Fourier-Transform Spectroscopy

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Winter Semester 2020-2021

1 Tasks

1. Investigate the relationship between the interferogram and spectrum's widths.
2. Investigate which quantities determine the spectrum's resolution.

2 Equipment, Procedure and Data

2.1 Equipment

Although the experiment was performed in an online simulation, a physical experiment would involve:

1. Light sources: helium-neon laser, a mercury bulb, and a white-light flashlight
2. Michelson interferometer with a movable mirror
3. Photodiode and ammeter to measure photocurrent

2.2 Procedure

Use the online `iPython` simulator to measure the interferogram of a helium-neon laser, a mercury bulb, and a white flashlight.

2.3 Data

Independent Variable: Displacement x of the interferometer's movable mirror relative to the equilibrium position.

Dependent: Photodiode current I_{diode}, which is proportional to the intensity of the detected light.

Parameters: Mirror displacement step size: $0.05\,\mu\mathrm{m}$.

3 Discussion

3.1 Interferogram and Spectrum

- A purely sinusoidal interferogram, corresponding to monochromatic light, (i.e. an ideal laser) has a spectrum with a single discrete component.

- An interferogram with a finite number of frequency components (e.g. a mercury lamp with emission peaks at a few specific frequencies) has a Fourier spectrum with finitely many discrete spikes at the corresponding frequencies.
- An interferogram with an appreciable non-zero response only in small displacement range (e.g. approximately white light) has a continuous Fourier spectrum.

3.2 Spectral Resolution

- The energy resolution in Fourier spectroscopy is proportional to the maximum displacement $x_{\max}$ of the movable interferometer mirror from the equilibrium position.
- This is the manifestation of a more general result, namely that the resolution of a discrete Fourier transform in the conjugate domain (e.g. frequency f, wave vector k, etc...) improves with the length of the input signal in the original domain (e.g. time t, position x, etc...).
- Meanwhile, the spectral bandwidth of the Fourier transform improves with the sample rate of the original signal.

4 Results

The simulated experiment produced the following emission line spectrum for the mercury lamp:

Wave vector $2\pi k$ [μm^{-1}]	Wavelength λ [nm]	Frequency f [THz]
1.78	562	534
1.83	546	549
2.30	435	690
2.48	403	744

Table 1: Measured emission line date for the mercury vapor lamp. The reference values for the same lines are 578.2, 546.1, 435.8 and 404.7 nm.

Meanwhile, the helium-neon laser showed the following spectral characteristics:

$$2\pi k = 1.58\,\mu\text{m}^{-1} \qquad \lambda = 633\,\text{nm} \qquad f = 474\,\text{THz}$$

The reference wavelength for a helium-neon laser is $\lambda_{\text{HeNe}} = 632.8\,\text{nm}$. Of course, the agreement between the measured and reference spectral data for both the mercury lamp and helium-neon laser is artificial, since the simulation was reverse-engineered to produce the reference values.

5 Interferogram and Spectrum Plots

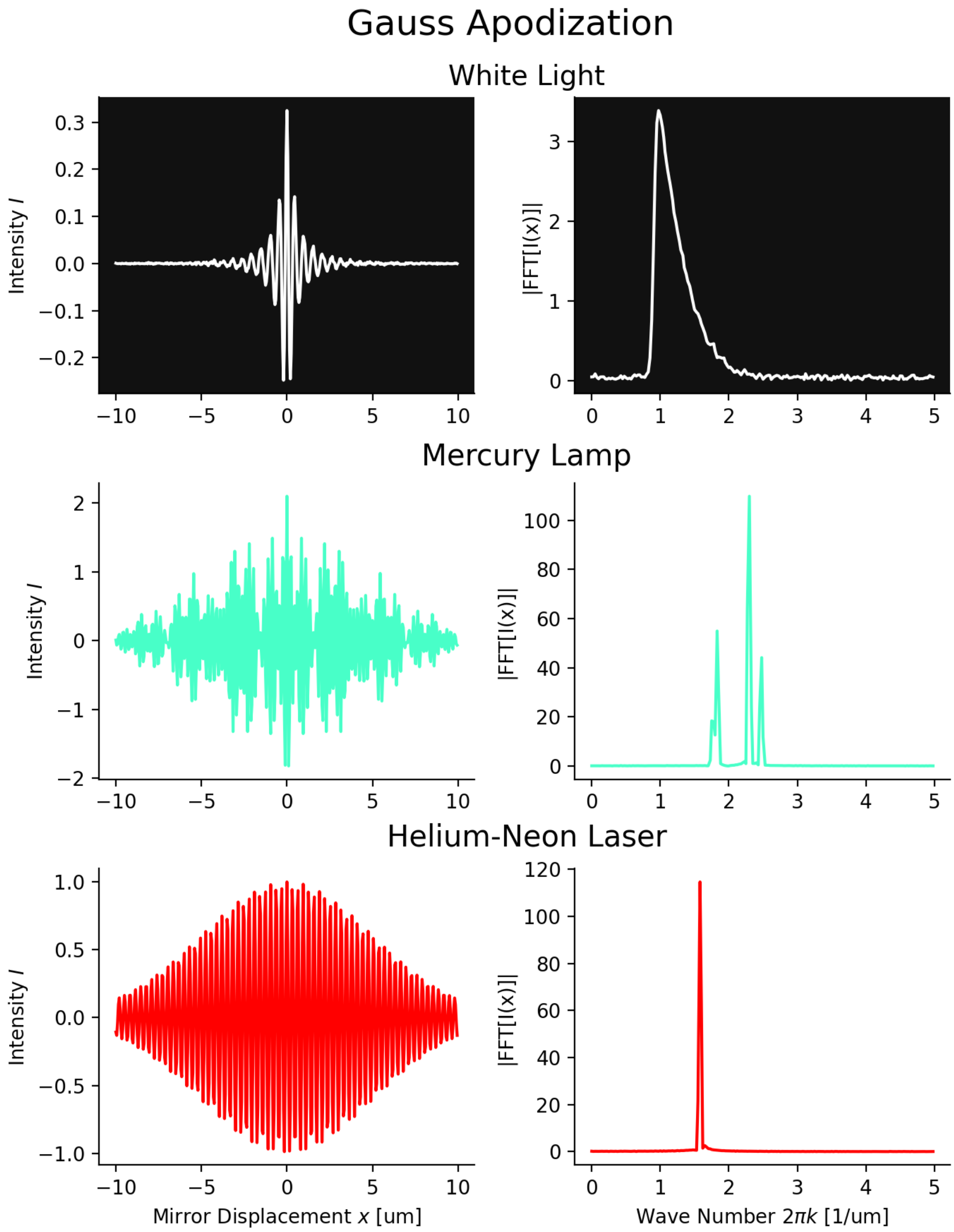

Figure 1: Michelson interferogram and Fourier spectra of a (computer simulated) white flashlight, mercury lamp and helium-neon laser. The interferogram is apodized with a Gauss filter.

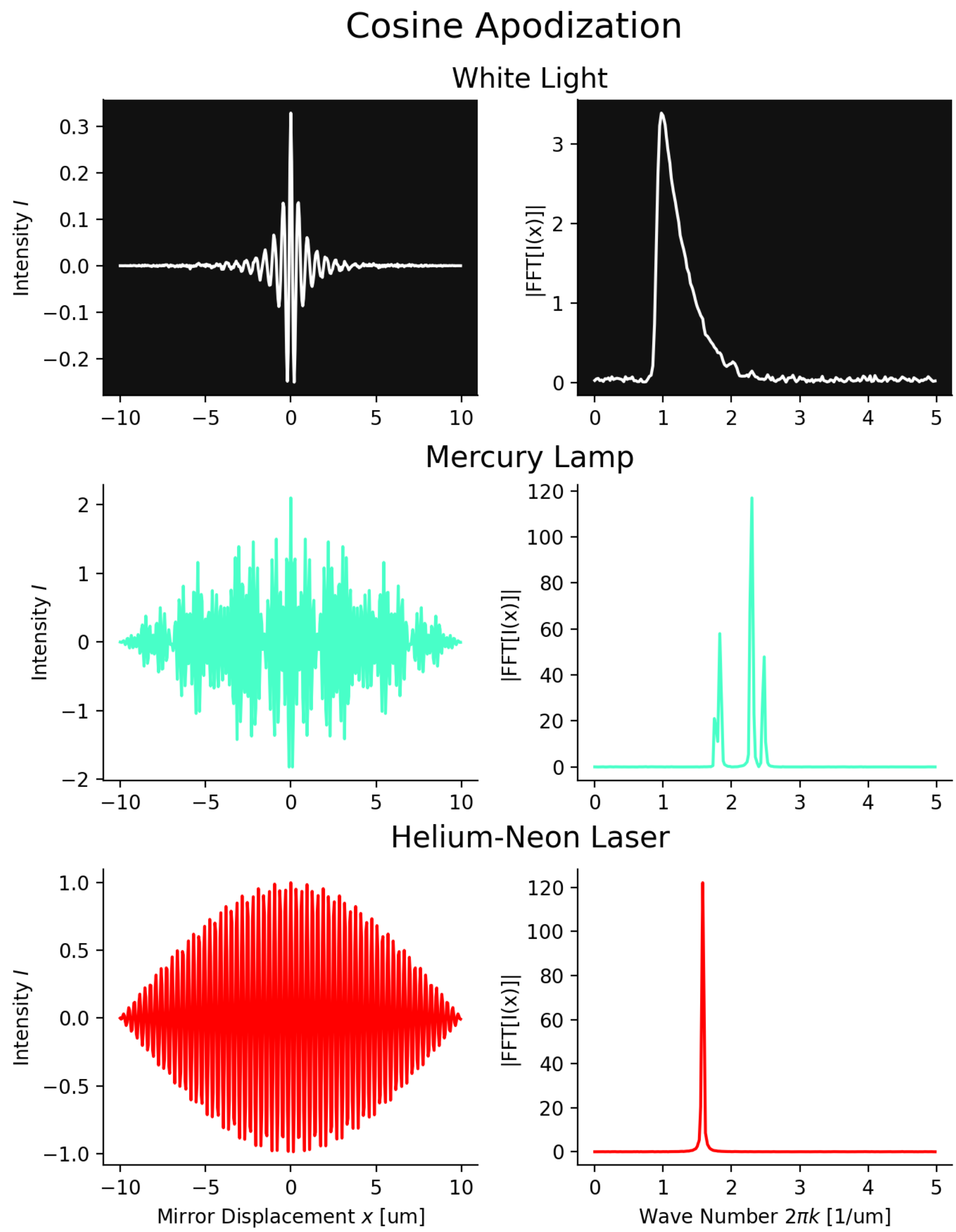

Figure 2: Michelson interferogram and Fourier spectra of a (computer simulated) white flashlight, mercury lamp and helium-neon laser. The interferogram is apodized with a cosine filter.

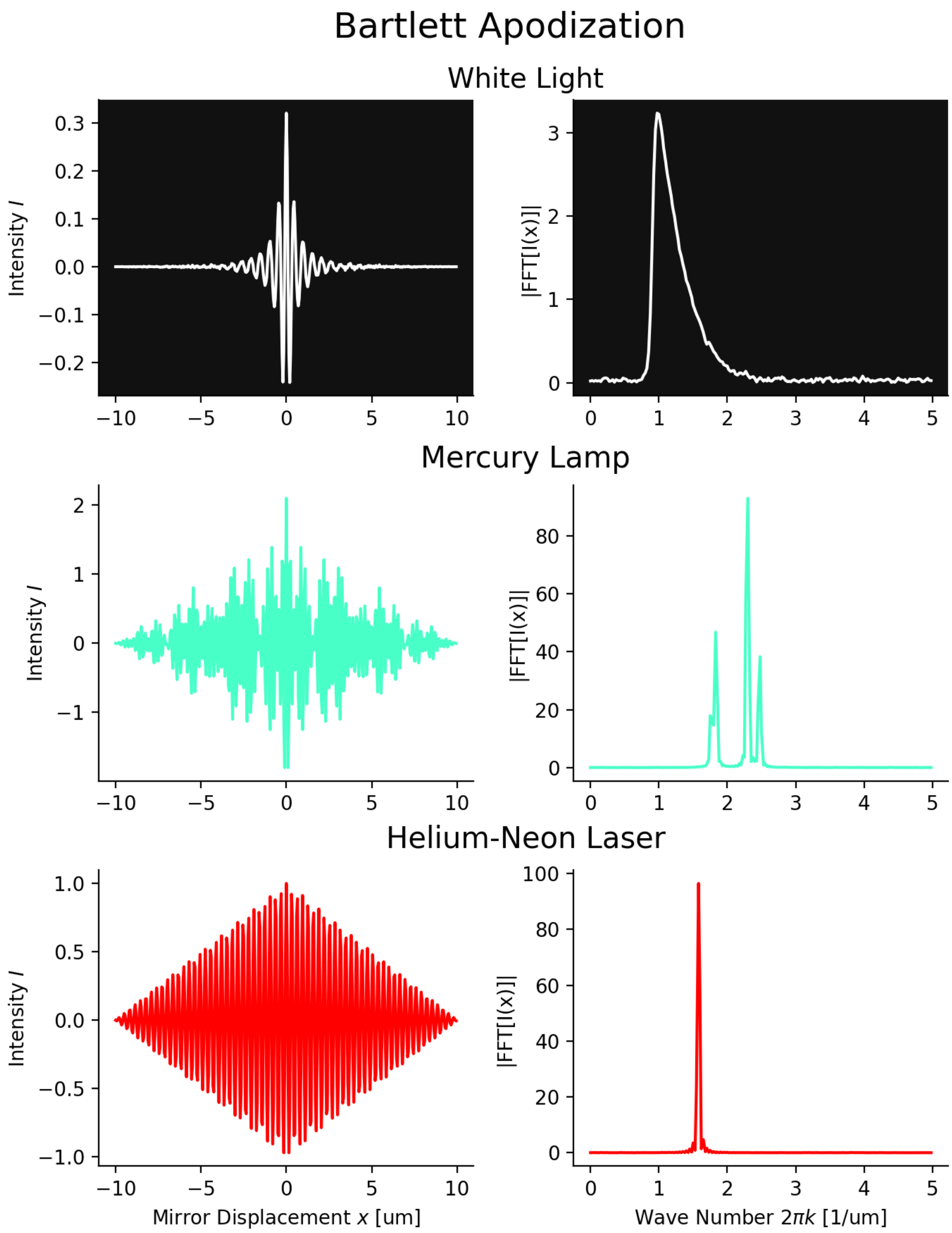

Figure 3: Michelson interferogram and Fourier spectra of a (computer simulated) white flashlight, mercury lamp and helium-neon laser. The interferogram is apodized with a Bartlett (absolute value) filter.

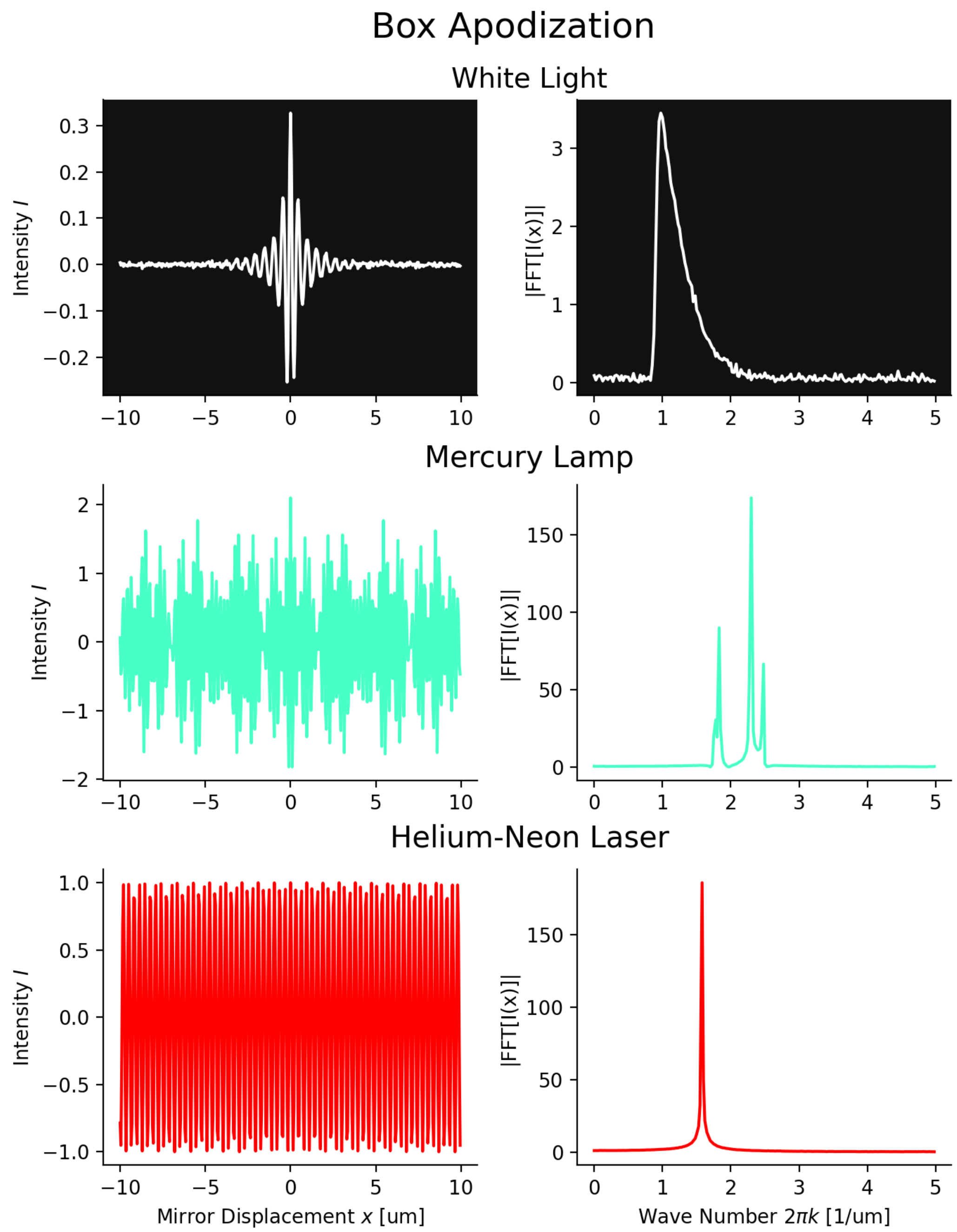

Figure 4: Michelson interferogram and Fourier spectra of a (computer simulated) white flashlight, mercury lamp and helium-neon laser. The interferogram is apodized with a box filter.

A Theory

The intensity (irradiance) of light incident on the detector is proportional to the time average of the energy:

$$I_{\mathrm{det}} \propto \left\langle |E_{\mathrm{det}}|^2 \right\rangle$$

When both split beams incident on the detector are monochromatic and equally intense, we can write the intensity on the detector as

$$I_{\mathrm{det}}(x) = I_0 \big[1 + \cos(2kx)\big]$$

where I_0 is the intensity of the incident light on the beam splitter, k is the light's wave vector and x is the displacement of the movable interferometer mirror. If the incident contain a spectral distribution $S(k)$ of many frequencies, the corresponding intensity on the detector is

$$I_{\mathrm{det}} = \int_0^\infty S(k)(1 + \cos(2kx))\,\mathrm{d}k$$

The convolution of the spectrum with a unit step function spreads out the spectral peaks and degrades their resolution.