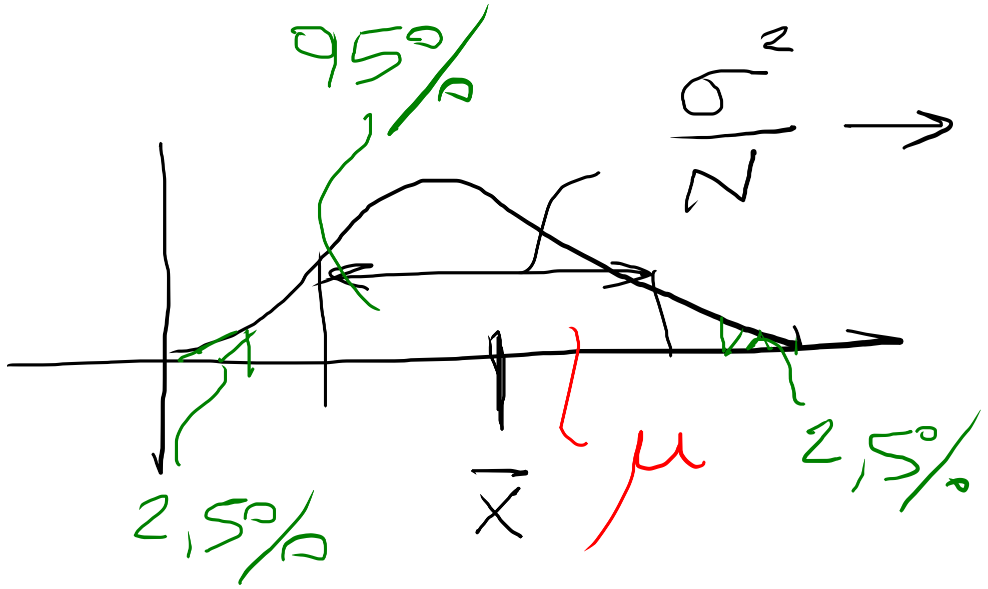
Ej 1 Clase 5

$N = 100$

$\bar{x} = 1.8\,m$

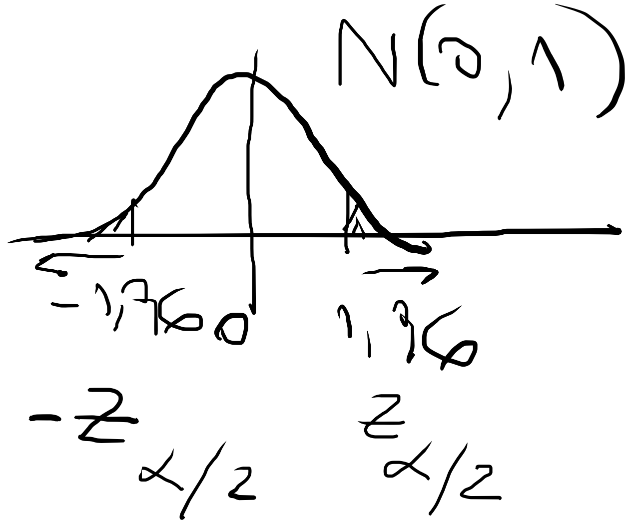
$\sigma^2 = 49\,cm^2 \leftarrow$



$\frac{\sigma^2}{N} \rightarrow \frac{\sigma}{\sqrt{N}}$

I.C. (95%)

$$\left(\bar{x} - 1{,}96\frac{\sigma}{\sqrt{N}},\ \bar{x} + 1{,}96\frac{\sigma}{\sqrt{N}}\right)$$

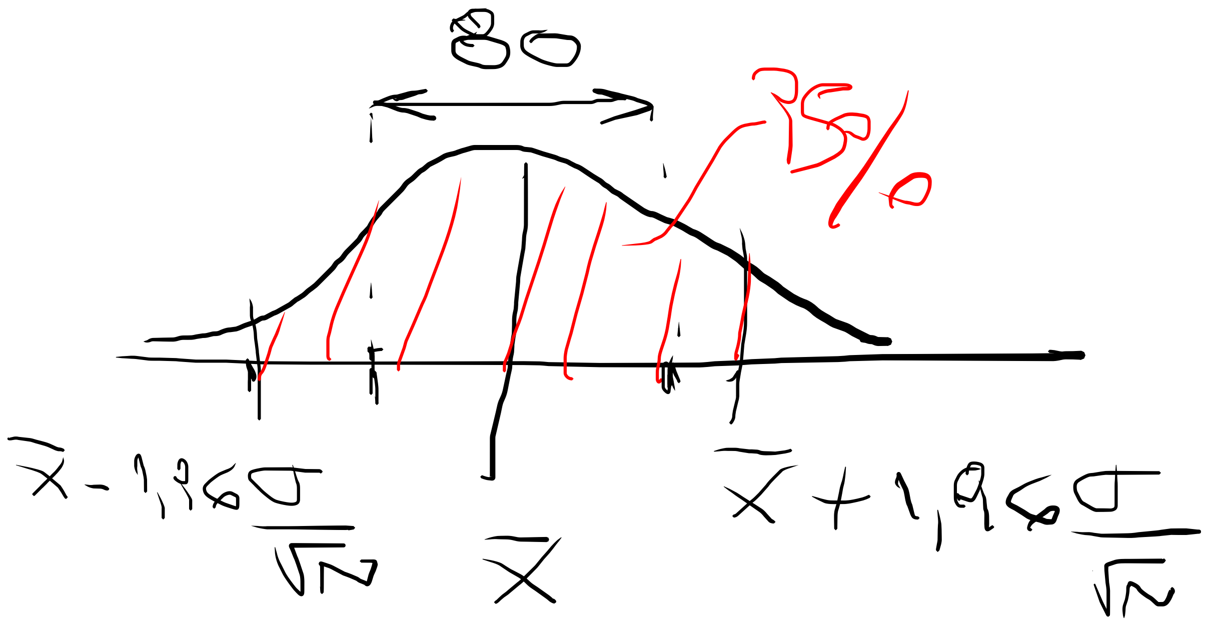


Ej 2 Clase 5

$N = 10$

$\bar{x} = 998{,}9$

$\sigma = 80$



$$\left(\bar{x} - 1{,}96 \frac{\sigma}{\sqrt{N}} , \bar{x} + 1{,}96 \frac{\sigma}{\sqrt{N}}\right)$$

$$\left(\bar{x} - z \frac{\sigma}{\sqrt{N}} , \bar{x} + z \frac{\sigma}{\sqrt{N}}\right) \rightarrow \text{Rango: } \bar{x} + z \frac{\sigma}{\sqrt{N}} - \left(\bar{x} - z \frac{\sigma}{\sqrt{N}}\right) =$$

$$2z \frac{\sigma}{\sqrt{N}} = 80 \Rightarrow \frac{2z}{\sqrt{10}} = 1 \Rightarrow z = 1{,}58$$

↑ enunciado

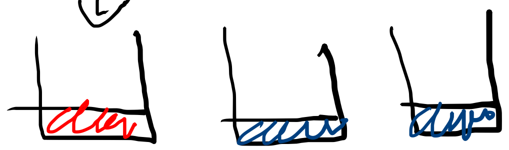
Ej 3 Clase 5

$H_0: p = 1/3$

$H_1: p > 1/3$

$n = 10 \qquad \alpha = 5\%$

P (quiero identificar)



A priori: $p = 1/3$

$\overline{Y}$: personas que aciertan la bebida

$$P(\overline{Y} = k) = \binom{n}{k} p^k (1-p)^{n-k} = B(n, p, k)$$

permite que falle más gente

$$P(\overline{Y} = 10) = \binom{10}{10} p^{10} (1-p)^0 = p^{10} = 1{,}7 \cdot 10^{-5} \ll \alpha = 0{,}05$$

$$P(\underline{Y} \geq 6) = 0{,}07 > \alpha$$

$$P(\underline{Y} \geq 7) \simeq 0{,}02 < \alpha$$

Mínima cant. personas que deben acertar es 7, para rechazar Ho.

Ej. 4 Clase 5

$N = 10$

Asumimos 2 opciones de pasta dental.

$H_0: P = 1/2$

$H_1: p > 1/2$

$K = 8 \quad \alpha = 0{,}05.$

$$P(X \geq 8) = \binom{10}{8}(1/2)^8\left(\frac{1}{2}\right)^2 + \binom{10}{9}(1/2)^9(1/2)^1 + \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0 = 0{,}054 > \alpha$$

No puedo rechazar H_0.

μ → constante, desconocida

$\hat{\mu}$ ↓

estimador: $\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$

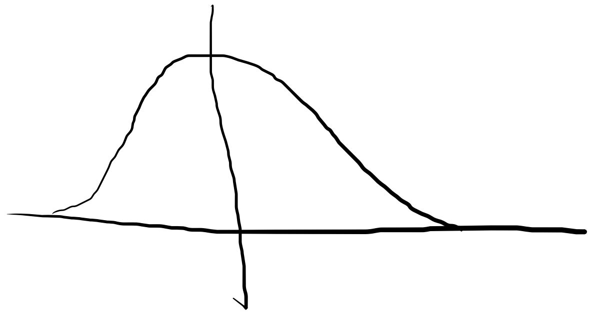
$\mu \rightarrow \theta$

$\theta \neq$ const. $\Rightarrow$ $\theta \sim \underbrace{\pi(\theta)}$

distribución a priori → antes de usar los datos

dato $\underline{\bar{X}} = x$ → Distribución a posteriori → después de usar datos

$\pi(\theta | x)$.



θ

Verosimilitud: (ejemplo Gaussiano)

$$f_{\underline{X}|\theta}(x|\theta) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2}\frac{(x-\theta)^2}{\sigma^2}\right]$$

x: datos , θ: no lo conozco

Si conociera θ, cómo se distribuiría $\underline{X}$.

$$\pi(\theta|x) = \frac{f_{\underline{X}|\theta}(x|\theta)\,\pi(\theta)}{\underbrace{f_{\underline{X}}(x)}_{\text{normalización}}} \propto f_{\underline{X}|\theta}(x|\theta)\,\pi(\theta)$$

Como es pdf:

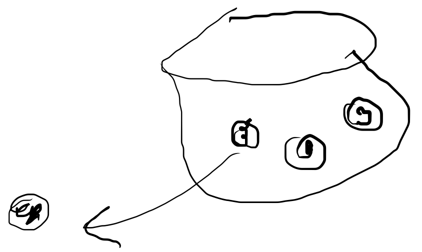
$$\int \pi(\theta|x)\,d\theta = 1$$

Ejemplo Monedas

$X = \{0, 1\}$

$X = 1$ evento cara

$X = 0$ " cece



$\pi(\theta) = 1/3$

θ : p. cara $\quad \theta \in (0,25 \; ; \; 0,5 \; ; \; 0,75)$

Aprori : $\pi(\theta = 0,25) = \pi(\theta = 0,5) = \pi(\theta = 0,75) = 1/3$

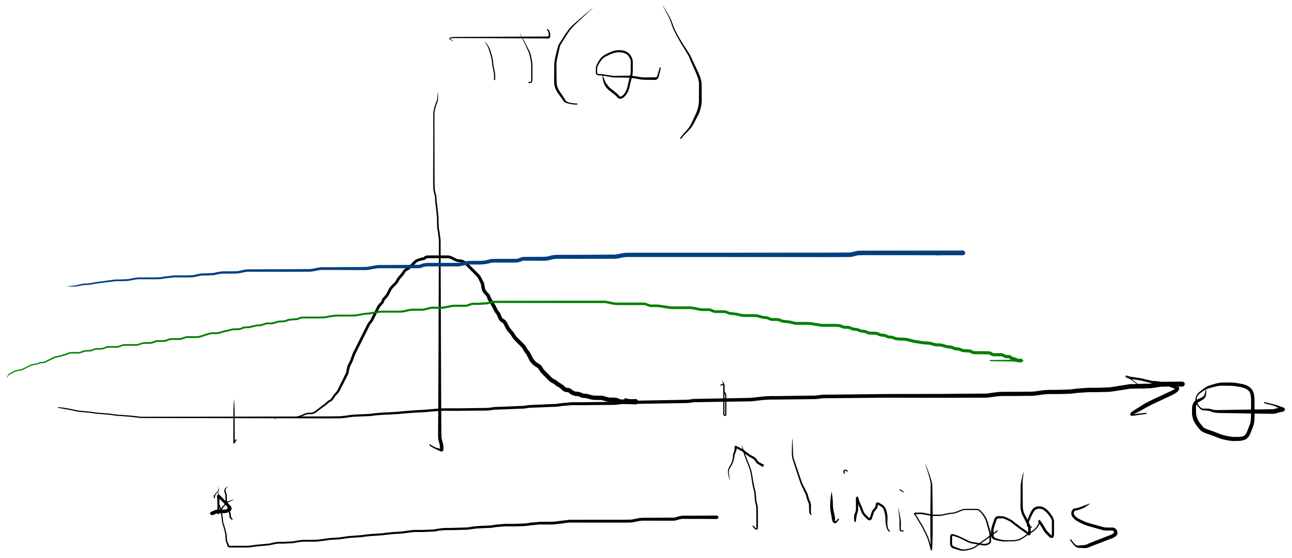
$P(X = 1 \mid \theta) = \theta$

$P(X = 0 \mid \theta) = 1 - \theta$

Moneda	θ	A priori ($\pi(\theta)$)	A posteriori (sin $/p(x)$) $P(x=1\mid\theta)\cdot\pi(\theta)$	A posteriori $P(x=1\mid\theta)\,\pi(\theta)/p(x)$
1	0,25	1/3	$0,25 \times 1/3 = 0,0833$	0,167
2	0,5	1/3	0,167	1/3
3	0,75	1/3	0,25	1/2
				Σ 1

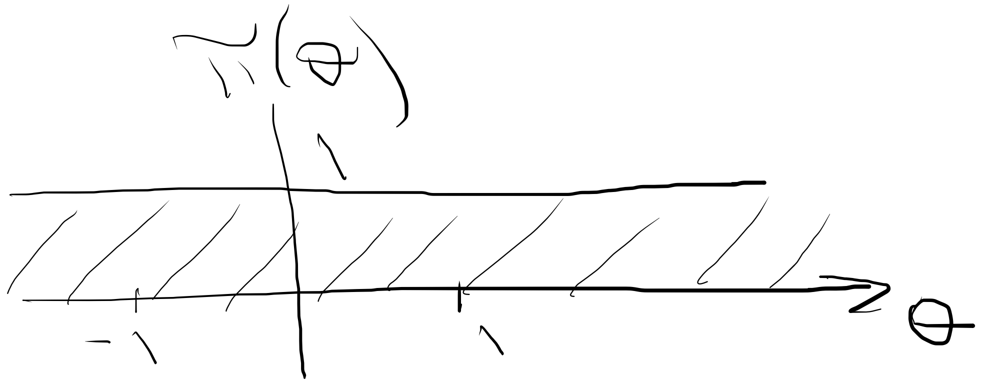
$$P(x) = \sum_{i=1}^{3} P(x\mid\theta_i)\,\pi(\theta_i) = 0,5$$

$$P(\theta=\theta_3 \mid X=1) = 1/2$$



$$\pi(\theta|x) = \frac{\overbrace{f_{\underline{X}|\theta}(x|\theta)}^{\text{clásica}}\ \overbrace{\pi(\theta)}^{\text{creencia}}}{f_{\underline{X}}(x)}$$

No informativa



$$\int_{-\infty}^{+\infty} \pi(\theta)\,d\theta > 1 \quad (\text{o incluso} \to \infty)$$

Lo importante es que

$$\int \pi(\theta|x) = 1.$$

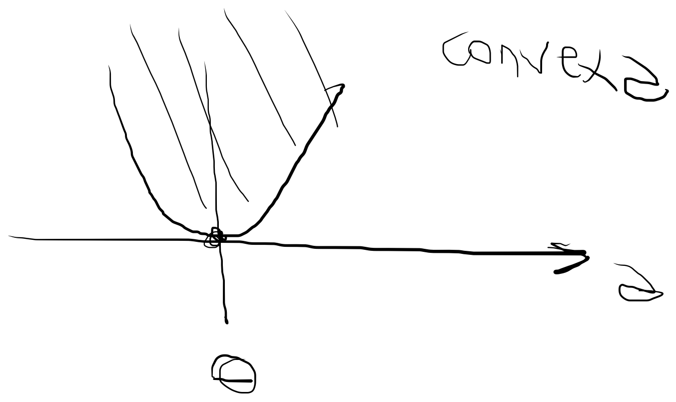
Distribución Beta =

$$f(x|\theta) = \theta^{x_1}(1-\theta)^{1-x_1} \cdots \theta^{x_2}(1-\theta)^{1-x_2} \cdots$$

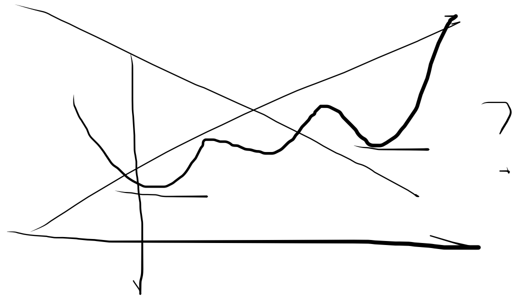
$$= \theta^{\sum_i x_i}(1-\theta)^{n-\sum_i x_i}$$

$$\underline{X} \sim B(\alpha, \beta)$$

$$E[\underline{X}] = \frac{\alpha}{\alpha+\beta}$$



$$L(\theta, a) = (\theta - a)^2$$



$$h(a) = \int (a-\theta)^2 \, \pi(\theta|x) \, d\theta$$

$$\frac{dh}{da} = 0 \Rightarrow a_{opt} = E[\theta|x] = \int \theta \, \pi(\theta|x) \, d\theta$$

Media a posteriori

Estimador Bayesiano óptimo.