X-Rays

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1 Tasks

1. Use the ionization cell to measure the average dose strength in an x-ray beam.
2. Measure the polarization of primary x-ray beams
3. Measure the polarization of scattered x-ray beams

2 Equipment and Procedure

2.1 Equipment

- Lehr und Didaktiksysteme 554811 x-ray apparatus, associated software, and computer.
- Ionization cell, with a power supply and voltage-regulation dials.
- Multimeters to measure voltage applied to ionization cell and ionization current.
- Polarization measurement module with scatterers Geiger-Muller counter

2.2 Sketch of Procedure

1. Set up the ionization cell in the main compartment of the X-ray apparatus; set x-ray anode voltage and current. Measure the the voltage drop across the output resistor as a function of voltage applied to the ionization cell for three difference anode voltages
2. Remove the ionization cell and set up the polarization module, initially with one scatterer; turn on the computer and open the x-ray apparatus software. Use the software to measure the number of radiation counts for two perpendicular orientations of the scatterer-Geiger apparatus.
3. Modify the polarization module to use two scatterers. Repeat the x-ray count measurement for two perpendicular orientations of the scatterer-Geiger apparatus.

2.3 Data

Ionization Measurements: Independent variable is the voltage U_{cell} applied to the ionization cell and the dependent variable is the voltage drop U_R across the output resistor, which is converted to ionization cell current I_{cell} with the known output resistor value R_{out}. The $U_R(U_{\text{cell}})$ characteristic is measured for three values of x-ray anode voltage.

Polarization Measurement: The independent variable is the spatial orientation of the scatterer relative to the incident x-ray beam (and the measurement time). The dependent variable is the number of GM radiation counts in the measurement period.

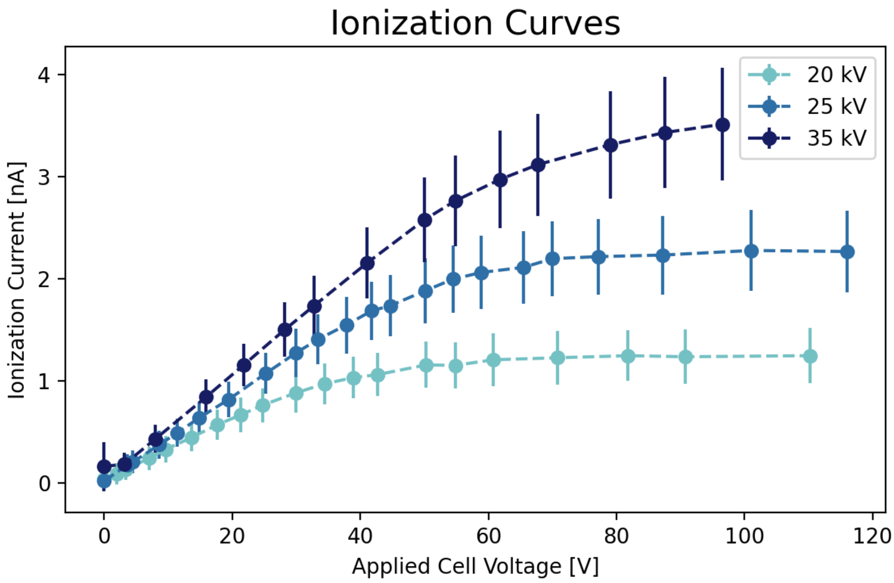


Figure 1: Ionization curve for three values of x-ray anode voltage.

3 Analysis

3.1 Ionization and Dose Rate

I first converted voltage drop U_R across the output resistor to current through the ionization cell with Ohm's law

$$I_{\text{cell}} = \frac{U_R}{R_{\text{out}}}$$

where $R_{\text{out}} = 1\,\text{G}\Omega$ is the output resistance. I then plotted the $I_{\text{cell}}(U_{\text{cell}})$ ionization curve for each value of the anode voltage, shown in Figure 1. Using the ionization current, I then estimated the exposition dose rate in the ionization cell as a function of applied cell voltage U_{cell} using

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{I_{\text{cell}}}{\rho V_{\text{cell}}}$$

where ρ and V_{cell} are the density and volume of the air in the ionization cell, respectively. The cell has a uniform trapezoidal base, so the cell volume is

$$V_{\text{cell}} = A_{\text{base}} \cdot h \approx 144\,\text{cm}^2 \cdot 3\,\text{cm} = 432\,\text{cm}^3$$

where A_{base} is the area of the trapezoidal base and h is the cell height. I took the density of air to be $1.23\,\text{kg}\,\text{m}^{-3}$. Figure 2 shows the results. The dose rate in the cell is of order $3\,\mu\text{A}\cdot\text{kg}^{-1}$ and increases with increasing cell voltage before plateauing at a saturated value. Logically, the dose rate increases with increasing anode voltage.

3.2 Polarization

This section uses a Cartesian coordinate system whose axes align with the axes of the cuboid x-ray apparatus. The x-rays propagate along the y axis, which points

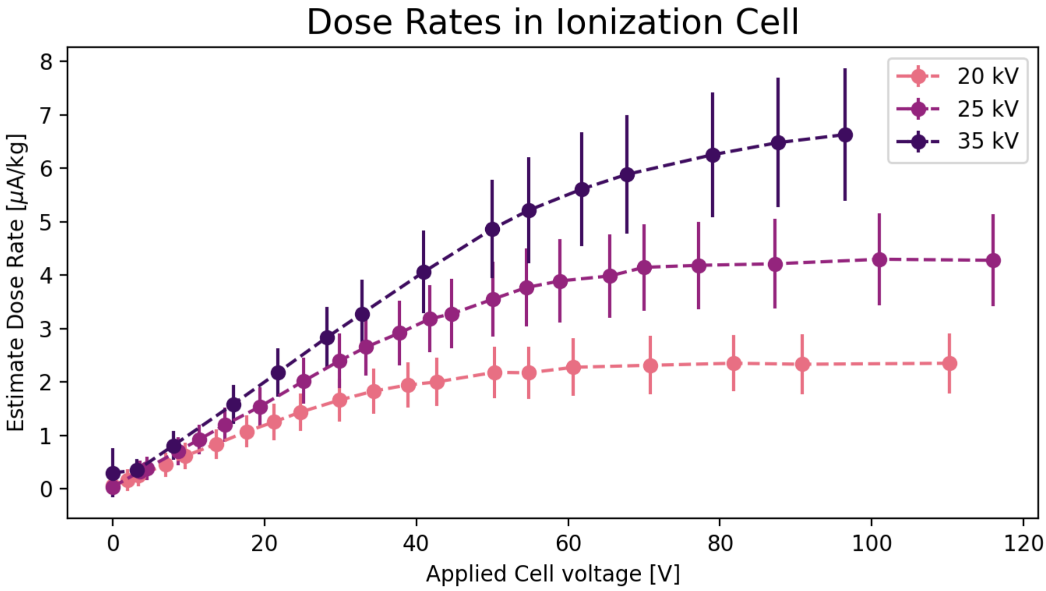


Figure 2: Dose rate in the ionization cell for three values of x-ray anode voltage.

toward the protective circular cover on the wall nearest the camera. The z axis points parallel to the apparatus's ceiling (i.e. "up") and the x axis points toward the sliding glass door. Table 1 shows the polarization measurement data.

One Scatterer: The x-rays are incident on the scatterer in the y direction; I measured the scattered radiation over a ten-second period using a cylindrical Geiger-Muller counter whose cross section was pointed in the x and z directions. First, I divided counts by measurement time to get two rates R_x and R_z. I then estimated the incident x-ray's polarization in the incident y direction using

$$\eta_y = \frac{R_z - R_x}{R_x + R_z} = \frac{296.6\,\mathrm{s}^{-1} - 267.3\,\mathrm{s}^{-1}}{296.6\,\mathrm{s}^{-1} + 267.3\,\mathrm{s}^{-1}} \approx 0.055$$

There is almost no polarization. This is, I believe, expected—the x-rays would be polarized in the z direction, the direction of electron incidence on the anode.

	N_1	N_2	N_3	N_4	N_5	$\bar{N}$	σ_N
Single scatter-x	2702	2683	2675	2659	2646	2673	22
Single scatter-z	2955	2979	2965	2951	2982	2966	14
Two scatters-x	1.8	1.1	2.1	1.4	1.5	1.58	0.38
Two scatters-y	7.2	7.0	6.3	6.2	7.3	6.80	0.51

Table 1: 5 sample runs, average, and standard deviation of X-ray counts in a ten-second period using a Geiger-Muller counter.

Two Scatterers: In this case, the secondary x-rays scattered from the first scatterer propagate in the z direction, and we use the Geiger-Muller counter with its cross section pointed in the x and y directions.

$$\eta_z = \frac{R_y - R_x}{R_x + R_y} = \frac{0.680\,\mathrm{s}^{-1} - 0.158\,\mathrm{s}^{-1}}{0.680\,\mathrm{s}^{-1} + 0.158\,\mathrm{s}^{-1}} \approx 0.623$$

In this case, the scattered radiation is partially polarized in the plane xy plane, perpendicular to its direction of incidence. Theory predicts a perfect $\eta_z = 1$, but because of non-ideal measurement conditions, we get roughly $\eta_z = 0.5$.

4 Error Analysis

4.1 Ionization Dose Rate

Current: Input data is voltage U_R across output resistor, converted to cell current I_{cell} using

$$I_{\mathrm{cell}} = \frac{U_R}{R_{\mathrm{out}}}$$

I estimated the error u_{R} as one half of difference between the maximum and minimum value plus $0.1\,\mathrm{V}$, which is the smallest significant digit reported by the multimeter. With no better options available, I estimated the error on R_{out} as 15 percent error, which is roughly typical for an average resistor. Sensitivity coefficients are

$$c_U = \frac{\partial I_{\mathrm{cell}}}{\partial U_R} = \frac{1}{R_{\mathrm{out}}} \qquad \text{and} \qquad c_R = \frac{\partial I_{\mathrm{cell}}}{\partial R_{\mathrm{out}}} = -\frac{U_R}{R_{\mathrm{out}}^2}$$

Error is

$$u_I = \sqrt{(u_U c_U)^2 + (u_R c_R)^2}$$

Dose Rate: Next, when calculating dose rate via

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{I_{\mathrm{cell}}}{\rho V}$$

Input quantities with error are I_{cell}, found above, V, which I'll assume to have about 25 percent error since I estimated it visually, and ρ, with e.g. 5 percent error. So $\rho = (1.23 \pm 0.06)\,\mathrm{kg\,m^{-3}}$, and volume $V = (430 \pm 100)\,\mathrm{cm}^3$. Sensitivity coefficients are

$$c_I = \frac{1}{\rho V} \qquad c_\rho = -\frac{I_{\mathrm{cell}}}{\rho^2 V} \qquad c_V = -\frac{I_{\mathrm{cell}}}{\rho V^2}$$

Error is

$$u_X = \sqrt{(u_I c_I)^2 + (u_\rho c_\rho)^2 + (u_V c_V)^2}$$

4.2 Polarization

Working in counts per ten seconds instead of rate, I introduced the quantities N_+ and N_- and rearranged the polarization formula to read

$$\eta = \frac{N_2 - N_1}{N_2 + N_1} \equiv \frac{N_-}{N_+}$$

The input data are N_+ and N_-, and the associated errors u_- and u_+ are the sum of the error on N_1 and N_2, which I estimated as $u_N = \frac{\sigma_N}{\sqrt{5}}$ (see Table 1).

Sensitivity coefficients are

$$c_+ = \frac{\partial \eta}{\partial N_+} = -\frac{N_-}{N_+^2} \qquad \text{and} \qquad c_- = \frac{\partial \eta}{\partial N_-} = \frac{1}{N_+}$$

and error is

$$u_\eta = \sqrt{(u_+ c_+)^2 + (u_- c_-)^2}$$

Single Scatterer

$$\begin{aligned} N_+ &= \bar{N}_x + \bar{N}_z = 2966 + 2673 = 5639 \\ N_- &= \bar{N}_z - \bar{N}_x = 2966 - 2673 = 293 \\ u_+ = u_- &= \frac{\sigma_x + \sigma_z}{\sqrt{5}} = \frac{22 + 14}{\sqrt{5}} \approx 16 \end{aligned}$$

The result—implemented in Python—is $u_\eta = 0.003$. Error is low because of small standard deviation.

Two Scatterers

$$\begin{aligned} N_+ &= \bar{N}_x + \bar{N}_z = 6.80 + 1.58 = 8.38 \\ N_- &= \bar{N}_z - \bar{N}_x = 6.80 - 1.58 = 5.22 \\ u_+ = u_- &= \frac{\sigma_x + \sigma_z}{\sqrt{5}} = \frac{0.38 + 0.51}{\sqrt{5}} \approx 0.40 \end{aligned}$$

The result—implemented in Python—is $u_\eta = 0.056$.

5 Results

A Theory

X-Ray Emission

- Electrons from a cathode are accelerated by a large potential difference into a target anode, where x-rays are emitted in the *brehmstrallung* interaction between the electrons and metal nuclei. This radiation corresponds to the continuous part of the x-ray spectrum.

- An accelerated electron passing a nucleus in the target anode loses speed as it emits electromagnetic radiation. The radiation's frequency ν is related to the kinetic energy ΔT lost by the electron via

  $$\Delta T = h\nu$$

  where h is Planck's constant. The max frequency occurs when the electron loses its entire kinetic energy T:

  $$h\nu_{\max} = T = e_0 V \implies \lambda_{\min} = \frac{e_0 V}{hc}$$

- If the accelerated electrons have large enough energies, they can also free atomic electrons from the nuclei's inner electron shells. Atomic electrons from the outer shells filling the holes in the inner shells then emit *characteristic x-ray* of discrete energy characteristic of the element and initial/final electron shells. Characteristic x-rays correspond to the discrete spikes in the x-ray spectrum.

Ionization Cell

- A basic ionization cell is a parallel-plate capacitor wired to a large potential difference. An x-ray incident on the space between the capacitor plates frees photoelectrons via the photoelectric effect, which ionizes the matter between the plates. The resulting ionized matter—pairs of molecular anions and free electrons—accelerates across the capacitor's potential difference, giving rise to an electric current pulse in the presence of an x-ray pulse.

  For x-ray photon currents of order greater than $\sim 10^9\,\mathrm{s}^{-1}$, the pulses average to a macroscopically measurable electric current.

- Of course, all ionized pairs don't reach the capacitor electrodes; some will recombine in the space between the plates. Recombination dominates at low electric field strengths and nearly disappears for large electric fields. This trend appears when measuring current as a function of capacitor voltage at a constant x-ray intensity: current initially grows with increasing capacitor voltage before reaching a saturated value. For vary large voltages current begins to increases again, but this effect is beyond the scope of this experiment.

- X-ray beams are usually characterized by the exposition dose X, defined as the electric charge ΔQ of a given sign freed by ionizing radiation in a mass Δm.

$$X = \frac{\Delta Q}{\Delta m}$$

  The SI units of exposition dose are $\mathrm{C\,kg^{-1}}$.

- The exposition dose rate is defined as

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{\Delta Q}{\Delta t \Delta m} = \frac{I}{\Delta m} = \frac{I}{\rho \Delta V}$$

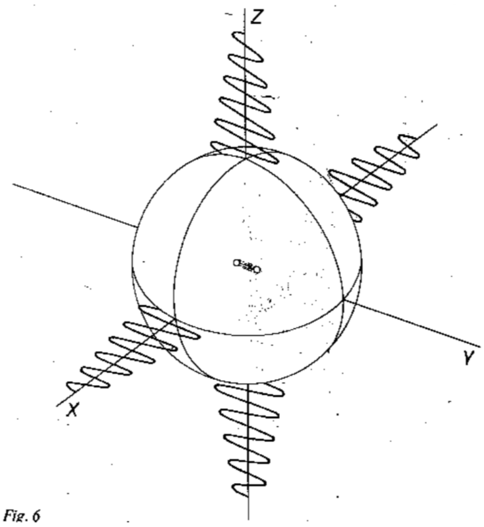
  where I is the current of charged particles freed by the ionizing radiation and $\rho = \frac{m}{V}$ is the density of the matter exposed to the radiation.

X-Ray Polarization

- We take a simplified classical approach and assume the accelerated electrons behave as harmonically oscillating classical charge. If the charge oscillates in e.g. the y direction, its position and acceleration are

$$y = A \sin \omega t \qquad \text{and} \qquad a_y = -A\omega^2 \sin \omega t$$

  The accelerating charge emits electromagnetic radiation, which we describe in terms of the electric field strength $\boldsymbol{E}$ and magnetic flux density $\boldsymbol{B}$; $\boldsymbol{E}$ is parallel to the charge's velocity and is perpendicular to the direction of wave propagation, while $\boldsymbol{B}$ is perpendicular to both $\boldsymbol{E}$ and the direction of wave propagation.



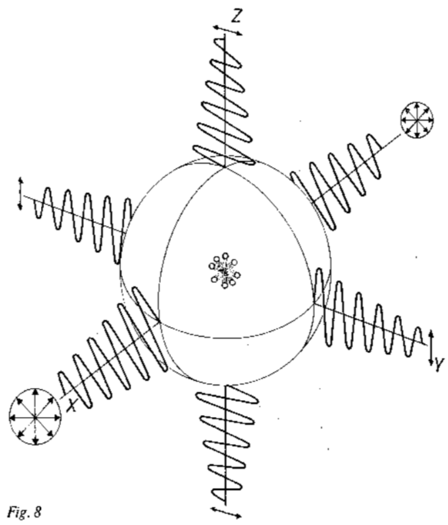


Figure 3: Understanding x-ray polarization with an oscillating electric charge. The left figure is linearly polarized in the y direction, while the right figure is only partially polarized.

- If the charge oscillates along the y axis, $\boldsymbol{E}$ will always point along the y axis, and we say the radiation is *linearly polarized* in the y direction. The energy current of linearly polarized radiation is anisotropic; it is largest in the equatorial plane of wave propagation and is zero along the axis of charge oscillation.

- Next, suppose we have multiple charges whose oscillation directions are uniformly distributed in the $y-z$ plane. In this case the electromagnetic radiation is linearly polarized in the y and z directions and is unpolarized in the x direction.

  In a similar scenario in which oscillation is *not* uniformly distributed in the $y - z$ plane, the radiation is still linearly polarized in the y and z directions and partially polarized in the x direction.

- If the electrons in the target anode decelerated only in a direction parallel to their initial velocity (e.g. the y direction), we would observe linearly polarized x-rays which would propagate in the $x-z$ plane. Because the incident electrons tend to deviate from their initial direction as they pass through the anode before the brehmstrallung process begins, we see only partial polarization in the $x - z$ plane.

Coherent X-Ray Scattering

- When x-rays interact with *bound* electrons, the photon energy remains unchanged[1]. We call this process elastic, or coherent scattering. Elastic scattering is used to measure x-ray polarization.

- In the classical model of elastic scattering, an electromagnetic wave of frequency ν incident on an electron excites the electron, causing it to oscillate with the

[1]When x-rays interact with free electrons, the photon energy decreases

same frequency ν. The oscillating electron then returns the absorbed energy by subsequently emitting electromagnetic radiation of frequency ν. This secondary radiation propagates through space as is characteristic for an oscillating classical charge.

- The excited electron oscillates in the same direction as the electric field vector $\boldsymbol{E}$ of the incident radiation, i.e. in a plane perpendicular to the direction of the incident wave's propagation.

  If the incident wave is polarized, the charge oscillates equally in all directions in the plane perpendicular to the propagation direction; if the the incident radiation is partially polarized, some oscillation directions are more pronounced than others; if the incident radiation is linearly polarized, the charge oscillates in a line.

- Elastically scattered electromagnetic radiation emitted from an oscillating charge that propagates in a plane perpendicular to the incident radiation is linearly polarized.

  The energy current density radiated by an oscillating charge varies as $\sim \sin^2\theta$, where θ is the angle between the direction of charge oscillation and the direction of the radiated wave. We thus see maximum radiation in the plane perpendicular to the charge's oscillation (where $\theta = \frac{\pi}{2}$) and no radiation along the direction of oscillation (where $\theta = 0$).

- The angular distribution of elastically scattered radiation depends on the polarization of the incident radiation.

  Suppose the incident radiation propagates in the e.g. y direction. If the incident radiation is polarized in the z direction, there is no scattered radiation in the z direction; if the incident radiation is partially polarized in the z direction, the strength of the scattered radiation is smaller in the z direction than in the x direction; only if the incident radiation is unpolarized is the scattered radiation equally strong in the x and z directions.

- By measuring the strength of elastically scattered radiation we can thus determine the polarization of the incident radiation. For an incident beam propagating in the y direction, we place a scatterer in the beam and then measure the angular distribution of scattered radiation in the $x-y$ plane with a counter for ionizing radiation (e.g. a Geiger-Muller counter). The distribution is circular for unpolarized radiation and elliptical for partially polarized radiation. In practice, we rarely measure the entire angular distribution but only the intensity values I_x and I_z. We then define polarization η as

$$\eta = \frac{I_z - I_x}{I_x + I_z}$$