Project 1

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Abstract

Introduction

Computing has had and still have an undeniable influence on science. has allowed scientist to explore everything from the tiniest scale of an atom,to tropical cyclones and galaxies. Therefore understanding the inner workings behind a computer program is critical in order to avoid unwanted errors. Errors which in the worst case can have catastrophic consequences [1].

Our aim is to investigate some of the common errors one might face if one doesn't think when developing code. To begin with we will look at a how to solve a second order differential equation, specifically the general one dimmensional Poisson's equation (2).

$$f(x) = -\frac{\partial^2 u}{\partial x^2} \tag{1}$$

Method

Numerical differentiation

Computers operate in discrete steps, which means that variables are stored as discrete variables. A discrete variable defined over a particular range would have step length h between each value and can not represent any values in between. This means that how well a discrete variable would represent the continuous variable depends on the size of the step length. The step length h can either be set manually or it can be determined based on the start and end point of our particular range, $h = \frac{x_n - x_0}{n}$. Where n is the number of points we choose to have in our range. The shorter the step length in the discrete variable the better it will approximate the continuous variable.

The simplest way to compute the derivate numerically is to use what is called forward difference method eq.(2) or equivalently backward difference method (eq.3). If we include the limit as$\lim_{h \to 0}$ we obtain the classic definition of the derivate.

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \tag{2}$$

$$f'(x) \approx \frac{f(x-h) + f(x)}{h} \tag{3}$$

Since numerical differentiation always will give an approximation of the derivate, we would like to quantify our error. The error can be derived if we do a taylor series expansion of the $f(x+h)$ term in around x.

$$f(x+h) = f(x) + h'f(x) + \frac{h^2 f''(x)}{2} + \frac{h^3 f'''(x)}{6} + \dots \quad (4)$$

If we next insert this taylor expansion into eq.(4) we get:

$$f'(x) = f'(x) + \frac{hf''(x)}{2} + \frac{h^2 f'''(x)}{6} + \dots \quad (5)$$

Our approximation of the derivate includes $f'(x)$ and some terms which are proportional to $h, h^2, h^3 \dots$ and since h is assumed to be small the h terms would dominate. The error is said to be of the order h.

To get a numerical scheme for the second derivate we would just take the derivate of eq. 2.

$$f''(x) \approx \frac{f'(x+h) - f'(x)}{h} \quad (6)$$

n	run time (s)
10	5.79e-05
100	2.91e-04
1000	1.71e-03
10000	1.97e-02
100000	1.55e-01
1000000	1.73e+00

n	run time (s)
10	7.10e-05
100	1.41e-04
1000	1.46e-03
10000	1.45e-02
100000	1.04e-01
1000000	1.07e+00

log_{10}(h)	max(relative error)
-1.04	0.03
-2.00	-0.15
-3.00	-0.02
-4.00	-0.00
-5.00	-0.00
-6.00	-0.00

References

1. Arnold, D. N. *The sinking of the Sleipner A offshore platform* `http://www-users.math.umn.edu/~arnold//disasters/sleipner.html`. (accessed: 01.09.2019).