1 Exercise: Basic Concepts

Task 1.1 Basic calculation and plotting

a) Consider a discrete random variable X that has a Poisson distribution with parameter $\lambda(>0)$. (i) Let $\lambda = 1$. Write a program to compute the probabilities $P(X = x)$ for $0 \leq x \leq 100$ and plot the probability mass function $P_X(x)$, i.e. $P(X = x)$, and the cumulative distribution function $F_X(x) = P(X \leq x)$. (ii) Now suppose $\lambda = 0.1$. Plot the probability mass function $P_X(x) = P(X = x)$ and the cumulative distribution function $F_X(x) = P(X \leq x)$.

b) Consider a continuous random variable Y that has an exponential distribution with parameter $\mu(>0)$. (i) Let $\mu = 1$. Plot the probability density function $f_X(x)$ and the cumulative distribution function $F_X(x)$. (ii) Now suppose $\mu = 10$. Plot the probability density function $f_X(x)$ and the cumulative distribution function $F_X(x)$.

c) Let W be a function of two parameters λ and μ as $W = \frac{1}{\mu-\lambda}$. (i) Calculate the value of W under two settings and compare under which case the value is larger. Case 1: $\lambda = 1$ and $\mu = 10$. Case 2: $\lambda = 0.1$ and $\mu = 1$. (ii) Fix $\lambda = 1$. Plot W as a function of μ for $1.1 \leq \mu \leq 10$. (iii) Fix $\mu = 1$. Plot W as a function of λ for $0 \leq \lambda \leq 0.9$.

Task 1.2 Probability Theory: Football Scenario

Figure 1 shows some recent football match results between Norway and Sweden.

Figure 1: Recent football match results between Norway and Sweden

For a 90-minute football match between Norway and Sweden, assume both Norway and Sweden score according to a Poisson process of intensity $\lambda = 0.5$ goal per hour.

a) What is the probability that 2 goals are scored during the match?

b) What is the probability that the result of a match between Norway and Sweden is 1:1?

Task 1.3 Network Dependability: Ranking nodes based on centrality measures

Consider four networks, G_c, G_r, G_d, and G_t, constructed as follows:

- G_c: It is a network of 7 nodes, $\{a, b, c, d, e, f, g\}$, in chain.
- G_r: It is based on G_c with an added link (a, g) between nodes a and g.
- G_d: It is based on G_c with two added links (a, c) and (e, g).
- G_t: It is based on G_d but removing two links (a, b) and (f, g).

a) What is the average node degree of each network?

b) For each network, rank nodes based on node degree centrality, betweenness centrality and closeness centrality, respectively. Compare the ranking results and discuss your findings.

Task 1.4 Poisson process: Bus Stop Scenario

Consider a bus stop which is served by a bus every 15 minutes. Passengers arrive at the bus stop following a Poisson process with an average of five passengers in a 15-minute interval. It is assumed that when a bus arrives, it has the capacity to carry all the passengers waiting at the bus stop.

a) What is the distribution of the number of passengers at the bus stop when a bus arrives. Plot it.

b) What is the inter arrival time distribution for the passengers at the bus stop. What is the average inter arrival time. Plot the distribution.

Task 1.5 Stochastic Process: Multiple Access

Consider a digital satellite communication system with constant packet length. The satellite is in a geostationary position about 36000 km above equator, so the round trip delay is about 280 ms. The time axes is divided into slots of fixed duration, h, corresponding to the packet length. The individual terminal (earth station) transmits packets so that they are synchronized with the time slots. All packets generated during a time slot are transmitted in the next time slot. The transmission of a packet is only correct if it is the only packet being transmitted in a time slot. If more packets are transmitted simultaneously, we have a collision and all packets are lost and must be retransmitted. All earth stations receive all packets and can thus decide whether a packet is transmitted correctly. Due to the time delay, the earth stations transmit packets independently.

a) If the total arrival process is a Poisson process (rate λ), which distribution does the number of packets in each time slot follow? Write down the pdf for the distribution.

b) What is the probability of correct transmission?

c) The probability of correct transmission has an optimum when the derivative with respect to λh is zero. Calculate the value of the product λh and use it to determine the maximum utilization of the channel.

Task 1.6 Little's Theorem: University Study Programs

A department at a university has five full-time degree programs: one 3-year bachelors program BDIGSEC, one 5-year integrated masters program MTKOM, two 2-year masters programs (MDIGSEC and MIS), and one 3-year PhD program. The average number of students admitted to each program is as listed in Table 1. Although each program has a defined time-frame, not all students manage to finish within that time-frame. Rather, the degree completion time can vary from one to one and the average completion time for each program is also shown in Table 1.

Table 1: Degree Programs at IIK

Program i	Avg. students per year λ_i	Avg. years for completion T_i
BDIGSEC	45	3.5
MTKOM	72	6
MDIGSEC	28	2.5
MIS	23	2.5
PhD	5	4

In order to allocate adequate physical and digital resources, the department needs to find a number of parameters which are asked in the following questions.

a) What is (on average) the total number of students enrolled in each program at any given time in a long run? Subsequently, what is the average number of students enrolled at the department across all degree programs?

b) What is the average number of students admitted at the department in any given year?

c) What is the average time spent by a student in the department?