3SAT Survey

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1 Schöning's Algorithm

The paper [2] presents a simple randomized algorithm for solving k-SAT

- Guess an initial assignment $a \in \{0,1\}^n$ uniformly at random
- Repeat $3n$ times:
  - If the formula is satisfied by a - stop and accept a.
  - Let C be some clause not being satisfied by a. Pick one of the $\leq k$ literals in the clause at random and flip its value in the current assignment.

1.1 Analysis for $k = 3$

We now wish to estimate the running time of this randomized algorithm, and upper bound it as p. The rationale behind this is that the expected number of repetitions before a satisfying assignment is $\dfrac{1}{p}$, and the probability that after t repeitions, the probability of not getting a satisfying assignment is at most $(1-p)^t \leq e^{pt}$.

We provide analysis for 3 sat for now. General arguments can be found in the paper but this is a nicer simplification for presentation and understanding for now. First, there are $2^n \binom{n}{u}$ possible assignments of a which differ from a^* in exactly u variables.

Next we consider the question of finding a solution from assignment a in exactly $3u$ steps. A good step is one which flips a variable in which the assignment between a and a^* differ. A bad step is one which flips a variable in which a and a^* are the same. This means that we need to perform exactly $2u$ good steps and u bad steps. The probability this occurs is

$$\binom{3u}{2u}\left(\frac{1}{3}\right)^{2u}\left(\frac{2}{3}\right)^{u}$$

We then apply Stirling's formula:

$$\binom{3u}{2u} = \binom{3u}{u} \geq \frac{1}{\sqrt{5u}} \frac{3^{3u}}{2^{2u}}$$

to obtain a nicer lower bound. We can hence put this together in order to calculate p, the expected probability of finding a satsifying assignments:

$$\begin{align}
p &= \sum_{u=0}^{n} 2^{-n} \binom{n}{u} \binom{3u}{2u} \left(\frac{1}{3}\right)^{2u} \left(\frac{2}{3}\right)^{u} \tag{1} \\
&\geq \frac{1}{\sqrt{5n}} 2^{-n} \sum_{u=0}^{n} \binom{n}{u} \frac{3^{3u}}{2^{2u}} \left(\frac{1}{3}\right)^{2u} \left(\frac{2}{3}\right)^{u} \tag{2} \\
&\geq \frac{1}{\sqrt{5n}} 2^{-n} \sum_{u=0}^{n} \binom{n}{u} \frac{1}{2^{u}} \tag{3} \\
&\geq \frac{1}{\sqrt{5n}} \left(\frac{1}{2} \cdot \left(1 + \frac{1}{2}\right)\right)^{n} \tag{4} \\
& \frac{1}{\sqrt{5n}} \cdot \left(\frac{3}{4}\right)^{n} \tag{5}
\end{align}$$

As the running time is just a constant factor of $\frac{1}{p}$ then the running time is $O\left(\frac{4}{3}\right)^{n}$.

1.2 Analysis for general k

2 PPSZ Algorithm

In paper [1], the authors present an algorithm based around *critical variables*.

References

[1] Ramamohan Paturi, P Pudlik, Michael E Saks, and Francis Zane. An improved exponential-time algorithm for k-sat. In *Foundations of Computer Science, 1998. Proceedings. 39th Annual Symposium on*, pages 628–637. IEEE, 1998.

[2] Uwe Schöning. A probabilistic algorithm for k-sat and constraint satisfaction problems. In *Proceedings of the 40th Annual Symposium on Foundations of Computer Science*, FOCS '99, pages 410–, Washington, DC, USA, 1999. IEEE Computer Society.