Solver as the 2D Acoustic Wave Propagation

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The wave propagation equation is solved using a finite difference scheme of 6th order in space and 2nd order in time. We apply a free surface at the top and a Convolutional Perfectly Matched Layers (CPML) scheme [Pasalic and McGarry 2010] to prevent artificial numerical reflections on the left, right, and bottom boundaries. The wave equation is defined differently outside and inside the PML layers. The acoustic wavefield propagation, denoted as $p(r,t)$, for a given source defined as a Ricker wavelet, $f_s(S,t)$, in a velocity model, $v(r,t)$, is defined as follows:

$$\frac{1}{v^2}\partial_t^2 p = \partial_k^2 p_k + f_\mathrm{s} + f_p^{\mathrm{PML}} \ , \tag{1}$$

$$f_p^{\mathrm{PML}} = \begin{cases} \partial_k \cdot \psi_k(p) + \xi_k(p), \ \mathbf{r} \in \mathrm{PML} \\ \\ 0, \ \mathbf{r} \notin \mathrm{PML} \end{cases} \tag{2}$$

The additional term for the PML layer f_p^{PML} is defined with the auxiliary variables $\psi_k(p)$ and $\xi_k(p)$, whose evolution for each component (k=x,z) and time step, n, is:

$$[\psi_k(p)]^n = a_k \cdot [\psi_k(p)]^{n-1} + b_k \cdot [\partial_k p_k]^n \ , \tag{3}$$

$$[\xi_k(p)]^n = a_k \cdot [\xi_k(p)]^{n-1} + b_k \cdot \left[\partial_k^2 p_k + \partial_k \psi_k(p)\right]^n \ , \tag{4}$$

$$\tag{5}$$

where the parameters a$_k$ and b$_k$ are:

$$a_k = e^{-(\sigma_k+\alpha_k)\cdot\Delta t} \quad , \quad b_k = \frac{\sigma_k}{\sigma_k+\alpha_k}\cdot(a_k-1) \ . \tag{6}$$

with Δt the time step, σ_k the absorption damping factor of the acoustic wave and α_k the real positive pole shifting factor [Zhang and Shen 2010].

References

[Pasalic and McGarry 2010] D. Pasalic and R. McGarry, "Convolutional perfectly matched layer for isotropic and anisotropic acoustic wave equations," in Proc. SEG Tech. Program Expanded Abstracts, 2010, pp. 2925–2929.

[Zhang and Shen 2010] W. Zhang and Y. Shen. Unsplit complex frequency-shifted PML implementation using auxiliary differential equations for seismic wave modeling. *Geophysics*, **75**, 4, 141-154. https://doi.org/10.1190/1.3463431.

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