R-C Circuits

PHYS 296

Your name____________________ **Lab section____________________**

PRE-LAB QUIZZES

1. What will we investigate in this lab?

2. For the R-C circuit shown in Figure 1 on Page 2, the capacitor is initially not charged and a DC voltage is applied to the circuit at $t = 0$, such that $V(t) = 0$ for $t < 0$ and $V(t) = V_0$ for $t \geq 0$. Obtain the formula for the charge $q(t)$ on the capacitor and the current, $I(t)=dq(t)/dt)$, in the circuit.

3. What is the time constant of the R-C circuit?

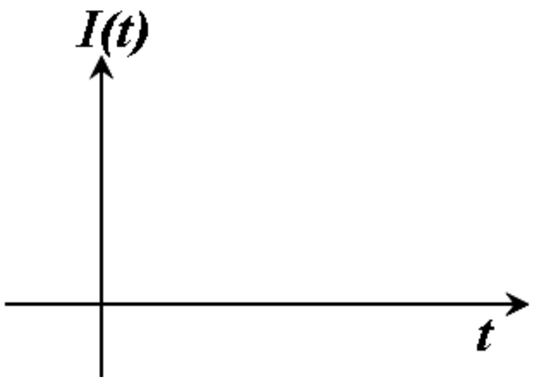
4. What is the current in the circuit at $t = 0$? Does it immediately reach the maximal value?

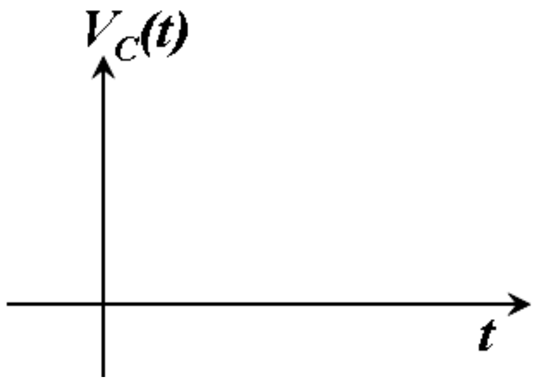
5. What is the magnitude of the current in the circuit?
   at $t = RC$: at $t = \infty$:

6. What is the voltage on the capacitor at $t = 0$? Does it immediately reach maximum?

7. What is the magnitude of the voltage $V_C(t)$ on the capacitor?
   at $t = RC$: at $t = \infty$:

8. Draw the $I(t)$ and $V_C(t)$ curves in the following graphs.





R-C Circuits
PHYS 296

Name_____________________________ **Lab section**_______________________

Lab partner's name(s)____

Objective

In this lab, we study the R-C circuits and simple application, and measure the time constant of the basic R-C circuit.

Background

When establishing an electric potential difference of V between the two terminals of a capacitor of capacitance C, the two terminals will accumulate charges of the opposite signs but of the same magnitude, $q = V/C$. In an idealized scenario for a completely isolated capacitor, q changes instantaneously when V changes, i.e., one can instantaneously charge or discharge a capacitor! In practice, one can never test on an isolated capacitor. Moreover, an isolated capacitor is not thrillingly useful!

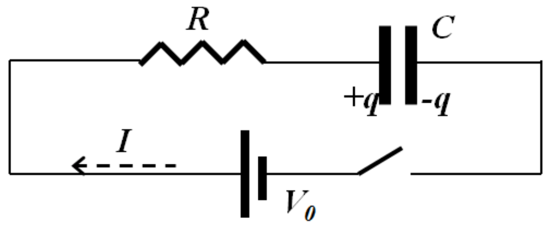


Figure 1. A basic R-C circuit.

Now consider a basic R-C circuit (Figure 1) consisting of one capacitor of capacitance C and one resistor of resistance R. Because of the resistor, the current (I) in the circuit cannot be infinitely large. Therefore, discharging/charging the capacitor (note: discharging/charging must proceed through current) does not occur instantaneously. For the same voltage, a larger R leads to a smaller I and thus leads to a longer discharging/charging time. To change the voltage across the capacitor by the same magnitude, a larger C leads to a larger change of q and thus also leads to a longer discharging/charging time. Therefore, larger R or C leads to larger time constant for the R-C circuit. Such relationships make the time constant a controllable and useful parameter!

For the shown R-C circuit, the sum of the voltage across the resistor, $V_R(t)=I(t)R$, and the voltage across the capacitor $V_C(t)=q(t)/C$, is:

$$V(t) = I(t)R + q(t)/C. \tag{1}$$

Because the conservation of charge, $I(t) = dq/dt$, Equation (1) can be rewritten as:

$$V(t) = \frac{dq(t)}{dt}R + \frac{q(t)}{C}. \tag{2}$$

Charging a Capacitor Turn on the switch at $t = 0$. The total DC voltage changes from 0 to V_0 at $t = 0$, such that $V(t) = 0$ for $t < 0$ and $V(t) = V_0$ for $t \geq 0$. Solving Equation (2), one has

$$q(t) = CV_0(1 - e^{-t/RC}). \tag{3}$$

Therefore, the current through the circuit is

$$I(t) = \frac{V_0}{R} e^{-t/RC}, \tag{3'}$$

and the voltage across the resistor and the voltage across the capacitor are respectively as

$$V_R(t) = \frac{dq(t)}{dt} R = V_0 e^{-t/RC} \quad (t \geq 0), \tag{4}$$

$$V_C(t) = \frac{q(t)}{C} = V_0 (1 - e^{-t/RC}) \quad (t \geq 0). \tag{5}$$

Discharging a Capacitor The switch is on for a much longer time period than RC such that the capacitor is approximately fully charged – we will now take this as our 'initial' condition. Turn off the switch at $t = 0$. The total DC voltage changes from V_0 to 0 at $t = 0$, namely, $V(t) = V_0$ for $t < 0$ and $V(t) = 0$ for $t \geq 0$. The solution of Equation (2) is:

$$q(t) = CV_0 e^{-t/RC} \quad (t \geq 0). \tag{6}$$

Therefore, the current through the circuit is

$$I(t) = -\frac{V_0}{R} e^{-t/RC}, \tag{6'}$$

and the voltage across the resistor and the voltage across the capacitor are respectively as

$$V_R(t) = \frac{dq(t)}{dt} = -V_0 e^{-t/RC} \quad (t \geq 0), \tag{7}$$

$$V_C(t) = \frac{q(t)}{C} = V_0 e^{-t/RC} \quad (t \geq 0). \tag{8}$$

Note the opposite signs in Equations (4) and (7). As explicitly shown by the above equations, one cannot instantaneously charge or discharge the capacitor in an R-C circuit. Instead, the voltage across the capacitor increases/decreases according to the time constant of the circuit, $\tau_0 = RC$.

Now imagine an AC wave with low frequency or high frequency modulation is applied in the R-C circuit, the effect will substantially depend on the relationship between the time constant of the circuit and the modulation frequency. Mathematically, we can solve the first-order differential equation (2) for a general case: $V(t) = 0$ for $t < 0$, $V = V(t)$ for $t \geq 0$, and initially the capacitor is not charged. The solution is:

$$q(t) = e^{-t/RC} \int_0^t \frac{V(t)}{R} e^{t/RC} dt\,. \tag{9}$$

Two extreme cases are of interest and will be studied in this lab:

(A) When $V(t)$ varies much slower than $e^{t/RC}$, the integral in Equation (9) behaves like $(e^{t/RC} - 1)$ and $q(t)$ behaves like $(1 - e^{-t/RC})$. The voltage on the capacitor will behave like $(1 - e^{-t/RC})$. The voltage on the resistor will behave like $e^{-t/RC}$ which is a much narrower pulse than $V(t)$. Therefore, a broad pulse of $V(t)$ applied to the circuit will result in a much narrower pulse on the resistor.

(B) When $V(t)$ varies much faster than $e^{t/RC}$, the integral in Equation (9) behaves like the average of $V(t)$. Therefore, a fast alternating AC wave of $V(t)$ applied to the circuit will result in a DC-like wave on the capacitor.

PART I Time Constant of the Basic R-C Circuit

In part I of the lab, we will measure the time constant of the basic R-C circuit consisting of a resistor and a capacitor.

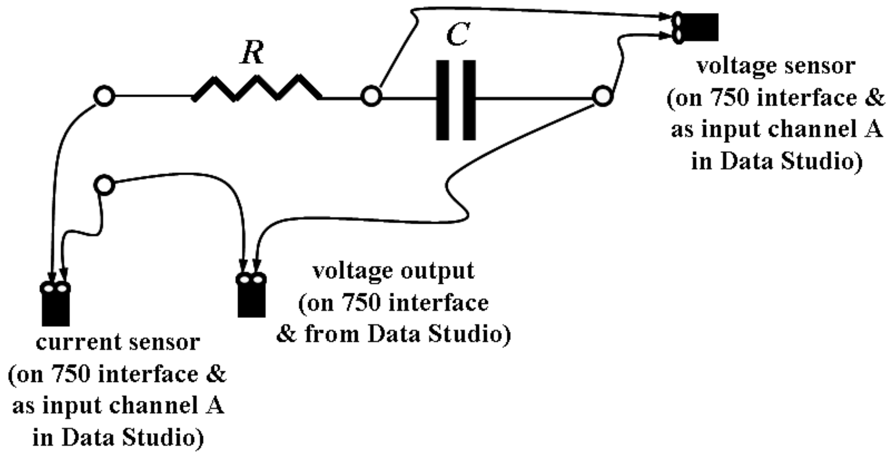


Figure 2. Setting up a basic R-C circuit.

PROCEDURES

1. Set up the R-C circuit as shown in Figure 2. In order to accurately measure the time constant, you should use the resistor on the supplied second board with the largest resistance (about *1000* ohm). In ***Data Studio***, set input *channel A* to measure the voltage on the capacitor (*330* μF) and input *Channel B* to measure the current in the circuit. A *DC voltage output* will be supplied by ***Data Studio*** for the circuit. Proper connections between ***Interface 750*** and the circuit are shown in Figure 2.
2. In ***Data Studio***, open a graph display and select voltage (*channel A*) for the y-axis and time (default) for the x-axis. Open a second graph display to display the current (*channel B*) as the y-axis and time as the x-axis. Label the axes and specify the scales and units.
3. Open the *signal generator* and set DC output at *5 V*.
4. Click on the *start* button and stop data collection when you have enough data points. To accurately measure the current curve, you may need to calibrate the current sensor. Ask your TA for help.
5. Fit the curves in both graphs and obtain the time constant of the circuit.

Data Analysis

1. Fit the *current-versus-time* curve and obtain the time constant of the circuit.

   Time constant =

2. On the *current-versus-time* curve, find the time at which the current decreased to 37% ($\approx e^{-1}$) of the initial current. Mark this point in the graph.

   The *time constant* determined by this method =

3. Fit the *voltage-versus-time* curve and obtain the time constant of the circuit.

Time constant =

4. On the *voltage-versus-time* curve, find the time at which the voltage has increased to *64%* ($\approx 1- e^{-1}$) of the maximal voltage. Mark this point in the graph.

The *time constant* determined by this method =

5. Using the values of the resistance and capacitance, calculate the *time constant* of the circuit:

R =

C =

Time constant =

6. Compare the calculated *time constant* using the R and C values with the obtained *time constants* from the measurement using the four different analysis methods. Calculate the % errors for the experimentally determined *time constants*.

PART II Applications of the R-C Circuit

In part II, we will use the R-C circuit (a) to shape electrical pulses and (b) to convert AC signal to DC signal.

II-A PROCEDURES

1. In this experiment, we demonstrate that the R-C circuit can be used to change a broad electric pulse into a narrow pulse. Set up the circuit as shown in Figure 3 to measure the voltage on the resistor of about 50 ohm. Select *square wave* as the output. Set the amplitude of the square wave to *2.0 V*. Set the period of the square wave somewhere in the range *10-100* times the time constant of the circuit. (Note: what you can change is actually the frequency of the square wave. So you may set the frequency for the square wave to 2 Hz.) Open the Scope display in *Data Studio*. Use scope to display both the output voltage and the voltage on the resistor. Set the scale of the horizontal axis at 200 ms/div. Select the output voltage as the trigger source and use the trigger tool to set trigger to "rising" and to "0.0 V". Click on the start button. Print out the graph.

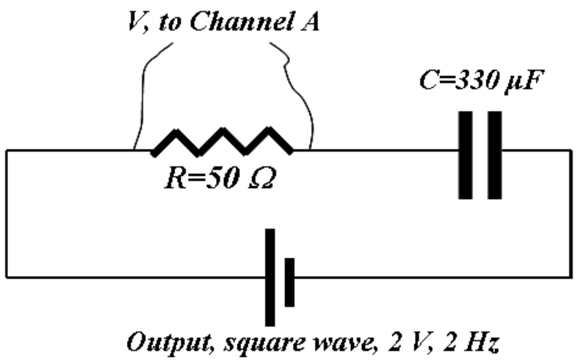


Figure 3

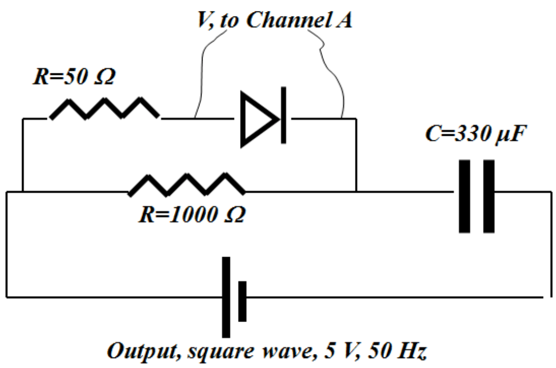


Figure 4

QUESTIONS

1. Why did the applied square wave lead to a series of periodic narrow pulses for the voltage on the resistor?

PROCEDURES

2. Set up the circuit as shown in Figure 4. Now, we are going to measure the voltage over the diode. Follow procedure 1 and observe the change for the series of periodic narrow pulses. Print out the graph. Reversing the connections for the diode, do the periodic narrow pulses change the sign?

II-B

PROCEDURES

Set up the circuit as shown in Figure 2 and change the resistor to the 200 ohm resistor. A DC power supply can convert high-frequency AC voltage into DC voltage using an R-C circuit. For this experiment, the time constant of the circuit should be much larger than the period of the AC wave. Open the *signal generator* and select *positive square wave* as output and the amplitude at *2.0 V*. Change the connections in Figure 2 to measure the voltage on the capacitor. Open oscilloscope to display both the output voltage and the voltage on the capacitor. Select the output voltage as the trigger source and use the trigger tool to set trigger to "rising" and to "0 V". Click on the start button. Observe how the wave shape of the voltage on the capacitor changes as you change the frequency of the AC output from 5 to 10, 50, 100, 200, 250, 500 Hz. Note that as the frequency increases the voltage over the capacitor approaches a constant value with small variations known as ripples.

QUESTIONS

1. Describe how the ripples in the voltage signal over the capacitor change with frequency.

2. Describe how the time constant of the circuit affects the ripples?