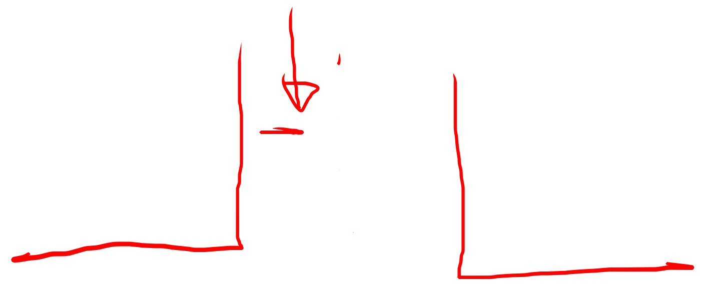
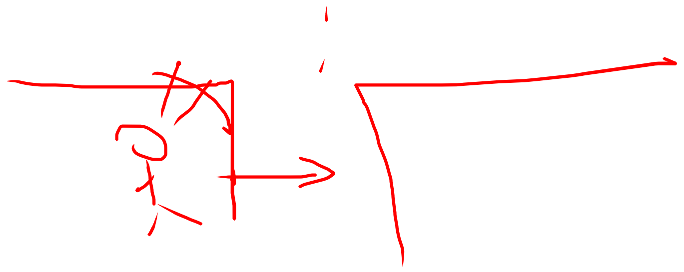




$S = \{H, T\}$

H T

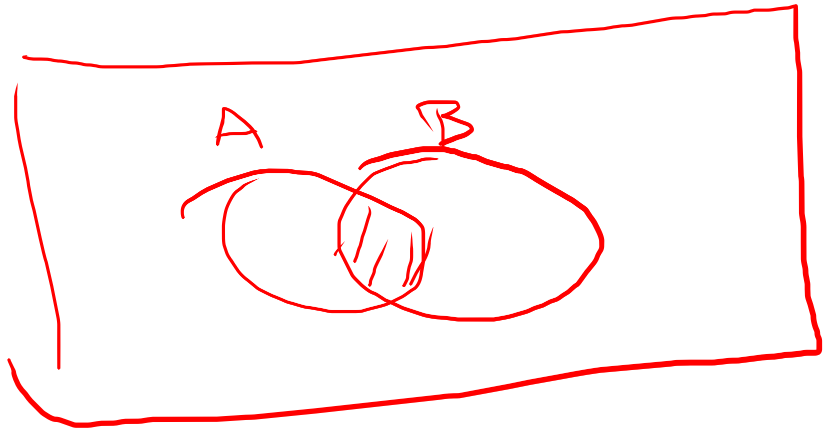
P $\quad 1-P$

Prob. $[0, 1]$

$P(E) \in [0,1]$

$P(S) = 1$

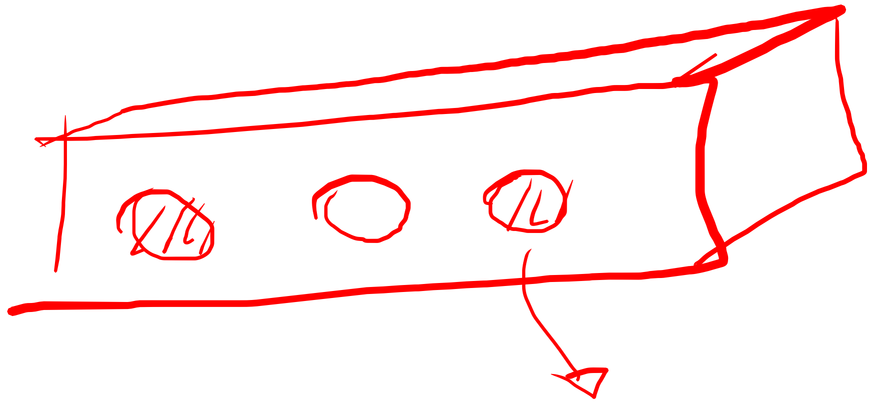
$P = \dfrac{1}{2}$ ← favorable
→ posible.



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cup B \quad - A \cap B$

Probabilidad Condicional



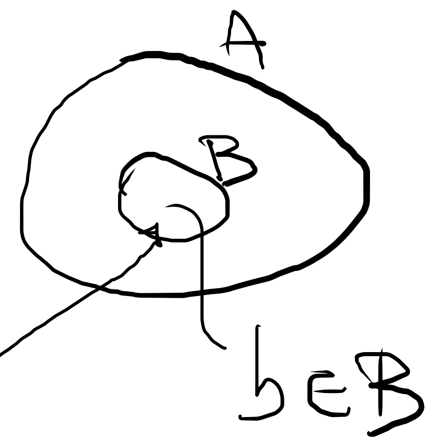
$$P(\text{blanca}) = \frac{1}{3} \quad (\text{a priori})$$

sin observación

$$P(\text{blanca} \mid \underbrace{\text{saque roja}}_{\text{observación}}) = \frac{1}{2}$$

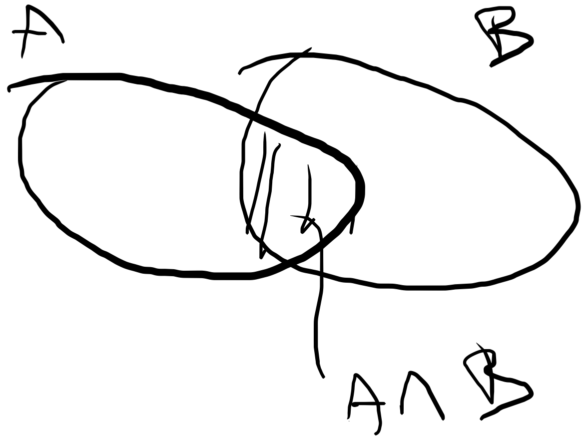
a posteriori

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$



$A \cap B \to B$

En este caso $P(A|B) = \dfrac{P(B)}{P(B)} = 1$



$$P(A|B) = \frac{\overbrace{P(A\cap B)}^{\text{intersección de evento.}}}{\underbrace{P(B)}_{\text{normalizo}}}$$

Independencia

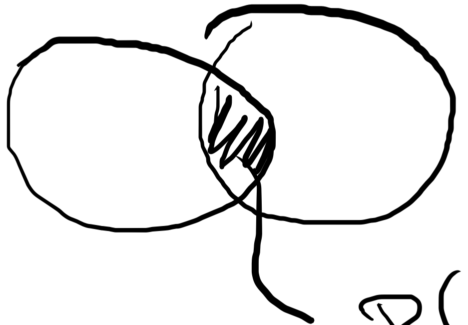
$P(A|B) = P(A)$ A es indep. de B.

$P(B|A) = P(B)$

$P(A\cap B) = P(A)P(B)$

$A\cap B = \emptyset$

$P(A\cap B) = 0$ No!



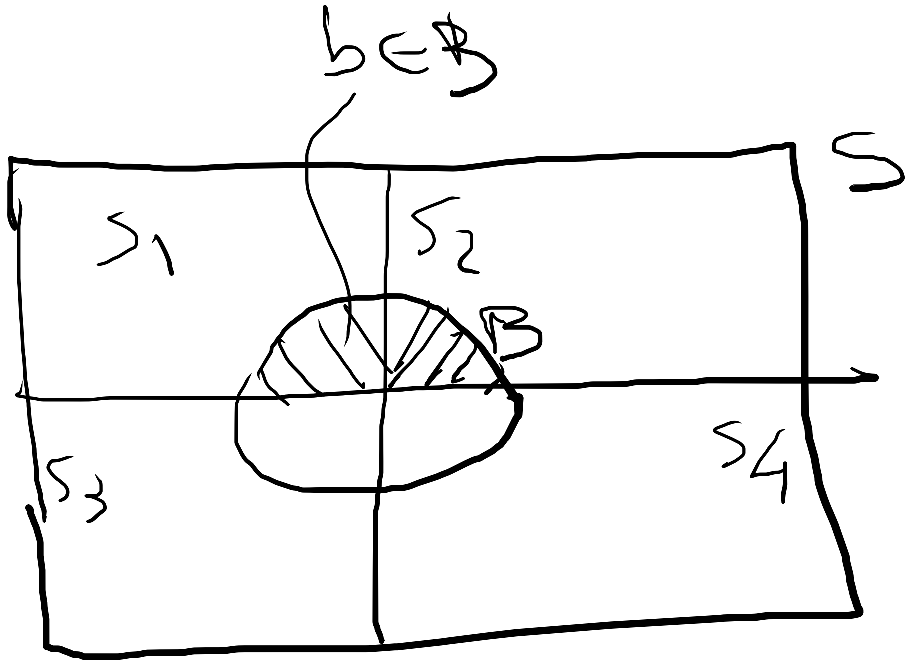
$P(A\cap B) = \underbrace{P(A)}_{\le 1}\underbrace{P(B)}_{\le 1} \le P(A)$

$\le P(B)$

Bayes

$$P(A|B) = \frac{\overbrace{P(B|A)}^{\text{verosimilitud}}\ \overbrace{P(A)}^{\text{a priori}}}{\underbrace{P(B)}_{\text{total } B}}$$

invirtiendo
el problema
en base a datos
y algunas hipótesis



$S = \cup S_i$

$\cap S_i = \emptyset$

$P(B) = P(B|S_1) \cdot P(S_1) + P(B|S_2)\, P(S_2) + \dots + P(B|S_4)\, P(S_4)$

Probabilidad total.

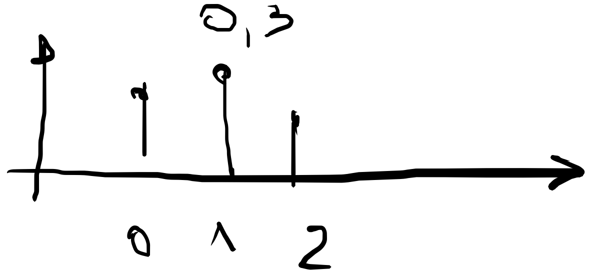
$$\overset{\text{v.a.}}{\underline{X}} \le \overset{\text{valor}}{x}$$

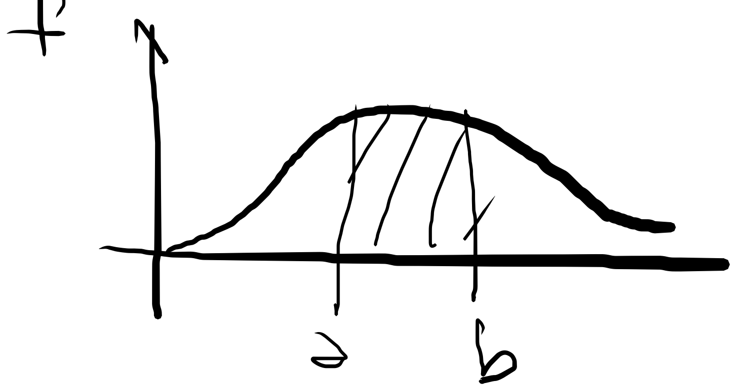
$$F(\underline{X} \le x) = \begin{cases} \ge 0 \ \wedge \le 1 \\ \text{monótona (no decreciente)} \end{cases}$$

$F(\underline{X} \le x)$

$$\underset{\text{cdf}}{F(x)} = \int_{-\infty}^{x} \underset{\text{pdf}}{f(t)}\, dt$$

$P(\underline{X} = x)$





$$\int_{-\infty}^{+\infty} f(t)\, dt = 1$$

$$F(b) - F(a)$$

$x, y \quad f_{\underline{X}\underline{Y}}$

$$P(\underline{X} \le x, \underline{Y} \le y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{\underline{X}\underline{Y}}(x,y)\,dy\,dx = F(x,y)$$

$$F_{\underline{X}}(x) = P(\underline{X} \le x, \underline{Y} \le +\infty) = \int_{-\infty}^{x} \underbrace{\int_{-\infty}^{+\infty} f_{\underline{X}\underline{Y}}(x,y)\,dy}_{f_{\underline{X}}(x)}\,dx$$

Distrib. Condicionales

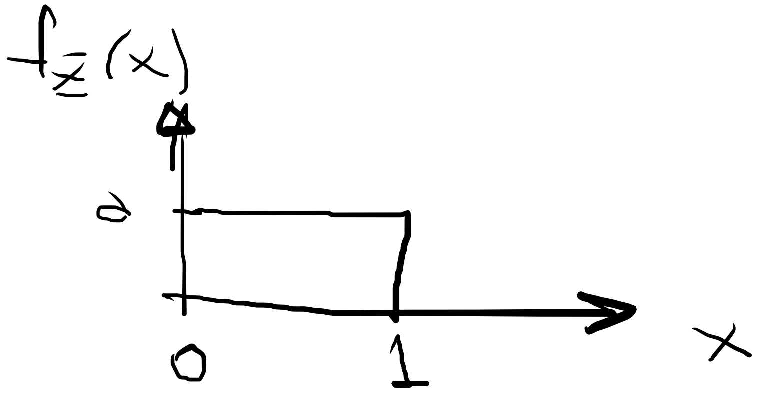
$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

X, Y independ. $\Rightarrow f_{X|Y} = f_X$

$$f_{XY} = f_X f_Y$$

Si $X = Y$ $\quad P_{X|X}(x|x) = 1 \qquad P_{X|X} = \frac{\overbrace{P(X \cap X)}^{P(X)}}{P(X)} = 1$

Distrib. Uniforme



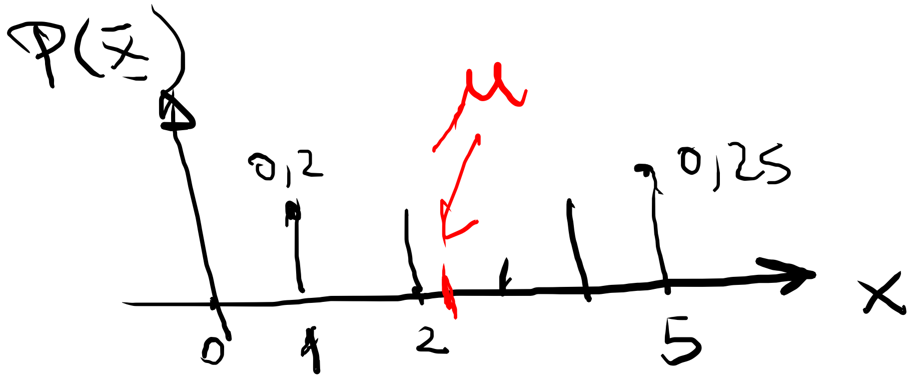
$$\int_{-\infty}^{+\infty} f_Z(x)\,dx = \cancel{\int_{-\infty}^{0} 0 \cdot dx} + \int_{0}^{1} a\,dx + \cancel{\int_{1}^{+\infty} 0 \cdot dx} \triangleq 1$$

$$\Rightarrow a \underbrace{\int_0^1 dx}_{1} = 1 \Rightarrow a = 1.$$

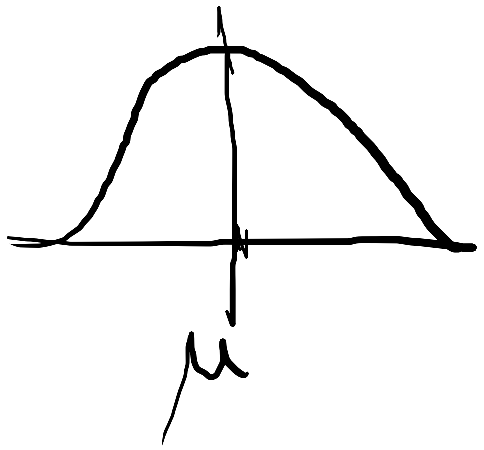
Esperanza

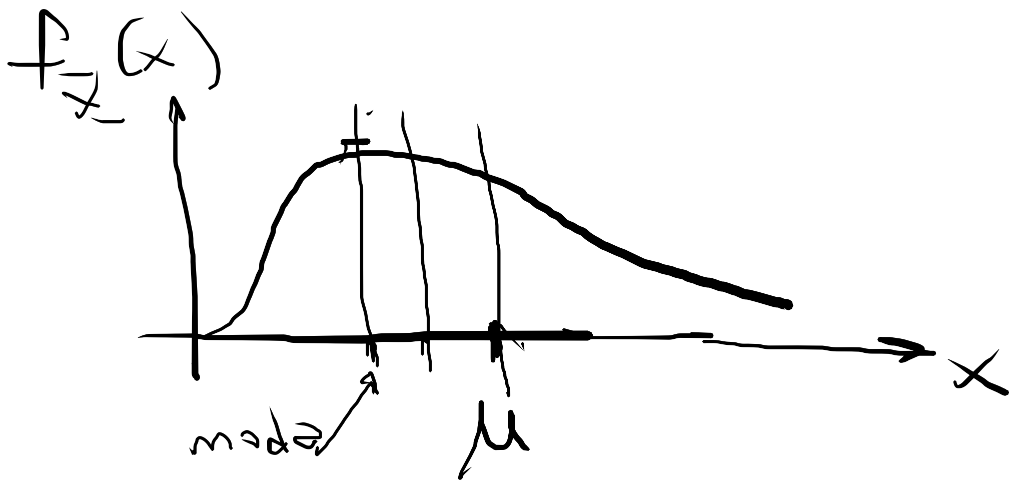
discreto

$$\mu = E[\underline{X}] = \sum_i x_i \, P(\underline{X} = x_i)$$



$$= 1 \times 0{,}2 + 2 \times 0{,}15 + \dots + 5 \times 0{,}25 = 0{,}21$$





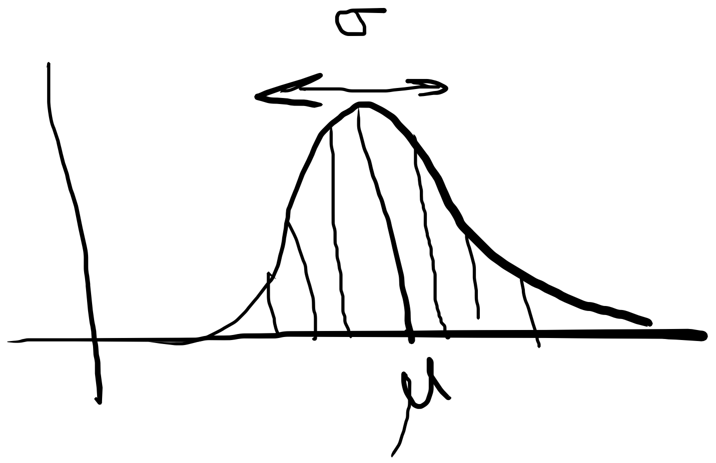
Continua

$$\mu = \int_{-\infty}^{+\infty} x \, f_{\underline{X}}(x) \, dx$$

Varianza

Variabilidad respecto de la media.

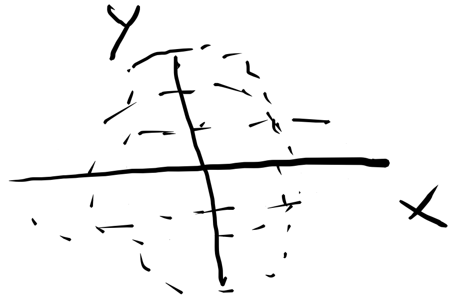
$$\text{var}[X] = \sigma^2_X = E[(x-\mu_X)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f_X(x)\,dx$$



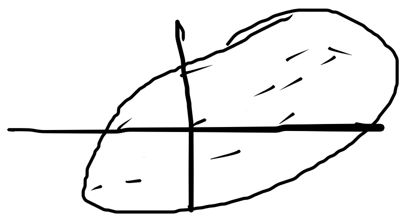
Índice de correlación

$$\rho_{\underline{X}\,\underline{Y}} = \frac{cov[X, Y]}{\sigma_{\underline{X}}\,\sigma_{\underline{Y}}} \in [-1, 1]$$

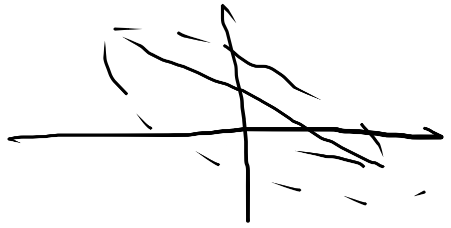
$\rho = 0 \rightarrow \underline{X}, \underline{Y}$ descorrelacionadas



$\rho = 1 \rightarrow \underline{X}, \underline{Y}$

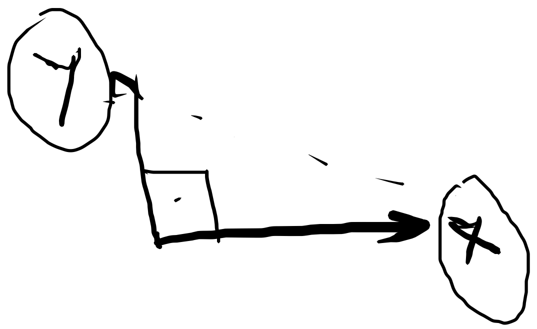


$\rho = -1 \rightarrow \underline{X}, \underline{Y}$



Correlación $\neq$ Independencia

Pero si es Gaussiano corr $\iff$ independencia



$E[XY] \rightarrow$ descorr.

Teorema Central del Límite

$x_1, \ldots, x_m$ $\qquad E[x_i] = \mu$

$\qquad\qquad\qquad var[x_i] = \sigma^2$

$$\hat{\mu} = \frac{\sum_{i=1}^{N} x_i}{N} \xrightarrow[N \to \infty]{} \mu$$

$$x_1 + \cdots + x_N \xrightarrow[N \to \infty]{} \mathcal{N}(N\mu, N\sigma^2)$$

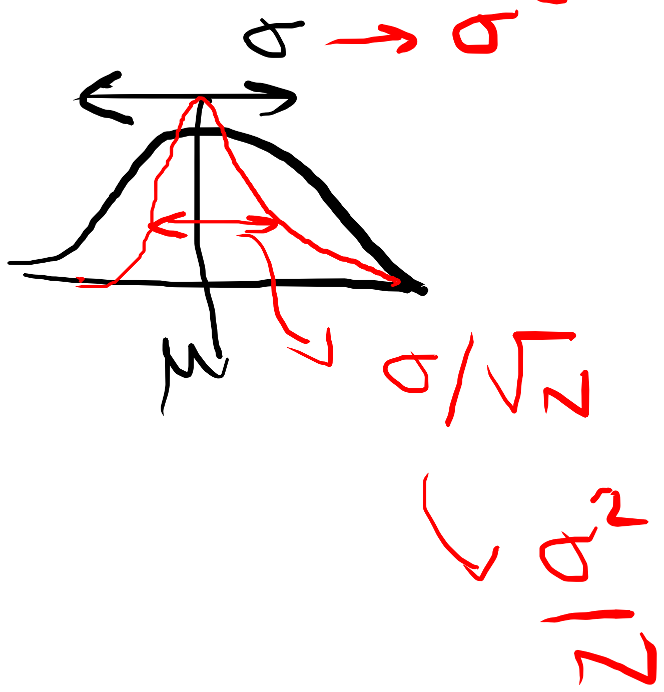
$\underline{X} \sim f_{\underline{X}}(\mu, \sigma^2)$

$$y = \frac{X_1 + X_2 + \dots + X_N}{N} \longrightarrow \mathcal{N}\left(\mu, \frac{\sigma^2}{N}\right)$$

$$E[y] = \frac{1}{N} E[\underbrace{X_1}_{\sim\mu} + \dots + X_N] = \frac{1}{N} \cdot N\mu = \mu \quad \checkmark$$

$$\mathrm{var}[y] = \mathrm{var}\left[\frac{1}{N}(X_1 + \dots + X_N)\right] = \frac{1}{N^2} N\sigma^2 = \frac{\sigma^2}{N}$$

Varianza atenuada por $1/N$



Bernoulli

H   T

p $1-p$

$x_1 = 0$

$x_2 = 1$

Tiro 10 veces la moneda $\rightarrow N = 10$

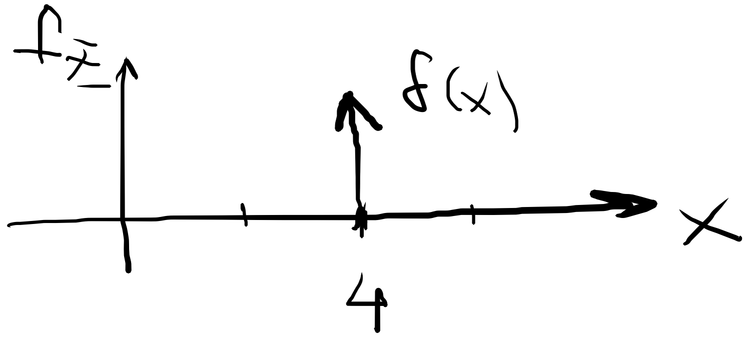
Prob. salgan exactamente 2 caras?

$p = 0{,}4 \rightarrow P(\text{cara})$

Binomial

$$P(X = 2) = \underbrace{\binom{10}{2}}_{\text{no me importa dónde salgan}} p^2 (1-p)^{10-2}$$

$$\underbrace{P(X \leq 1)}_{\text{acumulada}} = 1 - P(X \geq 2)$$

Distribución de una constante = 

$$\int_{-\infty}^{+\infty} 1 \cdot \underbrace{f_X(x)}_{\delta(x)}\, dx = 1$$

Normalización

$$X \sim N(\mu, \sigma^2)$$

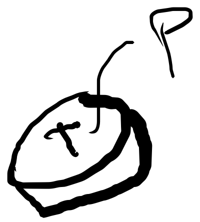
$$Z = \frac{X - \mu}{\sigma} \qquad E[Z] = \frac{1}{\sigma}\underbrace{E[X - \mu]}_{E[X] = \mu} = 0$$

normal

cero-uno.

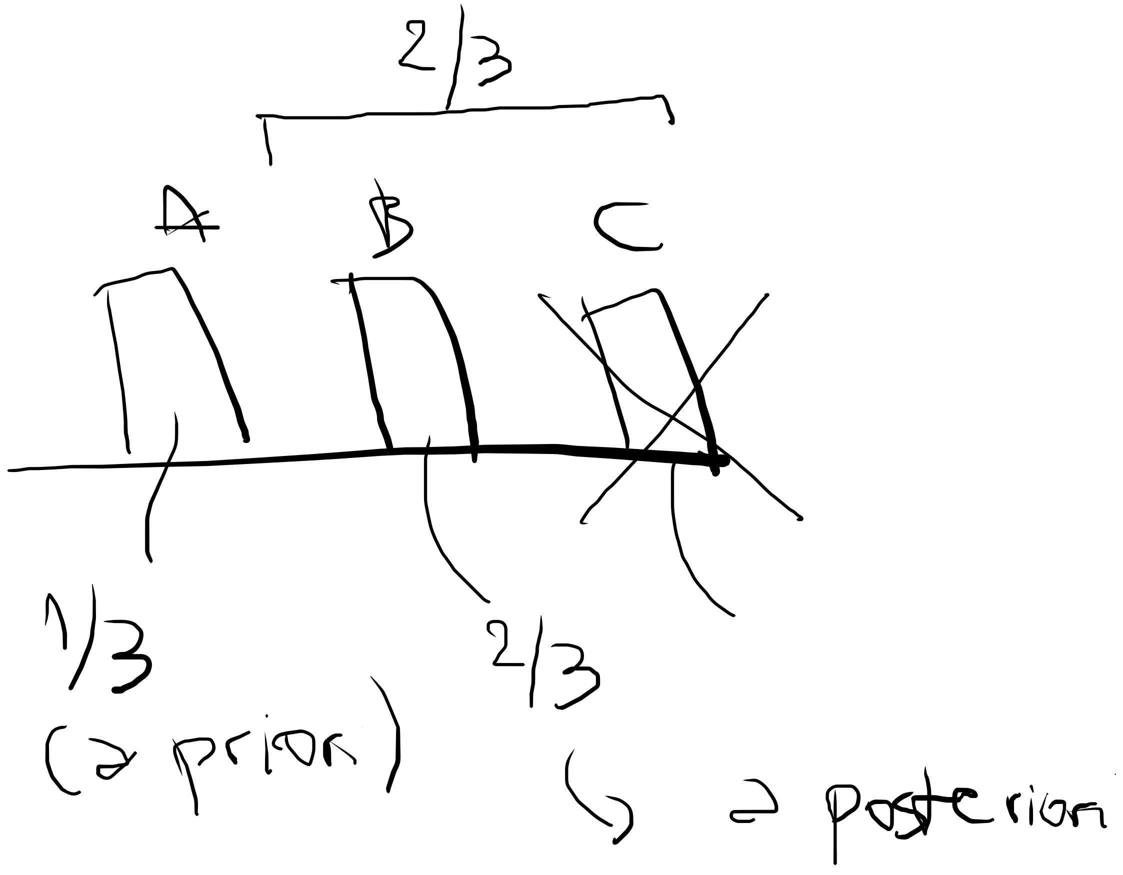
$$\mathrm{var}[Z] = \frac{1}{\sigma^2}\overbrace{\sigma[X]}^{\sigma^2} = 1$$

Cómo tiramos una moneda en la simulación



p = prob. de cara.

```
if(rand() ≤ p)
    cara
else
    ceca.
```



$$P(B|\overline{C}) = \frac{2}{3}$$