$$X = aU + bV \qquad U, V \sim N(0,1) \text{ iid}$$
$$Y = cU + dV$$

$$a, b, c, d \to X(U,V), \quad Y(U,V)$$

$$E[X] = E[Y] = 0 \qquad -1 \leq \rho \leq 1$$
$$\sigma_X^2 = a^2 + b^2 \qquad \sigma_Y^2 = c^2 + d^2$$

$$\operatorname{cov}[X,Y] = E[XY] - E[X]\,E[Y] = \rho_{XY}\,\sigma_X\sigma_Y =$$
$$= E[\underbrace{(aU+bV)}_{X}\underbrace{(cU+dV)}_{Y}] = ac\,\underbrace{E[U^2]}_{1} + bd\,\underbrace{E[V^2]}_{1} = ac+bd$$

$$\rho = \frac{ac+bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$$

$a=1,\ b=0,\ c=0,\ d=1$

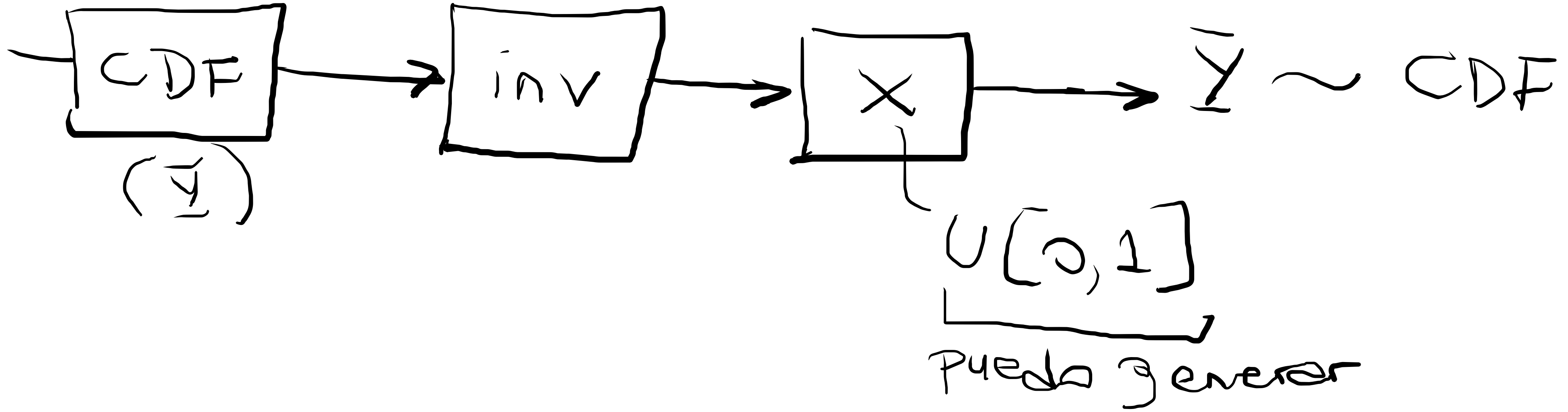
Transformaciones de V.A. (Ej 3, Clase 2)

$Y \sim$ exponencial par. λ $\quad f_Y(y) = \lambda e^{-\lambda y}$, $y \geq 0$; sino 0, $y < 0$.

→ (pdf)

Busco la CDF: $F_Y(y) = \int_{-\infty}^{y} f_Y(y)\,dy = \int_0^{y} \lambda e^{-\lambda y}\,dy = 1 - e^{-\lambda y}$, $y \geq 0$

Método de la transf. inversa



Busca la inversa: $x = \underbrace{1 - e^{-\lambda y}}_{h(y)} \Rightarrow y = \underbrace{-\frac{1}{\lambda} \ln(1-x)}_{g(x)}$

$h(y)$ es CDF rango $[0,1]$

$$\begin{tikzcd} X \arrow[r, bend left, "g = h^{-1}"] & Y \arrow[l, bend left, "h = g^{-1}"] \end{tikzcd}$$

$y \to f(x)$

pdf Y: $f_Y(y) = \underbrace{f_X(x = h(y))}_{1} \left| \frac{dh(y)}{dy} \right|$

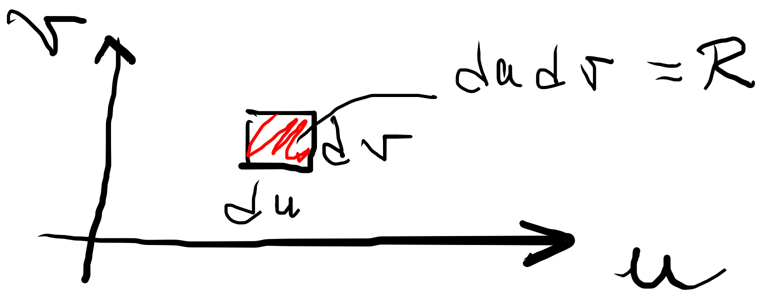
$X \sim U[0,1] \to 1$

$f_Y(y) = \lambda e^{-\lambda y}, \quad y \geq 0.$

Transformación de V. A. - Jacobiano

$$x = x(u, v)$$
$$y = y(u, v)$$

$u\text{-}v \longrightarrow x\text{-}y$

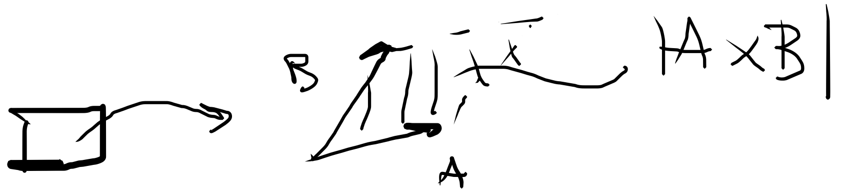


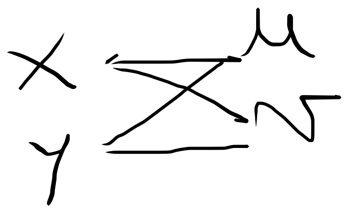
$R \neq S$

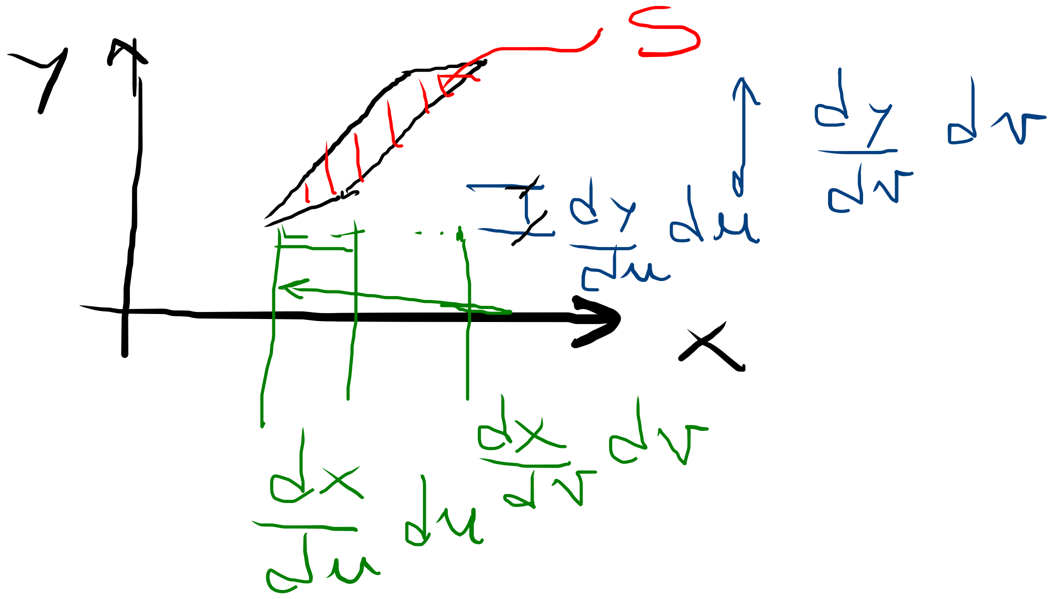
Al calcular las probabilidades (P) de un evento:

R, S deben dar la misma P.

$$S = |A \times B| = |J| \underbrace{du\,dv}_{R}$$







Jacobiano: det.

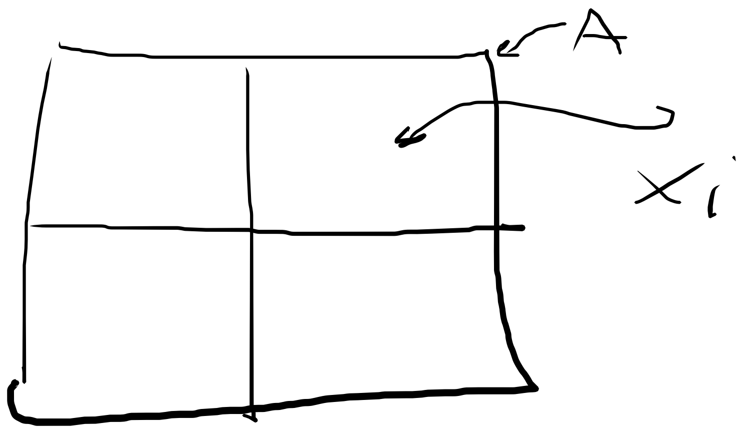
$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \begin{matrix} \leftarrow \nabla x \\ \leftarrow \nabla y \end{matrix}$$

$$\overbrace{f_{xy}\,dx\,dy}^{\partial P} = \overbrace{f_{UV}\,du\,dv}^{\partial P}$$

▱ $\square$

$$f_{UV} = f_{xy}\,|J| = f_{xy}\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$

$$f_{xy} = \frac{f_{UV}}{|J|} \rightarrow f_{XY}(x,y) = \left.\frac{f_{UV}(u,v)}{|J(u,v)|}\right|_{\substack{u=u(x,y)\\ v=v(x,y)}}$$



$$P(A) = \sum_i P(A \cap x_i)$$

$$= \sum_{x_i} P(A|x_i)\,P(x_i)$$

Dado con $\underline{X}$: número del dado

A: evento de número.

Dado a priori: $P(\underline{X} = i) = \frac{1}{6}$

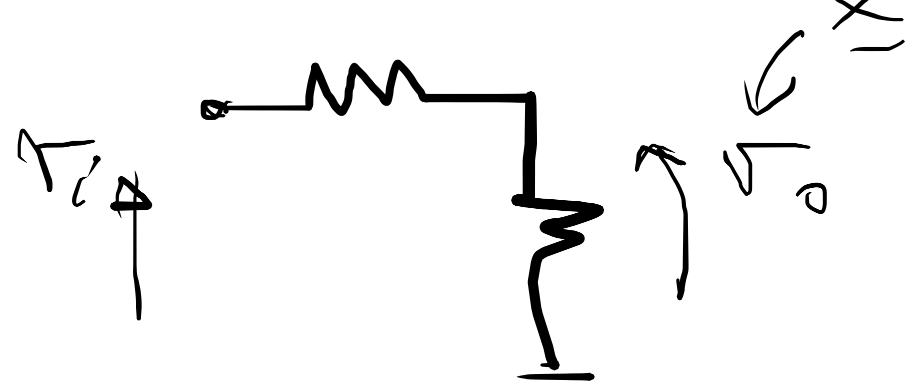
$$P(\underline{X} = i \mid A \text{ es par}) = \begin{cases} \frac{1}{3}, & x = 2,4,6. \\ 0, & x = 1,3,5 \end{cases}$$

Teorema de la Esperanza Total

$$E[\underline{X}] = \sum_{k} x_k \, P(\underline{X} = x_k)$$

$$E[\underline{X} / A] = \sum_{k} x_k \, P(\underline{X} = x_k \mid A)$$

$$E[\underline{X}] = \sum_{y} \underbrace{E[\underline{X} \mid \underline{Y} = k]}_{f(\underline{Y})} \, P(\underline{Y} = y)$$



$$\underline{Y} = \underline{X} + \underline{M}$$

Ley de Esperanzas Iteradas

$$E[X|Y] = \sum_{x} x \, P(X=x|Y=y)$$

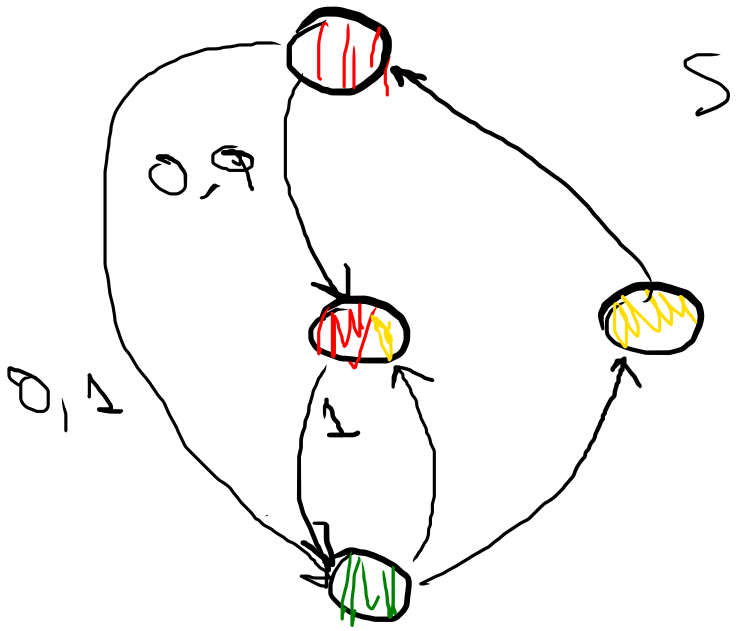
$$E\big[E[X|Y]\big] = \sum_{y} \underbrace{\sum_{x} x \, P(X=x|Y=y)}_{E[X|Y]} \, \overbrace{P(Y=y)}^{P(X=x,\,Y=y)}$$

$$\sum_{x} x \sum_{y} P(X,Y) = \sum_{x} x \, P(X=x) = E[X]$$

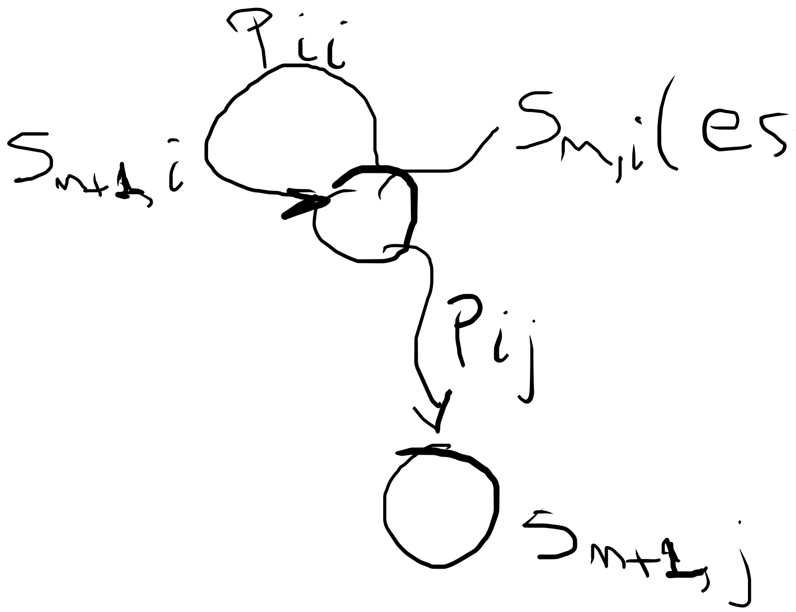
Proceso de Markov

$x(n) \quad x(n-1) \quad x(n-2) \ldots$

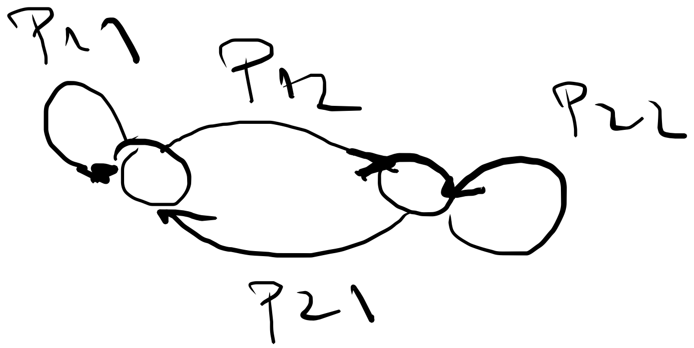
$\begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \end{bmatrix}$ ← memoria.

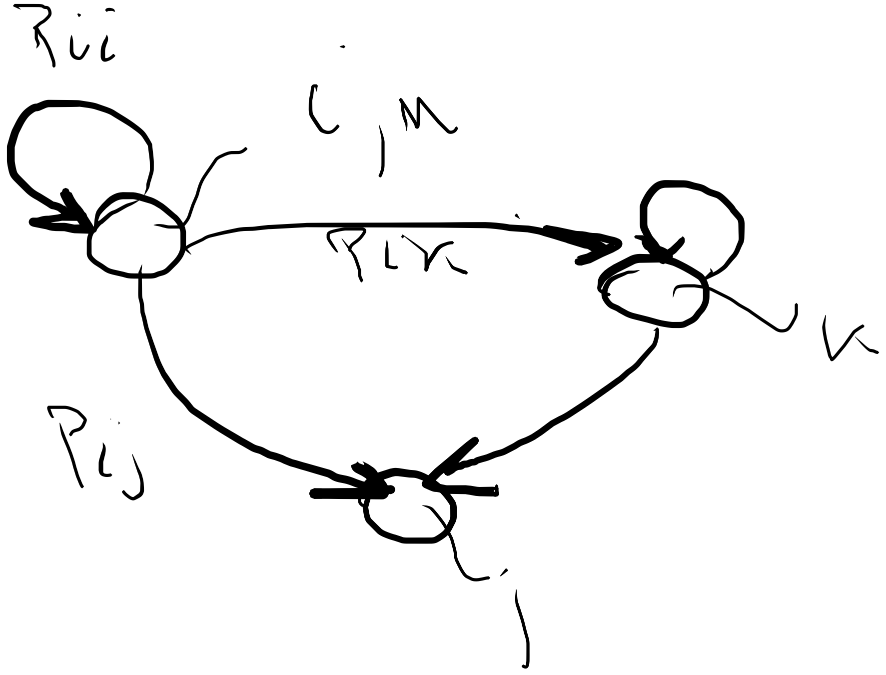


Semáforo funciona mal



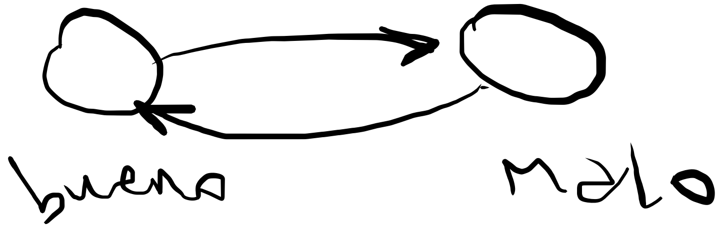
$$\sum P_{salientes} = 1$$

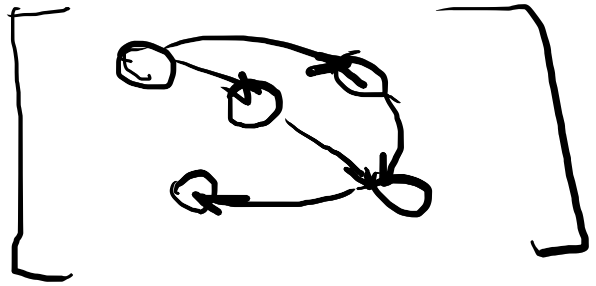




$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Cadenas de Markov





Ruido Blanco

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t) = 1$$

$$\int_{-\infty}^{+\infty} f(t)\,\delta(t)\,dt = f(0)$$

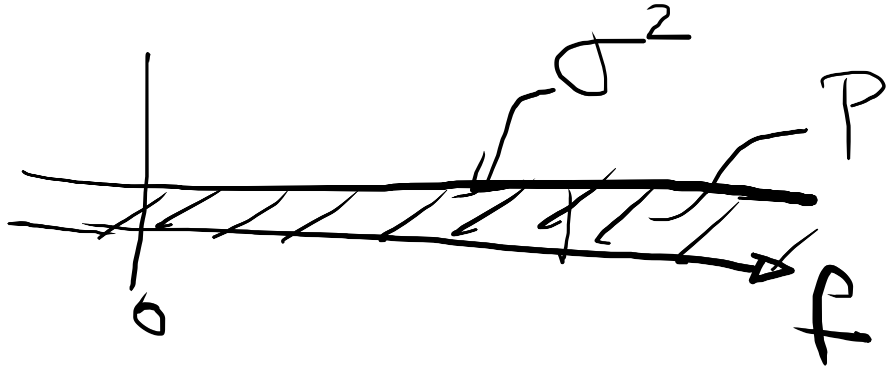
$$n - t \qquad = f(n)$$

Función de Autocorrelación

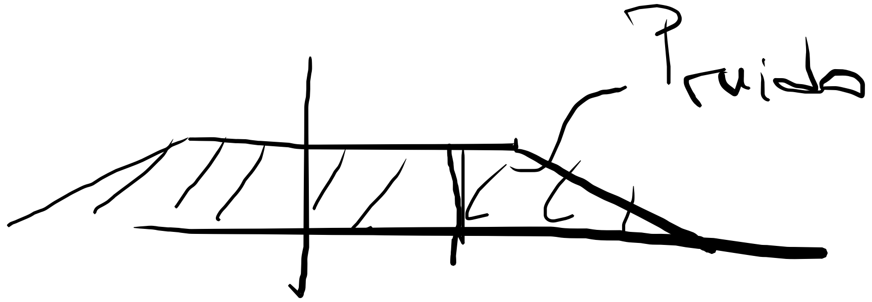
$$E\left[X(t)X(t+\tau)\right] = R(t, t+\tau)$$

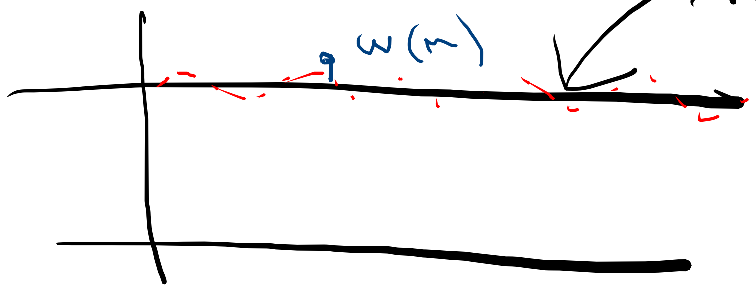
Ruido blanco : $R(\tau)$

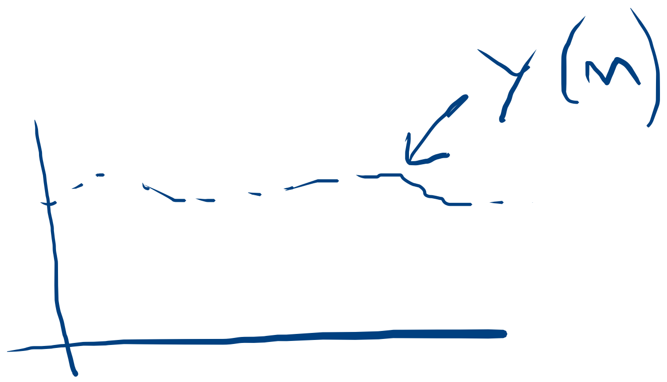
$$\sigma_{\underline{x}}^2 = \int_{-\infty}^{+\infty} x^2 f_{\underline{x}}(x)\,dx \longrightarrow 0$$



$$P(f) = \int_{-\infty}^{+\infty} \underset{\uparrow\,\sigma^2}{P_f}\, df \longrightarrow \infty$$







$$w \sim N(0, \sigma_w^2)$$

$$y \sim N(A, \sigma_w^2)$$

$$y = A + w$$

$$E[y] = \overset{A}{E[A]} + \overset{0}{E[w]} \quad \Big| \quad \sigma_y^2 = \underbrace{\sigma^2(A)}_{0} + \sigma_w^2$$

Proceso de Wiener

$W(t)$ es v.a. con distribución de Wiener

$E[W(t)] = 0$, $0<s<t<T$

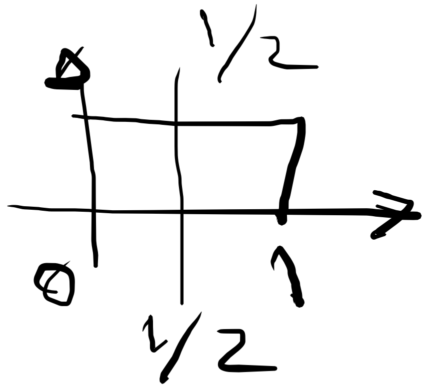
$W(t) - W(s) \sim Z$

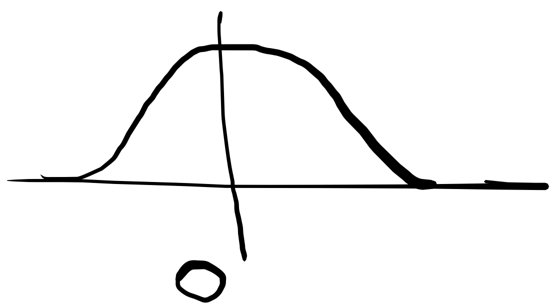
independientes

$W(u) - W(v) \sim Y$

$$W(t) - W(s) \sim \sqrt{t-s}\, N(0,1)$$
$$= N(0, t-s)$$

Ej 3





$X, Y \sim U[0,1]$ indeptes.

$\bar{X} = x$

$Z = X + Y$

$$\left[\, E[Z \mid X] = E[X+Y \mid X] = \underbrace{E[X \mid X]}_{\bar{X} = x} + \underbrace{E[Y \mid X]}_{E[Y]} = X + \frac{1}{2} \,\right]$$

$\bar{X} = x_k$

$\bar{X} \in I$

$P(\text{ahora está lloviendo} \mid \text{ahora está lloviendo}) = 1$

$$E[X \mid X] = \sum_k x_k \underbrace{P(X = k \mid X = k)}_{1} = X$$