3SAT Survey

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1 Schöning's Algorithm

The paper [Schöning(1999)] presents a simple randomized algorithm for solving k-SAT

- Guess an initial assignment $a \in \{0,1\}^n$ uniformly at random
- Repeat $3n$ times:
  - If the formula is satisfied by a - stop and accept a.
  - Let C be some clause not being satisfied by a: Pick one of the $\leq k$ literals in the clause at random and flip its value in the current assignment.

We now wish to estimate the running time of this randomized algorithm, and upper bound it as p. The rationale behind this is that the expected number of repetitions before a satisfying assignment is $\frac{1}{p}$, and the probability that after t repeitions, the probability of not getting a satisfying assignment is at most $(1-p)^t \leq e^{pt}$.

We provide analysis for 3 sat for now. General arguments can be found in the paper but this is a nicer simplification for presentation and understanding for now. First, there are $2^n \binom{n}{u}$ possible assignments of a which differ from a^* in exactly u variables.

Next we consider the question of finding a solution from assignment a in exactly $3u$ steps. A good step is one which flips a variable in which the assignment between a and a^* differ. A bad step is one which flips a variable in which a and a^* are the same. This means that we need to perform exactly $2u$ good steps and u bad steps. The probability this occurs is

$$\binom{3u}{2u}\left(\frac{1}{3}\right)^{2u}\left(\frac{2}{3}\right)^{u}$$

We then apply Stirling's formula:

$$\binom{3u}{2u} = \binom{3u}{u} \geq \frac{1}{\sqrt{5u}}\frac{3^{3u}}{2^{2u}}$$

to obtain a nicer lower bound. We can hence put this together in order to calculate p, the expected probability of finding a satsifying assignments:

$$
\begin{align}
p &= \sum_{u=0}^{n} 2^{-n} \binom{n}{u} \binom{3u}{2u} \left(\frac{1}{3}\right)^{2u} \left(\frac{2}{3}\right)^{u} \tag{1} \\
&\geq \frac{1}{\sqrt{5n}} 2^{-n} \sum_{u=0}^{n} \binom{n}{u} \frac{3^{3u}}{2^{2u}} \left(\frac{1}{3}\right)^{2u} \left(\frac{2}{3}\right)^{u} \tag{2} \\
&\geq \frac{1}{\sqrt{5n}} 2^{-n} \sum_{u=0}^{n} \binom{n}{u} \frac{1}{2^{u}} \tag{3} \\
&\geq \frac{1}{\sqrt{5n}} \left(\frac{1}{2} \cdot \left(1 + \frac{1}{2}\right)\right)^{n} \tag{4} \\
& \frac{1}{\sqrt{5n}} \cdot \left(\frac{3}{4}\right)^{n} \tag{5}
\end{align}
$$

As the running time is just a constant factor of $\frac{1}{p}$ then the running time is $O\left(\frac{4}{3}\right)^{n}$.

2 PPSZ Algorithm

[Paturi et al.(1998)Paturi, Pudlik, Saks, and Zane]

References

[Paturi et al.(1998)Paturi, Pudlik, Saks, and Zane] Ramamohan Paturi, P Pudlik, Michael E Saks, and Francis Zane. An improved exponential-time algorithm for k-sat. In *Foundations of Computer Science, 1998. Proceedings. 39th Annual Symposium on*, pages 628–637. IEEE, 1998.

[Schöning(1999)] Uwe Schöning. A probabilistic algorithm for k-sat and constraint satisfaction problems. In *Proceedings of the 40th Annual Symposium on Foundations of Computer Science*, FOCS '99, pages 410–, Washington, DC, USA, 1999. IEEE Computer Society. ISBN 0-7695-0409-4. URL `http://dl.acm.org/citation.cfm?id=795665.796524`.