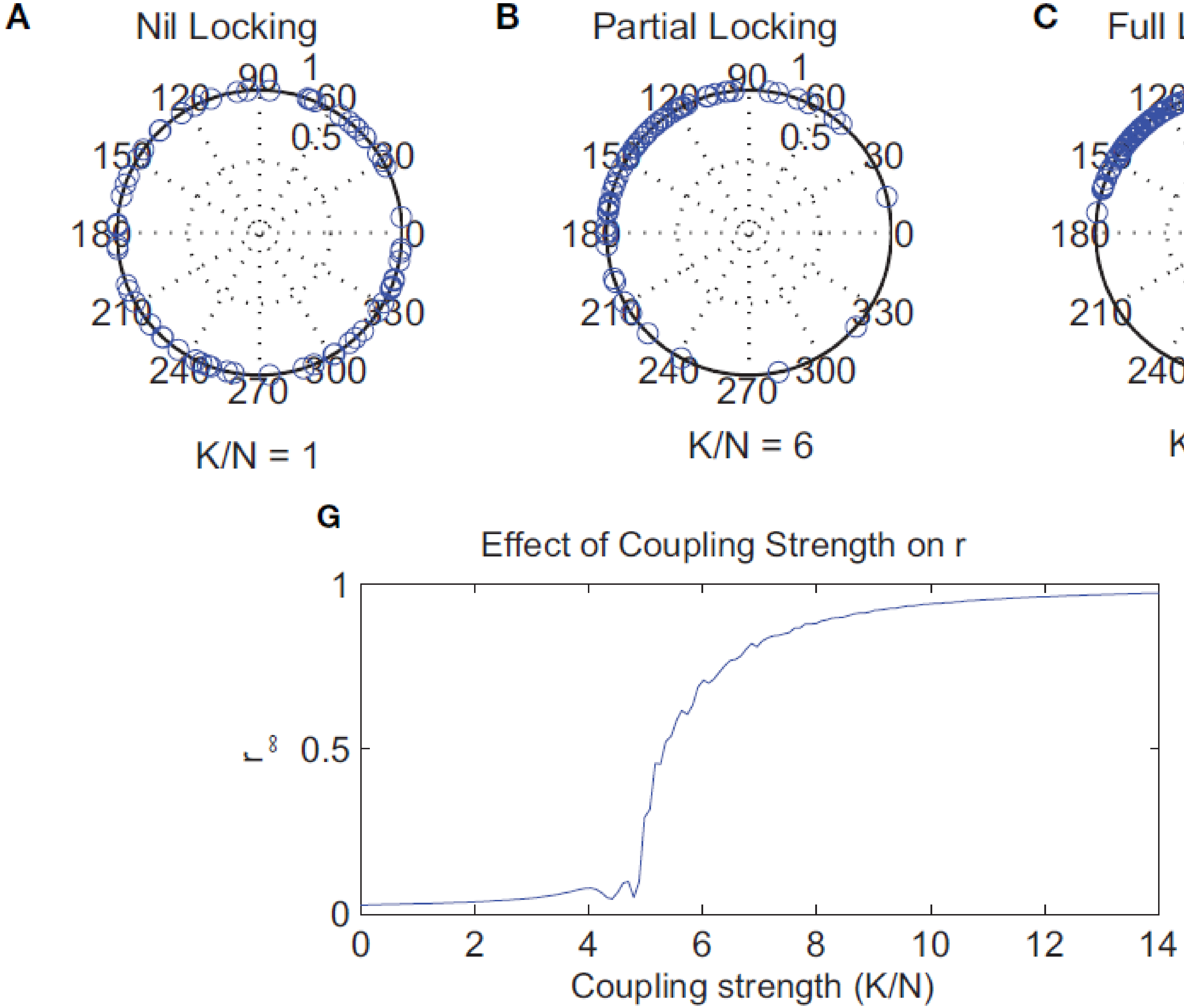


2 or 4 quadrants: -

Kuramoto model

$$\dot{\theta}_j(t) = \omega_j + \frac{K}{N}\sum_{k=1}^{N} A_{jk} \sin\Big(\theta_k(t-\tau) - \theta_j(t)\Big), \qquad j = 1,2,\dots,N.$$

$$Re^{\Theta} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$



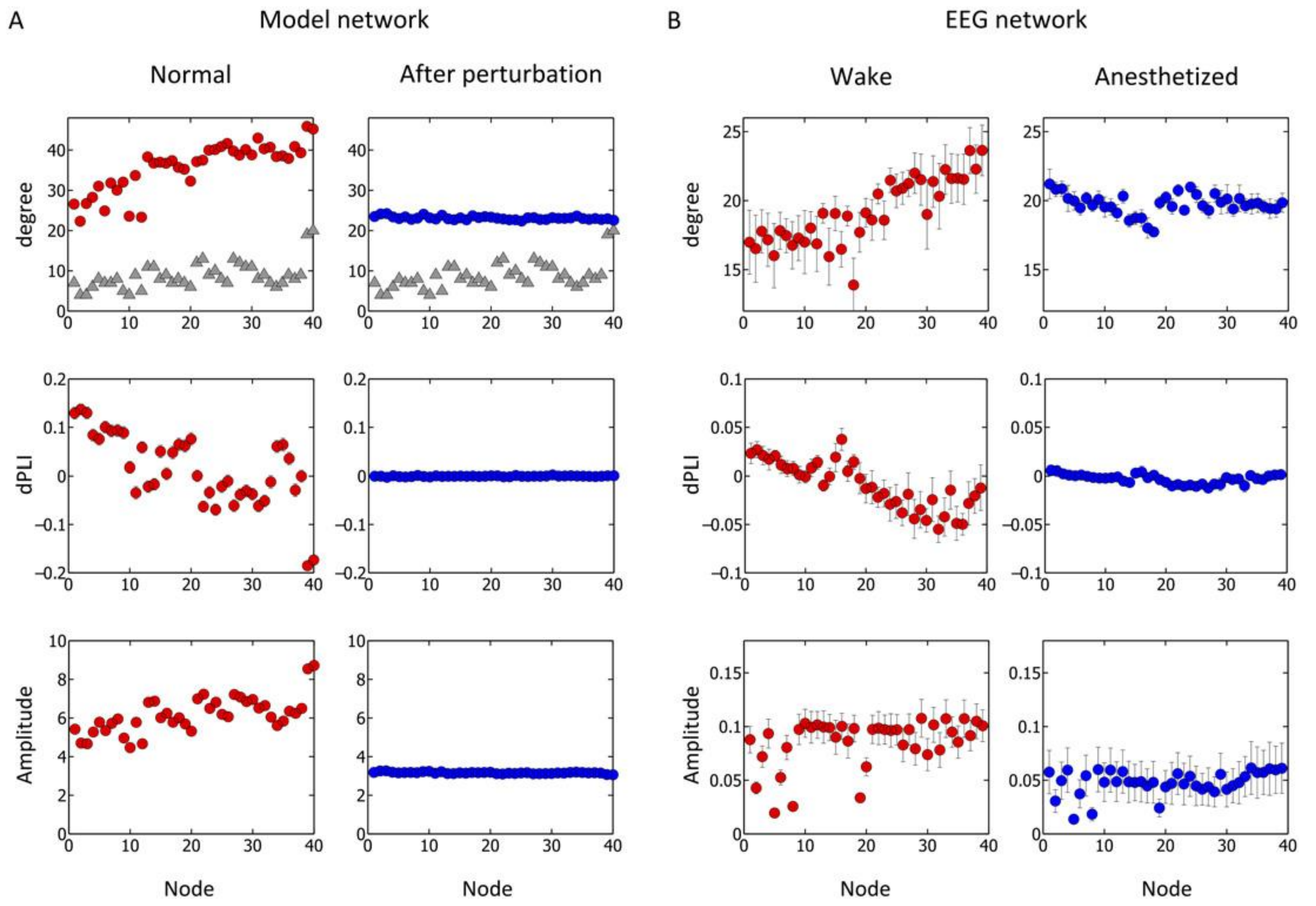
M. Breakspear, et al. Front. Human Neurosci. 2010

Red: degree defined by PLI of each node

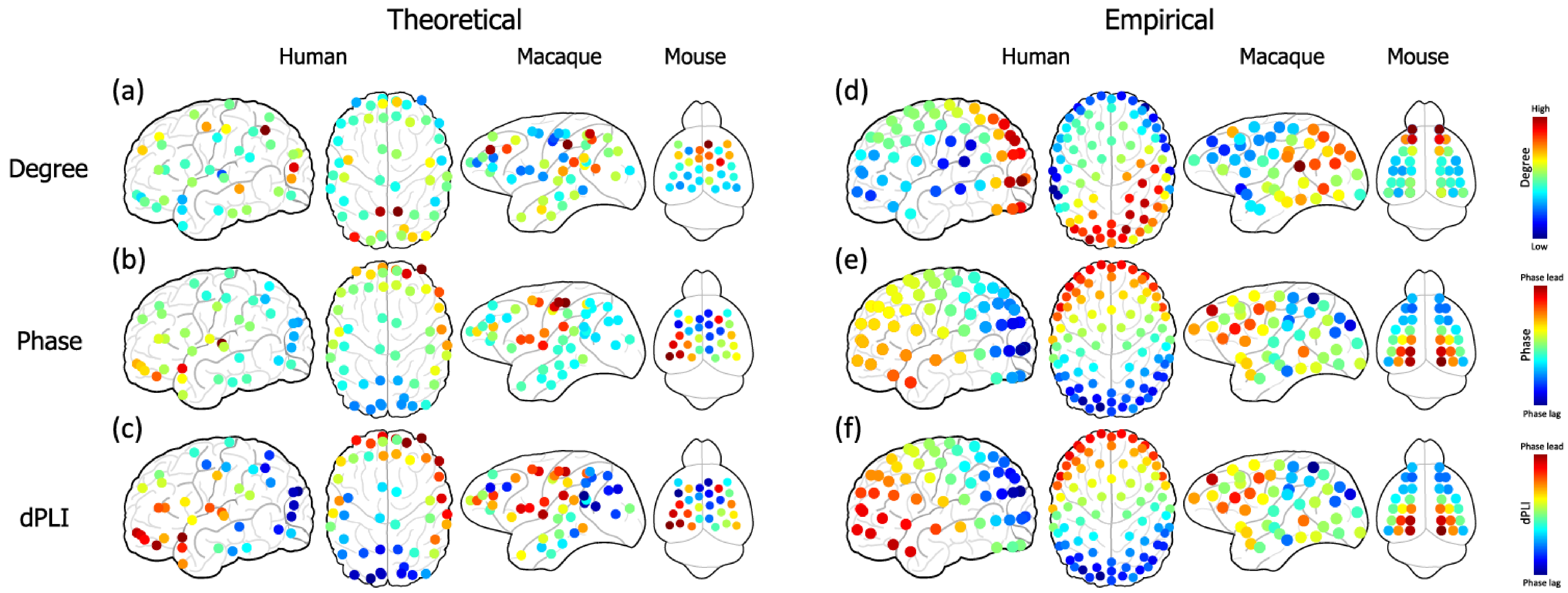
Grey: degree of nodes in str. network

Red: dPLI of each nodes

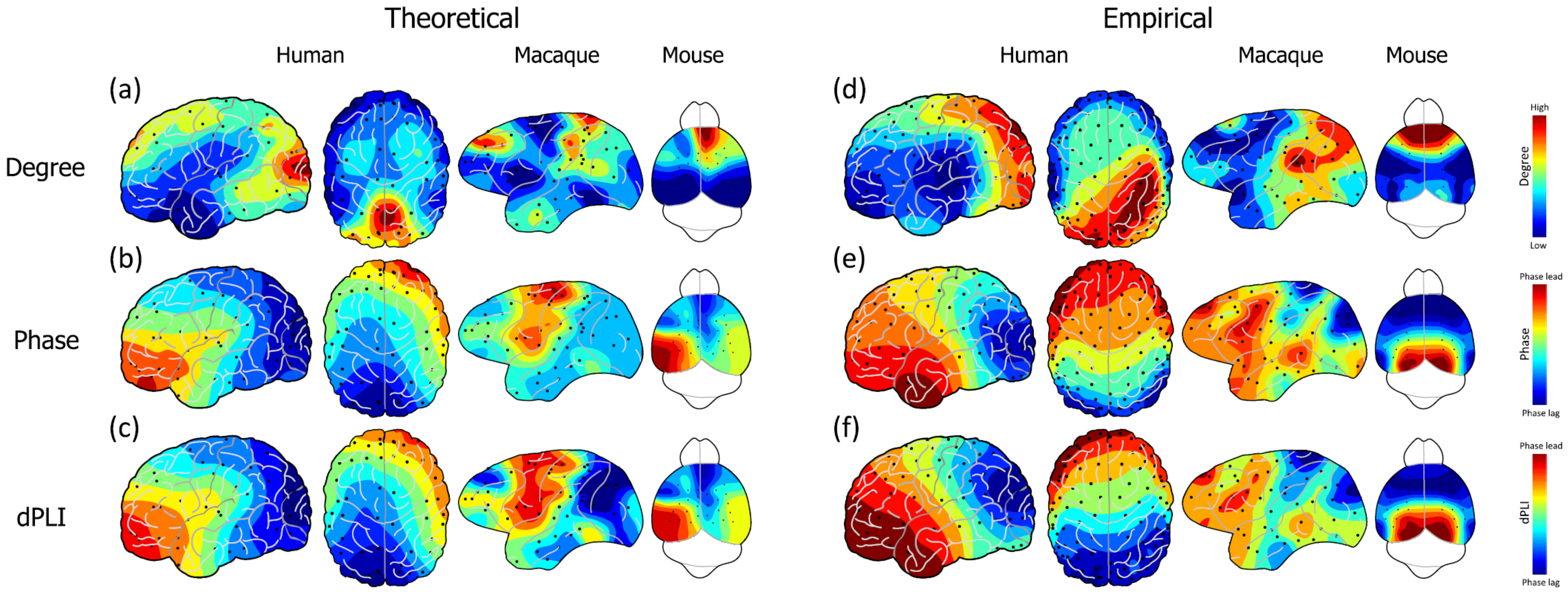
Red: amplitude of each nodes



Moon, et al., PLOS Comp. Biol. 2015



Moon et al., Sci. rep. 2017



Moon, et al., Sci. rep. 2017