OMEGA ACADEMY, NUMERICAL METHODS COURSE.

Erika Jissel Gutiérrez Beltrán

Daniel Fernandez Delgado

Frank Edward Daza González

Johanna Arias

Freddy Sebastian Garcia

Teacher:

Walter German Magaña

Matter:

Numerical Methods

Universidad de San Buenaventura Cali

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UNIT THREE

Bisection method.

This method consists of calculating roots that are not easily cleared by applying Bolzano theorem or intermediate-value theorem. This algorithm looks for dividing roots half interval sub-interval selecting the root.

- Bolzano theorem or intermediate value: a theorem on continuous functions defined on a real interval.

$$\frac{\exists\, C \in [a, b]}{f(c)} = 0$$
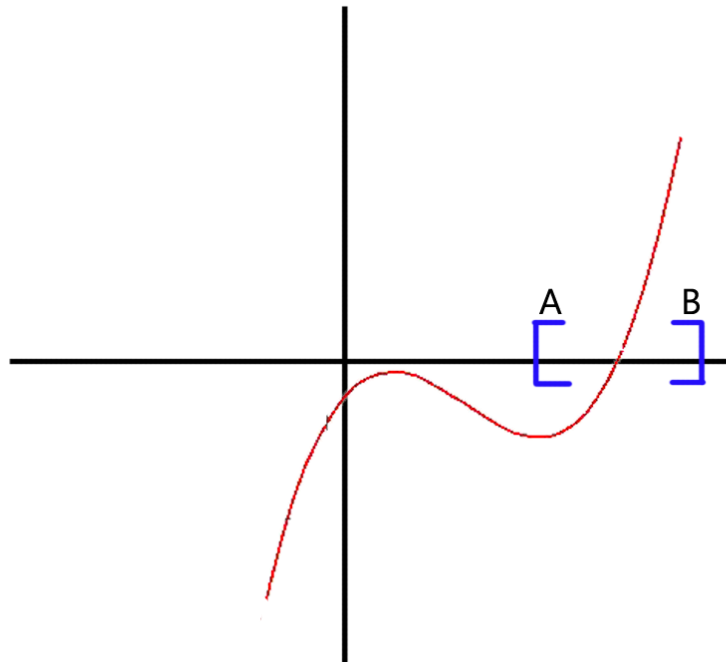
C is the root of the function.

Image 1: (x,y) graph intersection selection of intervals to find the root zeros included.

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If f(a) < 0   f(a) > 0
                Or
    f(b) > 0   f(b) < 0

Are changing

To find the root you must use the following procedure

$$x1 = \frac{a + b}{2}$$

Where this result is half the sum of the extremes.

f(x1) < 0 ➔ [x1,b]
f(x1) < 0 ➔ [x1,b]

and so on for all the values obtained until the value of the root is found or a zero (0) which is in the range.

Where

 f (x) is a function

- Define an interval [a, b]

- F (a) * f (b) <0 to ensure a root in the interval.

The formula to be used in this method to find the relative error is:

$$Er = \frac{|\,rNew - rold\,|}{rNew}$$

- **Calculate the roots for the following function $x^5 - 3 = 0$ ➜ $f(x) = x^5 - 3 - 0$**
  Interval [0, 4].

  $f(0) = -3 < 0$

  $f(4) = (4^5) - 3 = 1.021 > 0$

$$a_1 = 0 \qquad a_2 = 4$$
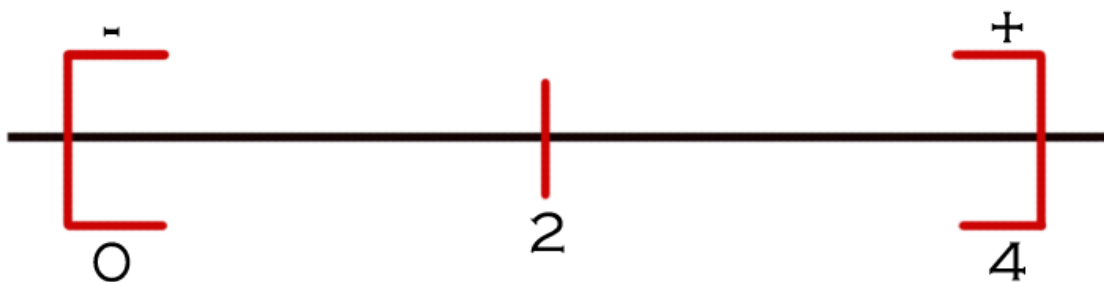
$$\frac{a1 + a2}{2}$$

$$\frac{0 + 4}{2} = 2$$

Image 2: Graphic demonstration of the procedure

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$F(2) = (2^5) - 3 = 29 > 0$

If f (2) * f(4) > 0 ➜ the root won't be here

But if f (0) * f(2) < 0 ➜ the root is here

New interval [0,2]

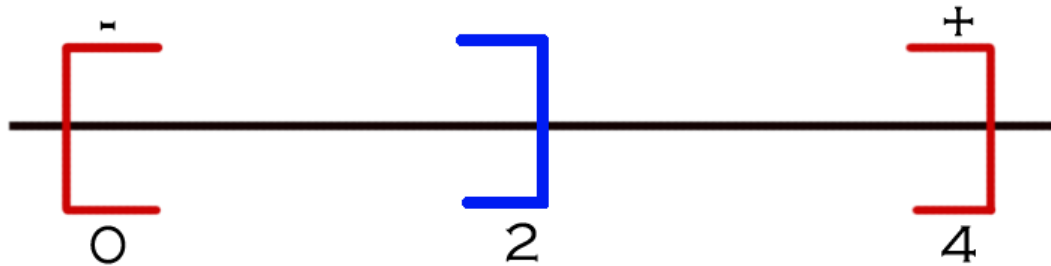*Image 3: Selection of the new interval [0, 2]*

$a_1 = 0$ $a_3 = 2$

$$\frac{a1 + a3}{2}$$

$$\frac{0 + 2}{2} = 1$$

$f(1) = (1^5) - 3 = -2 < 0$

New interval [1,2]

$$a_4 = 1 \qquad a_3 = 2$$

$$\frac{a4 + a3}{2}$$

$$\frac{1+2}{2} = 1.5$$

f(1.5) = (1.5⁵) − 3 = 4.5 >0

f(1) < 0

f(1.5) > 0

f(2) > 0

New interval [1,1.5]

$$a_4 = 1 \qquad a_5 = 1.5$$

$$\frac{a4 + a5}{2}$$

$$\frac{1+1.5}{2} = 1.25$$

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f(1.25) = (1.25^5) − 3= 0.05 > 0

New interval [1,1.25]

$$a_4 = 1 \qquad a_6 = 1.25$$

$$\frac{a4 + a6}{2}$$

$$\frac{1 + 1.25}{2} = 1.125$$

f(1.125) = (1.125^5) − 3 = -1.1 < 0

New interval [1.25, 1.125]

$$A_6 = 1 .25 \qquad a_7 = 1.125$$

$$\frac{a6 + a7}{2}$$

$$\frac{1.25 + 1.125}{2} = 1.187$$

f(1.187) = (1.187^5) − 3 = -0.64 < 0

The more iterations, the greater is the approximation to the result.

To find the relative error in each of the results found must perform the half-sum of the ends of the interval or of the new intervals found to be developed as follows:

Relative error 1:

$$Er1 = \frac{x2 - x1}{x2 + x1}$$

$$\frac{1 - 2}{1 + 2} = -0{,}33$$

Relative error 2:

$$Er2 = \frac{x3 - x2}{x3 + x2}$$

$$\frac{1.5 - 1}{1.5 + 1} = 0.2$$

Relative error 3:

$$Er3 = \frac{x4 - x3}{x4 + x3}$$

$$\frac{1.25 - 1.5}{1\,.25 + 1.5} = -0.09$$

Relative error 4:

$$Er4 = \frac{x5 - x4}{x5 + x4}$$

$$\frac{1.125 - 1.25}{1.125 + 1.25} = -0.05$$

Relative error 5:

$$Er5 = \frac{x6 - x5}{x6 + x5}$$

$$\frac{1.187 - 1.125}{1.187 + 1.125} = 0.02$$

Table of iterations.

Far Left	Far Right	Midpoint	Value f(x)	Relative Error
0	4	2	29	
0	2	1	-2	-0.33
1	2	1.5	4.5	0.2
1	1.5	1.25	0.05	-0.09
1	1.25	1.125	-1.1	-0.05
1.25	1.125	1.187	-0.64	0.02