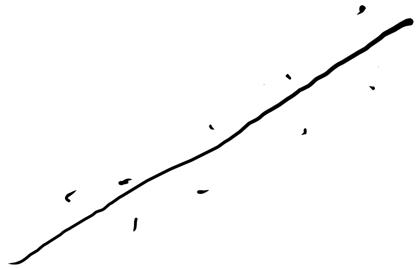
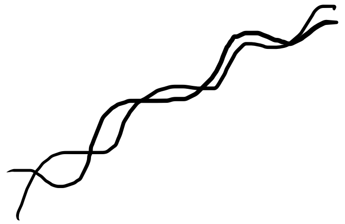
$$X = Y + W$$

$$E[(X-c)^2] \qquad E[(X-c)^2 \mid Y]$$

$$c = E[X] \qquad E[X \mid Y]$$

$X \qquad Y$





$\{x_i\} \; \{y_i\}$

$$y_i = a x_i + b$$

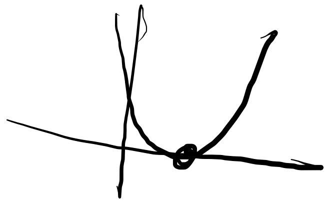
V grandes

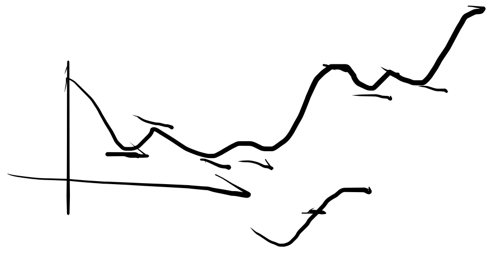
$$\frac{\sum_i y_i / v_i}{\sum_i 1/v_i} \qquad v_i = K \qquad \frac{\cancel{1/K} \sum y_i}{\cancel{1/K} \underbrace{\sum_i 1}_{m}} = \frac{1}{m}\sum_{i=1}^{m} y_i = \hat{y}$$

[$m > 30$

[$m > 100$

$$\boxed{a_i^* \quad b^*}$$





$$J = \frac{1}{2} E\left[(x - a_1 y_1 - a_2 y_2 - \cdots - a_n y_n - b)^2\right]$$

$$x = \sum_i a_i y_i + b$$

$$a_i^* = \frac{1/\sigma_i}{1/\sigma + \sum_i 1/\sigma_i} \qquad b^* = \frac{\mu/\sigma}{1/\sigma + \sum_i 1/\sigma_i}$$

$$\boxed{\frac{\partial J}{\partial a_i} = 0 \qquad \frac{\partial J}{\partial b} = 0}$$

$$\sum_i \frac{1/\sigma_i}{1/\sigma + \sum_i 1/\sigma_i} - 1 = \frac{-1/\sigma}{(\qquad)}$$

$$\sum_{i=1}^{n} a_i^* - 1 = -\frac{b^*}{\mu}$$

$$\frac{\partial}{\partial a_i}\left[E[x] = \int x f_X(x)\, dx\right] a_i$$

$$\left.\frac{\partial J}{\partial b}\right|_{a_i^*, b^*} = E\left[\left(-\frac{b^*}{\mu}\right) X + \underbrace{\sum_{i=1}^{m} a_i^* W_i}_{=0} + b^* \right] = 0$$

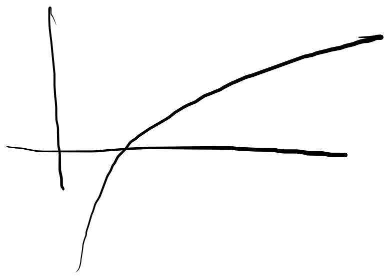
$$E\left[\left(-\frac{b^*}{\mu}\right) X + b^* \right] = 0 \quad \checkmark$$

$$E[X] = \mu$$

$$a_i^*, b^* \quad \checkmark \quad \text{LS}$$

$$\left.\frac{\partial J}{\partial a_i}\right|_{a_i^*, b^*} = \qquad = 0 \quad \checkmark$$

$\binom{m}{k} p^k (1-p)^{m-k}$

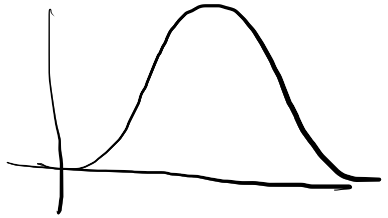


Máx. Verosimilitud

$m = 100 \quad k = 55$

$\frac{\partial}{p} \log[\quad] = \frac{k}{m} = 0{,}55$

$\mu, \sigma \to X$

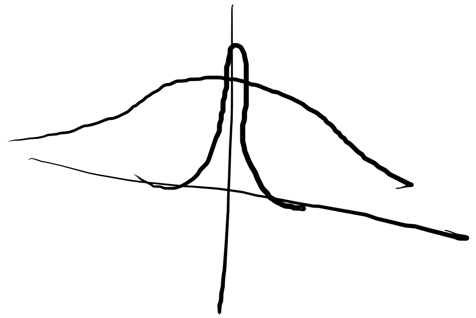


MV(ML) : dato X , variable p

μ, σ

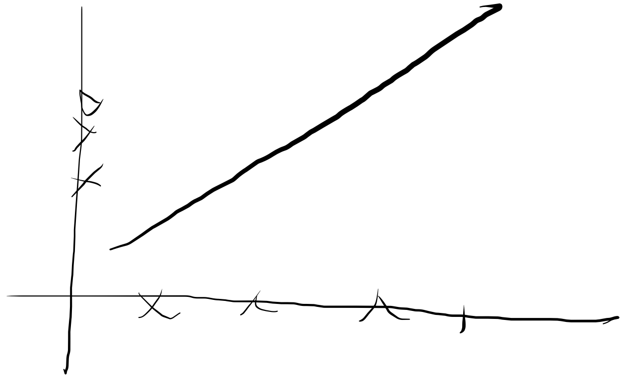
Kernel

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} x^2\right]$$



$$K\left(\frac{x_i - x}{h}\right) = \frac{1}{\sqrt{2\pi}\,\underset{\uparrow\,\sigma}{h}} \exp\left[-\frac{1}{2}\left(\frac{x - x_i}{\underset{\uparrow\,\sigma}{h}}\right)^2\right]$$

$$y_i = a x_i + b$$



$$\rightarrow E\left[(y - A\beta)^2\right]$$

$$\xrightarrow{\min} (y - A\beta)^2$$

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix} \qquad A = \begin{bmatrix} x(0) & 1 \\ \vdots & \vdots \\ x(m) & 1 \end{bmatrix}$$

$$\begin{cases} y(0) = a\,x(0) + b \\ \quad\vdots \\ y(m) = a\,x(m) + b \end{cases}$$

$$y = A\beta + \cancel{w}$$

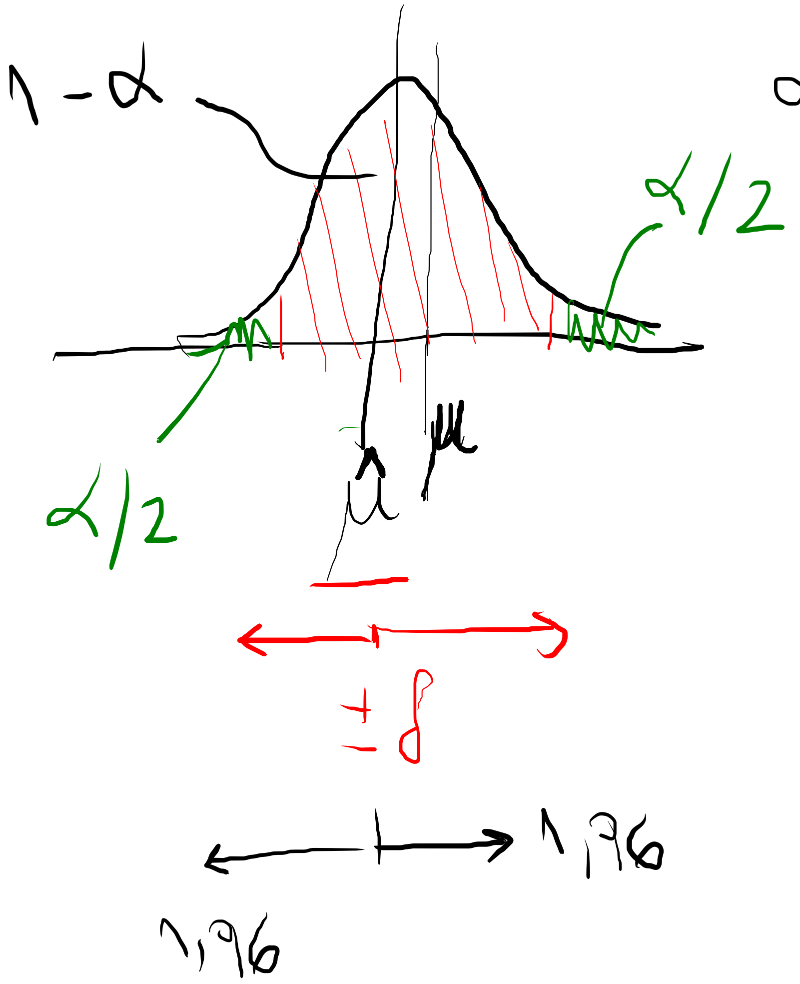
$$A^T y = A^T A \beta$$

$$\boxed{(A^T A)^{-1} A^T y = \beta.}$$

$\boxed{A \text{ inv.}}$

$$\beta = A^{-1} y.$$

Intervalo de confianza

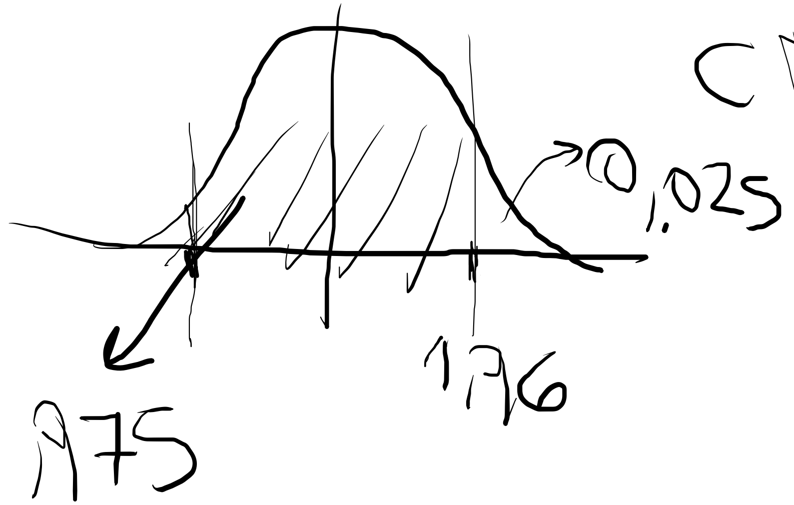


$\alpha = 0$

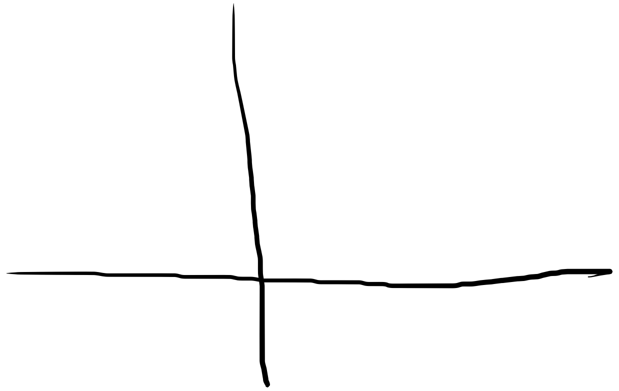
$\mu \in (-\infty, +\infty)$

$N(0,1)$: $\mu \in (-1{,}96;\ 1{,}96)$, $\alpha = 0{,}05$

CDF(1,96) N(0,1)



μ es constante (desconocida) $\rightarrow$ frecuentista



μ

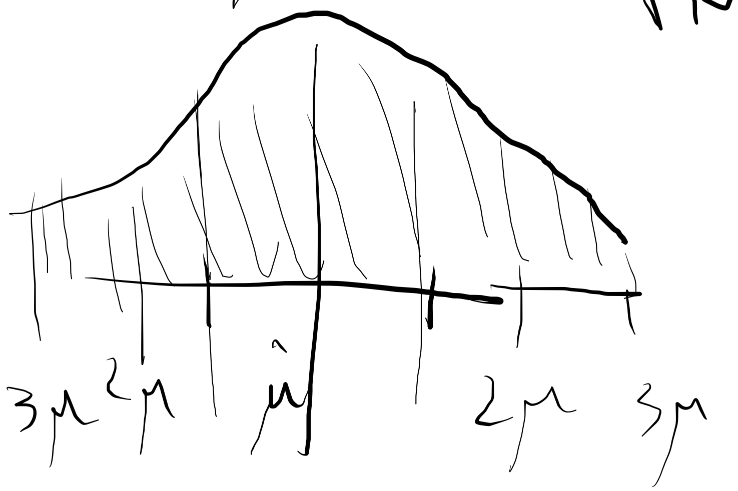
$\cancel{\hat{\mu}}$

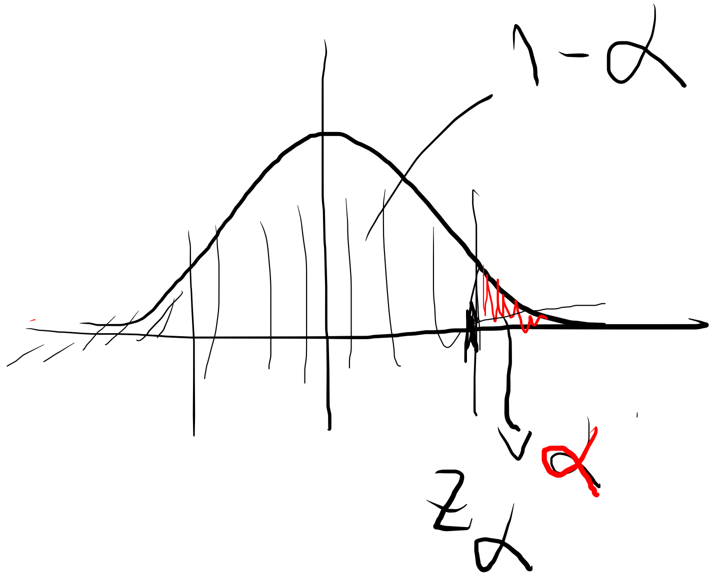
$1-\alpha$: nivel de confianza

μ con confianza $1-\alpha$

$\hat{\mu} - 1{,}96\frac{\sigma}{\sqrt{N}}$ $\qquad$ $\hat{\mu} + 1{,}96\frac{\sigma}{\sqrt{N}}$

$$\left.\begin{array}{l} 1\sigma \rightarrow 68\% \\ 2\sigma \rightarrow 95\% \\ 3\sigma \rightarrow 99\% \end{array}\right\} N(0,1)$$





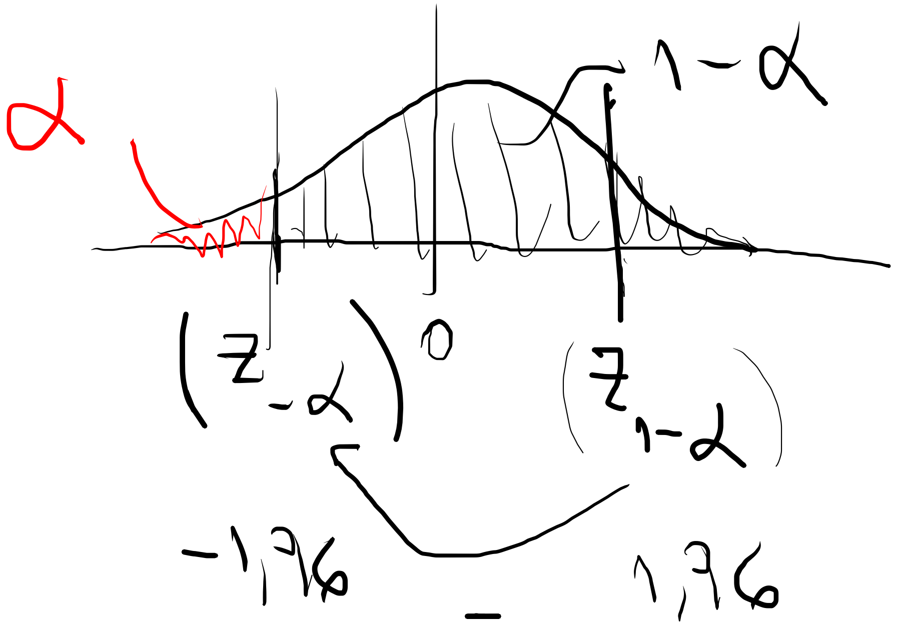
$$P(Z > \overset{>0}{z_\alpha}) = \alpha$$

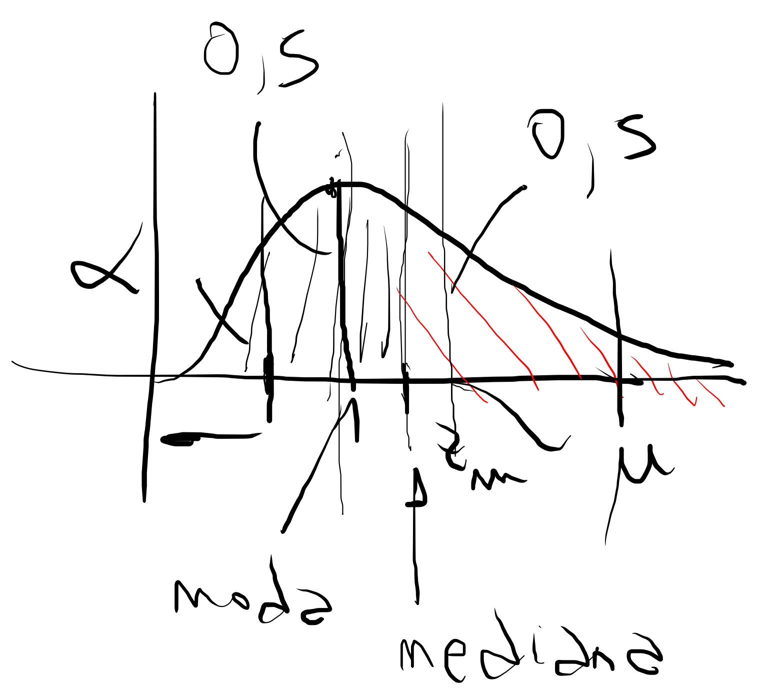
$$P(Z < z_{-\alpha}) = \alpha$$

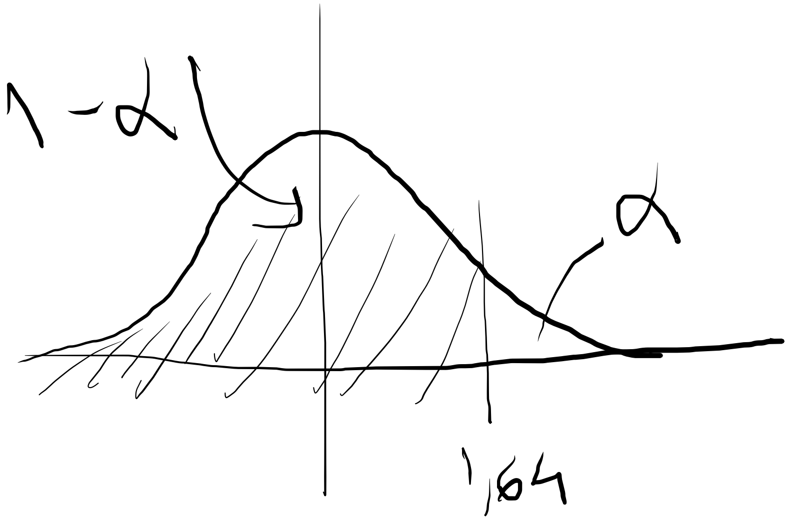
$$z_\alpha = -z_{-\alpha}$$

$$P(Z > z_{1-\alpha}) = \alpha$$

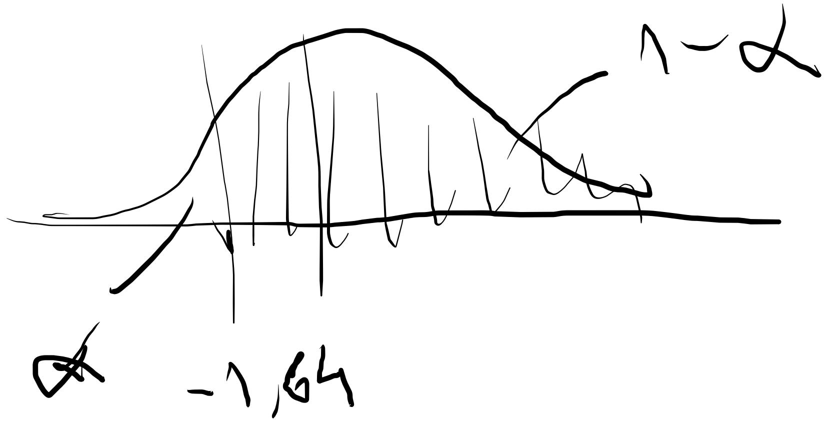
$$\boxed{z_{1-\alpha} = -z_\alpha}$$







unilateral inferior



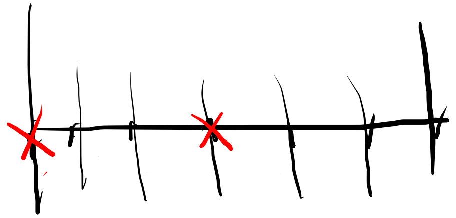
unilateral superior

$$\frac{\hat{\mu}-\mu}{\sigma/\sqrt{N}} = Z$$

Metodo exacto

$\boxed{\text{Clopper - Pearson}}$

Binomial $\rightarrow$ Beta

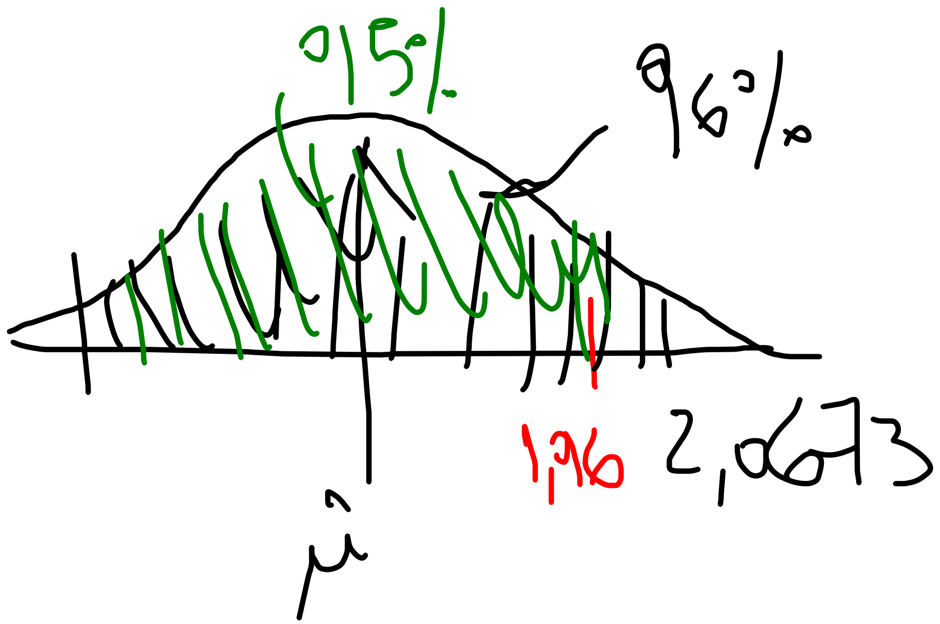


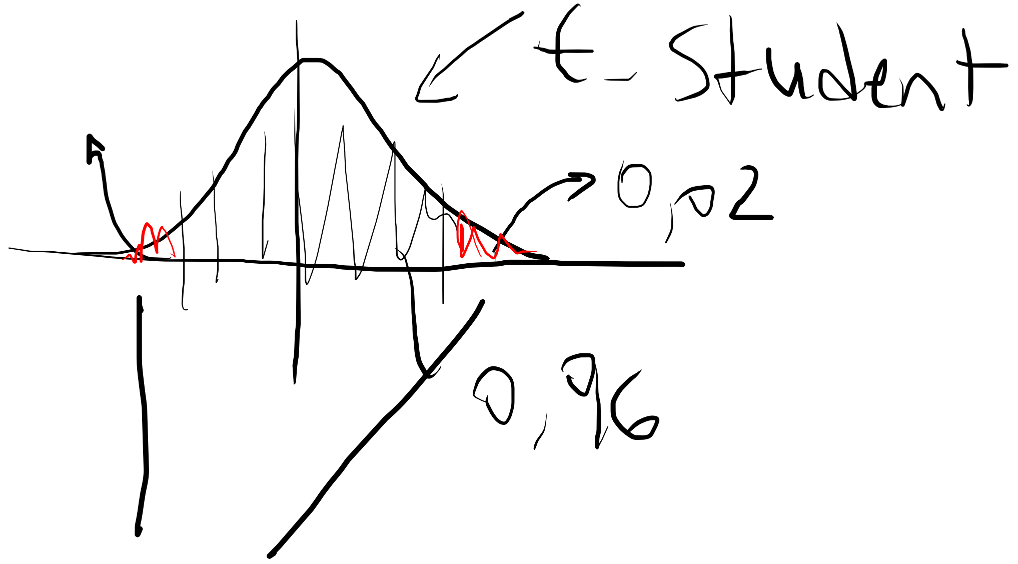
B $\binom{7}{2} p^2 (1-p)^5$

n chico $\leftarrow$

grande $\leftarrow$

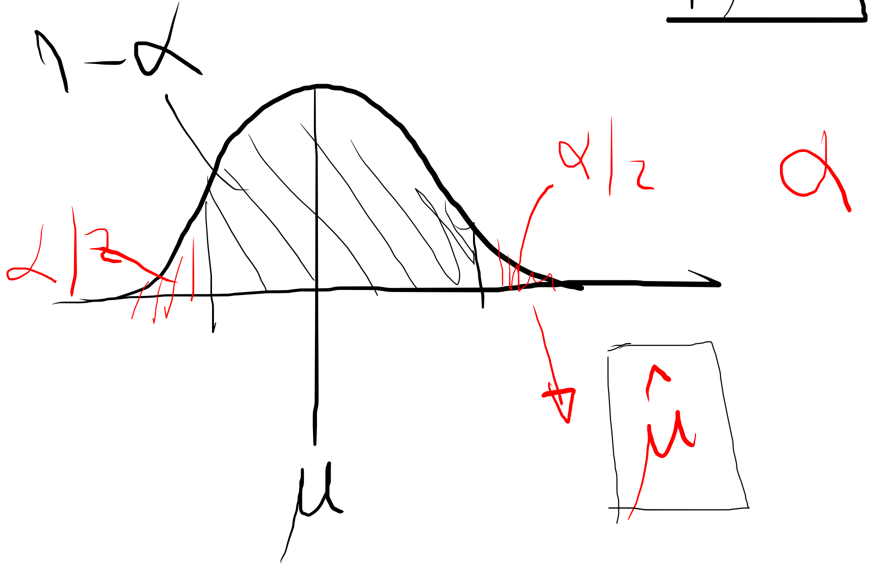
$N=200$ $\hat{\mu}=2{,}49$ $S=0{,}674$





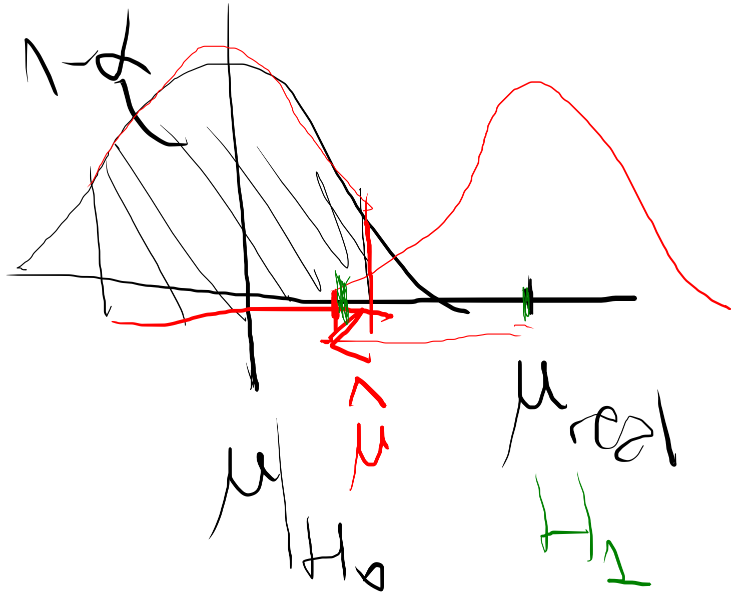
$\alpha=0{,}04$

H_0 es incorrecta

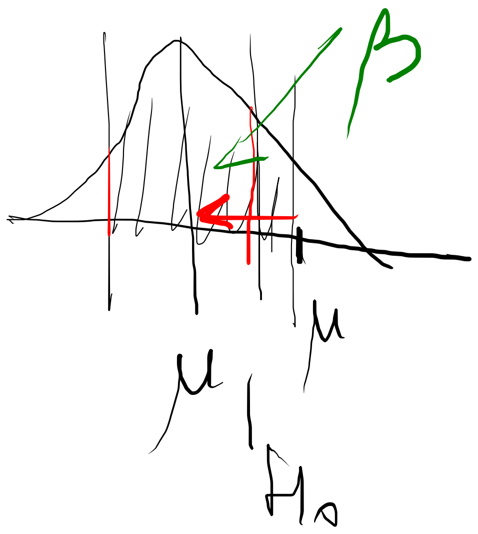


H_0 correcta

Error de tipo 1



H0 es correcta



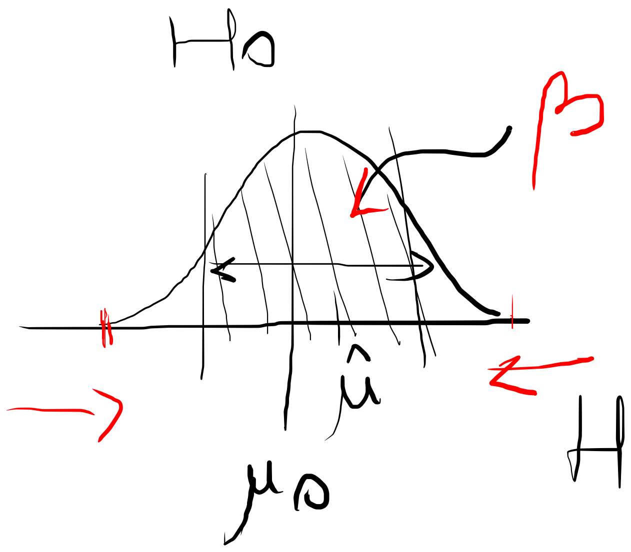
Rechazar H_0 cuando es incorrecta

$\boxed{1-\beta}$

Poder

Acepto H_0 cuando H_1 es correcta → error tipo 2

$\mu \to \mu_0$ error tipo 2 crece.



H_0 es incorrecta

(H_0)

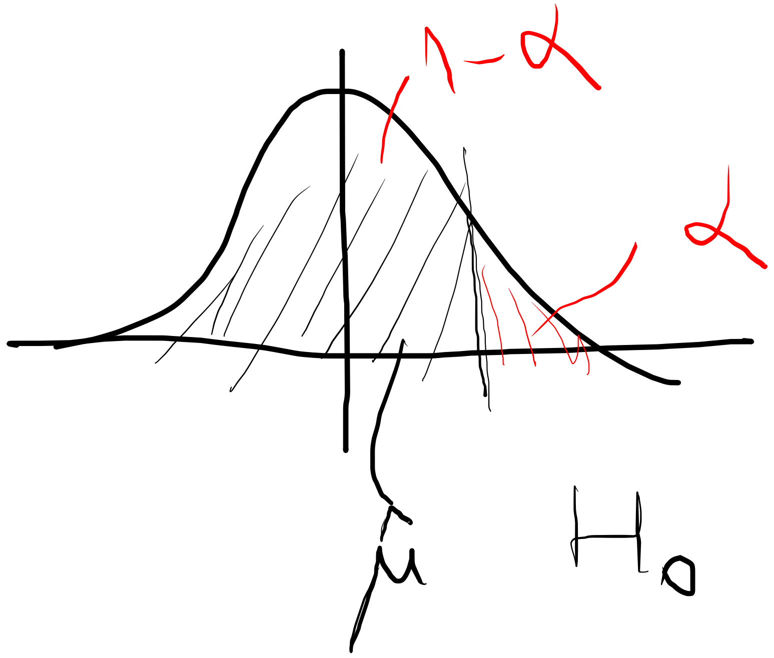
$\mu \neq \mu_0$

$\mu > \mu_0$

$$\Phi\left(\sqrt{N}\,\Delta - z_{1-\alpha/2}\right) = 1-\beta$$

$$\Phi^{-1} \to \Phi^{-1}(1-\beta) = z_{1-\beta}$$

$$\sqrt{N}\,\Delta - z_{1-\alpha/2} = z_{1-\beta} \Rightarrow N = \left(\frac{z_{1-\beta} + z_{1-\alpha/2}}{\Delta}\right)^2$$



1 - α
α
μ̂
H₀

Ej 1

μ

$N = 100$, $\hat{\mu} = 180\,cm$, $\sigma^2 = 49\,cm^2$ $\rightarrow$ $\sigma = \sqrt{49}\,cm = 7\,cm$

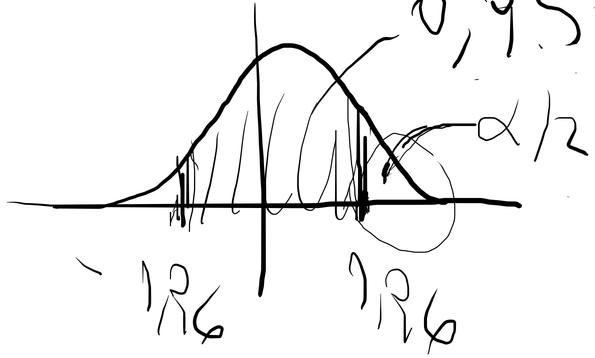
$\boxed{95\%}$ $\rightarrow$ $N = 100$ $\rightarrow$ normal

$\rightarrow$ $Z = 1{,}96$

$$\left(\hat{\mu} - 1{,}96\frac{\sigma}{\sqrt{N}},\ \hat{\mu} + 1{,}96\frac{\sigma}{\sqrt{N}}\right)$$

180

$$\boxed{(178{,}63\ ;\ 181{,}37)}$$

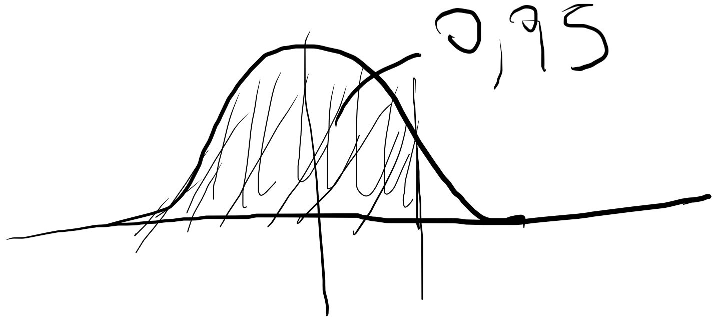


$0{,}95 = 1 - \alpha \Rightarrow \alpha = 0{,}05$

$\boxed{\alpha/2 = 0{,}025}$

$P(Z < z_{1-\alpha/2}) = 1 - \frac{\alpha}{2}$

$P(Z > z_{1-\alpha/2}) = \frac{\alpha}{2}$



$-z_{1-\alpha/2}$

$z_{\alpha/2}$

$P(Z < z_{\alpha/2})$

$$\left(\hat{\mu} - t_{\alpha/2,\,N-1}\frac{s}{\sqrt{N}},\ \hat{\mu} + t_{\alpha/2,\,N-1}\frac{s}{\sqrt{N}}\right)$$

$$\begin{bmatrix} x(1) \\ x(i) \\ \vdots \\ x(N) \end{bmatrix} \to \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \right\} \left\{ \frac{\sum}{N} \right\}$$

for m

end $\quad 1/m$

$E(\text{int térka})$

Ej 3

$H_0: p = 1/3$

$H_1: p > 1/3$

Ej 4

$H_0: p = 1/2$

$H_1: p > 1/2$