The Hall Effect

Elijan Mastnak

Winter Semester 2020-2021

Contents

1 Tasks

1. Measure the dependence of Hall voltage on temperature in a germanium n-type semiconductor in the temperature range 20 °C to 80 °C.

2. Plot the semiconductor's Ohmic resistance R and Hall coefficient R_H as a function of temperature T.

3. Use the dependence of charge carrier concentration on temperature to determine which charge carriers dominate in which temperatures. Experimentally verify the validity of theoretical semiconductor charge-carrier equations.

2 Equipment, Procedure and Data

2.1 Equipment

- Hall probe containing an n-type germanium semiconductor sample.
- Enclosure for the probe containing a known magnetic field
- 1.5 V battery serving as a voltage source for the Hall probe
- Temperature regulation mechanism for the Hall probe, in our case a water cooler/heater.
- Voltmeter and ammeter for measuring Hall voltage and current through sample

2.2 Procedure

1. Set up the electrical connections as shown in Figure 1 and place the Hall probe inside its enclosure containing the magnetic field.
2. Vary the sample's temperature from 20 °C to 80 °C in increments of 5 °C using a boiler, and wait about 5 min between increments for the temperature of the sample to stabilize.

   At each temperature, measure the resulting current I and Hall potential U_H on the Hall probe for both orientations of the Hall probe in the magnetic field.

2.3 Data

Independent Variable: Temperature, measured in the range range 20 °C to 80 °C and varied with uniform spacing 5 °C using a water-based heater.
Orientation of the Hall probe relative to the magnetic field

Dependent Variable: Hall voltage and current through the Hall probe.

Parameters: Semiconductor thickness $c = 0.95\,\mathrm{mm}$ with band gap $E_g \approx 0.66\,\mathrm{eV}$ and donor energy gap $E_d \approx 0.01\,\mathrm{eV}$, exposed to a magnetic field of magnitude $B = 0.173\,\mathrm{T}$.

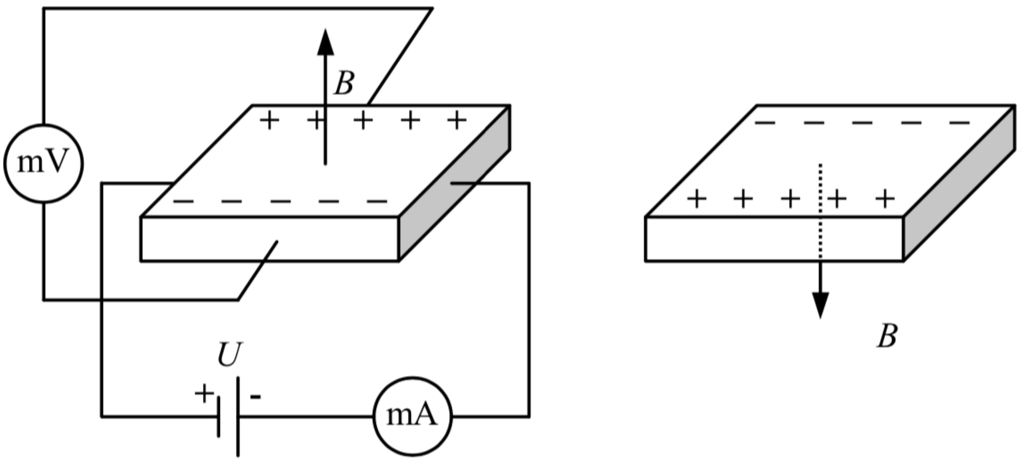

Figure 1: Schematic of the measurement set-up used in the experiment.

3 Analysis

3.1 Calculating Hall Voltage

Because of contact asymmetries, the measured potential difference across the sample (along the y axis) is the sum of the Hall potential U_H and an asymmetry term U_0. To solve for U_H, note that the sign of the Hall potential flips when the sample is flipped by 180° in the magnetic field. By measuring the potential difference with the sample facing up and facing down (flipping the orientation of the ab plane), we get the expressions

$$U_1 = U_0 + U_H \qquad \text{and} \qquad U_2 = U_0 - U_H$$

from which we get the Hall voltage

$$U_H = \frac{1}{2}(U_1 - U_2)$$

Figure 2 shows the dependence of Hall voltage on temperature—note that the voltage decreases in magnitude with increasing temperature. The fact that the Hall voltage is negative indicates the majority charge carriers through semiconductor are negative electrons.

3.2 Ohmic Resistance

From Ohm's law, the semiconductor's Ohmic resistance is

$$R = \frac{U_{\text{source}}}{I}$$

where $U_{\text{source}} = 1.5\,\text{V}$ is the voltage of the battery powering the Hall probe and I is the current through the semiconductor. Figure 3 shows the dependence of the semiconducting probe's Ohmic resistance on temperature. The resistance increases with temperature because exponentially more charge carriers are excited into the conduction band with increasing temperature, facilitating the flow of current.

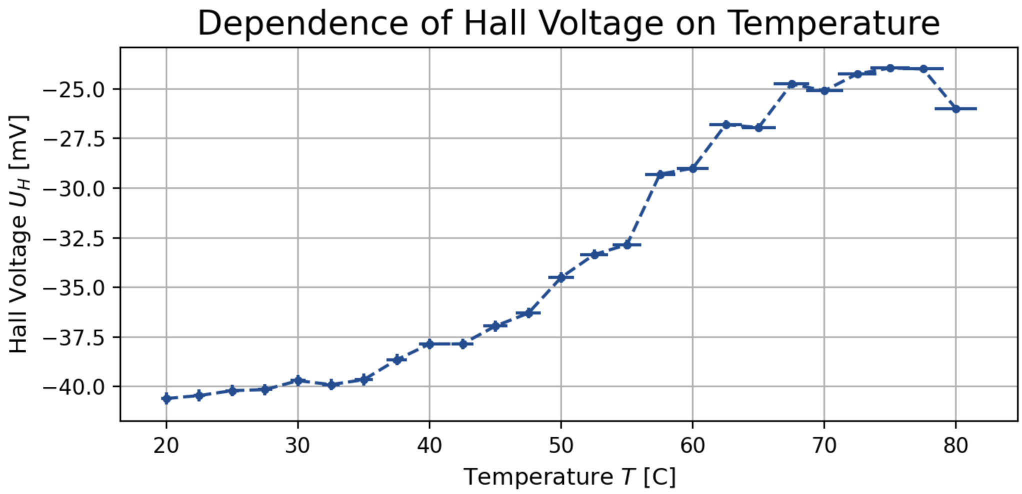

Figure 2: Experimental dependence of Hall voltage on the probe temperature T.

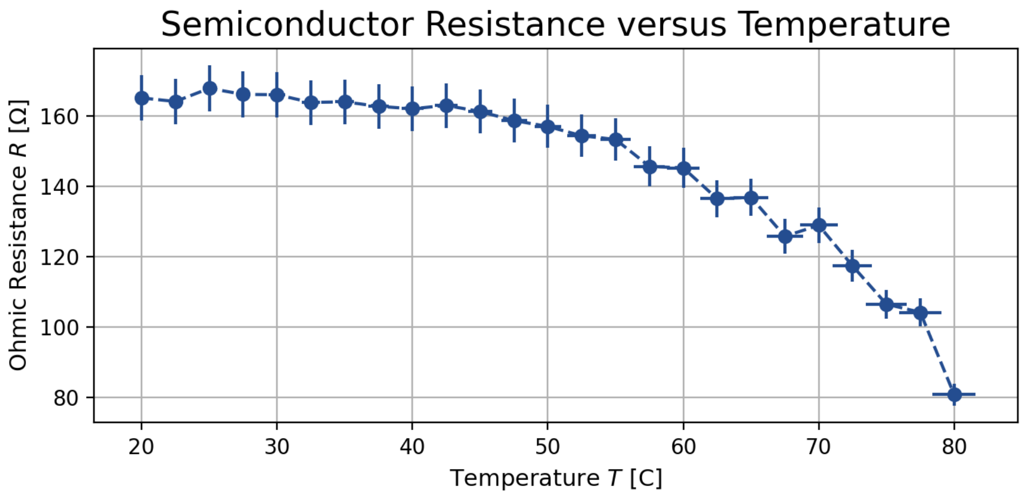

Figure 3: The dependence of the semiconducting probe's Ohmic resistance on temperature T. Note the decrease in resistance with temperature, which would not occur in a standard resistor circuit element.

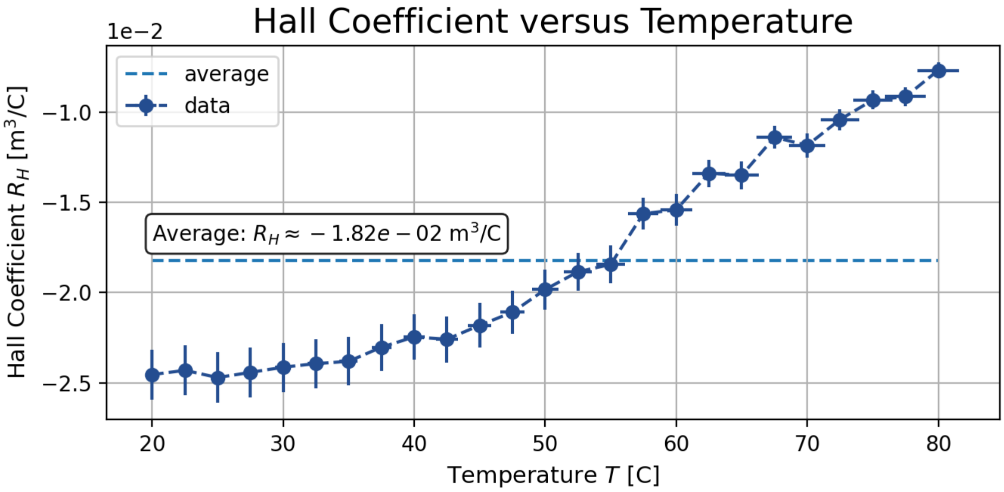

Figure 4: Dependence of the Hall coefficient on temerature..

3.3 Hall Coefficient

We calculate the Hall coefficient R_H with

$$R_H = \frac{U_H c}{IB}$$

Figure 4 show the Hall coefficient's dependence on temperature. The fact that the Hall coefficient is negative indicates the majority charge carriers are electrons, meaning the probe uses an n-type semiconductor. Although the extact value of R_H varies with temperature, the order of magnitude agrees with that given in [1] and the references therein.

3.4 Carrier Concentration

We calculate the number density n of charge carriers in the semiconductor with

$$n = -\frac{IB}{ce_0 U_H}$$

3.5 High Temperatures

In the high temperature limit $k_B T \gtrsim E_g$, both donor and valance electrons are thermally excited to the conduction band. Because there are so many more valence electrons than donor electrons for typical dopant concentrations, the contributions of the donor electrons is negligible, and we have the intrinsic dependence

$$n_e(T) = \frac{1}{4}\left(\frac{2m_e k_B T}{\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right)$$

where E_g is the energy band gap between the valence and conduction bands. The slope of the graph of $\ln n$ versus $\frac{1}{k_B T}$, shown in Figure 5, gives an estimate of the silicon band gap.

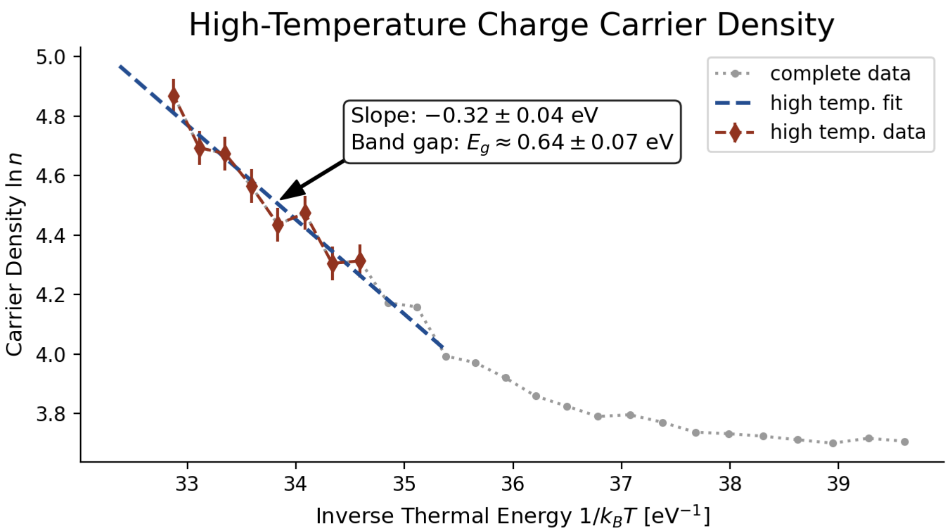

Figure 5: The bold data points show the high-temperature dependence of carrier concentration $\ln n$ on inverse thermal energy $\frac{1}{k_B T}$. The fitted line's slope gives an estimate of the germanium band gap E_g.

3.6 Moderate Temperatures

At temperatures $k_B T \gtrsim E_d$, essentially all donor electrons are excited to the conduction band, but the thermal energy is still negligible compared to the band gap E_g, so valence electrons remain frozen out. The carrier concentration is approximately

$$n(T) \approx N_d$$

where N_d is the number density of donor impurities. A plot of the charge carrier concentration at medium temperatures is shown in Figure 6

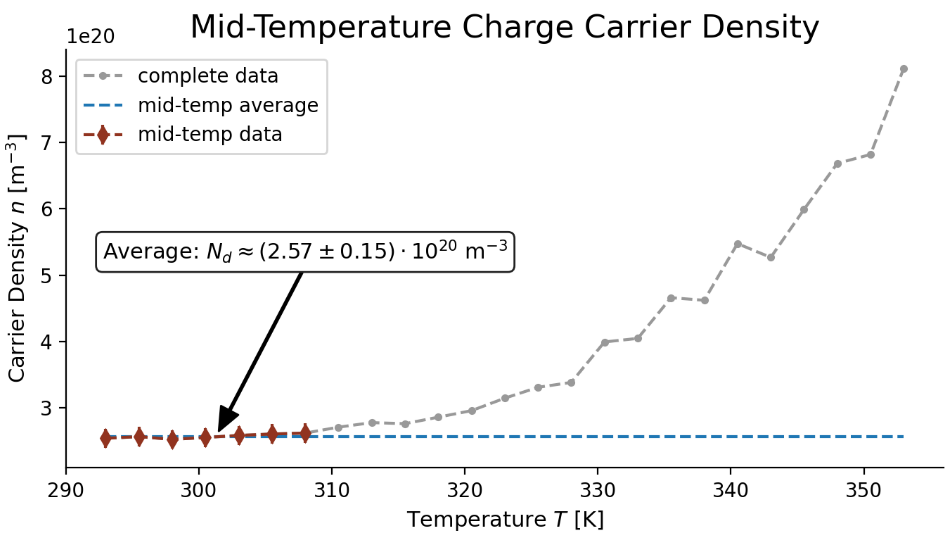

Figure 6: The bold data points show the mid-temperature dependence of carrier concentration n on temperature T, and give an estimate of donor concentration N_d.

4 Error Analysis

4.1 Hall Voltage

Hall voltage is found with

$$U_H = \frac{U_1 - U_2}{2}$$

The input quantities are the voltages U_1 and U_2 with the probe's normal parallel and anti-parallel to the magnetic field. Both quantities carry a one percent uncertainty. Sensitivity coefficients are

$$c_1 = \frac{\partial U_H}{\partial U_1} = \frac{1}{2} \qquad \text{and} \qquad c_2 = \frac{\partial U_H}{\partial U_2} = -\frac{1}{2}$$

The corresponding uncertainty in Hall voltage U_H is

$$\delta_H = \sqrt{(\delta_1 c_1)^2 + (\delta_2 c_2)^2}$$

4.2 Ohmic Resistance

The semiconducting probe's Ohmic resistance R is found according to

$$R = \frac{U_{\text{source}}}{I}$$

$U_{\text{source}} = (1.50 \pm 0.05)\,\text{V}$ is the battery voltage applied to the probe; I is the current through the probe and carries a one percent uncertainty. The sensitivity coefficients are

$$c_U = \frac{\partial R}{\partial U_{\text{source}}} = \frac{1}{I} \qquad \text{and} \qquad c_I = \frac{\partial R}{\partial I} = -\frac{U_{\text{source}}}{I^2}$$

The corresponding uncertainty in the probe resistance R is

$$\delta_R = \sqrt{(\delta_U c_U)^2 + (\delta_I c_I)^2}$$

4.3 Hall Coefficient

We calculate the Hall coefficient R_H using

$$R_H = \frac{U_H c}{IB}$$

Constant input quantities are semiconductor width $c = (0.95 \pm 0.05)\,\text{mm}$ and $B = (0.173 \pm 0.002)\,\text{T}$. The current carries a one percent uncertainty, and the uncertainty in the Hall voltage is calculated above. Sensitivity coefficients are

$$c_U = \frac{\partial R_H}{\partial U_H} = \frac{c}{IB} \qquad c_c = \frac{\partial R_H}{\partial c} = \frac{U_H}{IB}$$
$$c_I = \frac{\partial R_H}{\partial I} = -\frac{U_H c}{I^2 B} \qquad c_B = \frac{\partial R_H}{\partial B} = -\frac{U_H c}{IB^2}$$

The corresponding uncertainty in the Hall coefficient R_H is

$$\delta_{R_H} = \sqrt{(\delta_U c_U)^2 + (\delta_c c_c)^2 + (\delta_I c_I)^2 + (\delta_B c_B)^2}$$

4.4 Carrier Concentration

We calculate the number density n of charge carriers in the semiconductor with

$$n = -\frac{IB}{ce_0U_H}$$

where the argument is a positive quantity because U_H is negative. Constant input quantities are semiconductor width $c = (0.95 \pm 0.05)$ mm and $B = (0.173 \pm 0.002)$ T. The current carries a one percent uncertainty, and the uncertainty in the Hall voltage is calculated above. I assume uncertainty in the elementary charge e_0 is negligible.

The sensitivity coefficients for n are

$$c_U = \frac{\partial n}{\partial U_H} = \frac{IB}{ce_0U_H^2} \qquad c_c = \frac{\partial n}{\partial c} = \frac{IB}{c^2e_0U_H}$$
$$c_I = \frac{\partial n}{\partial I} = -\frac{B}{ce_0U_H} \qquad c_B = \frac{\partial n}{\partial B} = -\frac{I}{ce_0U_H}$$

High-Temperature Limit
When working with the quantity $\ln n$ in the high-temperature limit, sensitivity coefficients are

$$c_U = \frac{\partial \ln n}{\partial U_H} = -\frac{1}{U_H} \qquad c_c = \frac{\partial \ln n}{\partial c} = -\frac{1}{c}$$
$$c_I = \frac{\partial \ln n}{\partial I} = \frac{1}{I} \qquad c_B = \frac{\partial \ln n}{\partial B} = \frac{1}{B}$$

The uncertainties in n and $\ln n$ are

$$\delta n, \delta\big[\ln n\big] = \sqrt{(\delta_U c_U)^2 + (\delta_c c_c)^2 + (\delta_I c_I)^2 + (\delta_B c_B)^2}$$

Figure 5 and 6 are equipped with error bars showing the corresponding uncertainties. The error in the band gap E_g estimate in Figure 5 comes from the covariance matrix of the linear fit, where the ordinate data are weight with the error bars shown in the Figure.

5 Results

Ohmic Resistance
The semiconductor's Ohmic resistance R ranged from

$$R \approx 160\,\Omega \text{ at } 20\,^\circ\text{C} \quad \text{to} \quad R \approx 80\,\Omega \text{ at } 80\,^\circ\text{C}$$

Note that the resistance falls with increasing temperature, characteristic of a doped semiconductor at thermal energies near the donor energy gap.

Hall Coefficient
The Hall coefficient for the germanium semiconductor ranged from

$$R_H \approx -2.45 \times 10^{-2}\,\text{m}^3\,\text{C}^{-1} \text{ at } 20\,^\circ\text{C} \quad \text{to} \quad R_H \approx -0.77 \times 10^{-2}\,\text{m}^3\,\text{C}^{-1} \text{ at } 80\,^\circ\text{C}$$

The fact that the Hall coefficient is negative indicates the semiconductor's majority charge carriers are negative electrons, meaning the semiconductor is n-type.

Band Gap Estimate

The estimate for the semiconductor's band gap E_g is

$$\boxed{E_g \approx (0.64 \pm 0.07)\,\mathrm{eV}}$$

Donor Impurity Concentration

The estimate for the semiconductor's concentration of donor impurities N_d is

$$\boxed{N_d \approx (2.57 \pm 0.15) \times 10^{20}\,\mathrm{m}^{-3}}$$

A Background Theory

A.1 Hall Effect

- The Hall effect involves a metal conductor placed in an external magnetic field perpendicular to the motion of electrons through the conductor.

- In our experiment the conductor is a thin, rectangular metal strip with side of length a, b and c in the x, y and z directions. Current I flows in the positive x direction and the magnetic field B points in the positive z direction. The z dimension is much smaller than the other two dimensions, i.e. $c \ll a, b$.

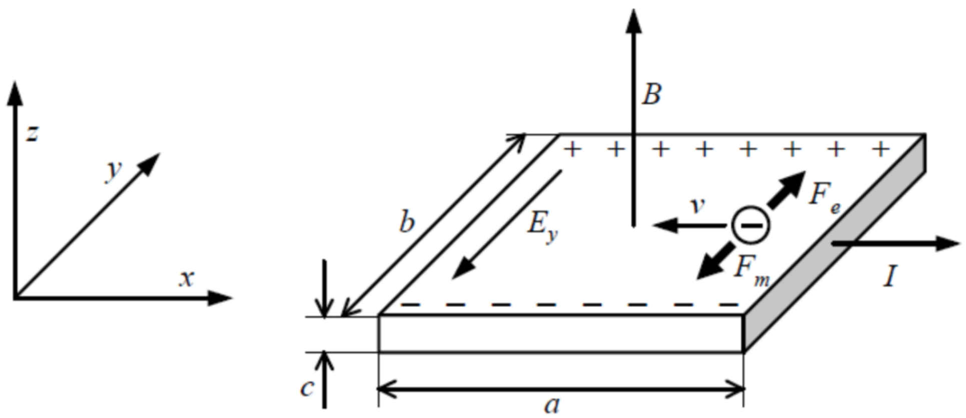

Figure 7: Coordinate system and relevant physical quantities for understanding the Hall effect in this experiment.

We assume change carriers are electrons with charge $-e_0$. Current density is

$$j = \frac{I}{bc} = -ne_0 v$$

where n and v are the electron number density and drift velocity, respectively.

- The negatively charged electrons experience a magnetic force of magnitude $F_m = -e_0 vB$ in the negative y direction and begin to accumulate at the edge of the metal strip. The accumulation of negative charge leads to an electric field $-E_y$ in the negative y direction and a corresponding electric force

$$F_e = qE = (-e_0)(-E_y) = e_0 E_y$$

in the positive y direction. In the stationary state (which occurs quickly, in times of order 10×10^{-12} s), the electric force balances the magnetic force:

$$e_0 E_y = e_0 v B \implies E_y = vB = -\frac{jB}{ne_0}$$

- The Hall voltage U_H is the voltage between the edges of the conductor resulting from the electric field E_y:

  $$U_H = E_y b = -\frac{jBb}{ne_0} = -\frac{IB}{ne_0 c}$$

  The quotient $\frac{E_y}{jB}$ is called the Hall coefficient and is denoted by R_H.

  $$R_H = \frac{E_y}{jB} = -\frac{1}{ne_0} = \frac{U_H c}{IB}$$

  The coefficient is characteristic of the material from which the conductor is made.

- We can rearrange the Hall voltage equation to measure magnetic field. In a Hall probe, a conducting strip is placed in an unknown magnetic field. We send a known current through the conductor and measure the Hall voltage, which is proportional to the magnetic field.

  $$B = -ne_0 c \frac{U_H}{I}$$

  The constant factor $ne_0 c$ is a property of the conductor and need be measured only once. We can also use the Hall coefficient to determine the sign and density of charge carriers in different materials. In this experiment, we will investigate the charge carriers in a germanium semiconductor.

A.2 Hall Effect in Semiconductors

- For an *intrinsic* semiconductor, the number density of charge carriers n excited from the valence to the conduction band is

  $$n_e(T) = \frac{1}{4}\left(\frac{2m_e k_B T}{\pi \hbar^2}\right)^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right)$$

  where E_g is the energy band gap between the valence and conduction bands.

- Hall effect in n-type semiconductor: add a dopant with five valence electrons. Creates small energy gap E_d between donor level and conduction band. For a donor level just below the conduction band, primary charge carriers are electrons, hence n-type semiconductor.

Three regimes for a *doped n-type* semiconductor:

1. Low temperatures, $k_B T \ll E_d$

$$n(T) = \sqrt{N_d}\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/4} e^{-\frac{E_D}{2k_B T}}$$

   where N_d is the density of donor impurities and m_e is the effective electron mass in the semiconductor.

   Only the donor electrons contribute appreciably to conduction because thermal energy is negligible compared to the band gap E_g. All valence electrons are completely frozen out.

2. At medium temperatures $k_B T \gtrsim E_d$, essentially all donor electrons are excited to the conduction band, but the thermal energy is still negligible to the band gap E_g, so valence electrons remain frozen out.

$$n(T) = N_d$$

3. High temperatures $k_B T \gtrsim E_g$, both donor and valance electrons are thermally excited to the conduction band. Because there are so many more valence electrons than donor electrons for typical dopant concentrations, the contributions of the donor electrons is negligible, and we have the intrinsic dependence

$$n(T) = \frac{1}{4}\left(\frac{2m_e k_B T}{\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right)$$

   where E_g is the energy band gap between the valence and conduction bands.

$k_B = 8.617 \times 10^{-5}\,\mathrm{eVK^{-1}}$. The temperatures range from $T_{\mathrm{low}} = 20\,°\mathrm{C} = 293\,\mathrm{K}$ to $T_{\mathrm{high}} = 80\,°\mathrm{C} = 353\,\mathrm{K}$. The corresponding $k_B T$ are

$$k_B T_{\mathrm{low}} \approx 0.025\,\mathrm{eV} \qquad \text{and} \qquad k_B T_{\mathrm{high}} \approx 0.030\,\mathrm{eV}$$

References

[1] Kuck, Andrew. "Measurement of the Hall Coefficient in a Germanium Crystal". Physics Department, The College of Wooster, Wooster, Ohio 44691. April 1998. http://physics.wooster.edu/JrIS/Files/Kuck.pdf