Ej 1 Clase 6

$\underline{X} = [X_1, X_2, \dots, X_n]$ $\quad n = 10$

$x_i = \begin{cases} 1 & \text{, si } i\text{-ésima paciente fallece en hospital } H \\ 0 & \text{si no.} \end{cases}$

$f_{\theta|x} \propto f_{x|\theta}\, \pi(\theta)$

Verosimilitud: $f_{x_j|\theta}(x_j|\theta) = \theta^{i}(1-\theta)^{1-i}$

$i = 1 \rightarrow$ éxito $\Rightarrow \theta$

$i = 0 \rightarrow$ fracaso $\Rightarrow 1-\theta$

$\Big\}$ forma compacta

Bernoulli multivariable $f_{\underline{x}|\theta}(x|\theta) = \theta^{\sum x_i}(1-\theta)^{n-\sum x_i}$

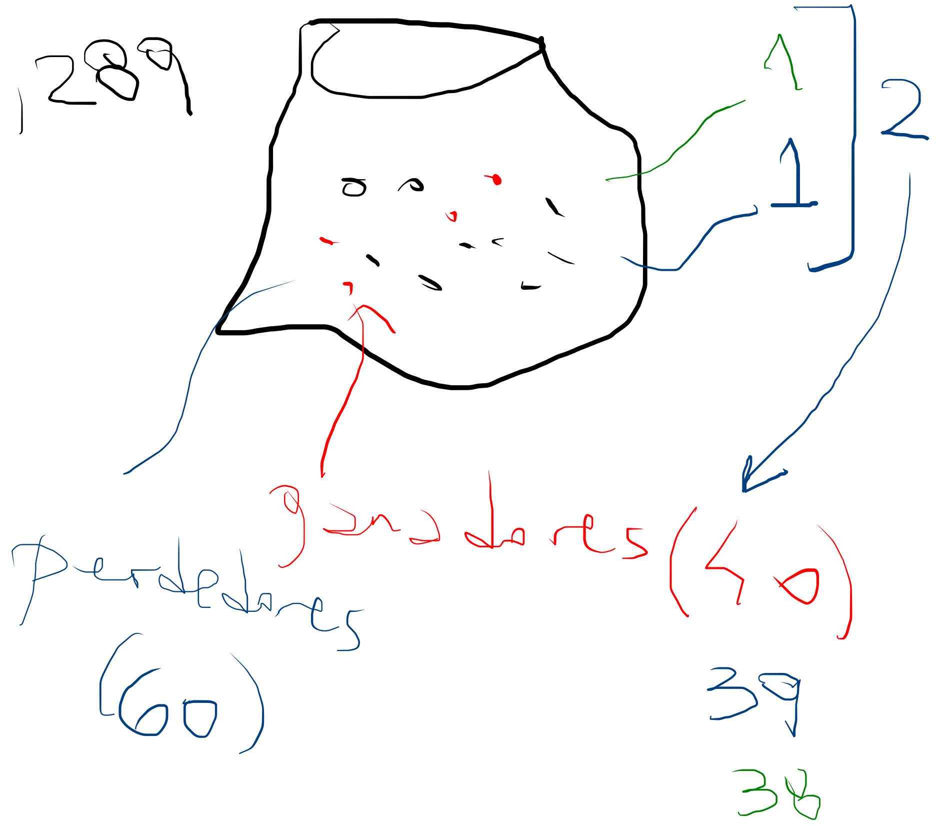
Ej 1 Clase 7

a) X: cantidad de premios

cuántos hay

$N = 100$ $\quad$ $n = 3$ ← compramos 3

$$P(X=2) = \frac{\text{casos favorables}}{\text{casos posibles}} = \frac{\underbrace{\binom{40}{2}}_{\text{ganadores}} \underbrace{\binom{60}{1}}_{\text{Perdedor}}}{\binom{100}{3}} = 0{,}289$$



Ej 2

$P(A) = 0{,}3 \quad P(B) = 0{,}5 \quad P(C) = 0{,}2$; V: votar

$P(V|A) = 0{,}65 \quad P(V|B) = 0{,}82 \quad P(V|C) = 0{,}5$

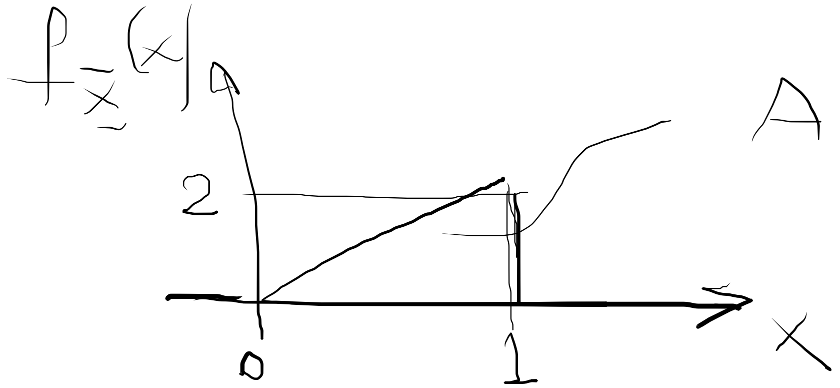
$$P(B|V) = \frac{P(V \cap B)}{P(V)} = \frac{P(V|B)\,P(B)}{P(V)} = 0{,}58$$

dato: persona al azar vota

$$P(V) = P(V|A)P(A) + P(V|B)P(B) + P(V|C)P(C).$$

Ej 3

$$X \to f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{si no} \end{cases}$$



$$A = \frac{1 \times 2}{2} = 1 \checkmark$$

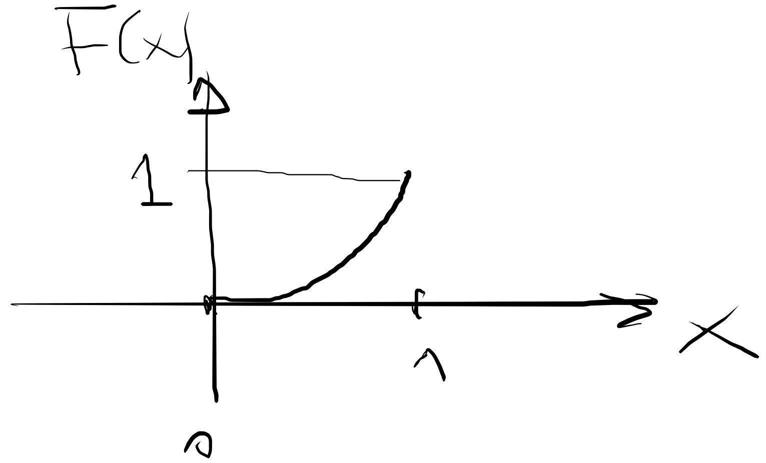
$$E[X] = \int_{-\infty}^{+\infty} x \, f_X(x) \, dx = \int_0^1 x \, 2x \, dx = \int_0^1 2x^2 \, dx = \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3}$$

$$var[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 f_X(x) \, dx = \int_0^1 \left(x - \frac{2}{3}\right)^2 2x \, dx = 0{,}055$$

$$Y = -2X + 3 \qquad E[Y] = -2E[X] + 3 = \frac{5}{3}$$

$$var[Y] = 4 \, var[X] = 0{,}88.$$

CDF: $F_{X}(x) = \int_{-\infty}^{x} f_{X}(x)\,dx = x^2, \quad 0 \le x \le 1$



$$F(x) = x^2 \rightarrow F^{-1}(x) = \sqrt{x}$$

Metodo transf. inversa: Quiero generar X a partir de $U \sim \text{Unif}[0,1]$

$$X = F^{-1}(U).$$

$$U \rightarrow \boxed{F^{-1}(x)} \rightarrow X \sim F(x) \text{ cdf}$$

U Unif[0,1]

$f(x)$ pdf

$$f(x) = \frac{dF(x)}{dx}$$

$$\pi(\theta) = \text{Beta}(\underset{a}{3}, \underset{b}{27})$$

$$\pi(\theta|x) \propto f_{X|\theta}(x|\theta)\,\pi(\theta) = \theta^{\overbrace{\sum x_i}^{0,\ \text{por enunciado}} + a - 1}(1-\theta)^{\overbrace{n}^{10} - \sum x_i + b - 1}$$

$$\pi(\theta|x) = \text{Beta}\Big(\underbrace{\sum x_i}_{0} + 3,\ \underset{10}{n} - \underbrace{\sum x_i}_{0} + 27\Big) = \text{Beta}(\underset{\alpha}{3}, \underset{\beta}{37})$$

$$E[\theta|x] = \frac{\alpha}{\alpha+\beta} = \frac{3}{40} = 0{,}075.$$

Ej 5

$\mu = 100, \ \sigma = 15$

$\underline{X} \sim N(\mu, \sigma^2)$

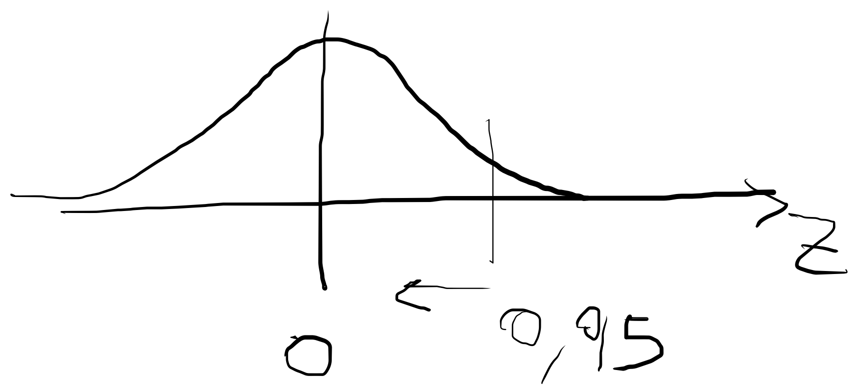
$P(\underline{X} > 125) = 1 - \underbrace{P(\underline{X} \leq 125)}$

$z \sim N(0,1)$

$x = \sigma z + \mu \Rightarrow z = \dfrac{x - \mu}{\sigma}$

$P(\underline{X} \leq 125) = P\left(z \leq \dfrac{125 - 100}{15}\right) = P(z \leq 1{,}67) = 0{,}952$

$P(\underline{X} > 125) = 1 - 0{,}952 = 0{,}047$



$z = 1{,}64$

$x = \sigma z + \mu = 124{,}6$

1,64

(redondear hacia arriba si es necesario).

↳ 125

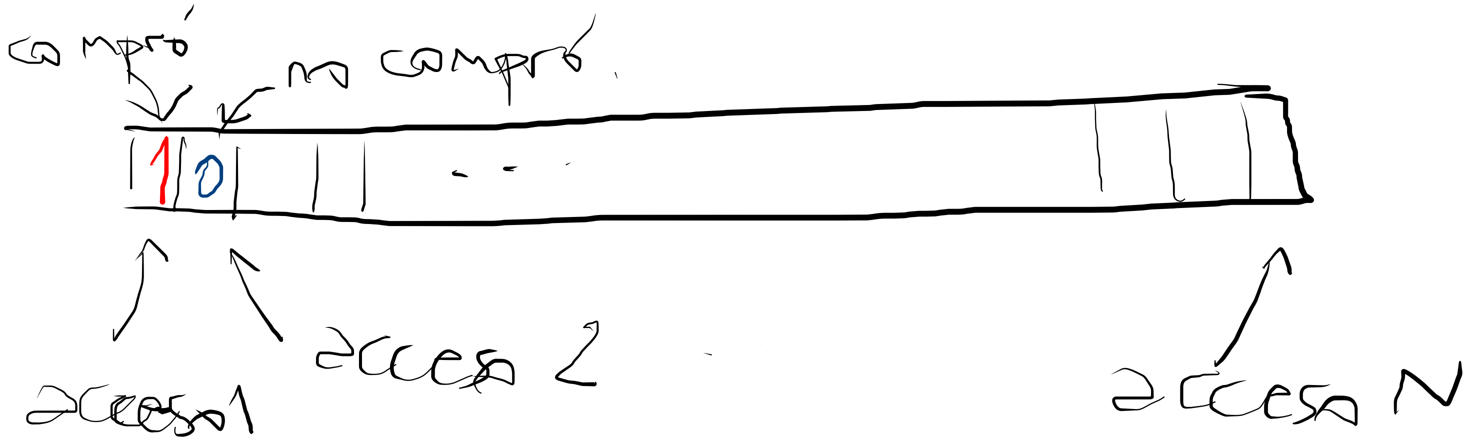
Ej 9

p: prob compra

$N = 500$ visitas

$K = 15$ compras.

$$\hat{p} = \frac{15}{500} = 0{,}03$$



Quiero saber cuántos compraron:

$p \sim \quad \hat{p},$

$$x_i = \begin{cases} 1, & \text{si compra} \\ 0, & \text{si no} \end{cases}$$

$$E\left[\sum x_i\right] = Np$$

$$var\left[\sum x_i\right] = Np(1-p)$$

$$\hat{p} = \frac{\overbrace{\sum x_i}^{\text{binomial}}}{N}$$

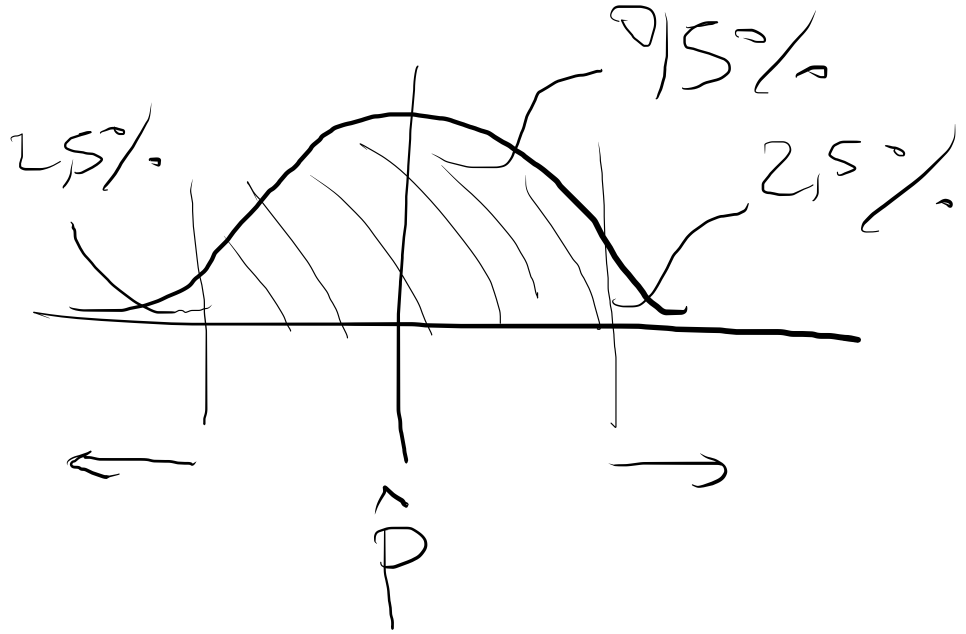
$$E[\hat{p}] = \frac{\cancel{N}p}{\cancel{N}} = p$$

$$var[\hat{p}] = \frac{\cancel{N}p(1-p)}{N^2} = p(1-p)/N$$

I.C. 95% p:

$p \in (p_{min} , p_{MAX})$

Podemos usar t-Student o Normal (N es grande).



$$\hat{p} \pm 1{,}96 \underbrace{\sqrt{\frac{\hat{p}(1-\hat{p})}{N}}}_{\sqrt{\sigma^2_{\hat{p}}}} = \hat{p} \pm 1{,}96 \cdot 0{,}0076$$

$$= 0{,}03 \pm 0{,}015$$

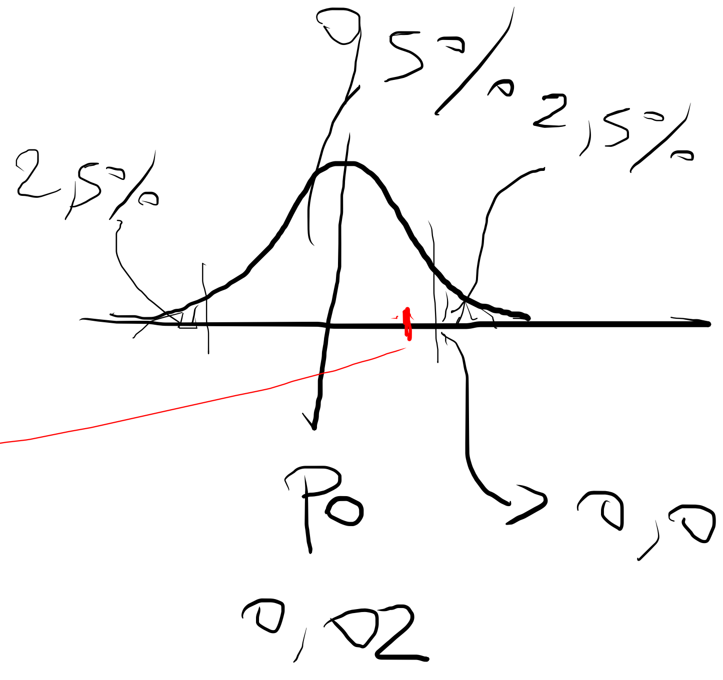
$$IC_{95\%} = (0{,}015 \; ; \; 0{,}045)$$

Test de Hipótesis

$H_0: p = 0{,}02 = P_0$

$H_1: p \neq 0{,}02$

$\hat{P}_{(dato)} = 0{,}03$



$$0{,}02 \pm \underbrace{1{,}96 \cdot \sqrt{\frac{0{,}02\,(1-0{,}02)}{N}}}_{0{,}012}$$

$$(0{,}02 - 0{,}012 ; 0{,}02 + 0{,}012) = (0{,}008 ; 0{,}032)$$

No podemos rechazar Ho.