Project 1

Ove Haugvaldstad, Eirik Gallefoss

September 4, 2019

Abstract

Introduction

Computing has had and still have an undeniable influence on science. has allowed scientist to explore everything from the tiniest scale of an atom,to tropical cyclones and galaxies. Therefore understanding the inner workings behind a computer program is critical in order to avoid unwanted errors. Errors which in the worst case can have catastrophic consequences [1].

Our aim is to investigate some of the common errors one might face if one doesn't think when developing code. To begin with we will look at a how to solve a second order differential equation, specifically the general one dimmensional Poisson's equation (2).

$$f(x) = -\frac{\partial^2 u}{\partial x^2} \tag{1}$$

Method

Numerical differentiation

Computers operate in discrete steps, which means that variables are stored as discrete variables. For a discrete variable over a particular range there is a minimum step length between each value that variable can take in that range. Compared to a continuous variable which can take on any value in that particular range. The step length h can either be set manually or it can be determined based on the start and end point our particular range, $h = \frac{x_n - x_0}{n}$. Where n is the number of points we choose to have in our particular range. The smaller the step length in of the discrete variable the better it will approximate the continuous variable.

The simplest ways to compute the derivate numerically is to use what is called forward difference method eq.(2) or equivalently backward difference method eq.(3). We also see that if we include the limit $\lim_{h\to 0}$ we obtain the definition of the derivate.

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \tag{2}$$

$$f'(x) \approx \frac{f(x-h) + f(x)}{h} \tag{3}$$

Since numerical differentiation always will give an approximation of the derivate, we would like to quantify our error. The error can be derived if we do a taylor series expansion of the $f(x+h)$ term in around x.

$$f(x+h) = f(x) + h'f(x) + \frac{h^2 f''(x)}{2} + \frac{h^3 f'''(x)}{6} + \ldots \tag{4}$$

If we next insert this taylor expansion into eq.(4) we get:

$$f'(x) = f'(x) + \frac{hf''(x)}{2} + \frac{h^2 f'''(x)}{6} + \ldots \tag{5}$$

Our approximation of the derivate includes $f'(x)$ and some terms which are proportional to $h, h^2, h^3 \ldots$ and since h is assumed to be small the h terms would dominate.

To find the numerical differentiation for the second derivate we would

n	run time (s)
10	1.21e-04
100	3.63e-04
1000	4.33e-03
10000	3.76e-02
100000	4.06e-01
1000000	3.13e+00
10000000	3.09e+01

n	run time (s)
10	1.03e-04
100	2.67e-04
1000	2.48e-03
10000	1.93e-02
100000	2.22e-01
1000000	2.15e+00
10000000	1.94e+01

log_{10}(h)	max(relative error)
-1.04	0.03
-2.00	-0.15
-3.00	-0.02
-4.00	-0.00
-5.00	-0.00
-6.00	-0.00
-7.00	-0.00

References

1. Arnold, D. N. *The sinking of the Sleipner A offshore platform* `http://www-users.math.umn.edu/~arnold//disasters/sleipner.html`. (accessed: 01.09.2019).