IN5270
MANDATORY EERCISE 3

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1. Description of the problem

The goal is to compute deflection of a cable with sine functions. We have a hanging cable with tension, and the cable has a deflection $\omega(x)$ which is governed by:

$$T\omega''(x) = l(x), \tag{1}$$

where the variables are:
- L: Length of cable - T: Tension on cable - $\omega(x)$: Deflection of cable - $l(x)$: Vertical load per unit length

Cable is fixed at $x = 0$ and $x = L$, and the boundary conditions are $\omega(0) = w(L) = 0$. Deflection is positive upwards and l is positive when it acts downwards.

Assuming $l(x) = \text{const}$, the solution is symmetric around $x = L/2$. For a function $\omega(x)$ that is symmetric around a point x_0, we have that

$$\omega(x_0 - h) = \omega(x_0 + h), \tag{2}$$

which means that

$$(3) lim_{h->0}(w(x_0 + h) - w(x_0 - h))/(2h) = 0. \tag{3}$$

We can therefore halve the domain, since it is symmetric. That limits the problem to find $\omega(x)$ in $[0, L/2]$, with boundary conditions $\omega(0) = 0$ and $\omega'(L/2) = 0$.

Scaling of variables:

$$x_{\_} = x/(L/2) \quad \text{(setting } x = x_{\_} \text{ in code for easier notation)} \tag{4}$$

$$u = \omega/\omega_c \quad \text{(where } \omega_c \text{ is a characteristic size of } \omega\text{)} \tag{5}$$

By putting this into the original equation we get

$$(4T\omega_c)/L^2 * u''(x_) = l = \text{const}. \tag{6}$$

We set $|u''(x_)| = 1$, and we get $\omega_c = 0.25lL^2/T$, and the scaled problem is

$$u'' = 1, x_{\in}(0,1), u(0) = 0, u'(1) = 0. \tag{7}$$