Ej 1

$P = 0,5$

$N = 10$

$k = 3$

$$\binom{10}{3} p^3 (1-p)^{10-3}$$

Binomial media muestral : $\underset{10}{n} \underset{0,4}{p} = 4$

" **varianza**

$N p (1-p)$

.

Ej. 2

$$f_{\underline{X}\underline{Y}} = f_{\underline{X}} \, f_{\underline{Y}}$$

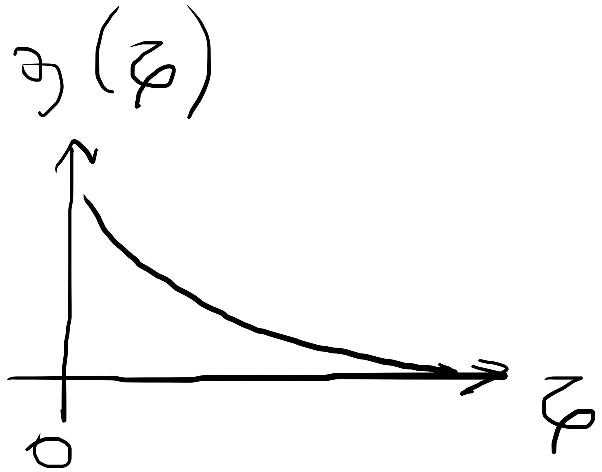
indep.

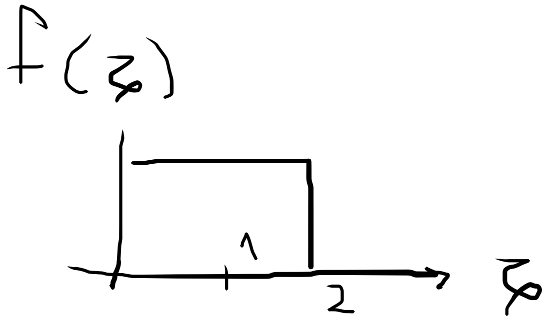
Producto de Convolución

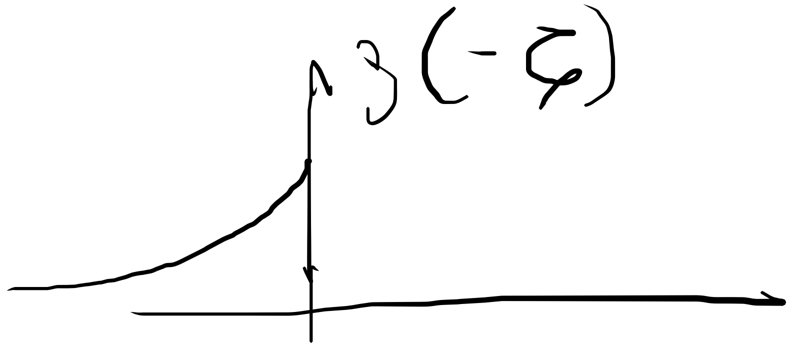
$f(t), g(t)$ reales

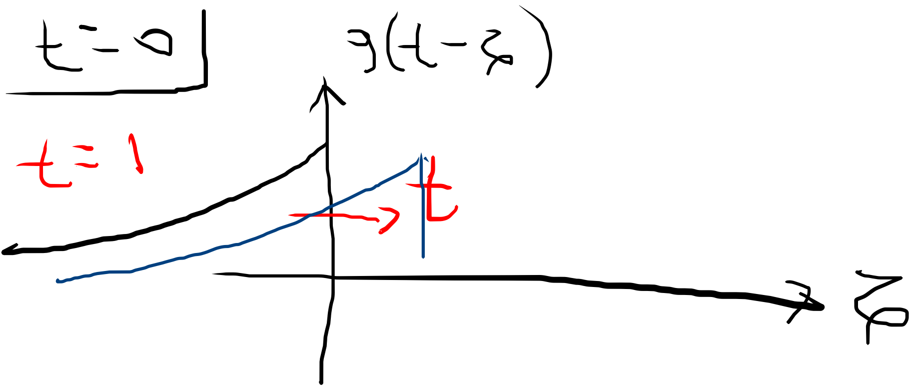
desplazamiento

$$(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)\, g(t-\tau)\, d\tau$$









Lineal $(\lambda f) * g = \lambda (f * g)$; $(f_1 + f_2) * g = f_1 * g + f_2 * g$

Asociativa $(f * g) * h = f * (g * h)$

Conmutativa $f * g = g * f$.

$x = 1\ 2\ 3\ 4$ $\qquad$ $x^e = 4\ 3\ 2\ 1$

conv x, x

$1\ 2\ 3\ 4$

$m = 1|$ $\quad 4\ 3\ 2\ 1$

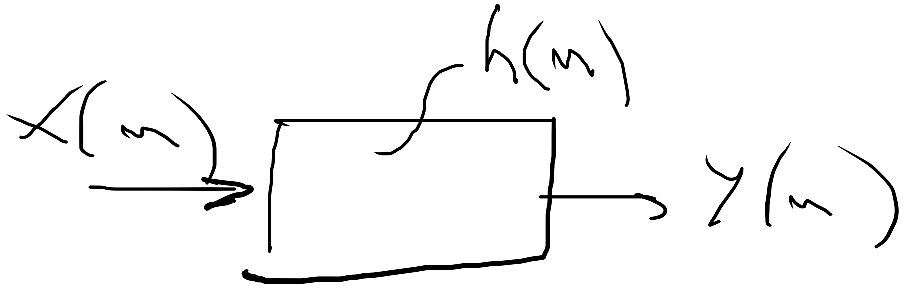
$m = 2|$ $\quad 4\ 3\ 2\ 1$

$m = 4|$ $\quad 4\ 3\ 2\ 1$

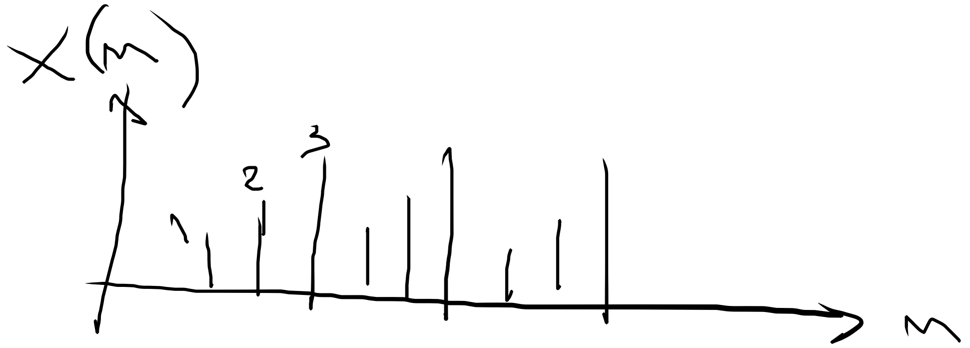
$x \rightarrow n$

$y \rightarrow m$

$x * y \longrightarrow n + m - 1$



$$y(n) = x(n) * h(n)$$



$$x(n) * h(n) = \ ?$$

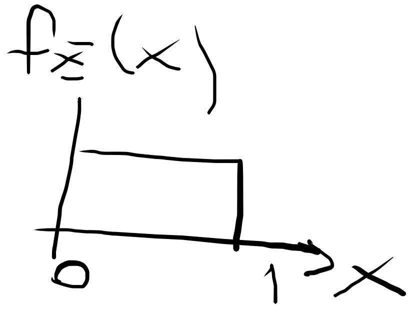
Promediador móvil.

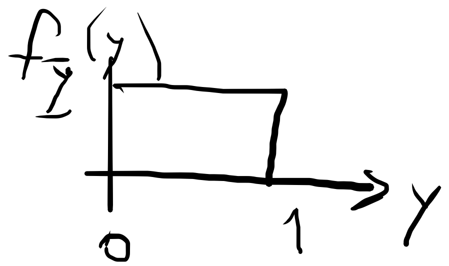
Bolsa.

$$Z = X + Y \qquad X, Y \sim U[0,1]$$

$$P(Z \le z) = P(X + Y \le z) = P(X \le z - y,\ Y \le y)$$

Indep. $\rightarrow$ $P(Z \le z) = P(X \le z-y) \cdot P(Y \le y)$

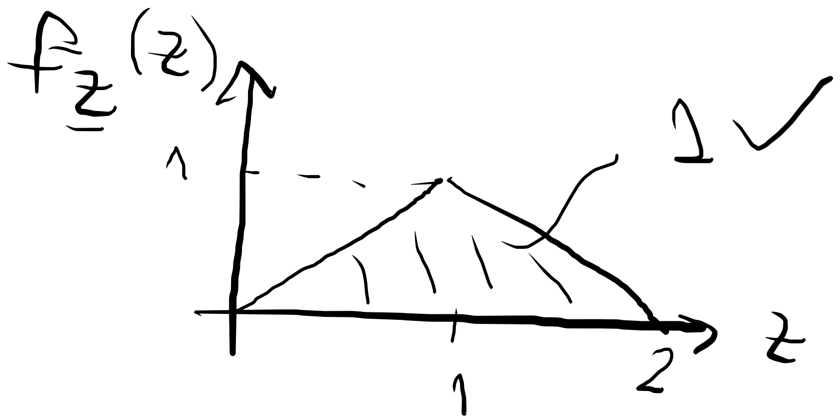




$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(z-y)\, \underbrace{f_Y(y)}_{[0,1]\neq 0 \,\to\, 1}\, dy$$

1

$$f_Z(z) = \int_0^z 1 \cdot 1\, dy = z, \qquad z \in [0,1]$$



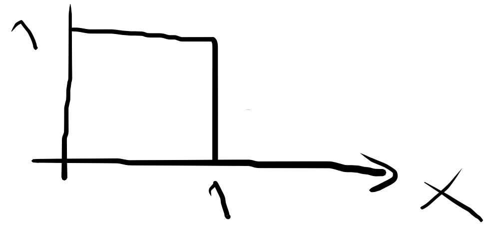
2

$$f_Z(z) = \int_{z-1}^{1} 1 \cdot 1 \cdot dy = 1 - (z-1) \qquad z \in [1,2]$$

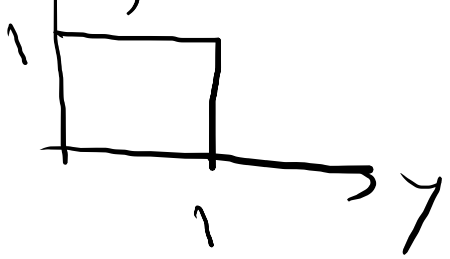
$$= 2 - z$$

$$\frac{\text{Base} \times \text{altura}}{2} = 1$$

$f_X(x)$

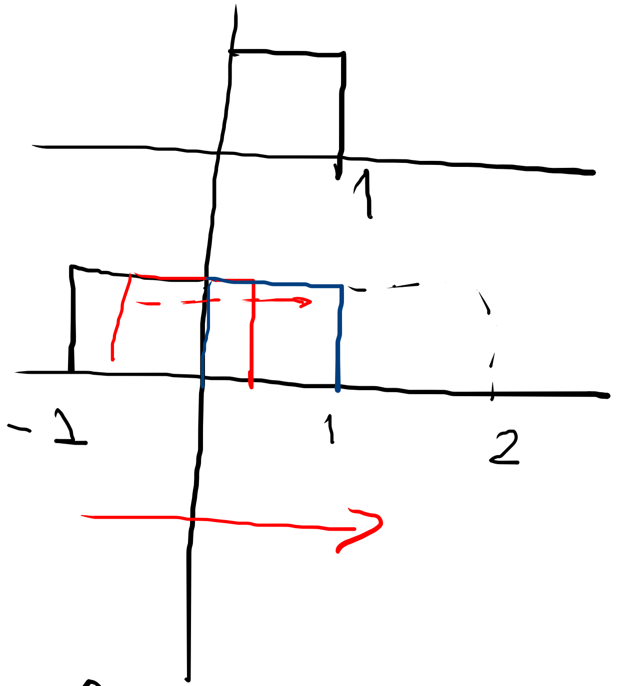


$f_Y(y)$

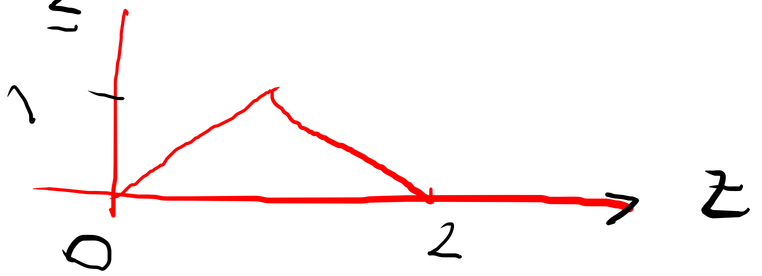


$f_X * f_Y$

$t<0$



f_Z



$X \sim N(\overset{\mu}{0}, \overset{\sigma^2}{1})$

$Y \sim N(0,1)$

iid

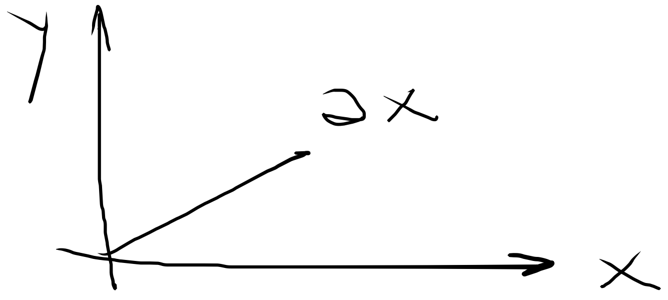
$\underline{Z} = \underline{X} + \underline{Y}$

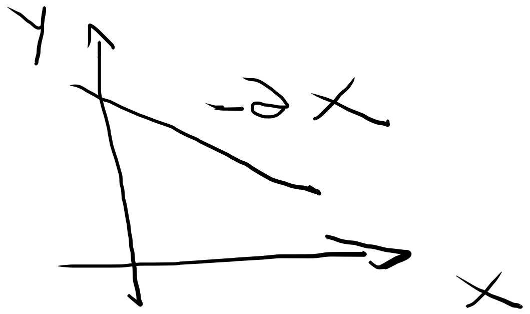
$E[\underline{Z}] = 0 = \mu$

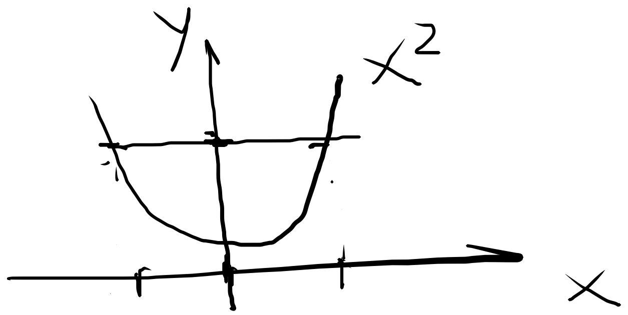
$\mathrm{var}[\underline{Z}] = \mathrm{var}[\underline{X}] + \mathrm{var}[\underline{Y}] = 2 = \sigma^2$

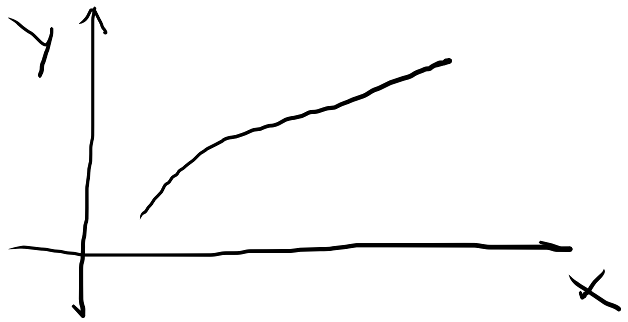
$\sigma_z = \sqrt{2} = \sqrt{\mathrm{var}[\underline{Z}]}$

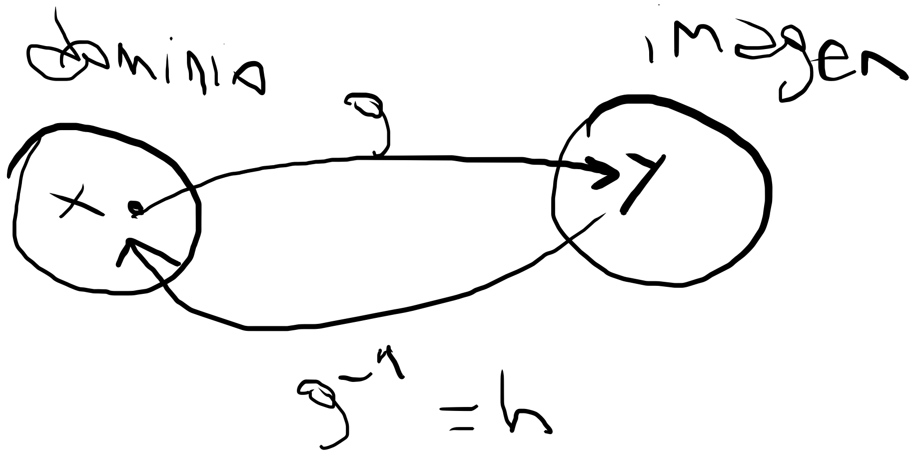
$$f_{\underline{Z}}(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{4\pi}} e^{-\frac{z^2}{4}}$$


y
2x
x


y
-2x
x


y
x^2
x


y
x


dominio
imagen
g
x
y
$g^{-1} = h$

$$\underline{Y} = g(\underline{X})$$

$$f_{\underline{Y}}(y) = f_{\underline{X}}(h(y)) \left| \frac{dh}{dy}(y) \right|$$

$$\begin{tikzcd} X \arrow[r, bend left, "g"] & Y \arrow[l, bend left, "h"] \end{tikzcd}$$

$$\underline{Y} = g(\underline{X}) \longrightarrow \underline{X} = h(\underline{Y}) \qquad h(x) = g^{-1}(x)$$

$$P(\underline{X} \le x) = \int_{-\infty}^{x} f_{\underline{X}}(x)\,dx$$

$$P(\underline{Y} \le y) = \left[\int_{-\infty}^{h(y)} f_{\underline{X}}(x)\,dx\right] \leftarrow P(\underline{X} \le h(y))$$

$$h'(y) = \frac{dh}{dy}(y)$$

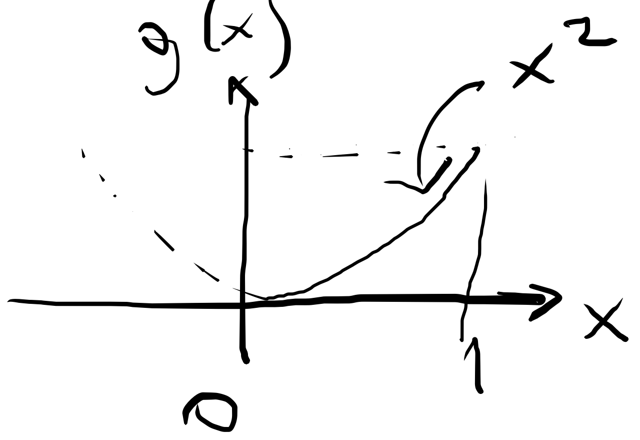
$$\frac{dP}{dy} = f_{\underline{Y}}(y) = \frac{d}{dx}\left(P(\underline{X} \le h(y))\right)$$

Regla de la cadena: $f_{\underline{Y}}(y) = \overbrace{f_{\underline{X}}(h(y))}^{\geq 0}\,\overbrace{|h'(y)|}^{\geq 0}$

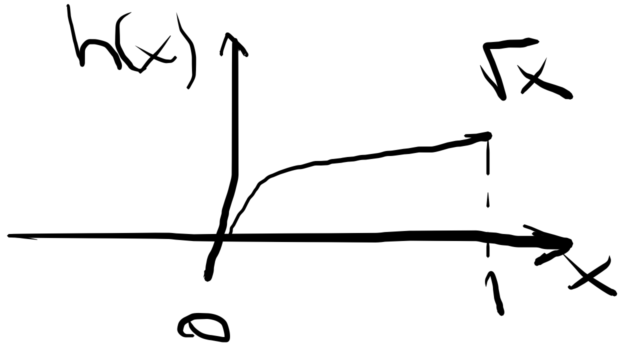
$$X \sim U[0,1] \rightarrow f_X = 1 \;, \; x \in [0,1]$$

$$Y = X^2 \qquad g(x) = x^2$$

$$h(x) = \sqrt{x}$$



$$h(y) = \sqrt{y}$$



$$\frac{dh}{dy} = \frac{1}{2\sqrt{y}} \;, \qquad f_X(\underset{\uparrow [0,1]}{\sqrt{y}}) = 1$$

$$f_Y(y) = \underbrace{f_X(h(y))}_{1} \cdot \left| \frac{dh}{dy}(y) \right| = \frac{1}{2\sqrt{y}} \qquad \int_0^1 \frac{1}{2\sqrt{y}} \, dy = \sqrt{y} \Big|_0^1 = 1$$

Distribución Gaussiana Bivariable

Matriz de Covarianza:

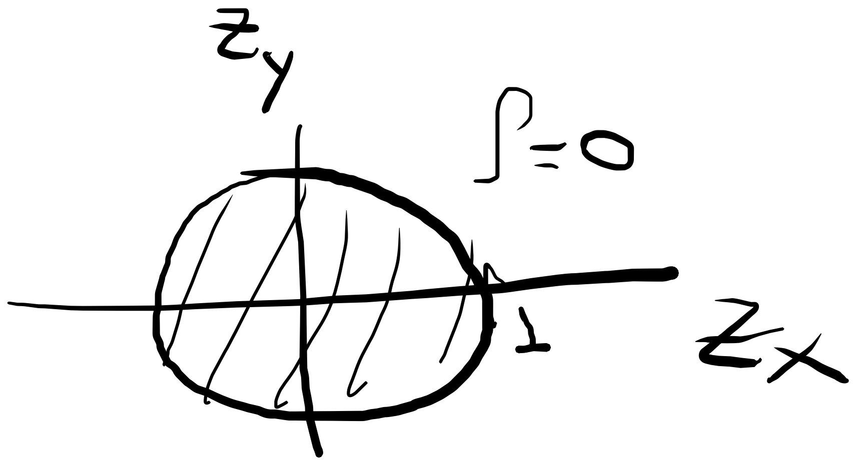
$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

$$|\Sigma| = \sigma_x^2\sigma_y^2 - \rho_{xy}^2\sigma_x^2\sigma_y^2$$

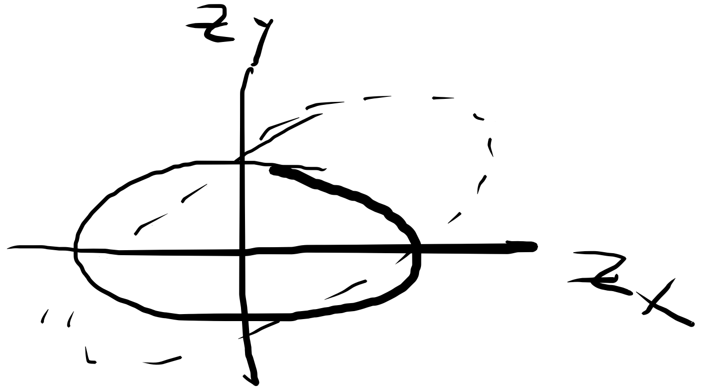
$|\rho_{xy}| = 1 \Rightarrow \rho_{xy}^2 = 1 \Rightarrow |\Sigma| = 0 \Rightarrow$ Singular

$\rho_{xy} = 0$

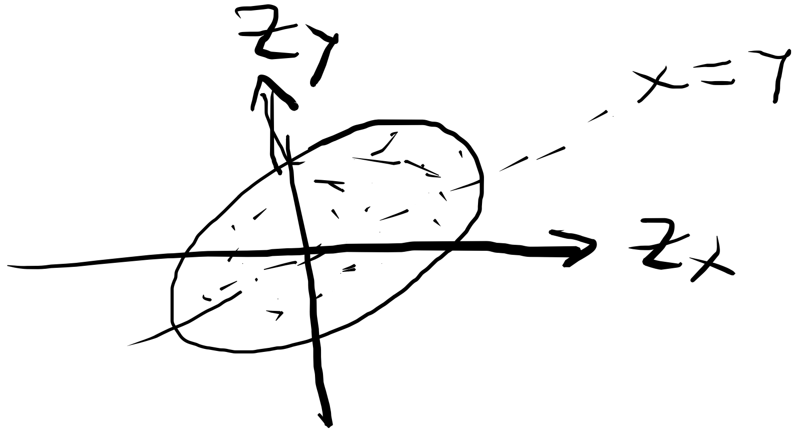
$z_x^2 + z_y^2 \le 1$



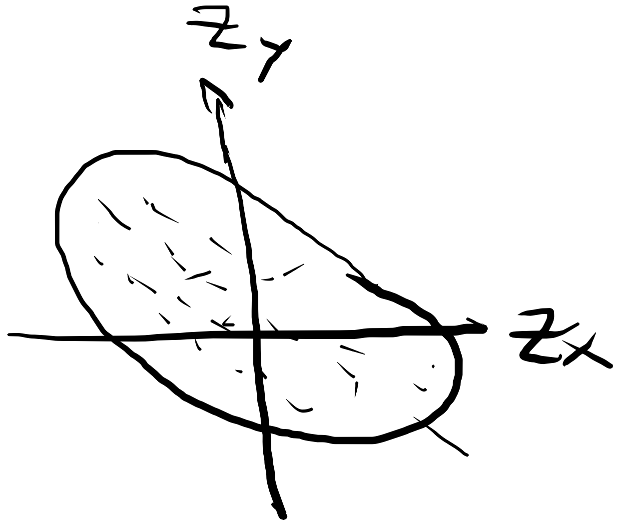
$z_x^2 - 2\rho_{xy} z_x z_y + z_y^2 \le k$



$\rho > 0$



$\rho < 0$



$$\mathrm{cov}[X, X] = \underbrace{\rho_{XX}}_{1} \sigma_X \sigma_X = \sigma_X^2$$

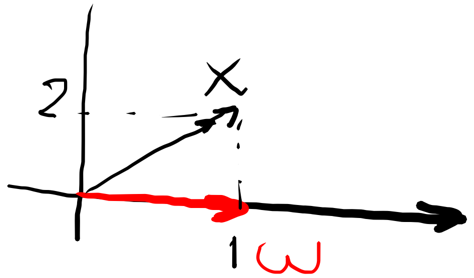
$$\mathrm{cov}[X, Y] = \rho_{XY} \sigma_X \sigma_Y$$

descorrelacionadas

$$\begin{bmatrix} \ddots & 0 \\ 0 & \ddots \end{bmatrix} \qquad \rho_{X_i X_j} = 0$$

Matriz covarianza

varianzas

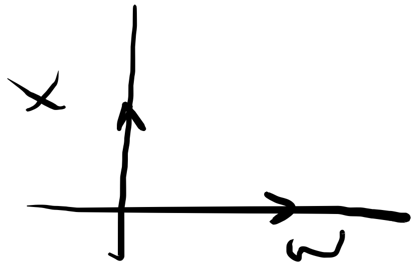


$$w = [1, 0]^T = \begin{bmatrix}1\\0\end{bmatrix}$$

$$x = [1, 2]^T = \begin{bmatrix}1\\2\end{bmatrix}$$

$$w^T \cdot x = [1\ 0] \cdot \begin{bmatrix}1\\2\end{bmatrix} = 1$$

Normalización de vectores $\rightarrow \dfrac{w}{\|w\|_2}$



$$w^T x = 0$$

$$\underline{\bar{z}} = \underline{w}^T \underline{\bar{x}}$$

$$\mu_{\underline{z}} = \underline{w}^T \mu_{\underline{x}}$$

(sólo para escalares) $$\sigma_{\underline{z}}^2 = w^2 \sigma_{\underline{x}}^2$$

$$w^T \cdot w = \|w\|^2$$

$$w^T \Sigma \, w$$