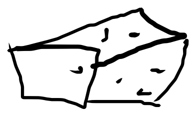
Ej 1

Juan	Pedro
2	1
3	1
3	2
4	1, 2, 3
5	1, 2, 3, 4
6	1, 2, 3, 4, 5



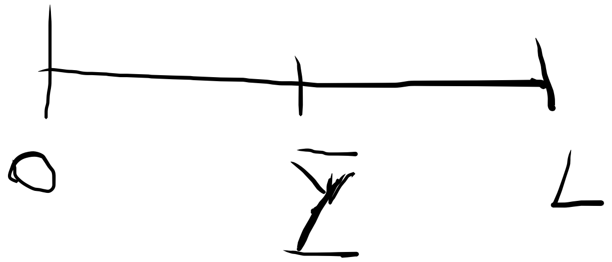
$$P(X = i) = \frac{1}{6}$$

$$\{1, 2, 3, 4, 5, 6\}$$

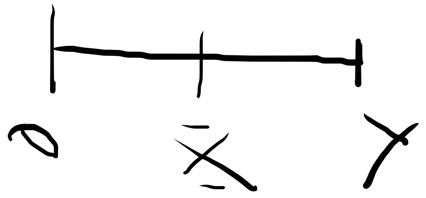
$\boxed{\text{Ganó Juan.}}$

$$\hat{p} = \frac{\text{casos favorables}}{\text{casos posibles}} = \frac{4}{15}$$

Ej 2



$$Y \sim U[0, L] \qquad E[Y] = L/2$$



$$X \sim U(0, Y] \Rightarrow E[X \mid Y = y] = y/2$$

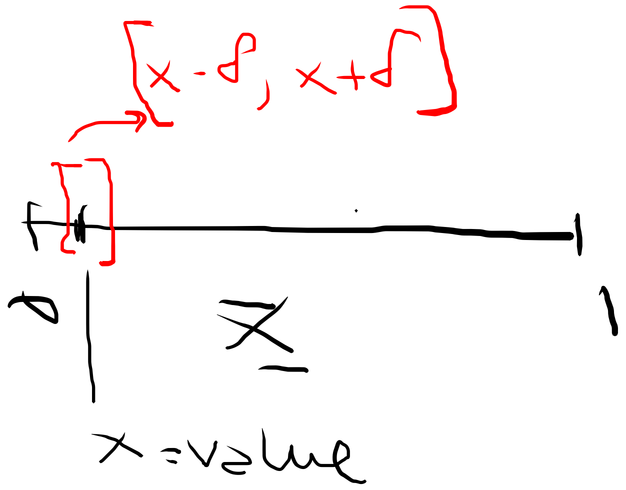
$$E[X] = E\big[E[X \mid Y]\big] = E[Y/2] = \frac{E[Y]}{2} = \frac{L}{4}$$

Ej 3

$X, Y \sim U[0,1]$ iid

$Z = X + Y$

$$E[Z \mid X] = E[x + y \mid X = x] = \underbrace{E[X \mid X = x]}_{x} + \underbrace{E[Y \mid X]}_{E[Y]} = x + \frac{1}{2}$$

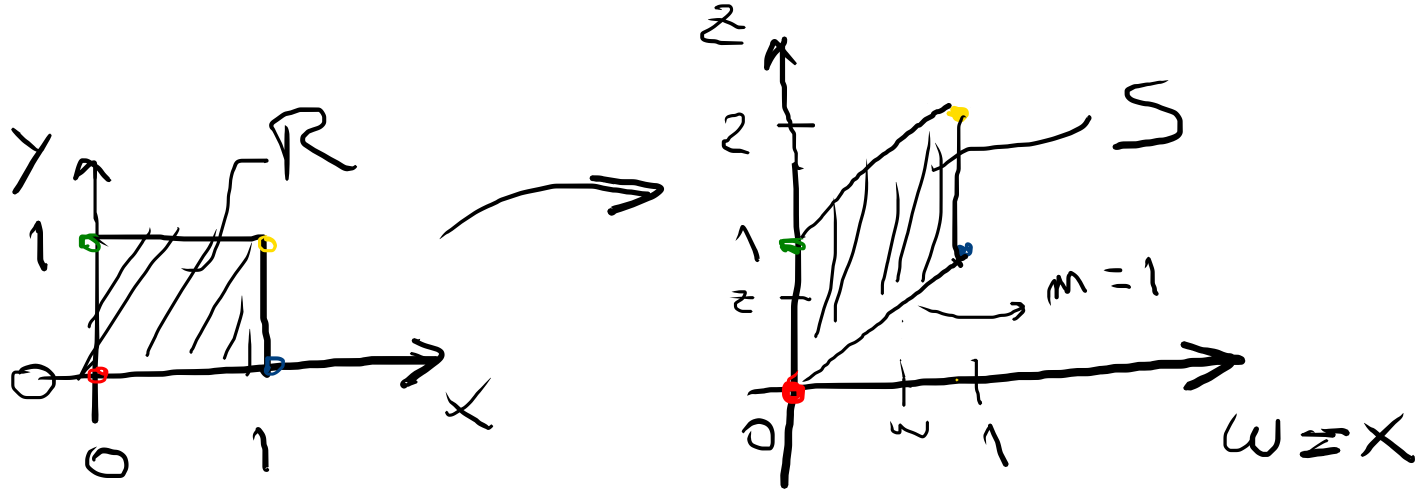


$$E[\underline{X}\underline{Z} \mid \underline{X}=x] = x\,E[\underline{Z} \mid \underline{X}] = x\left(x+\frac{1}{2}\right)$$

$$E[XZ \mid Z] = Z\,E[X \mid Z]$$

$$\boxed{E[\underline{X} \mid \underline{Z}]}$$

$$\begin{cases} Z = X + Y \\ W = X \end{cases} \qquad X, Y \sim U[0,1] \text{ iid}$$



$$(x,y) = (0,0) \longrightarrow (w,z) = (0,0)$$
$$= (1,0) \longrightarrow \quad = (1,1)$$
$$= (0,1) \longrightarrow \quad = (0,1)$$
$$= (1,1) \longrightarrow (1,2)$$

$$J = \begin{vmatrix} \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$E[\underline{W} \mid \underline{Z} = z] = E[\underline{X} \mid \underline{Z} = z]$$

Por transformación $(w = x)$

$$z \in [0,1] \Rightarrow w \in [0,z] \Rightarrow \underbrace{x \in [0,z]}_{\text{unif.}} \Rightarrow E[\underline{X} \mid \underline{Z} \in [0,1]] = \frac{z}{2}$$

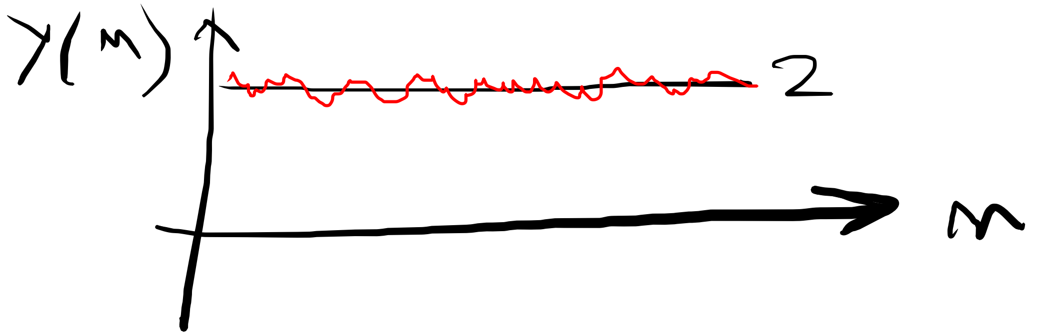
$$z \in [1,2] \rightarrow x \in [z-1, 1] \Rightarrow E[\underline{X} \mid \underline{Z} \in [1,2]] = \frac{z-1+1}{2} = \frac{z}{2}$$

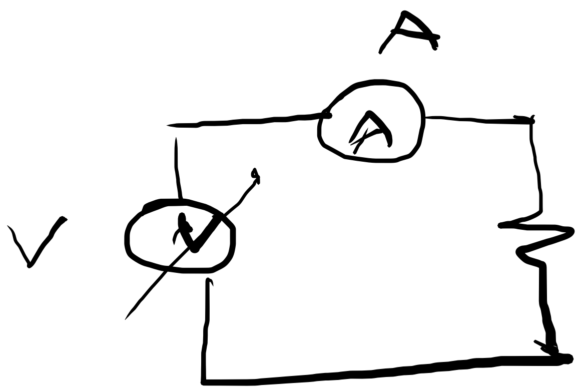
$$E[\underline{X} \mid \underline{Z}] = \frac{Z}{2} \quad w, z \in S.$$

$$\Rightarrow E[XZ \mid Z] = Z\, E[X \mid Z] = \frac{Z^2}{2}$$

Ej 4

$$y(n) = 2 + w(n) \quad , \quad w(n) \sim N(0,1)$$





$$\hat{R} = \frac{V}{I} + w(n)$$

Media muestral

$$\hat{y} = \bar{y} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) \rightarrow E[w[n]] = 0 .$$

$$E[\hat{y}] = \frac{1}{N} \sum_{N} \underbrace{E[y]}_{2} = \frac{1}{\cancel{N}} \cdot \cancel{N} \cdot 2 = 2 . \leftarrow$$ estimador insesgado de la media

$$\mathrm{var}[y] = 1$$

$\hat{x}$ estimador de un parámetro de $\underline{X}$ (por ej. la media)

$\hat{x} = g(\underline{X}) = g(x_i)$ x_i surgen de $\underline{X}$

$\hat{x}$ es una v.a. $\longrightarrow E[\hat{x}]$

$\text{var}[\hat{x}]$

parámetro a estimar

bias (sesgo) $\rightarrow$ $b = E[\hat{x}] - \mu$ $= 0$ $\leftarrow$ insesgado

$\neq 0$ $\leftarrow$ sesgado.

$$\text{var}[\hat{x}] = E\left[(\hat{x} - E[\hat{x}])^2\right]$$

$$MSE[\hat{x}] = \text{var}[\hat{x}] + (b[\hat{x}])^2$$

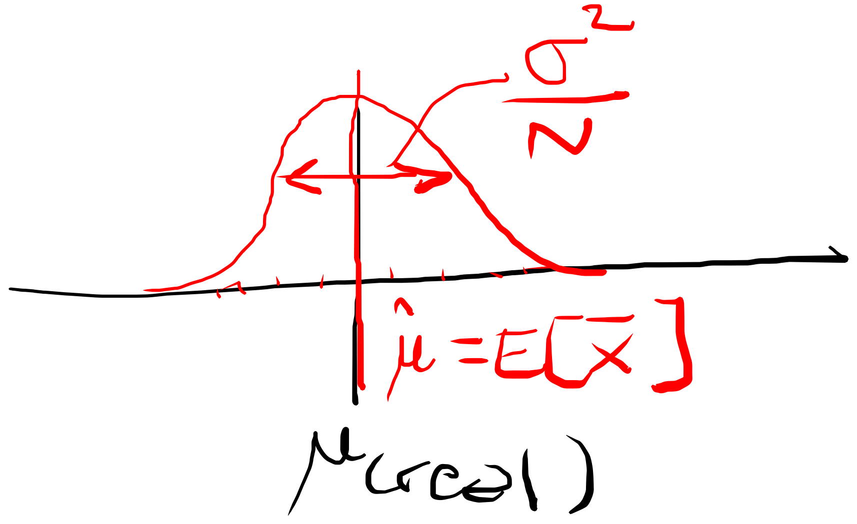
Estimador media muestral:

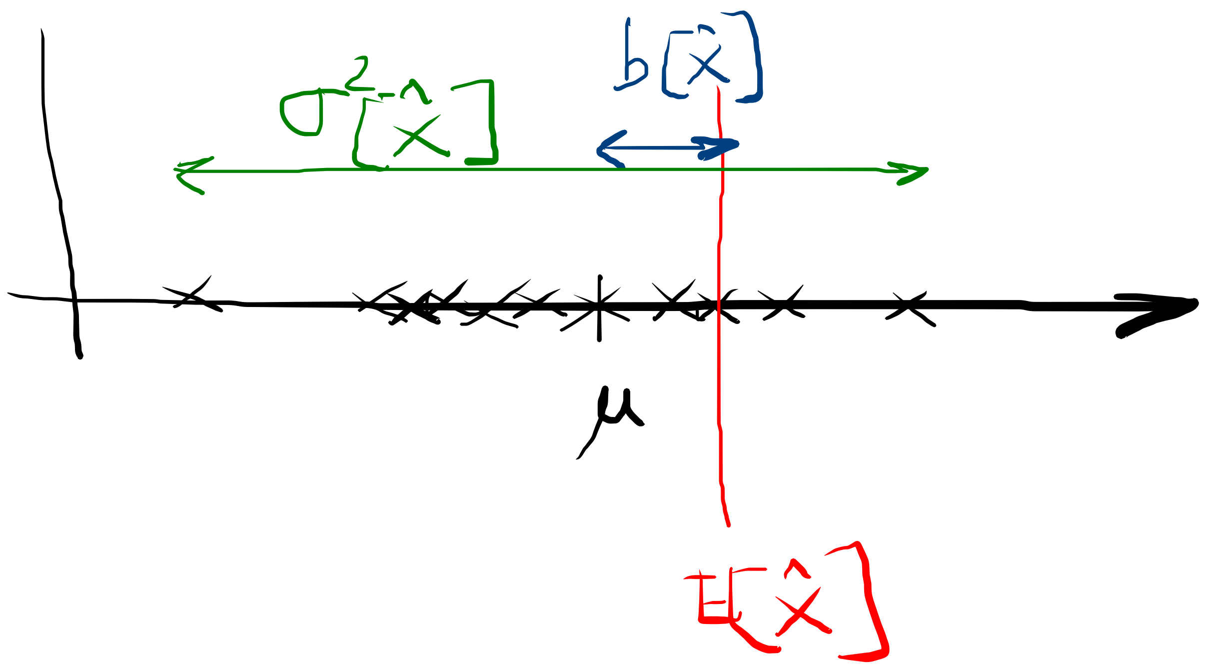
$$\bar{x} = \frac{1}{N} \sum_{i=0}^{N-1} x_i$$

$$\boxed{\begin{array}{l} x_i \sim \mu, \sigma^2 \\ x_i \text{ iid} \end{array}}$$

$$E[\bar{x}] = \frac{1}{N} \sum_{i=0}^{N-1} \underbrace{E[x_i]}_{\mu} = \frac{1}{N} \underbrace{\sum_{i=0}^{N-1} \mu}_{N\mu} = \mu \leftarrow \text{insesgado}$$

$$\text{var}[\bar{x}] = \frac{1}{N^2} \underbrace{\sum_{i=0}^{N-1} \sigma^2}_{N\sigma^2} = \frac{\sigma^2}{N}$$





$$MSE = var + b^2$$

$X \to x_i$ (muestras)

$\hat{X} \to \hat{x}_i = g(x_i)$

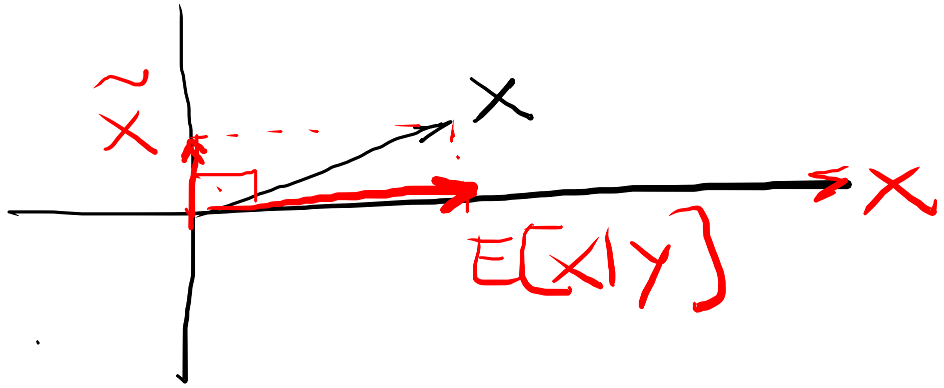
Quiero estimar μ con $\hat{x}$

$\tilde{x} = x - \hat{x}$ $\qquad$ $\hat{x} = E[x|y] \Rightarrow E[\hat{x}] = E\big[E[x|y]\big] = E[x]$

$E[\tilde{x}] = E[x] - \underbrace{E[\hat{x}]}_{E[x]} = 0 \rightarrow$ esperanza del error

$E[\hat{x}\,\tilde{x}] = 0 \rightarrow$ ortogonales

$\Rightarrow$ descorrelacionadas



$\hat{x} = ay + b$ $\qquad E[(x - \hat{x})^2]$ min $\rightarrow E[(x - ay - b)^2]$

fijo 'a' $\Rightarrow$ nueva v.a. $x - ay \Rightarrow \cancel{b}$ $\qquad b = E[x - ay]$

$b = E[x] - a\,E[y]$

$$\frac{\partial E[\cdot]}{\partial a} = 0 \Rightarrow a.$$

$y = x + w$

$E[x]$, $\mathrm{cov}[x, y]$, $\mathrm{var}[y]$.

$E[y]$

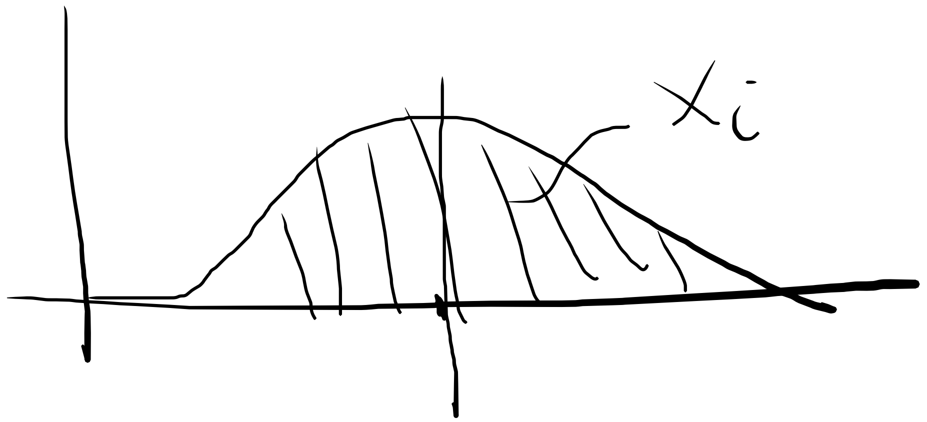
$$y(n) = a\,x(m) + b \cancel{+\varepsilon}$$

$$\left[\begin{aligned} y(1) &= a\,x(1) + b + \varepsilon(1) \\ y(2) &= a\,x(2) + b + \varepsilon(2) \\ &\;\vdots \\ y(m) &= a\,x(m) + b + \varepsilon(m) \end{aligned}\right.$$

$$y = \begin{bmatrix} y(1) \\ \vdots \\ y(m) \end{bmatrix} \qquad \underset{\sim}{x} \qquad A = \begin{bmatrix} x(1) & 1 \\ \vdots & \vdots \\ x(m) & 1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\min \|y - x\|^2 \to \beta \qquad y = A\beta \;\Rightarrow\; \beta = \underbrace{(A^T A)^{-1} A^T}_{\mathrm{Pinv}(A)}\, y.$$



$\theta \sim$ máxima $p(x|\theta)$.

$\mu = ?$

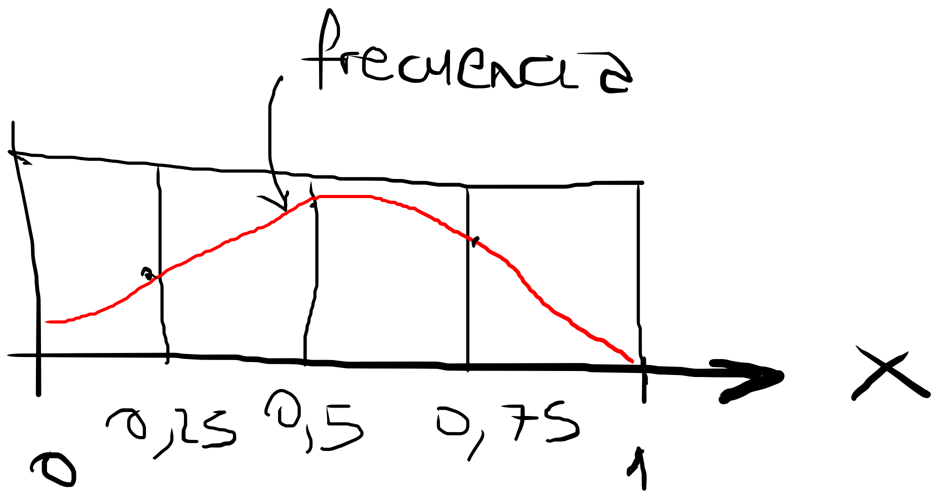
$\rightsquigarrow \mu = 1$ (por ej.)

σ^2 es conocida

$x_1, \dots, x_m \sim$ iid $\quad P_{x_1 x_2 \dots x_m|\theta} = \prod P_{x_i|\theta}$

$$\prod_{i=1}^{N} p(x_i|\mu) = \frac{1}{(2\pi)^{N/2}} \frac{1}{\sigma^N} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{N} (x_i - \mu)^2\right]$$

Histograma



$M = 4$

$n = 100$

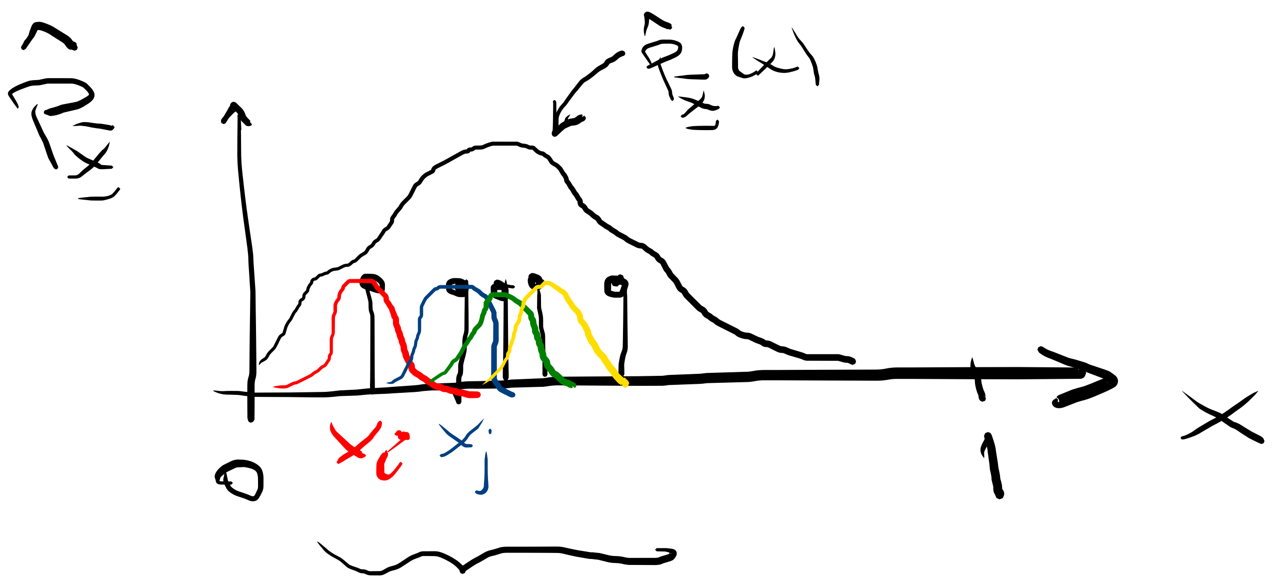
$$B_1 = \left[0, \frac{1}{M}\right]$$

$$B_2 = \left[\frac{1}{M}, \frac{2}{M}\right]$$

$$\vdots$$

$$B_M = \left[\frac{M-1}{M}, 1\right]$$

Estimación de densidad de Kernel



'm' muestras

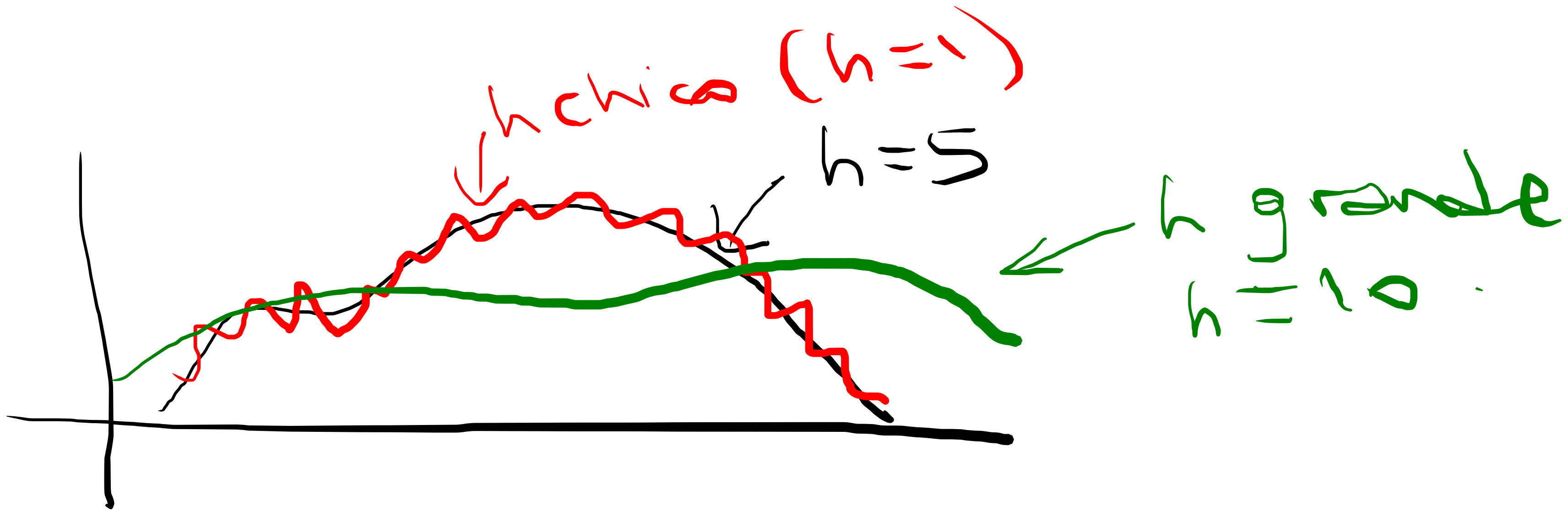
Σ distribuciones

muestra x_i

$$\hat{p}_{\underline{x}}(x) = \frac{1}{mh} \sum_{i=1}^{m} K\left(\frac{x_i - x}{h}\right)$$

parámetro.

$$\int \hat{p}_{\underline{x}}(x)\,dx = 1$$


h chica (h=1)
h=5
h grande
h=10