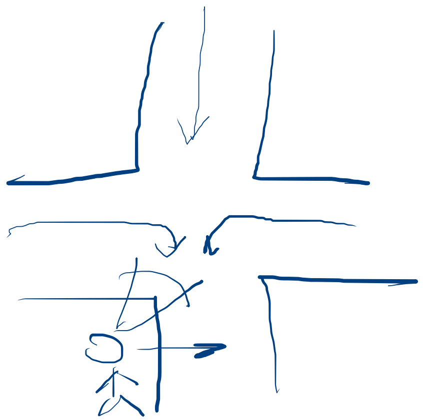


H   T

p $1-p$

$$S = \{H, T\}$$

evento cara $\in S$

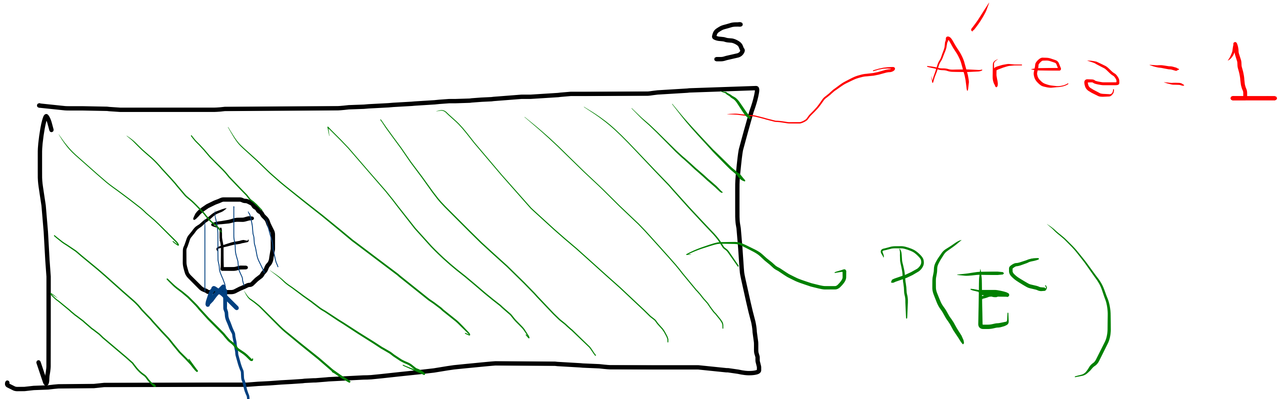
$$P(E) \in [0, 1]$$

$$P = \frac{\text{casos favorable}}{\text{casos posibles}}$$

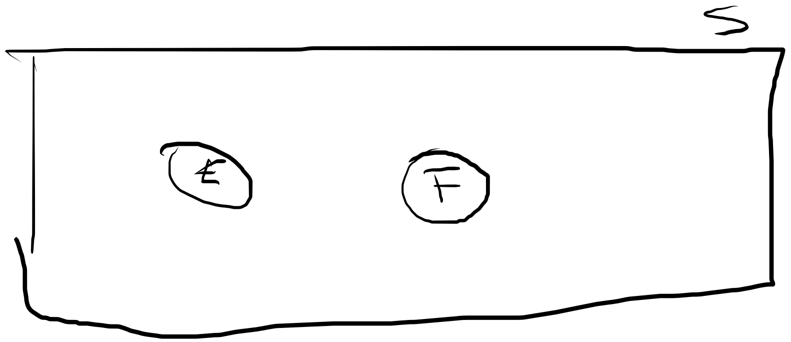
estadíst.

$$P_{\text{moneda balanceada}} = \frac{1}{2} = \frac{\sum \text{cecas}}{\sum \text{total}} = \frac{\sum \text{caras}}{\sum \text{total}}$$

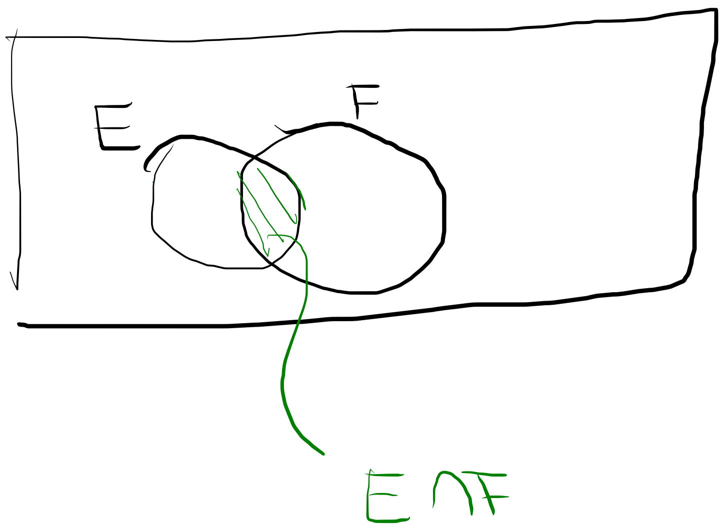
$$P(S) = 1$$



$$P(E) \leq 1$$

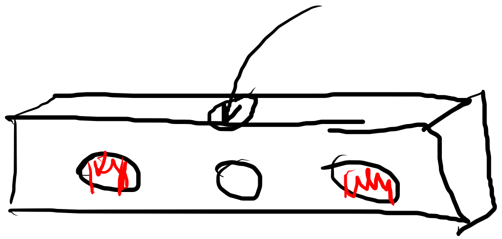


$$P(E \cup F) = P(E) + P(F) - \underbrace{\overbrace{P(E \cap F)}^{\emptyset}}_{0}$$



$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

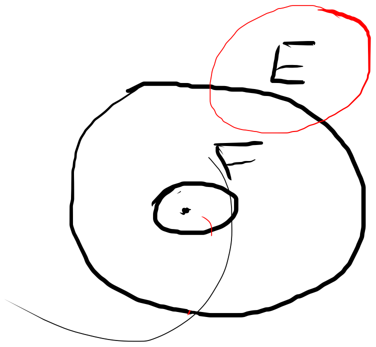
Probabilidad Condicional



$$P(\text{blanca}) = \frac{1}{3} \rightarrow \text{a priori}$$

$$P(\text{blanca} \mid \underbrace{\text{saqué roja}}_{\substack{\text{condición} \\ \text{u observación}}}) = \frac{1}{2} \rightarrow \text{a posteriori}$$

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$



En este caso: $P(E|F) = 1$

$F \subset E$

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(\cancel{F})}{P(\cancel{F})} = 1$$

$P(A\cap B)$

$$P(A|B) = \frac{\overbrace{P(A\cap B)}^{\text{intersección}}}{\underbrace{P(B)}_{\text{normalización}}}$$

Independencia estadística

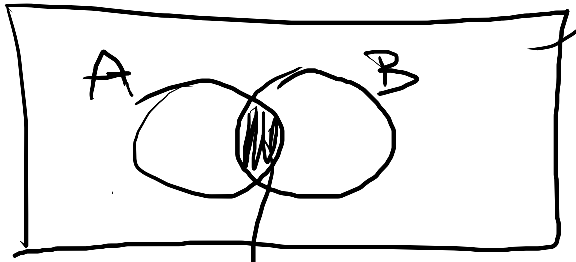
$P(A|B) = P(A)$ ← definición

$$P(A\cap B) = P(A)P(B)$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$ ← Bayes

$$\underbrace{P(A|B)}_{P(A)}P(B) = P(A\cap B) \iff$$ A, B son independientes.

$P(S) = 1$



$P(A)P(B)$

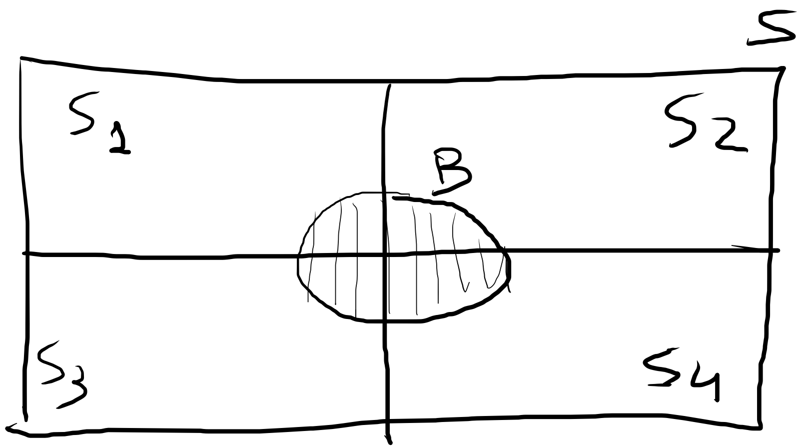
inversión del problema inicial

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\underbrace{P(F|E)}_{\text{verosimilitud}}\, P(E)}{P(F)}$$

a priori

total de F

verosimilitud



$$S = \cup S_i \quad i = 1, 2, 3, 4$$

$$\cap S_i = \emptyset$$

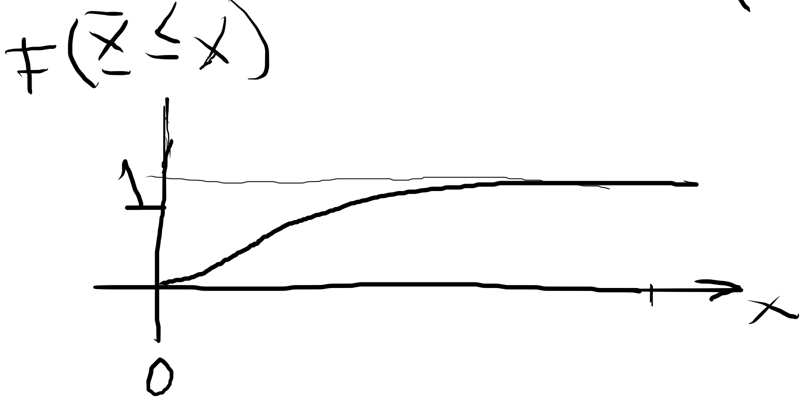
$$P(B) = P(B|S_1)P(S_1) + P(B|S_2)P(S_2) + P(B|S_3)P(S_3) + P(B|S_4)P(S_4)$$

Probabilidad total.

X es la v.a.

x es el valor que toma X

$$F(X \le x) = \begin{cases} \ge 0 \wedge \le 1 \\ \text{no decreciente, monótona.} \end{cases}$$



$$F_X(x) = \int_{-\infty}^{x} f(t)\,dt$$

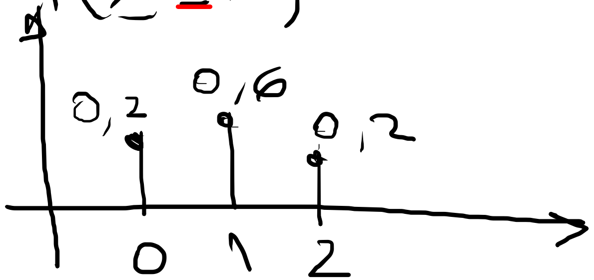
probability density function (pdf)

$$\frac{dF}{dx} = f(x)$$

pdf.

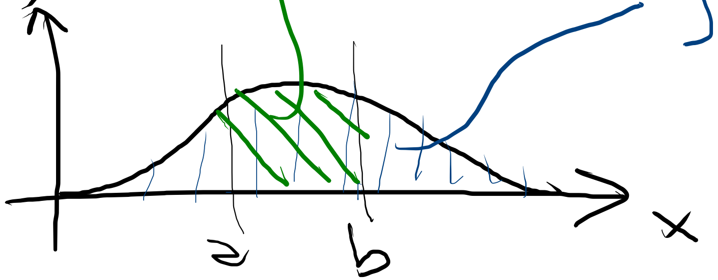
PDF

$P(\underline{X}=x)$



$\underline{X}$ es discreta

$P(\underline{X}\in[a,b])$



$$\int_{-\infty}^{+\infty} p(x)\,dx = 1$$

$\underline{X}$ es continua

$$P(\underline{X}\in[a,b]) = F(b) - F(a) =$$

$$= \int_a^b p(x)\,dx$$

Distribución conjunta

$\underline{X}, \underline{Y} \to f_{\underline{X}\underline{Y}}$

$$P(\underline{X} \leq x, \underline{Y} \leq y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{\underline{X}\underline{Y}}(x,y)\,dy\,dx = F(x,y)$$

Distribución Marginal

$$F_{\underline{X}}(x) = P(\underline{X} \leq x, \underline{Y} \leq +\infty) = \int_{-\infty}^{x} \underbrace{\int_{-\infty}^{+\infty} f_{\underline{X}\underline{Y}}(x,y)\,dy}_{f_{\underline{X}}(x)}\,dx$$

Distrib. Condicionales

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

X, Y indeptes $\Rightarrow$ $f_{X|Y} = f_X \leftarrow$ marginal

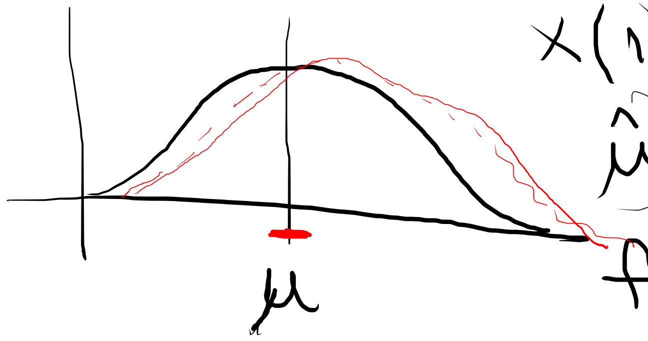
$$\underset{\substack{\uparrow \\ \text{conjunta}}}{f_{XY}} = \underbrace{f_X \cdot f_Y}_{\text{marginales}}$$

Si $\underline{X} = \underline{Y}$

$P_{\underline{X}|\underline{Y}} = P_{\underline{X}|\underline{X}} = 1$

está lloviendo

esté lloviendo



$x(1) \ldots x(m)$

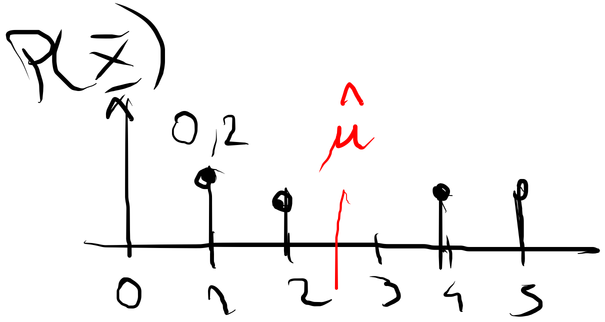
$\hat{\mu} \to \mu$ real ?

$f_{\underline{X}} \to$ conocida $\to \overbrace{x(1) \ldots x(m)}^{\checkmark} \mid \not{\mu}, \overbrace{f_{\underline{X}}}^{\checkmark}$

$f_{\mu} \to ?$

Esperanza

$$\mu = E[X] = \sum_i x_i \, P(X = x_i)$$



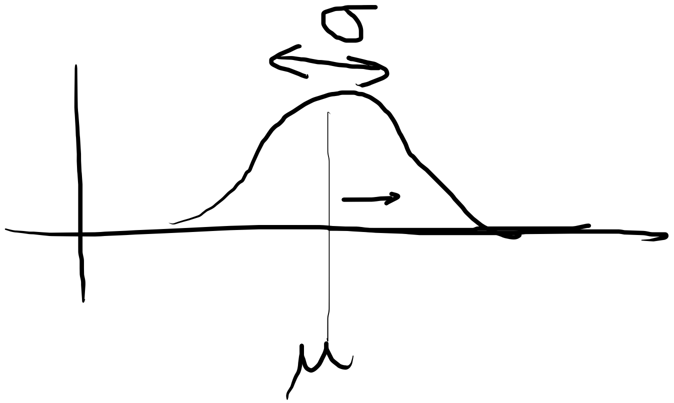
$$\mu = \int_{-\infty}^{+\infty} x \, f_X(x) \, dx$$

Centro de masa

Varianza

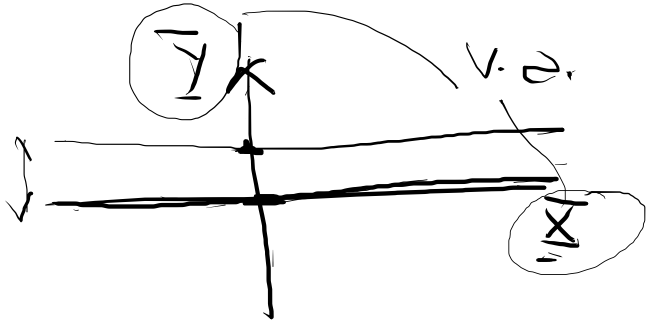
Variabilidad respecto de la media.

$$\operatorname{var}[X] = \sigma_X^2 = E\left[(x-\mu_X)^2\right] = \int_{-\infty}^{+\infty} \underbrace{(x-\mu)^2}_{\text{costo}} \underbrace{f_X(x)}_{\text{peso}} \, dx$$

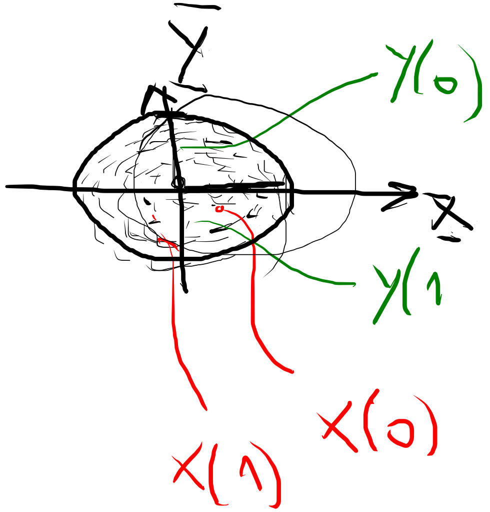


Dispersión $\Rightarrow \sigma = \sqrt{\sigma^2}$

Índice de correlación

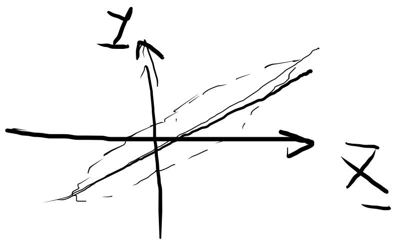


$$\rho_{XY} = \frac{cov[X,Y]}{\sigma_X \, \sigma_Y} \in [-1,1]$$

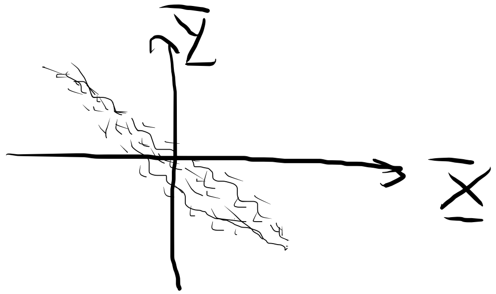


X, Y descorrelacionados $\Rightarrow \rho = 0$

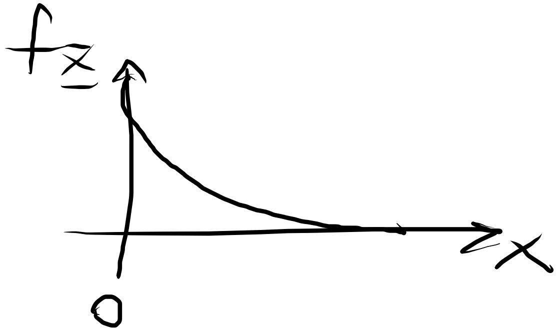
$\rho \to 1$



$\rho \to -1$

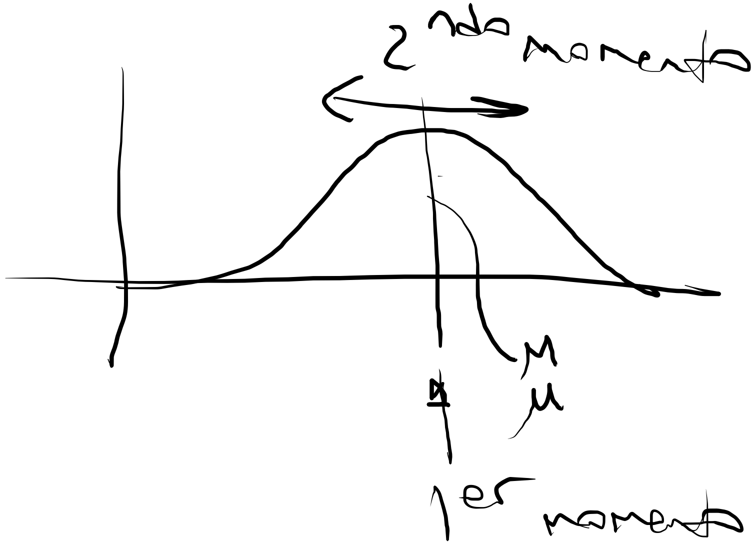


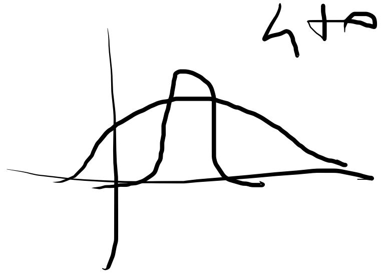
$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



$$E[X] = \frac{1}{\lambda}$$

$$var[X] = \frac{1}{\lambda^2}$$





4to momento

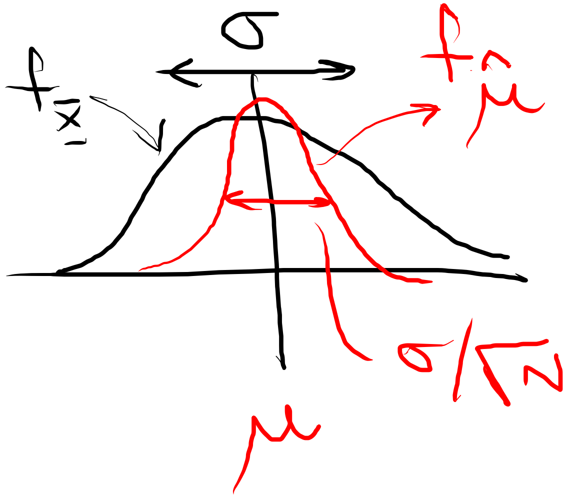
$$\underline{X}_i \sim f_{\underline{X}}(\mu, \sigma^2) \quad \boxed{iid}$$

$$Y = \frac{\underline{X}_1 + \underline{X}_2 + \cdots + \underline{X}_N}{N} = \hat{\mu} \longrightarrow \mathcal{N}\left(\mu, \frac{\sigma^2}{N}\right)$$

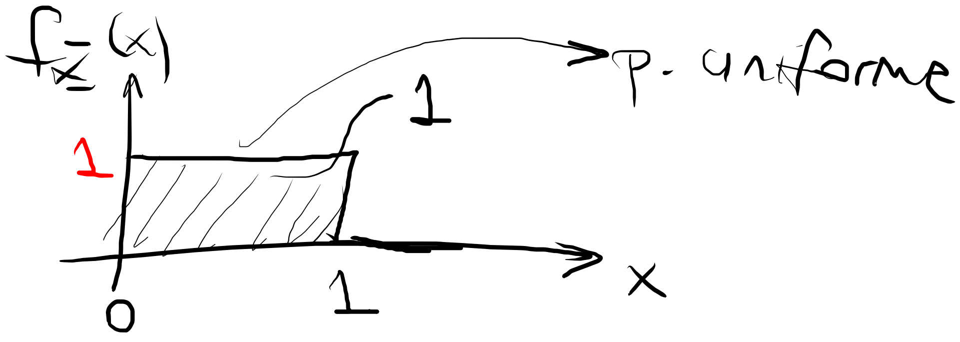
$$E[Y] = \frac{1}{N} \underbrace{E[\underline{X}_1 + \cdots + \underline{X}_N]}_{N \cdot \mu} = \mu$$

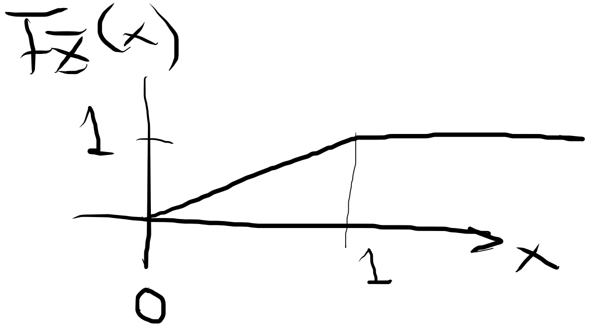
$$\mathrm{var}[Y] = \mathrm{var}\left[\frac{1}{N} \cdot (x_1 + \cdots + x_N)\right] = \frac{1}{\cancel{N^2}} \cancel{N} \sigma^2 = \frac{\sigma^2}{N}$$

var. atenuada por $1/N$

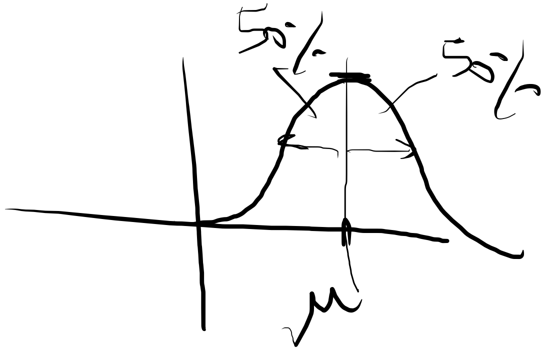


Distrib. Uniforme





Distrib. Normal



$$Z \sim N(\overset{\mu_Z}{0}, \overset{\sigma_Z^2}{1})$$

$$\boxed{X = \sigma Z + \mu}$$

$$E[X] = \mu$$

$$\mathrm{var}[X] = \sigma^2$$

Cómo tiramos una moneda en la simulación

p: prob cara $\leftarrow$ parámetro

```
if ( rand() ≤ p )
    cara
else
    ceca
```