I. ANOMALY CANCELATION

From [1]: física por la mañana We use f (f) to denote the general $\mathrm{U}(1)_X$ generation-independent charge assignments of the field f_R (F_L). The three linear anomalies in $\mathrm{U}(1)_X$ [2]

$$\begin{aligned}
[\mathrm{SU}(3)_C]^2\,\mathrm{U}(1)_X: \qquad & [3u+3d]-[3\cdot 2q]=0,\\
[\mathrm{SU}(2)_L]^2\,\mathrm{U}(1)_X: \qquad & -[2l+3\cdot 2q]=0,\\
[\mathrm{U}(1)_Y]^2\,\mathrm{U}(1)_X: \qquad & \left[(-2)^2e+3\left(\tfrac{4}{3}\right)^2u+3\left(-\tfrac{2}{3}\right)^2d\right]-\left[2(-1)^2l+3\cdot 2\left(\tfrac{1}{3}\right)^2q\right]=0,
\end{aligned} \tag{1}$$

allows to express three X-charges in terms of the other two

$$u=-e+\frac{2l}{3}\,,\qquad d=e-\frac{4l}{3}\,,\qquad q=-\frac{l}{3}\,. \tag{2}$$

The quadratic anomaly condition is automatically satisfied, while the mixed gauge-gravitational and cubic anomalies depend of any extra singlet quiral fermions of zero hypercharge, like the right-handed counterpart of the Dirac neutrinos. For N extra quiral fields with X-charge n_α, these conditions read

$$[\mathrm{Grav}]^2\,\mathrm{U}(1)_X:\ \sum_{\alpha=1}^{N}n_\alpha+3(e-2l)=0\,,\qquad [\mathrm{U}(1)_X]^3:\ \sum_{\alpha=1}^{N}n_\alpha^3+3(e-2l)^3=0\,. \tag{3}$$

We choose the solutions with $r\equiv e-2l$, such that

$$\sum_{\alpha=1}^{N}n_\alpha=-3r\,,\qquad \sum_{\alpha=1}^{N}n_\alpha^3=-3r^3\,. \tag{4}$$

The full set of anomaly free SM X-charges in terms of two parameters [2–4] that we choose as l and r, is just

$$u=-r-\frac{4l}{3}\,,\qquad d=r+\frac{2l}{3}\,,\qquad q=-\frac{l}{3}\,,\qquad e=r+2l\,,\qquad h=-r-l\,. \tag{5}$$

where the condition in the charged lepton Yukawa couplings have been used to fix h, and is automatically consistent with the conditions in the quark Yukawa couplings. By setting $l=0$ in the previous equations, we can define the Abelian symmetry in which only the right-handed charged fermions have non-vanishing X-charges as $\mathrm{U}(1)_R$. Then the general anomaly free two-parameter solution can be written as

$$X(r,l)=rR-lY\,. \tag{6}$$

If we now change $f\to f'=f/r$ for all the charged fermion X-charges [4], the first set of anomaly cancellation conditions Eq. (1) remains invariant, and without lost of generality it is always possible

Fields	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$	$\mathrm{U}(1)_{B-L}$	$\mathrm{U}(1)_R$	$\mathrm{U}(1)_D$	$\mathrm{U}(1)_G$	$\mathrm{U}(1)_{\mathcal{D}}$
L	$\mathbf{2}$	-1	l	-1	0	$-3/2$	$-1/2$	0
d_R	$\mathbf{1}$	$-2/3$	$1+2l/3$	$1/3$	1	0	$2/3$	0
u_R	$\mathbf{1}$	$+4/3$	$-1-4l/3$	$1/3$	-1	1	$-1/3$	0
Q	$\mathbf{2}$	$1/3$	$-l/3$	$1/3$	0	$1/2$	$1/6$	0
e_R	$\mathbf{1}$	-2	$1+2l$	-1	1	-2	0	0
H	$\mathbf{2}$	1	$-1-l$	0	-1	$1/2$	$-1/2$	0
N_α	$\mathbf{1}$	0	n_α	n_α	n_α	n_α	n_α	n_α

TABLE I: General one-parameter solution with some examples of rational solutions ($X = B-L, R, D, G$ and $\mathcal{D}$) for the radiative type-I seesaw realization of the effective operator $\mathcal{O}_{6D}$ for Dirac neutrino masses. The last column requieres the condition $\sum_{\alpha=1}^{N} n_\alpha = 0$.

to normalize the solutions such that the last set Eq. (4) is just

$$\sum_{\alpha=1}^{N} n'_\alpha = -\,3\,, \qquad\qquad \sum_{\alpha=1}^{N} n'^3_\alpha = -\,3\,. \tag{7}$$

For example, the solution with $r=3$: $n_\alpha = (-2,-2,-4,-1)$ [3] can be easily normalized to the form in Eq. (7) with $f \to f/3$ to $n'_\alpha = (-2/3,-2/3,-4/3,-1/3)$ as used in Ref. [5]. In this way, without lost of generality, we will work with the normalized solution in terms of a single parameter [6–8] that we choose to be l, by setting $r=1$ as summarized in column $\mathrm{U}(1)_X$ of Table I, which is just

$$X(l) = R - l\,Y\,. \tag{8}$$

In particular, this includes the solution $n_\alpha = (-4,-4,+5)$ [3].

II. DISCRETE SYMMETRIES

The possible charge assignments to a field ϕ under a Z_N symmetry are

$$1, w, w^2, ..., w^{N-1}, \ \text{ with } \ w = \exp(i2\pi/N). \tag{9}$$

We assume the existence of k scalar complex fields ϕ_α, $\alpha = 1,2,...,k$, each transforming as

$$\phi_\alpha \sim w^\alpha, \ \text{ with } \ k \leq N/2. \tag{10}$$

Note that $(w^\alpha)^* = w^{-\alpha} = w^{N-\alpha}$.

Consider one $\mathrm{U}(1)_X$ which is broken by a singlet scalar field S_N and one dark scalar field ϕ_1 where the subscript indicates the X-charge of the field. In addition to the self-conjugate terms the gauge symmetry allows the following term [9]

$$\mathcal{L} \propto S_N^\dagger \phi_1^N + \text{h.c}\,. \tag{11}$$

When S_N acquires a vev, the Lagrangian preserves a discrete Z_N subgroup of $\mathrm{U}(1)_X$ under which the dark scalar field transform as

$$\phi \to \phi' = \mathrm{e}^{2\pi i k/N}\,\phi_1\,, \qquad\qquad k = 0, 1, 2, \ldots N-1\,. \tag{12}$$

Then the remnant symmetry will be exactly conserved at all orders including non-renormalizable terms. Note that the Z_N symmetry can also appears accidentally. In this case, in contrast to the discrete gauge symmetries, these accidental symmetries may be violated at the non-renormalizable level, and consequently, the DM stability can be only granted at the renormalizable level [9].

For the Z_2 imposed usually by hand, we can identify this a gauge discrete symmetry associated to Lagrangian of the Singlet Scalar Dark Matter model (SSDM), ϕ_1, supplemented by the interaction with the singlet scalar S_2 which spontaneously breaks some $\mathrm{U}(1)_X$ gauge symmetry. The relevant term in the Lagrangian is

$$\mathcal{L} \subset \lambda S_2^* \phi_1 \phi_1 + \text{h.c.}\,. \tag{13}$$

Note that in this case ϕ_1 needs to be complex. However, this Lagrangian term induces a mass splitting between the real an imaginary components of ϕ_1. In this way we can have the usual SSDM with a real scalar.

For Z_4 as remnant of the spontaneous breaking of some $\mathrm{U}(1)_X$, we can have two-component dark matter depending on the mass hierarchy of the dark sector [9, 10]. For larger Z_N see [9]. In [11] is claimed that the heterogeneous self-interaction is a natural consequence of any 2DM or nDM models. In nDM models there can be various dark matter (DM) annihilation processes that are different from the standard DM annihilation process [12]. In particular it is possible to have an assisted freze-out which further reduce the direct detection signals [13]

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