# 3D Viewing (part I): Projections

Body implies a front view but the feet and head imply side view (cubism?)

# Projection in Drawing

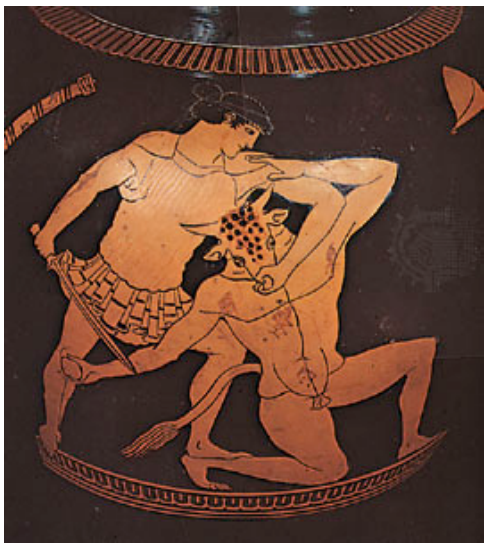
- Painting based on mythical tale as told by Pliny the Elder: Corinthian *wo*man traces shadow of departing (*male*) lover

Detail from **The Invention of Drawing**, 1830
K.F. Schinkle

# Early Examples of Projection

- Plan view (orthographic projection) from Mesopotamia, 2150 BC: earliest known technical drawing in existence

Carlbom Fig. 1-1

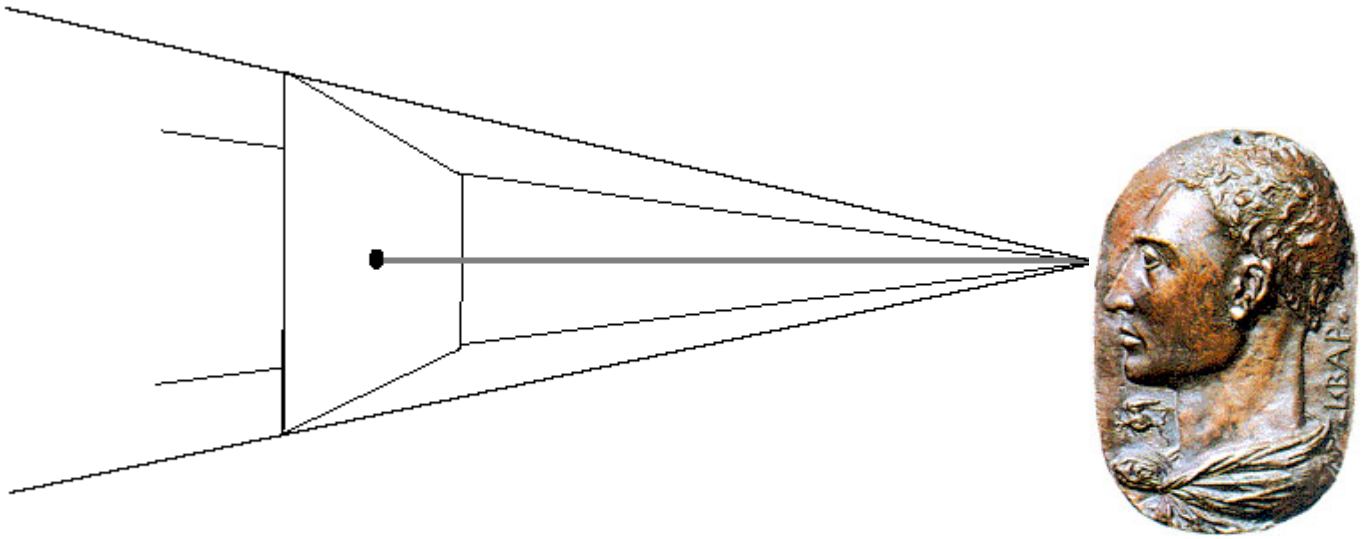


Theseus Killing the Minotaur by the Kleophrades Painter

- Greek vases from late 6th century BC show projection perspective(!)

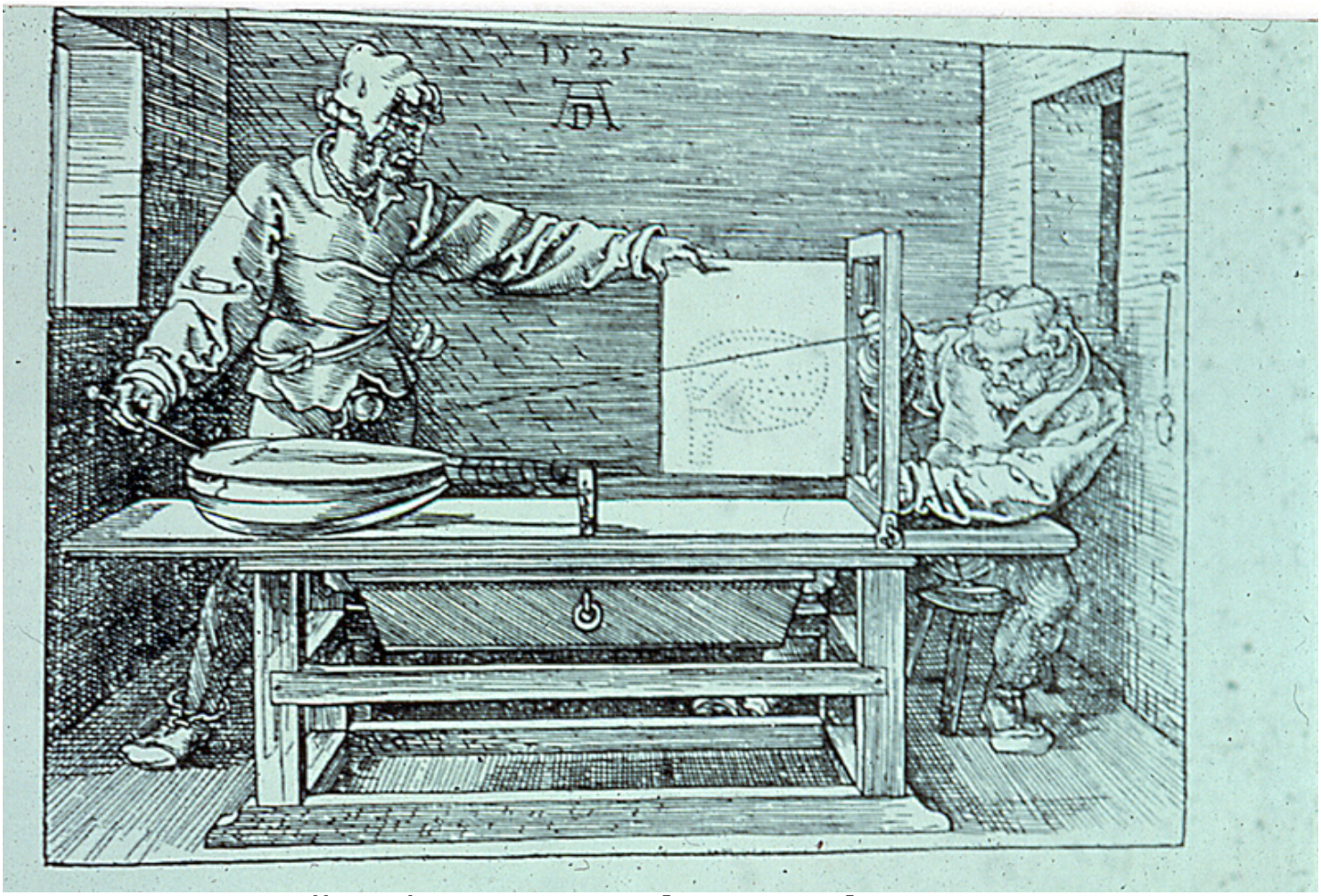
# How Do We Paint? (1435)

- "A painting **[the projection plane]** is the intersection of a visual pyramid **[view volume]** at a given distance, with a fixed center **[center of projection]** and a defined position of light, represented by art with lines and colors on a given surface **[the rendering]**." (Leono Battista Alberti (1404-1472), *On Painting*, pp. 32-33)



# Dürer's "Renderer" (1525)

- Concept of similar triangles described both geometrically and mechanically in widely read treatise by Albrecht Dürer (1471-1528)



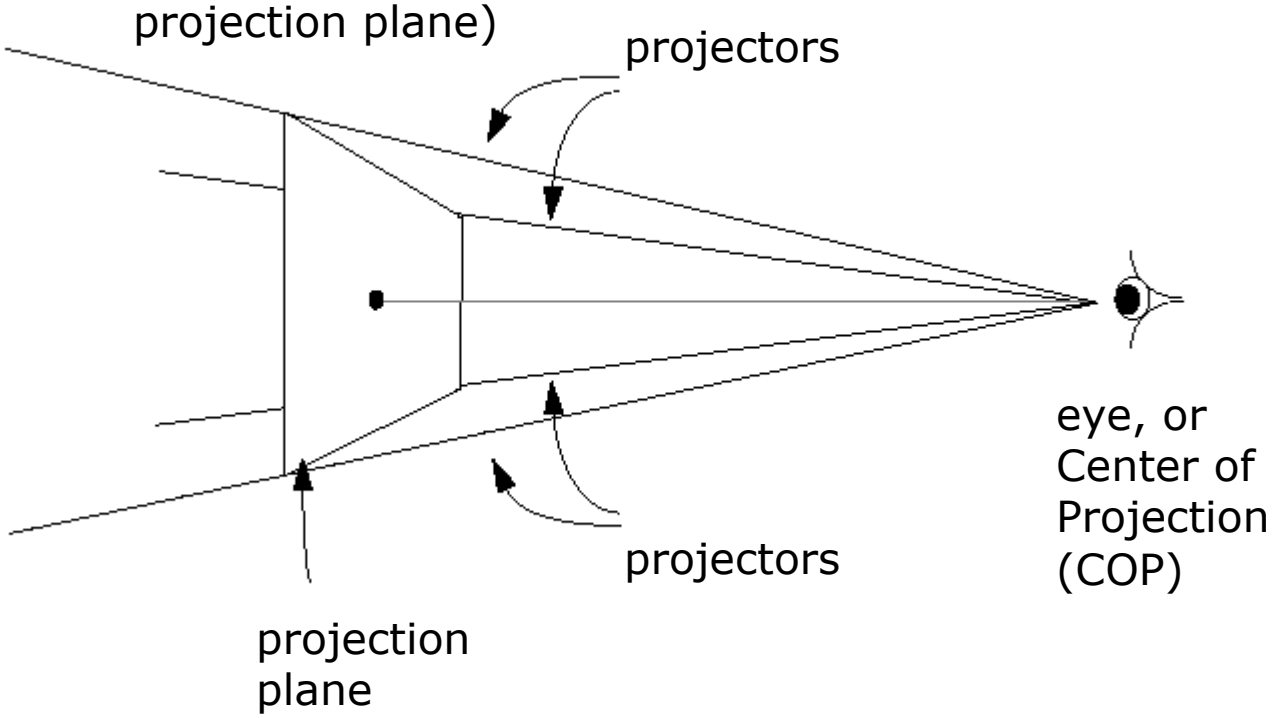

Albrecht Dürer, **Artist Drawing a Lute**

Woodcut from Dürer's work about the Art of Measurement. 'Underweysung der messung', Nurenberg, 1525

# Planar Geometric Projection

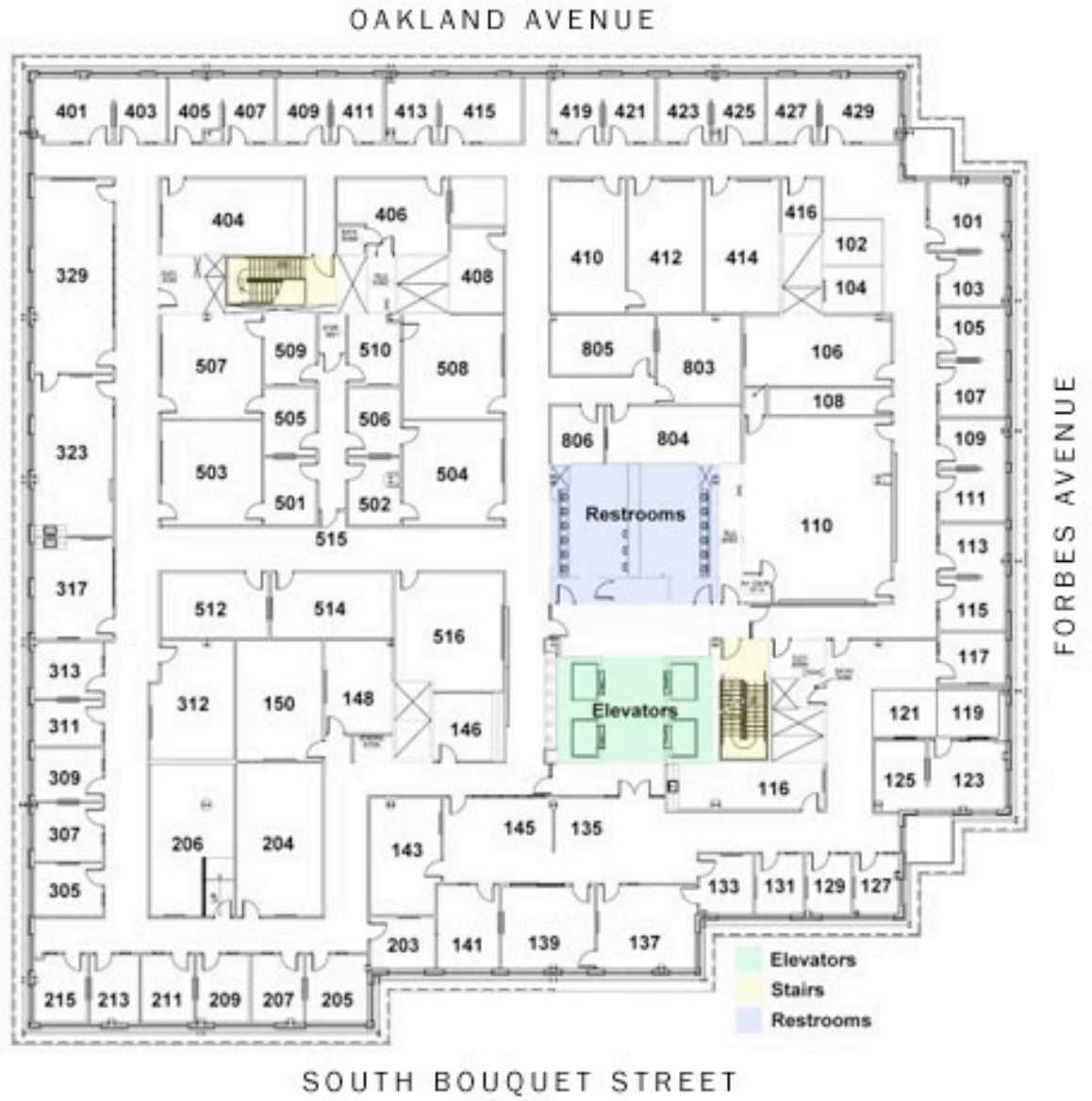
- Projectors are straight lines, like the string in Dürer's "Artist Drawing a Lute".

- Projection surface is plane (picture plane, projection plane)




- This drawing itself is perspective projection
- What other types of projections do you know?

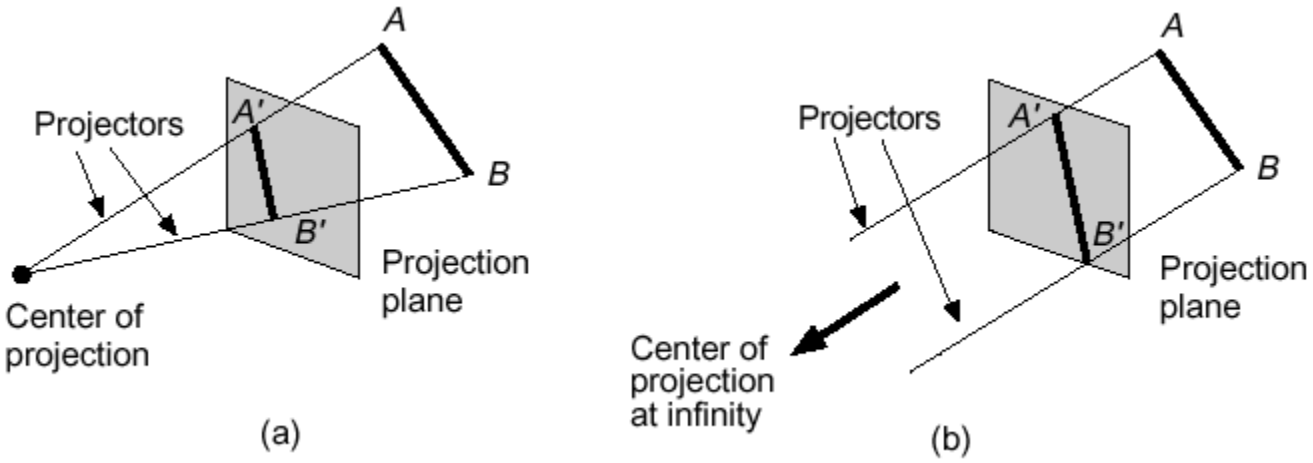
# SENSQ floor maps



# Main Classes of Planar Geometrical Projections

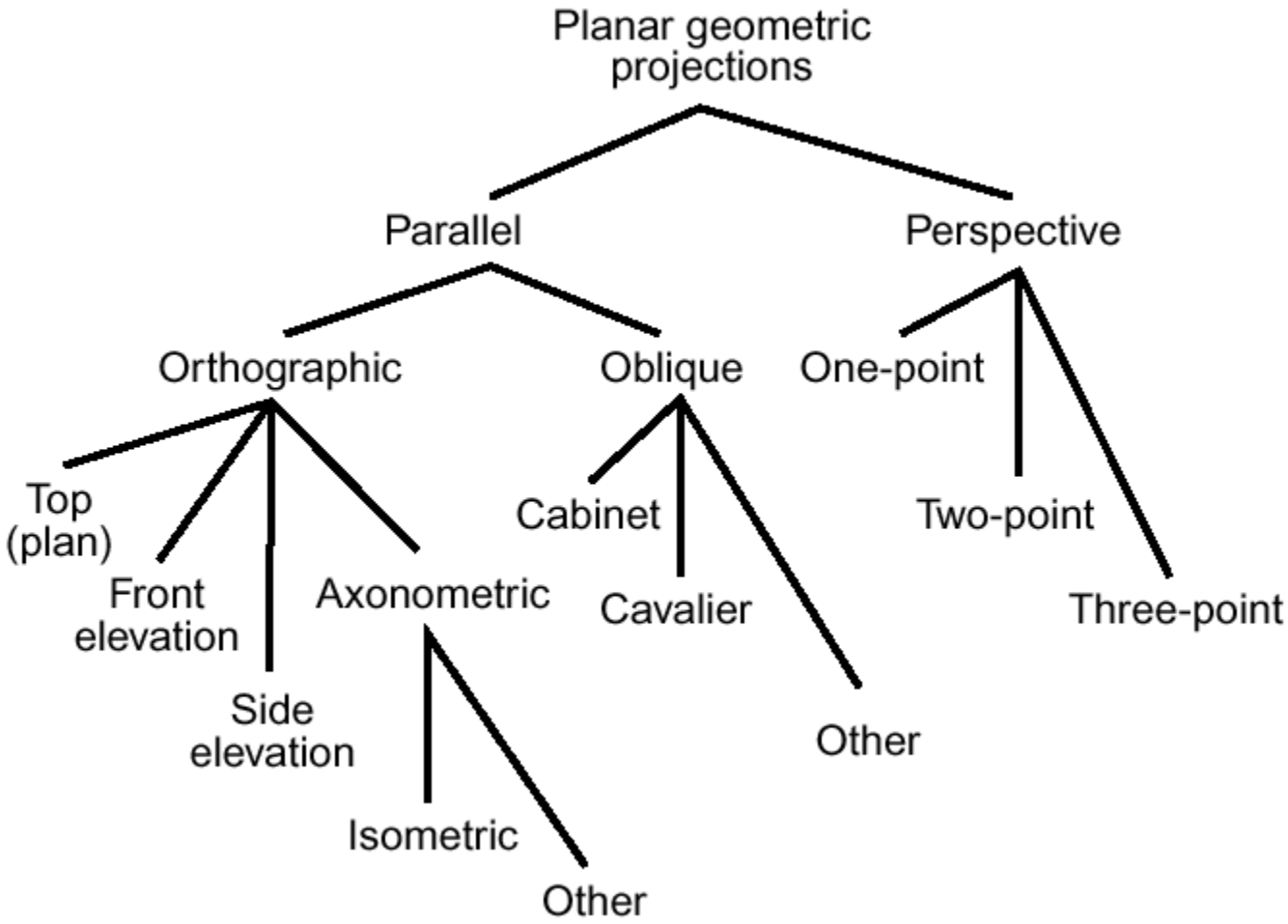
(a) Perspective (*Durer-style)*: COP is at the "eye"; projectors converge at COP

(b) Parallel (*SENSQ map-style)*: COP is at infinity; projectors are parallel




In general, the type of a projection is determined by where we place the projection plane relative to principal axes of object (relative angle and position), and what angle the projectors make with the projection plane

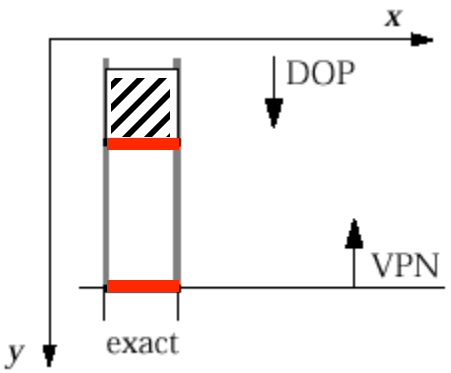
# Logical Relationship Between Types of Projections



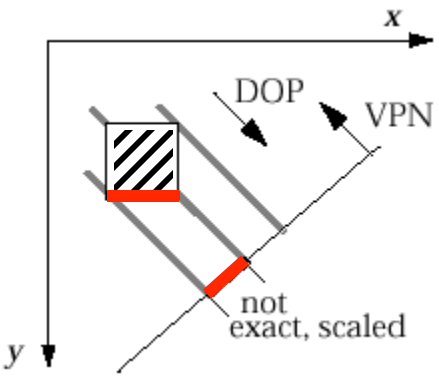

- Parallel projections used for engineering and architecture because they can be used for measurements
- Perspective imitates eyes or camera and looks more natural

# Overview of Parallel Projections

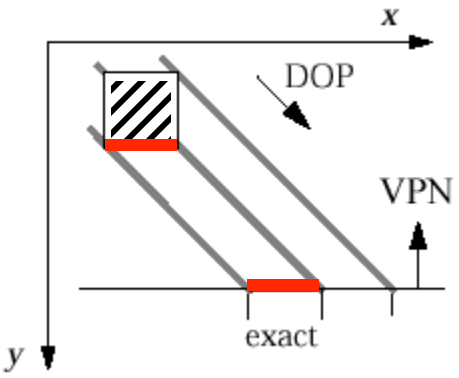
- Assume object face-of-interest lies in principal plane, i.e., parallel to *xy, yz,* or *zx* planes.
- DOP = Direction of Projection, VPN = View Plane Normal




1) Multiview Orthographic
   - VPN || a principal coordinate axis
   - DOP || VPN
   - shows single face, exact measurements



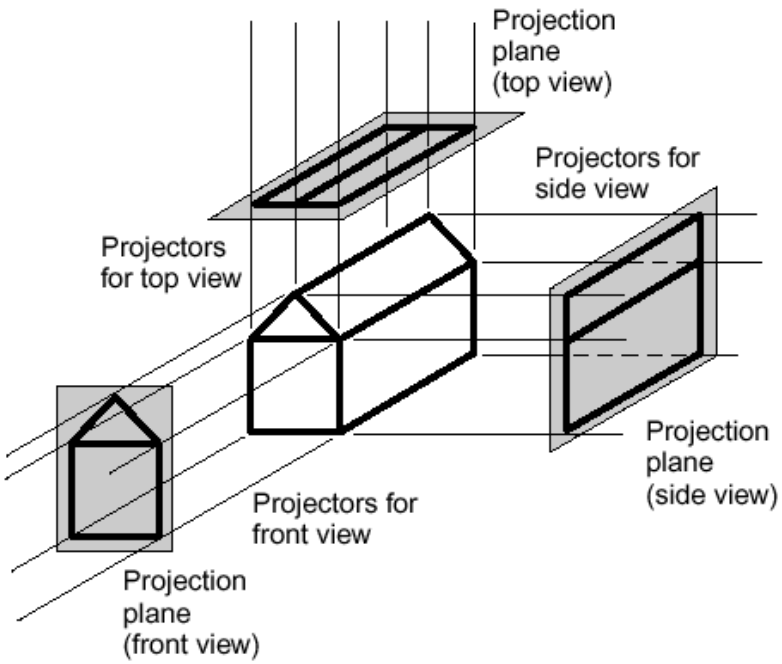

2) Axonometric
   - VPN ∦ a principal coordinate axis
   - DOP || VPN
   - adjacent faces, none exact, uniformly foreshortened (function of angle between face normal and DOP)




3) Oblique
   - VPN || a principal coordinate axis
   - DOP ∦ VPN
   - adjacent faces, one exact, others uniformly foreshortened

# Multiview Orthographic

- Used for:
  - engineering drawings of machines, machine parts
  - working architectural drawings
- Pros:
  - accurate measurement possible
  - all views are at same scale
- Cons:
  - does not provide "realistic" view or sense of 3D form; usually need multiple views to get a three-dimensional feeling for object

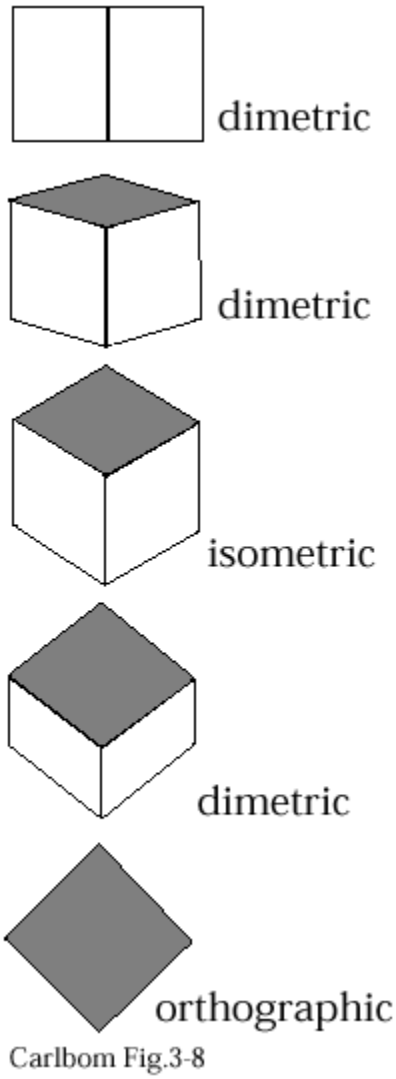


# Axonometric Projections

Same method as multiview orthographic projections, except projection plane, in general, is not parallel to any of **coordinate planes**

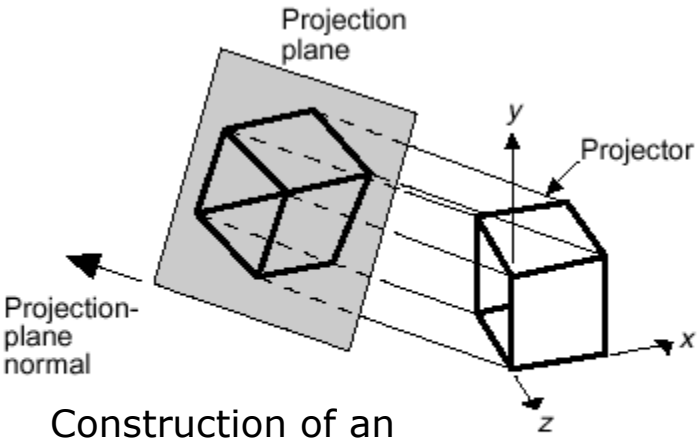
- *Isometric*: Angles between all three principal axes equal (120º). Same scale ratio applies along each axis
- *Dimetric:* Angles between two of the principal axes equal; need two scale ratios
- *Trimetric:* Angles different between three principal axes; need three scale ratios

Note: different names for different views, but all part of a continuum of parallel projections of cube; these differ in where projection plane is relative to its cube

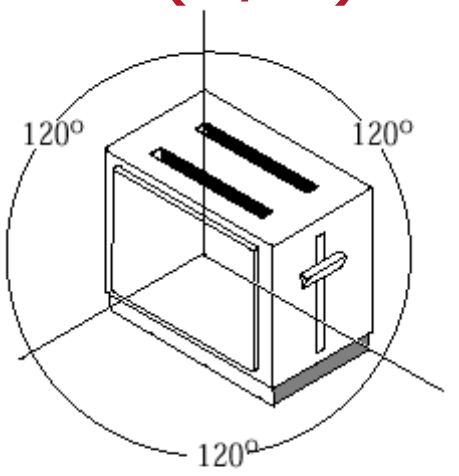


Carlbom Fig.3-8

# Isometric Projection (1/2)




Construction of an isometric projection: projection plane cuts each principal axis by 45°



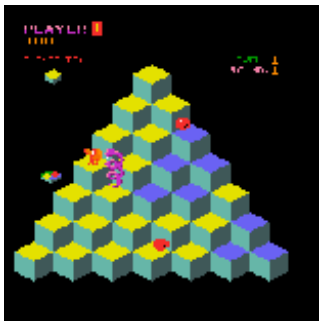

Example

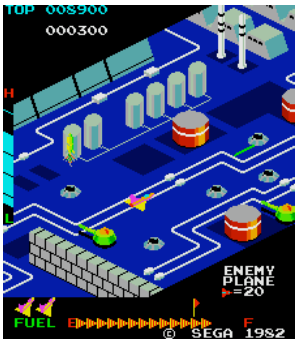
Carlbom Fig.2.2

- Used for:
  - catalogue illustrations
  - patent office records
  - furniture design
  - structural design
  - 3d Modeling in real time (Maya, AutoCad, etc.)
- Pros:
  - don't need multiple views
  - illustrates 3D nature of object
  - measurements can be made to scale along principal axes
- Cons:
  - lack of foreshortening creates distorted appearance
  - more useful for rectangular than curved shapes

# Isometric Projection (2/2)

- Video games have been using isometric projection for ages. It all started in 1982 with *Q*Bert* and *Zaxxon*






- Still in use today when you want to see things in distance as well as things close up (e.g. strategy, simulation games)

SimCity IV (Trimetric)

StarCraft II

- *Technically* some games today aren't isometric but instead are a general axonometric (trimetric) with arbitrary angles, but people still call them isometric to avoid learning a new word. Other inappropriate terms used for axonometric views are "2.5D" and "three-quarter."
  http://simcity.ea.com/about/inside_scoop/3d1.php

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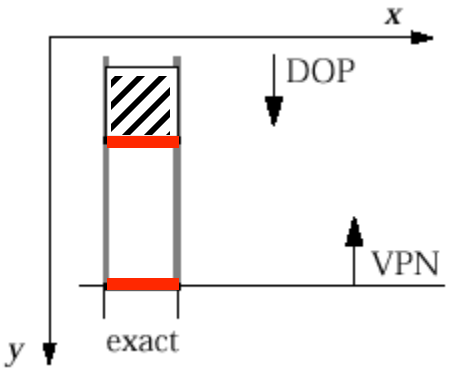


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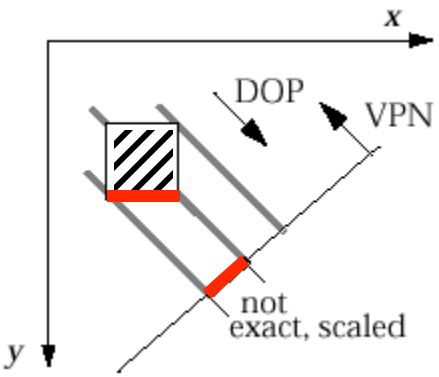


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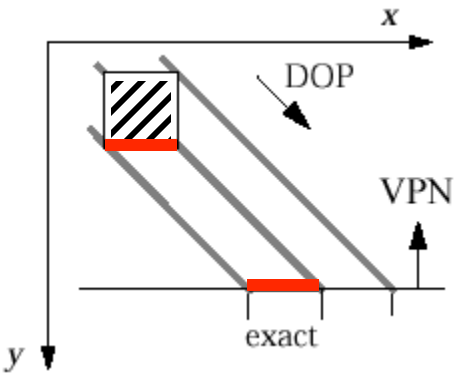



3) Oblique
   - VPN || a principal coordinate axis
   - DOP ∦ VPN
   - adjacent faces, one exact, others uniformly foreshortened

# Oblique Projections

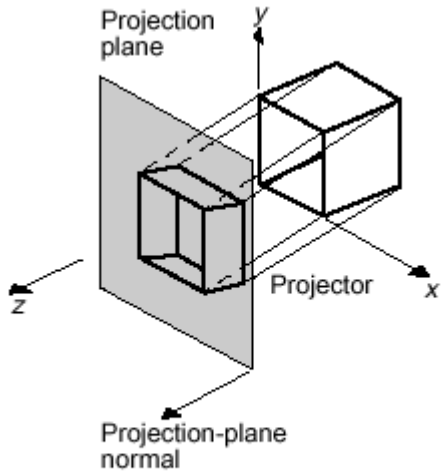
- Projectors at oblique angle to projection plane; view cameras w/ accordion housing,

- Pros:
  - can present exact shape of one face of an object (can take accurate measurements)
  - lack of perspective foreshortening makes comparison of sizes easier
  - displays some of object's 3D appearance
- Cons:
  - objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)
  - lack of foreshortening (not realistic looking)

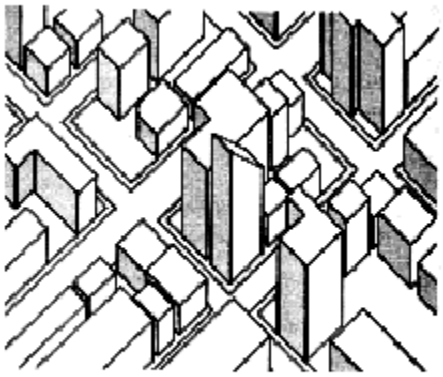
perspective

oblique

# Examples of Oblique Projections

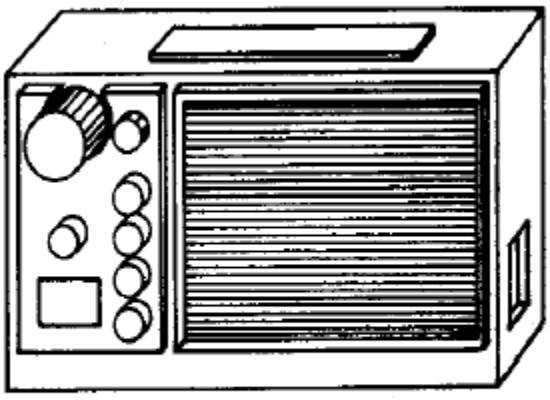



Construction of
oblique parallel projection



(Carlbom Fig. 2-6)

Plan oblique projection of city

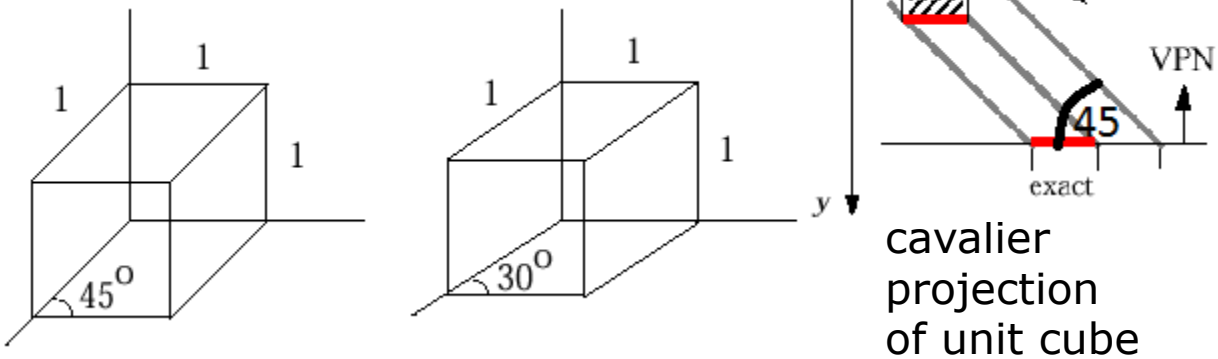


(Carlbom Fig. 2-4)

Front oblique projection of radio

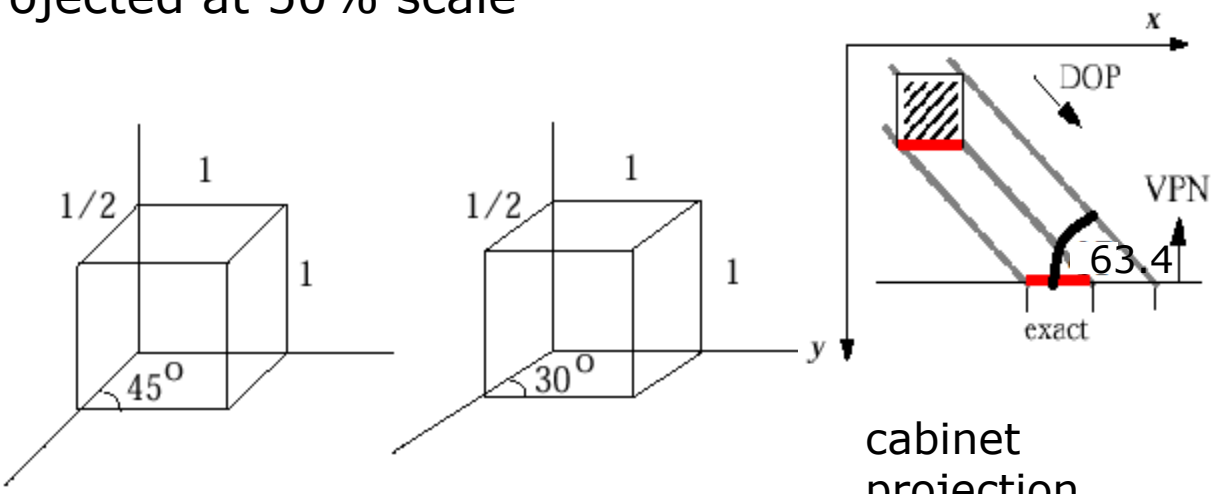
# Main Types of Oblique Projections

- *Cavalier:* Angle between projectors and projection plane is 45º. Perpendicular faces projected at full scale



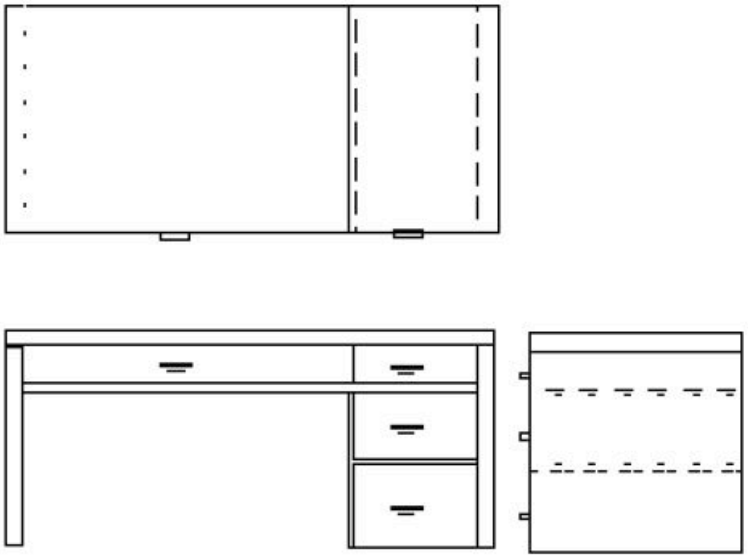

cavalier projection of unit cube

- *Cabinet:* Angle between projectors & projection plane: arctan(2) = 63.4º. Perpendicular faces projected at 50% scale

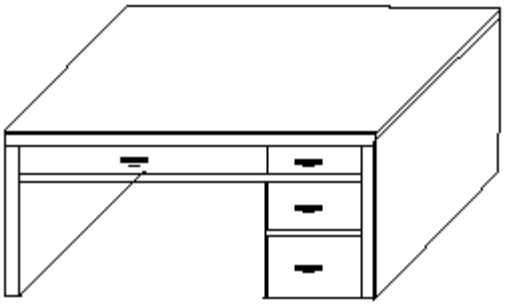



cabinet projection of unit cube

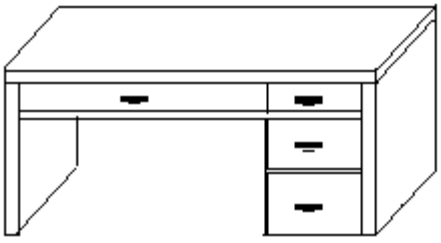
# Examples of Orthographic and Oblique Projections



multiview orthographic



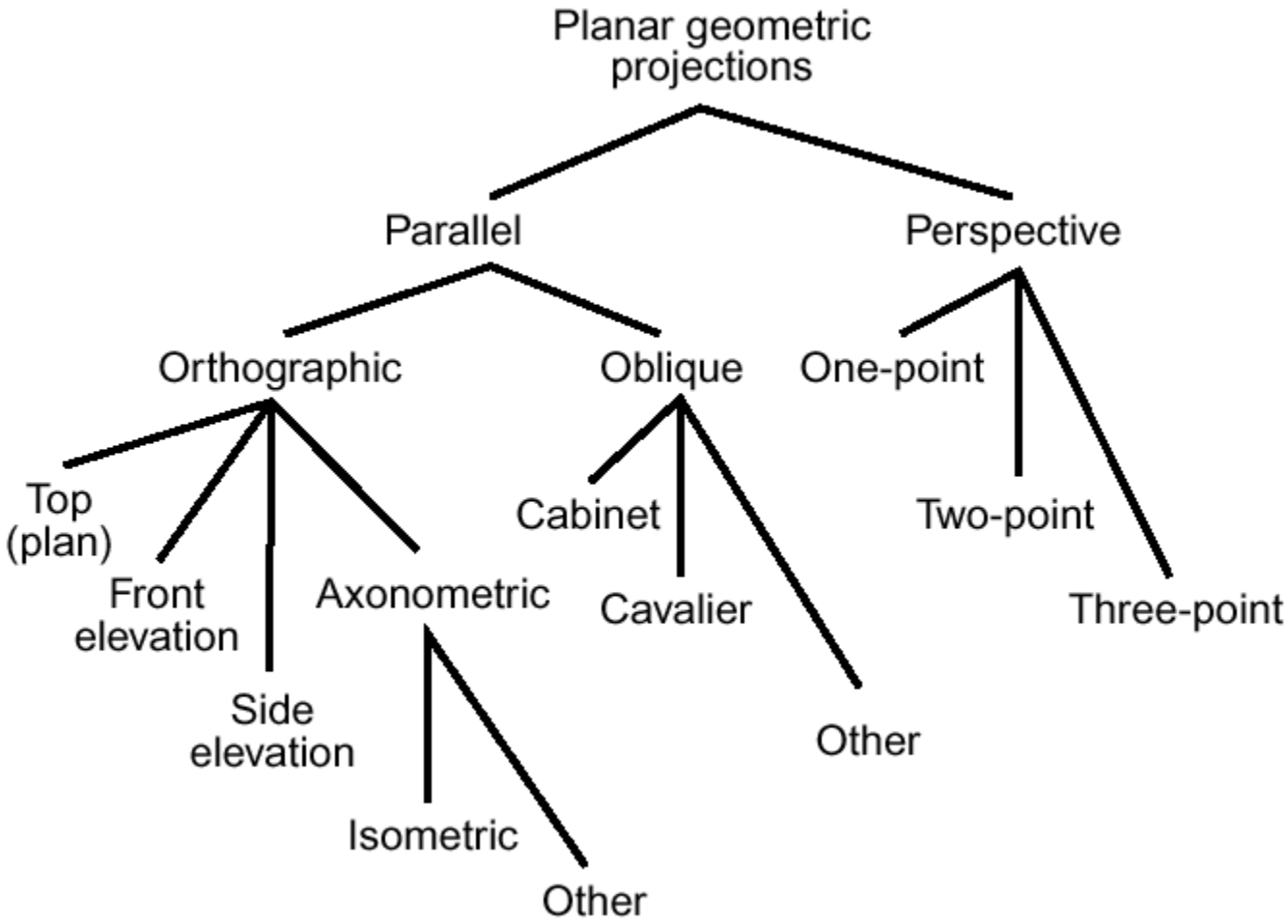
cavalier



cabinet

Carlbom Fig. 3-2

# Logical Relationship Between Types of Projections




- Parallel projections used for engineering and architecture because they can be used for measurements
- Perspective imitates eyes or camera and looks more natural

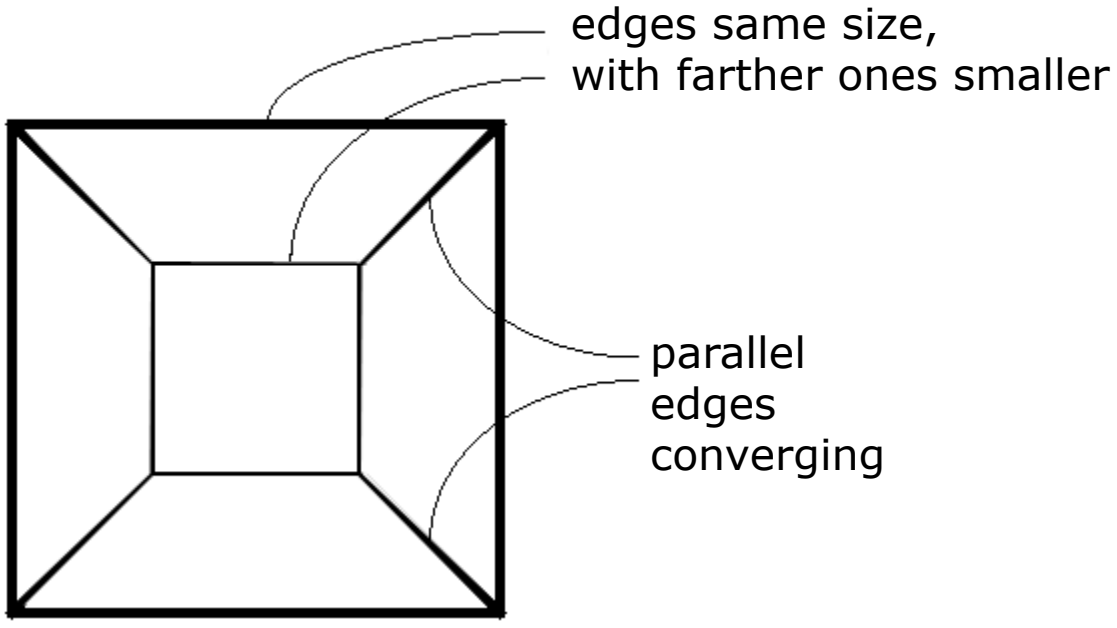
# Perspective Projections

If we were viewing this scene using parallel projection, the tracks would not converge

- Used for:
  - advertising
  - presentation drawings for architecture, industrial design, engineering
  - fine art
- Pros:
  - gives a realistic view and feeling for 3D form of object
- Cons:
  - does not preserve shape of object or scale (except where object intersects projection plane)
- Different from a parallel projection because
  - parallel lines not parallel to the projection plane converge
  - size of object is diminished with distance
  - foreshortening is not uniform

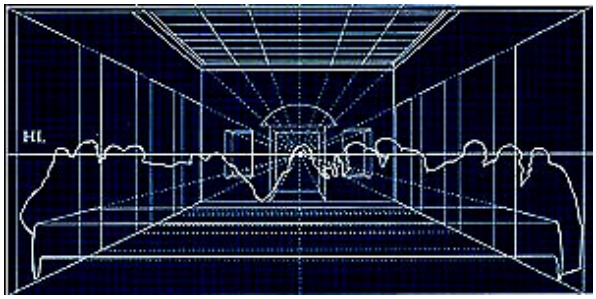
# Striking Features of Perspective Projection

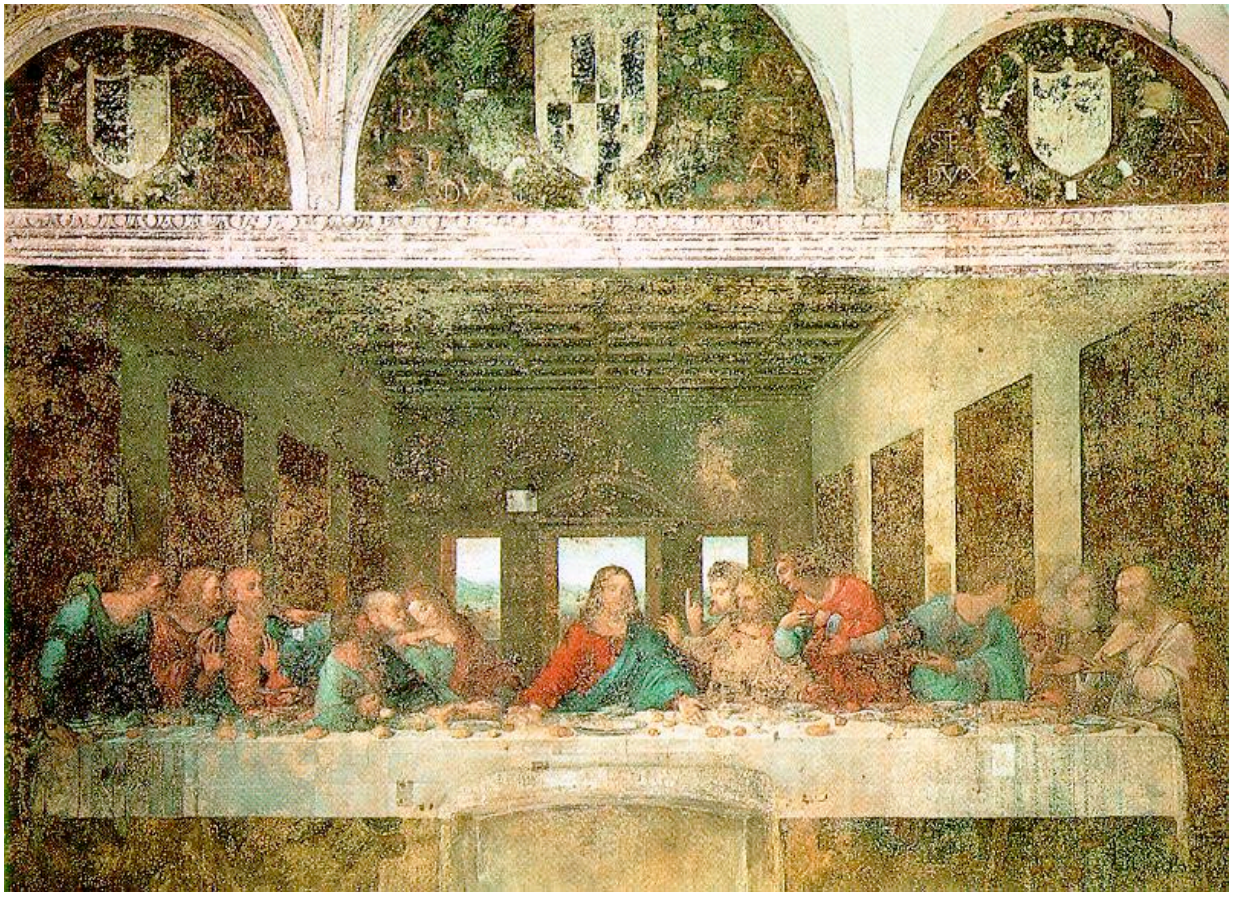
- || lines converge (in 1, 2, or 3 axes) to *vanishing point*
- Objects farther away are more *foreshortened* (i.e., smaller) than closer ones
- Example: perspective cube



# Perspective: Vanishing Point

- Leonardo da Vinci The Last Supper (1495)
- Perspective plays very large role in this painting





# Recap

- Drawing as projection
- Durer's renderer
- Planar geometric projection
  - center of projection (eye or camera)
  - projectors (direction: parallel or not)
  - projection plane (angle with projectors)
- Types of projection
  - Parallel
    - orthographic: a) multi, b) axonometric (iso, di, tri)
    - oblique: Cabinet, Cavalier
  - Perspective
    - foreshortening
    - vanishing point

# Sources


- Carlbom, Ingrid and Paciorek, Joseph, "Planar Geometric Projections and Viewing Transformations," *Computing Surveys*, Vol. 10, No. 4 December 1978
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