OMEGA ACADEMY, NUMERICAL METHODS COURSE.

Erika Jissel Gutiérrez Beltrán

Daniel Fernandez Delgado

Frank Edward Daza González

Johanna Arias

Freddy Sebastian Garcia

Teacher:

Walter German Magaña

Matter:

Numerical Methods

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Guide numerical methods.
Multimedia Engineering and Systems Engineering

UNIT THREE

Bisection method.

This method consists of calculating roots that are not easily cleared by applying Bolzano theorem or intermediate-value theorem. This algorithm looks for dividing roots half interval sub-interval selecting the root.

- Bolzano theorem or intermediate value: a theorem on continuous functions defined on a real interval.

$$\frac{\exists\ C \in [a,b]}{f(c)} = 0$$

C is the root of the function.

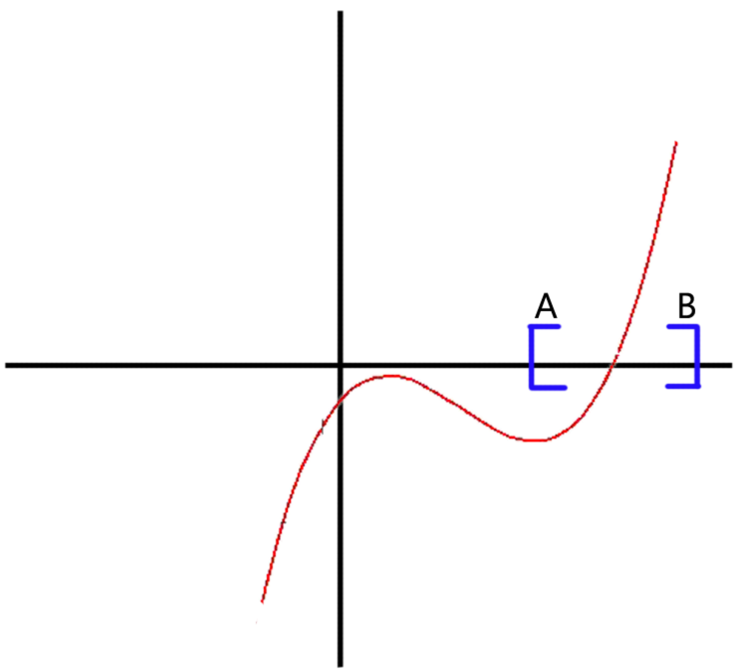

Image 1: (x,y) graph intersection selection of intervals to find the root zeros included.

If $f(a) < 0$ Or $f(a) > 0$

$f(b) > 0$ $f(b) < 0$

Are changing

To find the root you must use the following procedure

$$x1 = \frac{a+b}{2}$$

Where this result is half the sum of the extremes.

$f(x1) < 0 \rightarrow [x1,b]$
$f(x1) < 0 \rightarrow [x1,b]$

and so on for all the values obtained until the value of the root is found or a zero (0) which is in the range.

Where

f (x) is a function

- Define an interval [a, b]

- F (a) * f (b) <0 to ensure a root in the interval.

The formula to be used in this method to find the relative error is:

$$Er = \frac{|\,\mathrm{rNew} - \mathrm{rold}\,|}{\mathrm{rNew}}$$

- **Calculate the roots for the following function $x^5 - 3 = 0$ ➔** $f(x) = x^5 - 3 - 0$ Interval [0, 4].

$f(0) = -3 < 0$

$f(4) = (4^5) - 3 = 1.021 > 0$

$a_1 = 0$ $a_2 = 4$

$$\frac{a1 + a2}{2}$$

$$\frac{0 + 4}{2} = 2$$

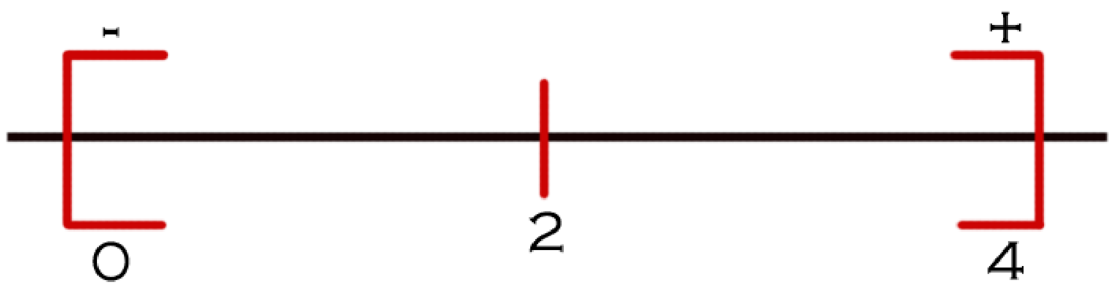

Image 2: Graphic demonstration of the procedure

$F(2) = (2^5) - 3 = 29 > 0$

If $f(2) * f(4) > 0$ ➔ the root won't be here

But if $f(0) * f(2) < 0$ ➔ the root is here

New interval [0,2]

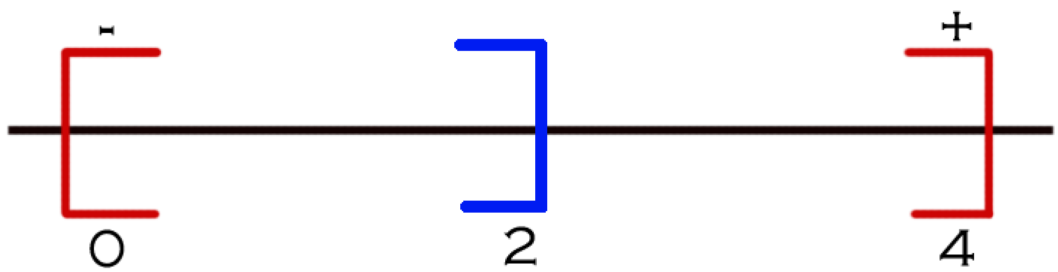

Image 3: Selection of the new interval [0, 2]

$a_1 = 0$ $a_3 = 2$

$$\frac{a1 + a3}{2}$$

$$\frac{0 + 2}{2} = 1$$

$f(1) = (1^5) - 3 = -2 < 0$

New interval [1,2]

$a_4 = 1$ $a_3 = 2$

$$\frac{a4 + a3}{2}$$

$$\frac{1 + 2}{2} = 1.5$$

$f(1.5) = (1.5^5) – 3 = 4.5 >0$

$f(1) < 0$

$f(1.5) > 0$

$f(2) > 0$

New interval [1,1.5]

$a_4 = 1$ $a_5 = 1.5$

$$\frac{a4 + a5}{2}$$

$$\frac{1 + 1.5}{2} = 1.25$$

f(1.25) = (1.25^5) – 3= 0.05 > 0

New interval [1,1.25]

$a_4 = 1$ $a_6 = 1.25$

$$\frac{a4 + a6}{2}$$

$$\frac{1 + 1.25}{2} = 1.125$$

f(1.125) = (1.125^5) – 3 = -1.1 < 0

New interval [1.25, 1.125]

$A_6 = 1 .25$ $a_7 = 1.125$

$$\frac{a6 + a7}{2}$$

$$\frac{1.25 + 1.125}{2} = 1.187$$

f(1.187) = (1.187^5) – 3 = -0.64 < 0

The more iterations, the greater is the approximation to the result.

To find the relative error in each of the results found must perform the half-sum of the ends of the interval or of the new intervals found to be developed as follows:

Relative error 1:

$$Er1 = \frac{x2 - x1}{x2}$$

$$\frac{1 - 2}{1} = -1$$

Relative error 2:

$$Er2 = \frac{x3 - x2}{x3}$$

$$\frac{1.5 - 1}{1.5} = 0.33$$

Relative error 3:

$$Er3 = \frac{x4 - x3}{x4}$$

$$\frac{1.25 - 1.5}{1\ .25} = -0.02$$

Relative error 4:

$$Er4 = \frac{x5 - x4}{x5}$$

$$\frac{1.125 - 1.25}{1.125} = -0.11$$

Relative error 5:

$$Er5 = \frac{x6 - x5}{x6}$$

$$\frac{1.187 - 1.125}{1\ .187} = 0.05$$

Table of iterations.

Far Left	Far Right	Midpoint	Value f(x)	Relative Error
0	4	2	29	
0	2	1	-2	-1
1	2	1.5	4.5	0.33
1	1.5	1.25	0.05	-0.02
1	1.25	1.125	-1.1	-0.11
1.25	1.125	1.187	-0.64	0.05