MA22004 Statistics II - Lecture Notes

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Contents

1 Preliminaries: Special distributions

1.1 Normal distribution

A random variable X with this distribution takes the form of a symmetric bell-shaped curve. The *location* (position) and *dispersion* (spread) of the distribution depends on the mean μ and variance σ^2, respectively. We write $X \sim \mathcal{N}(\mu, \sigma^2)$. Recall the *standard deviation* is the square root of the variance, i.e., σ.

1.2 Computations with normals

The *standard normal* variable Z has mean $\mu = 0$ and variance $\sigma^2 = 1$. Probability values such as $P(Z \leq z)$ can be looked up in standard normal tables.

A random variable $X \sim \mathcal{N}(\mu, \sigma^2)$ can be transformed into a standard normal using the transformation,

$$Z = \frac{X - \mu}{\sigma}\,. \tag{1}$$

Example 1.1. *Compute the probability that the random variable* $X \sim \mathcal{N}(5, 9)$ *exceeds* 5.5*.*

We first transform $X \mapsto Z$ using (1) and then look up the probability value up in a table of standard normal values (Z-score).

$$\begin{aligned} P(X \geq 5.5) &= P\left(Z \geq \frac{5.5 - 5}{3}\right) \\ &= P(Z \geq 0.167) \\ &= P(Z \leq -0.167) \\ &= 0.4364 - 0.0028 \\ &= 0.4346\,. \end{aligned}$$

Alternatively, we can use the code:

```
pnorm(5.5, mean=5, sd=9, lower.tail=FALSE)
```

```
[1] 0.4778479
```

Here we use the option `lower.tail=FALSE` as we are interested in the *upper tail* probability in this instance.

Example 1.2. *Compute the probability that the random variable* $X \sim \mathcal{N}(5, 9)$ *is between. . . .*

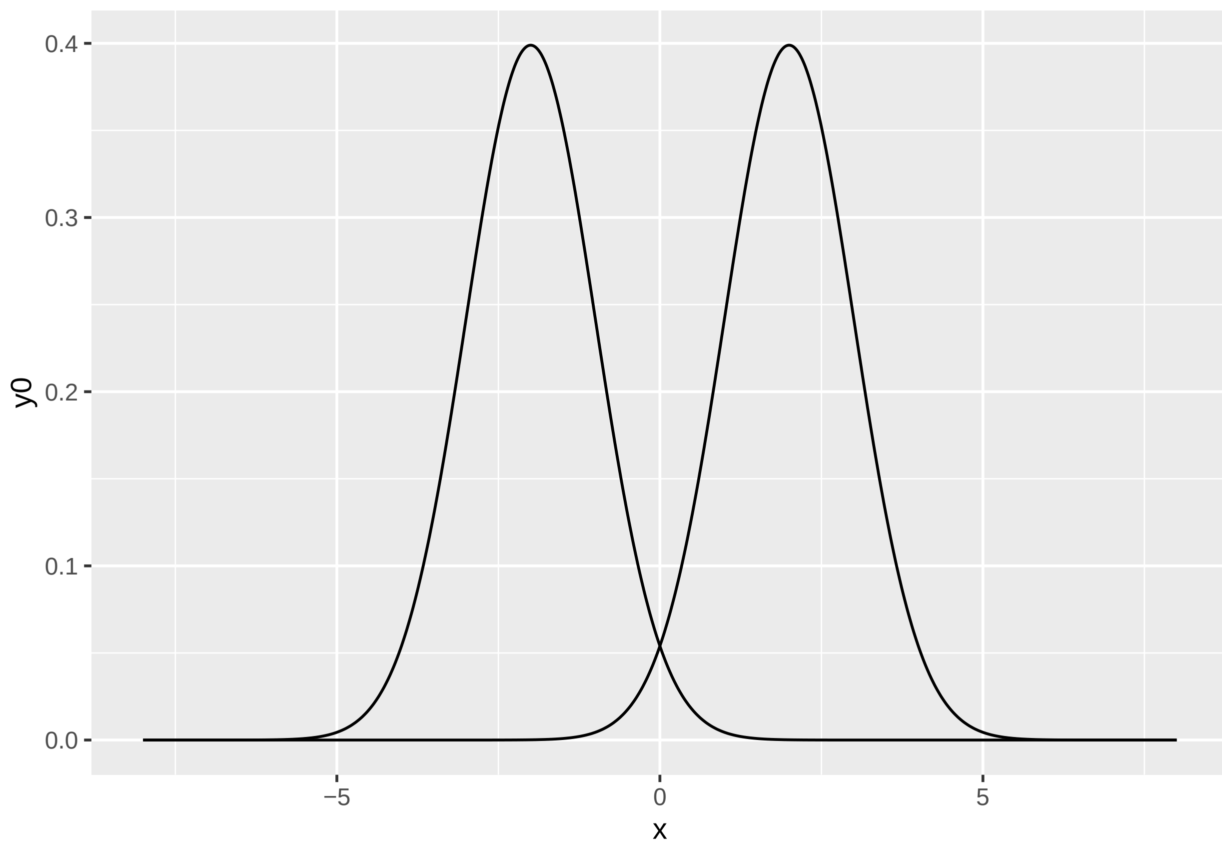

Figure 1: Two normal variates with different means and same variance.

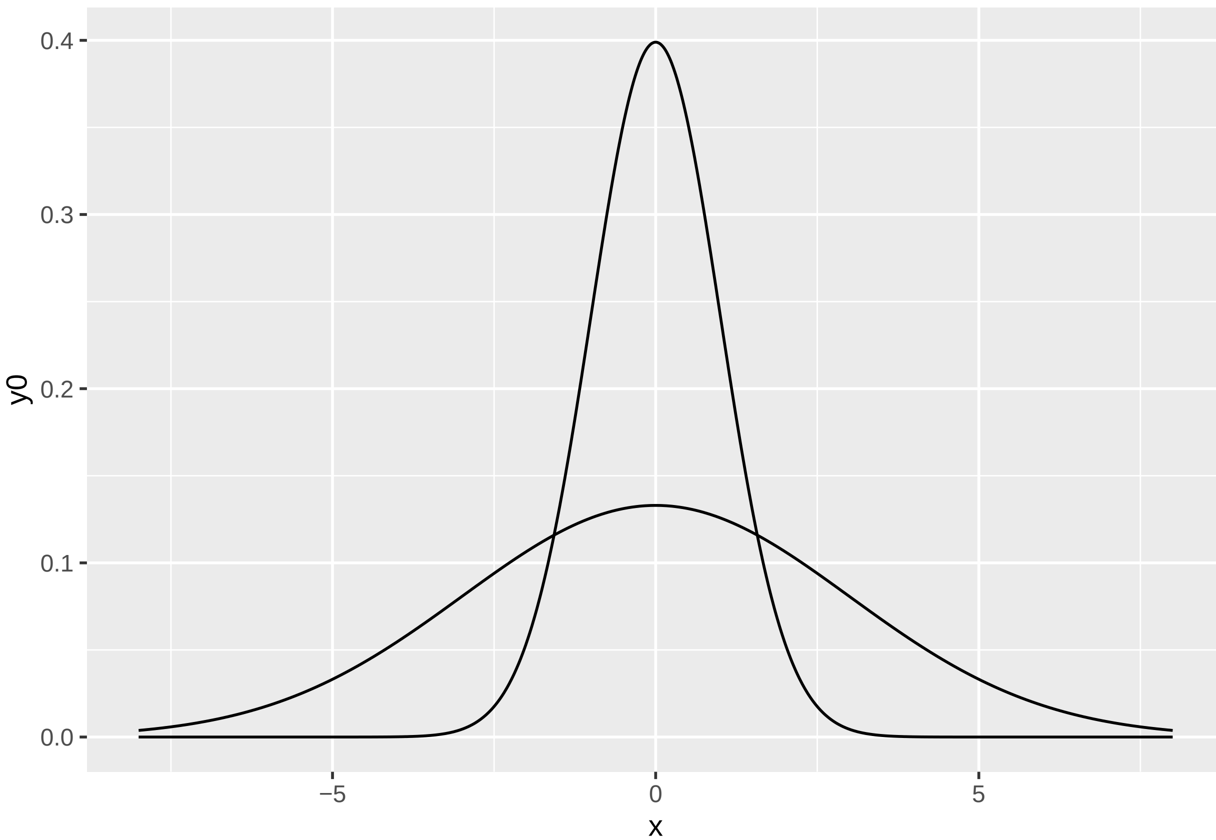

Figure 2: Two normal variates with different variance and same means.

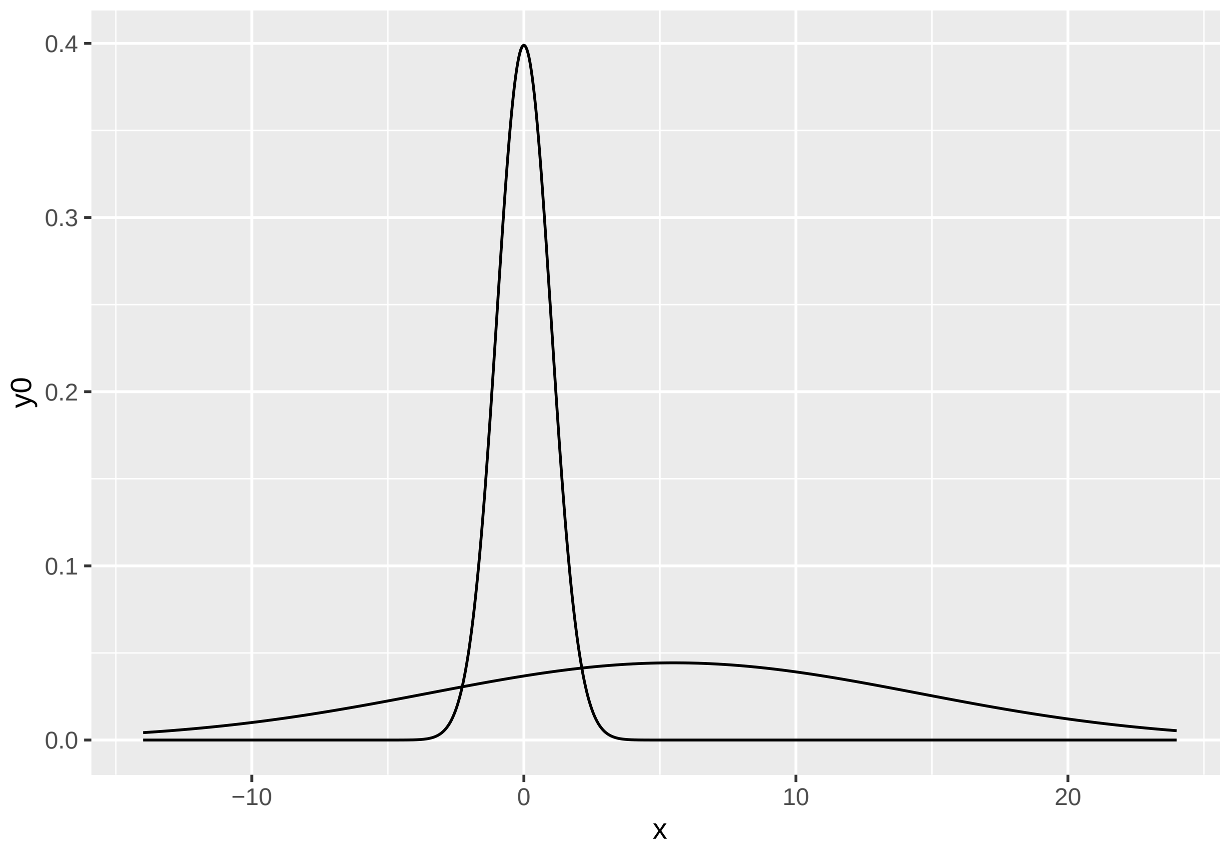

Figure 3: Standard normal Z and the X

1.3 Properties of normals