Electromagnetic Field Oral Exam Questions

Questions from the oral exam for the course *Electromagnetno polje* (Electromagnetic Field), compiled from exams from previous years and supplemented with additional course material from the winter semester 2020-2021.

1 Electrostatics

1. Write the first two Maxwell equations for a static electric field. State the equation for electric field lines and derive the Poisson equation.

2. Provide examples of a few common charge distributions.

   Derive the magnitude of the perpendicular component of the electric field for a surface charge distribution.

3. State the solution to the Poisson equation for a point particle and derive the associated Green's function.

4. Derive the electrostatic energy of a charge distribution in an external electric field.

5. Derive the total electrostatic energy in an electric field. Give the expression for total electoric field energy in terms of both the scalar potential ϕ and the electric field $\boldsymbol{E}$.

6. Derive the force on a charge distribution in an external electric field in terms of the electrostatic stress tensor.

   Use the result to calculate the electric force between two point charges of a.) equal and b.) opposite charge.

7. State and derive the multipole expansion of the electric potential up to the second order term, and use the result to define the dipole moment, electric potential and electric field of an electric dipole.

8. State and derive the multipole expansion of electrostatic energy up to the second order term. Use the the result to derive the force and torque on an electric dipole in an external electric field.

2 Magnetostatics

1. Define electric current density. State the electric current densities for current along a current-carrying wire and for a continuous charge distribution moving through space.

2. State the relationship between magnetic flux density (the $\boldsymbol{B}$ field) and electric current density. State and derive the field lines for both vectors.

3. Derive the expression for the magnetic vector potential and derive the magnetic analog of the Laplace equation relating magnetic potential and current density.

4. State and derive the magnetic vector potential inside and outside a long, straight inductor. Explain how it is possible to simplify the magnetic field outside the inductor using gauge transformations.

5. Derive the Biot-Savart law for the magnetic field of a continuous charge distribution. Use the result to derive the magnetic field of a straight conducting wire.

6. Derive the expression for the magnetic energy of a charge distribution in an external magnetic field.

   Then, explain the differences between finding the total magnetic field energy of a magnetic field and finding the total electric field energy of an electric field.

7. Derive the expression for the total magnetic field energy associated with a magnetic field. Give the result both in terms both in terms of magnetic field and magnetic vector potential.

8. Derive the magnetic force on current distribution in terms of the magnetostatic stress tensor. Use the result to calculate the magnetic force between two straight, parallel conducting wires carrying an electric current in (a) equal directions and b.) opposite directions.

9. Derive the multipole expansion of the magnetic vector potential to dipole term. Use the result to define magnetic dipole moment.

10. State and derive the magnetic vector potential and magnetic field of a magnetic dipole.

    Explain the concept of Ampere equivalence.

11. Derive the multipole expansion of magnetic energy up to the dipole term. Use the result to derive the force and torque on a magnetic dipole in an external electric field.

3 Quasistatic Electromagnetic Fields

1. State the Maxwell equations in the regime of quasi-static electromagnetic fields. Show that in the quasistatic regime, the Maxwell equations correspond to closed current loops.

2. Derive the relationship between electric field strength and the electric and magnetic potentials. Use the result to calculate the curl of the electric field and interpret the result.

3. State Ohm's law in terms of electric field and current density. Use the result to explain the behavior of the electric field within a conductor and the electric potential on the conductor's surface.

4. Derive Ohm's law from the Drude model of electrical conduction. Discuss the Drude model's prediction for a material's electrical conductivity.

5. Use Ohm's law to derive the expression for the Ohmic resistance of a conductor.

6. Derive the expression for dissipation of electric energy in a conductor with Ohmic resistance.

7. What is the skin effect? State the equations needed to analyze the skin effect, give their solutions in the case of a cylindrical conductor, and discuss the results. Discuss the limit cases of complex impedance for small and large frequencies.

4 Maxwell Equations

1. State the Maxwell equations in free space. Use the Maxwell equations to derive the continuity equation encoding conservation of electric charge.

2. Derive the Poynting theorem for conservation of electromagnetic energy in free space. Under what conditions is electromagnetic field energy conserved?

3. Derive the continuity equation encoding conservation of electromagnetic momentum.

5 Electromagnetic Fields in Matter

1. Define bound charge, polarization and electric susceptibility. State the definition of the $\boldsymbol{D}$ field, discuss some of its important properties, and explain its role in the study of electric fields in matter.

2. Show that electric polarization equals the volume density of electric dipoles in matter.

3. Define bound current density, magnetization and magnetic susceptibility. State the definition of the $\boldsymbol{H}$ field, discuss some of its important properties, and explain its role in the study of magnetic fields in matter.

4. Show that magnetization equals the volume density of the magnetic dipole moment in matter, and interpret the result in terms of Ampere equivalence.

5. State Maxwell's equations in matter. State and discuss the constitutive relations for the electric and magnetic fields in the linear regime.

6. State and derive the boundary conditions for the Maxwell equations along the boundary between two materials with different electromagnetic properties.

6 Frequency Dependence of the Dielectric Constant

1. State and derive the relationship between the electric field, polariation, and the dielectric function in both the time and frequency domains. Discuss the physical significance of the dielectric function's real and imaginary components in the frequency domain.

2. State and discuss the Kramers-Kronig relations for the dielectric function.

3. State and derive the expression for the dissipation of electric field energy in matter in terms of the dielectric function in the frequency domain.

4. Discuss the Debye regime of dielectric relaxation and derive and sketch the associated expressions for the real and imaginary components of the dielectric function as a function of electric field frequency.

5. Discuss the Lorentz model of dielectric relaxation and derive and sketch the associated expressions for the real and imaginary components of the dielectric function as a function of electric field frequency.

6. Discuss the plasma regime of dielectric relaxation and derive and sketch the associated expressions for the real and imaginary components of the dielectric function as a function of electric field frequency.

7 Hamiltonian Formalism for the Electromagnetic Field

1. State and derive the Lagrange function for a charged particle in an electromagnetic field.

2. State and derive the Hamiltonian function for a charged particle in an electromagnetic field.

3. State and derive the complete Lagrangian density function of a continuous charge distribution in an external electromagnetic field. Discuss the associated action, Euler-Lagrange equations, and the resulting Riemann-Lorenz equations.

8 Introduction to Special Relativity

1. Briefly discuss the Lorentz transformation between two frames of reference moving relative to eachother along a mutual x axis.

   State and sketch the derivation of the Lorentz transformation of the electric and magnetic field between two frames of reference. Demonstrate the implications of the above transformations on how the quantities $\boldsymbol{E} \cdot \boldsymbol{B}$ and $E^2 - c^2 B^2$ transform between frames of reference, and give a physics interpretation of the results.

2. Briefly discuss Minkowski space, the concept of covariant and contravariant vectors, and the generalization of the scalar product to Minkowski space.

   Discuss conservation of charge in Minkowski space and derive the associated expression for the current density four vector. Show that the magnitude of the current density four vector is invariant under Lorentz transformations.

3. Derive the expression for the electromagnetic potential four vector in terms of the Riemman-Lorenz equations and the current density four vector. Show that the magnitude of the electromagnetic potential four vector is invariant under Lorentz transformations.

4. State the electromagnetic tensor in both component and matrix form and sketch the motivation for its definition.