Lab 7

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11:59PM April 22, 2021

We will get some experience with speeding up R code using C++ via the `Rcpp` package.

First, clear the workspace and load the `Rcpp` package.

```
pacman::p_load(Rcpp)
```

Create a variable `n` to be 10 and a variable `Nvec` to be 100 initially. Create a random vector via `rnorm` `Nvec` times and load it into a `Nvec` x `n` dimensional matrix.

```
n = 10
Nvec = 100
X = matrix(data = rnorm(Nvec*n), nrow = Nvec)

X
```

```
##                [,1]        [,2]         [,3]        [,4]        [,5]
##   [1,]  1.164226646  1.64087030  0.329555090  0.17424788  1.61256886
##   [2,]  1.098877323 -0.40367207 -1.521493375 -2.95563964 -2.24063237
##   [3,]  0.750927790  0.89047029 -0.537885475 -1.43553507 -0.67517300
##   [4,] -1.046691331  2.18753435  0.878966085 -1.10845396  1.15478210
##   [5,]  0.005991113  1.65773762 -0.454295900  0.62749381 -1.55240542
##   [6,] -1.068537994 -1.22649890 -0.094449560  0.13233642 -0.60233906
##   [7,]  1.037456632 -0.60275962  0.217930924 -0.16859284 -0.90300274
##   [8,] -1.274415338 -0.21121084  0.362452381 -1.02116227  1.29419514
##   [9,] -0.684762316  1.47053598 -0.128770690  0.32007260  2.28432157
##  [10,]  0.014455520 -0.26857697 -0.315992637 -1.42621195 -1.98451243
##  [11,]  1.608224902 -0.44488602 -0.602099466  0.41262465 -0.50499907
##  [12,] -2.576310074  1.21440665 -1.588153484  0.58072380 -0.97776418
##  [13,]  0.907215278 -0.46887338  1.118165221 -0.02429242  0.15909429
##  [14,] -1.924244911  0.45151422 -0.326598793  0.48950352 -0.91436347
##  [15,]  0.433881561 -0.37398777 -0.222420175  0.16931681  0.18987432
##  [16,] -0.588757801  0.14570598  1.578976610  0.12195588  0.70869724
##  [17,]  1.001072929 -1.27892540  0.970455686 -0.73324489  1.32347603
##  [18,] -0.577299846  0.68896090  0.629638015  1.12708785  1.75361042
##  [19,] -0.482683125  0.53677552  0.234997379  0.44709185 -1.10418264
##  [20,]  0.693760783  0.60972418 -0.211643853  1.27801507  0.13293657
##  [21,]  0.851552875 -2.05859065 -1.597043669  1.43727138 -0.96116595
##  [22,] -1.001418346  0.72632401 -1.930544006 -0.24445942  1.60162489
##  [23,] -1.724324427  0.26242302  2.265502464 -1.22634282 -1.90984122
##  [24,] -2.046150445 -0.30505676 -0.132024821  1.12109252  0.78456539
##  [25,]  0.722666072 -0.81878928  0.576542743 -0.28245282 -1.65550188
```

```
##  [26,]  1.207925736  0.52260973 -0.890671699 -0.45803869 -0.70378368
##  [27,] -0.134306359  1.35422174  2.437827880  1.08119211 -1.01446814
##  [28,] -0.911698355  0.98042305  2.118498835 -1.49295843  2.73883527
##  [29,] -1.802694531 -1.12863054  0.974673818 -0.70442875  0.26583070
##  [30,] -0.421204294 -0.95392007 -0.840257677 -1.44409598  2.10001669
##  [31,] -0.393341696 -1.38421552  1.631537997  0.41895545 -0.09678185
##  [32,] -0.603191187  1.01825561  0.821593769 -0.58190661  0.46663532
##  [33,] -0.104516410  1.86577505 -0.006466095  0.09833930 -0.20765149
##  [34,] -0.885339350  0.95961725  2.633890844 -0.94795755 -1.08915131
##  [35,]  1.190317751 -1.06915715 -0.098966548 -0.83150418  1.85431927
##  [36,]  0.713496089  0.14881374  1.205733962 -0.62211982  0.91877387
##  [37,]  0.214091153 -0.70517491 -0.180390686  0.10951836  1.53207496
##  [38,] -2.877905996  0.58416876  0.221180873 -0.21732408 -0.05643093
##  [39,]  2.618111311  1.64481047  0.883799817 -1.20428535  0.05299025
##  [40,] -0.707002087 -0.93622362 -0.570387917  0.41987643 -0.60740648
##  [41,] -0.567213851  0.46000018  1.688674799 -0.51887904  0.14798614
##  [42,]  1.039786829 -0.97348711  0.476888642 -0.60810184  0.41764658
##  [43,] -0.590463568  0.25504078 -0.312336262  0.68713932  0.86562487
##  [44,]  0.379695957 -0.42137252 -2.027495442 -0.79289398 -0.41650982
##  [45,]  0.697907245 -0.33139367  0.218677302 -0.45414954 -1.22325252
##  [46,] -0.910626231 -0.19171873  1.075250651  1.44824086 -1.45821523
##  [47,] -0.521776149  0.35869371  0.670878026 -1.78950762 -0.05666223
##  [48,]  0.474869564  0.11547932  1.192990969  0.43148746 -1.10646258
##  [49,]  0.036353705 -0.43666215 -1.882213903 -0.72057437 -1.40691805
##  [50,]  1.739630836  1.73917629 -0.316052948 -0.82250297  0.58892830
##  [51,]  0.511064844 -1.19967610  0.286837346 -0.66187669  1.14729019
##  [52,] -1.759524906 -1.96283845 -0.441019387 -1.56216676  0.75961668
##  [53,]  0.569773897 -0.24449925  0.368285545  0.38476575  0.69181764
##  [54,] -0.462233151 -1.10110370 -0.283651750  0.90491778 -0.01781437
##  [55,] -0.304551099 -0.52466768  0.674905484  0.47652316 -0.54782743
##  [56,]  1.251834699 -0.32148055  0.396088290  0.14714373  1.12371388
##  [57,] -1.402871228  0.66079427 -1.926491851  0.36530165  0.02951909
##  [58,]  1.456074698 -1.14443449 -1.708876807 -0.61903683  0.12818922
##  [59,]  1.806905254 -1.61717735 -1.724258888  1.35565523  1.24763133
##  [60,]  2.069393798  0.95512172  0.266624804  1.91153977 -1.19776479
##  [61,]  0.314784819  0.20646674 -0.832335519  0.10261738  0.33243629
##  [62,] -1.239470083 -2.14674163 -0.244154070 -1.06129008 -1.26534464
##  [63,] -1.416714700  0.45585597 -0.113222390  1.16486441  0.77644232
##  [64,] -0.722681977 -0.32226304  0.379669676 -0.42257441  0.62254001
##  [65,]  0.815430902 -1.14670133 -1.256564604  1.83406893  0.48314190
##  [66,]  0.823977133 -0.42569348 -1.193574366  0.01809122 -0.72616480
##  [67,]  0.472694373  0.34909430 -0.443187539  0.33630730 -1.68474511
##  [68,] -1.196424795 -0.62092869 -0.950425791  0.81104916  0.84885268
##  [69,]  0.060259271 -1.34650782  0.689750560 -1.78220035  0.03124273
##  [70,] -0.955231654  0.73741475 -1.449463591 -1.67457056 -1.68347494
##  [71,]  0.111561218 -0.72390382 -2.124035708  0.33813494 -0.02079190
##  [72,]  0.741967293 -0.11821483  0.104081056 -1.02012901 -0.35485049
##  [73,]  1.282693085  1.09864814 -0.229233485 -0.96874053 -0.04078252
##  [74,]  1.323911310 -1.69880685 -1.026994946  1.03820138  0.26101095
##  [75,]  0.239563240 -2.09123760  0.353521436  0.46674818  0.59282169
##  [76,]  0.182437436 -1.46636228 -1.064136548  1.29774414  0.68160274
##  [77,] -1.547399003  2.42863856 -0.072386523  0.30237990  0.17725200
##  [78,] -0.553215416  1.57439196 -0.028536743 -0.15574978 -1.20725906
##  [79,]  1.030429250 -0.59574127  1.246514214  0.30157131 -0.07509579
```

```
##  [80,] -0.507734746 -1.53527117  0.473712769  0.18213081  0.33279989
##  [81,] -0.670000467 -0.40801384 -0.604158754  0.22595702  1.18018418
##  [82,] -0.201582957 -0.22084915  0.137483745 -2.97207166  0.08112054
##  [83,] -0.127578604 -1.03369398  0.303691235 -1.63916056 -0.26023707
##  [84,] -0.789500861 -0.04323479 -0.009851166  1.71063903 -0.09470256
##  [85,]  0.189206445  1.55228759  2.082840220  1.76858213  0.21036262
##  [86,]  0.742088166  2.41638864  0.961585643  0.69126734  0.71772386
##  [87,]  0.583042048 -0.37282372  1.041094226  1.19628252 -1.03418828
##  [88,] -0.910330858 -0.73404246 -0.091407521  0.29000803  1.19980220
##  [89,] -1.632022576 -0.16165151 -0.341832747 -0.94017454 -0.84964734
##  [90,] -0.841790430 -1.18802491  0.619802994 -0.91900459  0.62704333
##  [91,] -0.524945068 -1.66101464  1.499230341 -1.20392900  0.38843514
##  [92,] -1.072490617  0.02814696  1.336722461 -0.71455912 -0.38194808
##  [93,]  1.997223963 -0.07287641  1.350988158 -1.71253325 -0.51366577
##  [94,]  1.183830930 -1.57843622  1.119016991  2.47319212  1.38852196
##  [95,]  0.828626729  0.35736790 -0.065396307 -2.34206755  0.38364069
##  [96,] -0.374113408 -0.48924659  1.062248124 -0.61361182  2.04735817
##  [97,] -1.706020312  0.24575871  0.322574162  0.37778879  0.14355211
##  [98,] -1.286827967  2.26377891  1.334853996  1.54565390  0.53594998
##  [99,] -0.473119143  0.09365109 -1.469543350 -0.19871347  0.19225283
## [100,]  0.961377263 -1.46310723  2.202786399 -1.39631688 -0.50425979
##                [,6]         [,7]        [,8]         [,9]        [,10]
##   [1,]  0.142753776  2.258955308  2.02392415  0.763709856 -0.622665382
##   [2,]  0.551056675  0.780263233 -0.60413613 -1.137543750  0.958647317
##   [3,] -0.992968852 -0.030132864  0.88208713 -0.716051596  0.298910154
##   [4,] -0.192164887 -0.342356977 -1.94957602 -0.906691983 -1.590044216
##   [5,] -2.813784016  1.176967071  0.49128929 -0.683699865 -2.241998093
##   [6,] -0.298622476  0.581667542 -0.35565487 -0.666510853  0.416554012
##   [7,] -0.311324120 -1.703627689 -0.51236243  0.651105371  0.730414458
##   [8,] -0.208959104  0.353664367  0.44765612 -0.084531082 -0.394263149
##   [9,] -0.855229092  0.297836707 -0.82589426  1.003188234 -0.868237482
##  [10,]  0.253578887  1.198246386 -1.14081717 -0.503340888 -0.142971869
##  [11,] -0.630500536  0.235044239 -0.92261581 -1.997728013 -0.014359055
##  [12,]  0.049385740 -0.746911726 -1.31782141 -1.554888208  2.285262204
##  [13,] -0.355962879 -0.915560425 -1.42620974  2.323832955  2.931345773
##  [14,] -0.835664295 -0.738286293 -1.36908231 -1.382208924  0.150784873
##  [15,]  1.884571337  1.537261512 -1.87129241  1.001067040  0.023547193
##  [16,]  2.441185100 -0.204019209 -0.41859613  0.813954725 -0.691323672
##  [17,] -0.466079264  0.174540740  0.46197192 -0.313742793 -0.194684335
##  [18,]  0.736437052 -0.093212029  0.15279833  0.755908400 -0.073264677
##  [19,]  1.347236252  0.908020032 -0.13439501  0.933963316  0.325723219
##  [20,] -1.382931927 -0.329603053 -0.38444160 -0.207802780  0.926195366
##  [21,] -0.007262878  0.364791696 -1.31798198  0.823937148  0.079221479
##  [22,]  2.093244122 -0.386785357 -1.45216919  0.362196514 -0.055653022
##  [23,]  0.711996620 -1.839380076 -0.55894127  0.264028659  0.574439843
##  [24,] -1.064880866 -1.398668180  0.03867284  2.132174645 -0.002721245
##  [25,]  0.156291873 -0.535961753  1.16660288  0.064970306  0.711833222
##  [26,] -0.552023981  2.041965953  1.09893559  0.263726318  1.660392691
##  [27,] -0.877636662  0.001292197  2.12205656 -1.201223395 -0.220243980
##  [28,] -1.828853029 -0.747974834 -1.60566021 -1.306624241 -1.065098358
##  [29,]  1.239374920 -1.126731487  0.04799119  0.755860289  0.152471852
##  [30,] -0.972564615 -0.038494397 -1.27146543  0.066130972  1.824544925
##  [31,]  0.635029515 -0.242800804 -0.86405868 -0.323242771 -0.514653631
##  [32,]  0.559006278  0.020175052 -0.02798474 -0.257409336  1.487216110
```

```
##  [33,] -1.760547685 -0.466439574 -1.16256746 -0.217752963 -0.970026289
##  [34,]  0.359917180  0.238253656 -1.39836932 -0.170193169  1.963861096
##  [35,] -1.510926237 -0.453579069  1.13391865  0.265635417  0.036325010
##  [36,]  1.763031739 -0.715960961  0.42245315  0.194924219 -0.946822337
##  [37,]  0.527469350 -2.280654736  0.40068683 -1.748539429  0.579623316
##  [38,]  0.629129521 -0.415015479 -0.93167925  0.355570525 -0.722075206
##  [39,]  0.481015261  0.359935602  0.78105475 -1.128869135  0.058525033
##  [40,]  1.303062042 -0.859483126 -0.22176577  1.046628849 -0.667566242
##  [41,]  0.261587370 -0.125319422  0.11260548  0.846159168 -0.490326991
##  [42,] -1.047716193 -0.452728587  0.67760880 -1.388567734 -0.037250091
##  [43,]  0.371713608  2.081284163 -1.96831614 -0.480109307 -0.268437608
##  [44,] -0.241194601  1.246189538 -1.58295286 -1.316711410  1.437666166
##  [45,] -0.469767129 -0.917378965 -0.31599325  0.786805962 -0.576669068
##  [46,]  0.811295258 -1.186160469  0.63767934 -1.862012152 -0.681548731
##  [47,]  0.246012348 -0.509327136 -0.16412059  1.317021333  0.914165593
##  [48,]  1.249431451 -0.584477509 -0.35139505  0.413775845 -0.947149456
##  [49,] -0.247898784  2.323934661  0.66602021  1.297090382 -1.037519724
##  [50,]  1.948820361  1.231013945 -2.00805464  0.012700455  0.132889913
##  [51,]  0.297805110 -1.996436743  0.59664955 -0.161643683 -0.257770713
##  [52,] -0.204085008 -0.606025036 -1.46864429 -0.431848601 -0.232692928
##  [53,]  0.958446654  1.173832365 -2.06372786 -0.032466477 -1.177242571
##  [54,] -1.982540986 -0.042741432  0.13254847 -0.731936111  1.451980983
##  [55,]  0.081742912  0.982646960 -1.02298628 -0.321915424  0.883401273
##  [56,] -0.542260710  0.035377298  1.05913418  2.008230449  0.542468085
##  [57,]  0.793531200  0.488731224 -0.22143761  0.743440712  0.640823616
##  [58,]  0.619685667  0.557532905 -0.23757013 -0.212588947  1.681674669
##  [59,]  0.430404088  1.546200689 -1.19709390  1.258691632 -0.635476348
##  [60,]  0.689825750 -0.510268573 -1.62529926  1.784507103 -0.477123493
##  [61,] -2.573639350 -0.152700082 -1.07262957 -0.296693775 -0.630141118
##  [62,] -0.187176431 -0.122575685  2.45376871 -0.700215853 -1.408150839
##  [63,]  0.113021886  0.015711570 -1.23171548  1.809761445  0.161352135
##  [64,] -1.467772744 -1.749863822 -1.25660609  2.728596076 -1.178850822
##  [65,]  1.737046148 -0.779223689  2.11432287  0.706666061 -0.981900495
##  [66,] -0.432184915 -0.477431437  0.31975640  1.443590362  0.845793433
##  [67,] -0.151752378 -0.780996796  0.37656922 -0.736464784  0.924380508
##  [68,] -0.303411922  0.798661947 -0.40159206 -1.492759616 -0.928121866
##  [69,] -0.939448397  1.349053130 -0.90768443  0.283829988 -1.100681828
##  [70,]  1.948898847  2.272897213 -0.63590046  1.213370192  0.352872352
##  [71,] -0.593301371 -1.106987954  0.79537114 -1.338848756  0.655428120
##  [72,]  0.442284247  1.558651664 -0.52017945  0.310367631 -1.421201039
##  [73,] -0.112268838  0.463208990  0.47820070 -0.672941064  0.349073717
##  [74,] -2.631602540 -1.022341426  1.24627907  0.495115951 -1.539878898
##  [75,]  0.717115561 -0.027552265  0.10910207  0.401112383  0.459347318
##  [76,]  1.536378774 -0.031413208  1.19207782 -0.974624982  0.771539710
##  [77,] -0.428196918  0.943452431 -1.29335672  0.220396171  0.335166530
##  [78,]  0.907868546 -1.049858192 -0.13581424  0.005751365 -0.866856851
##  [79,] -1.255994914  0.339718444 -0.64313131 -0.100174305  0.071519100
##  [80,] -0.410810535 -0.273517025 -0.28418469  1.123212009 -0.690299904
##  [81,] -0.574633778  0.613847442 -1.21501695  0.637081352 -1.343839912
##  [82,]  0.665112488 -0.434618643 -0.90786347  0.151884881 -0.543980719
##  [83,] -0.115366101 -1.467960998  1.52470161  1.268859108  0.507981393
##  [84,] -0.950844334 -0.687364893 -2.14531620  0.137721178 -0.286380549
##  [85,] -0.399061813  0.413324191 -0.44373468 -0.351321899 -1.397053120
##  [86,] -0.596932839 -0.190539032  0.18268883  1.043668723 -1.263719888
```

```
##  [87,] -1.931996499 -0.969508056 -0.69746456  0.329722136  1.673143159
##  [88,] -0.064757602 -0.475400410  0.51448725  0.217824330 -0.082672554
##  [89,]  1.775806410 -0.777673898 -0.18487040 -1.262893542  1.686936276
##  [90,] -0.289477943 -0.794001080 -0.35119382 -1.391897531  2.387831377
##  [91,] -1.786592814  0.648515471 -0.05477210  1.235723762 -2.879086800
##  [92,]  0.610771611  0.017622847  1.03945910  0.480181217  0.094189340
##  [93,]  2.144978811  0.170479577 -0.03168096 -1.417064973 -1.366248285
##  [94,]  0.347517093  0.578971908  0.99573764  1.276799337 -0.950815247
##  [95,]  0.865927927 -1.151348549 -0.64373811  0.363834455 -1.476078056
##  [96,]  0.083425287 -0.787731080  0.50699956 -0.314524012 -1.721792987
##  [97,] -0.590707898 -0.158648847  0.89029920 -0.863378354  1.217004931
##  [98,]  0.424635090  1.552699908 -0.17390444  0.538010726 -0.572498035
##  [99,] -0.544483174  0.465712963 -0.19372023  1.090810065 -2.139916651
## [100,] -0.174285748  0.779718223 -1.08641730  1.194387436  0.530330557
```

Write a function `all_angles` that measures the angle between each of the pairs of vectors. You should measure the vector on a scale of 0 to 180 degrees with negative angles coerced to be positive.

```
angle = function(u, v){
  acos(sum(u*v) / sqrt(sum(u^2) * sum(v^2)))
}

all_angles = function(X){
  A = matrix(NA, nrow = nrow(X), ncol = nrow(X))
  for (i in 1:(nrow(X)-1)){
    for (j in (i+1):nrow(X)){
      A[i,j] = angle(X[i,], X[j,]) * (180/pi)
    }
  }

  A
}

all_angles(X)
```

```
##        [,1]     [,2]     [,3]     [,4]     [,5]      [,6]      [,7]      [,8]
##   [1,]   NA 104.9391 80.91954 90.40854 76.74411 121.79427 125.22366  77.18143
##   [2,]   NA       NA 52.99121 97.32177 91.78042  77.16420  78.46431 102.01240
##   [3,]   NA       NA       NA 89.13886 65.22303 101.66790  84.40677  92.13116
##   [4,]   NA       NA       NA       NA 77.22942 103.85385 107.79148  67.01382
##   [5,]   NA       NA       NA       NA       NA  90.65347 106.21719  95.63627
##   [6,]   NA       NA       NA       NA       NA        NA  97.81415  81.60507
##   [7,]   NA       NA       NA       NA       NA        NA        NA 124.07444
##   [8,]   NA       NA       NA       NA       NA        NA        NA        NA
##   [9,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [10,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [11,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [12,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [13,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [14,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [15,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [16,]   NA       NA       NA       NA       NA        NA        NA        NA
##  [17,]   NA       NA       NA       NA       NA        NA        NA        NA
```

```
##  [18,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [19,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [27,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [32,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [37,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [38,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [46,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [47,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [48,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [62,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [68,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [69,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [70,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [71,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [72,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [74,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [78,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [79,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [80,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [81,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [82,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [83,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [85,]   NA       NA       NA       NA       NA       NA       NA       NA
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##  [96,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [97,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [98,]   NA       NA       NA       NA       NA       NA       NA       NA
##  [99,]   NA       NA       NA       NA       NA       NA       NA       NA
## [100,]   NA       NA       NA       NA       NA       NA       NA       NA
##             [,9]     [,10]     [,11]     [,12]     [,13]     [,14]     [,15]
##   [1,]  66.53401 107.86426 102.91161 121.13973 103.24454 131.55603  86.37744
##   [2,] 125.61146  34.37226  66.52552  79.85131  91.94395  88.78260  81.25450
##   [3,] 103.52921  72.35137  73.85957  87.91352 100.64864  91.92541 122.59653
##   [4,]  52.73974  86.01514  92.61741  81.60147 104.21919  66.25271  88.38079
##   [5,]  79.60033  78.81830  74.69668  93.02273 121.52934  73.89290 112.92749
##   [6,] 113.13998  64.78420  78.46346  64.77786  95.38429  58.22006  87.76482
##   [7,] 114.38055  88.72784  81.32715  91.80437  48.96742  91.17373  97.84144
##   [8,]  63.86860 100.37818 119.27706  95.68702 105.63397  90.69387  99.82296
##   [9,]        NA 117.20724 109.43112  93.70813  90.40183  88.92811  84.57124
##  [10,]        NA        NA  71.10167  80.56528  97.07910  76.78717  69.49017
##  [11,]        NA        NA        NA  85.51582 100.81510  78.19131  90.75056
##  [12,]        NA        NA        NA        NA  87.82682  36.23003  95.20453
##  [13,]        NA        NA        NA        NA        NA  99.55980  76.69783
##  [14,]        NA        NA        NA        NA        NA        NA 104.06177
##  [15,]        NA        NA        NA        NA        NA        NA        NA
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##  [18,]        NA        NA        NA        NA        NA        NA        NA
##  [19,]        NA        NA        NA        NA        NA        NA        NA
##  [20,]        NA        NA        NA        NA        NA        NA        NA
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##  [22,]        NA        NA        NA        NA        NA        NA        NA
##  [23,]        NA        NA        NA        NA        NA        NA        NA
##  [24,]        NA        NA        NA        NA        NA        NA        NA
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##  [25,]       NA       NA       NA       NA       NA       NA       NA
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##  [29,]       NA       NA       NA       NA       NA       NA       NA
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##  [42,]       NA       NA       NA       NA       NA       NA       NA
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##  [62,]       NA       NA       NA       NA       NA       NA       NA
##  [63,]       NA       NA       NA       NA       NA       NA       NA
##  [64,]       NA       NA       NA       NA       NA       NA       NA
##  [65,]       NA       NA       NA       NA       NA       NA       NA
##  [66,]       NA       NA       NA       NA       NA       NA       NA
##  [67,]       NA       NA       NA       NA       NA       NA       NA
##  [68,]       NA       NA       NA       NA       NA       NA       NA
##  [69,]       NA       NA       NA       NA       NA       NA       NA
##  [70,]       NA       NA       NA       NA       NA       NA       NA
##  [71,]       NA       NA       NA       NA       NA       NA       NA
##  [72,]       NA       NA       NA       NA       NA       NA       NA
##  [73,]       NA       NA       NA       NA       NA       NA       NA
##  [74,]       NA       NA       NA       NA       NA       NA       NA
##  [75,]       NA       NA       NA       NA       NA       NA       NA
##  [76,]       NA       NA       NA       NA       NA       NA       NA
##  [77,]       NA       NA       NA       NA       NA       NA       NA
##  [78,]       NA       NA       NA       NA       NA       NA       NA
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##  [79,]        NA        NA        NA        NA        NA        NA        NA
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##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
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##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
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##  [89,]        NA        NA        NA        NA        NA        NA        NA
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##  [93,]        NA        NA        NA        NA        NA        NA        NA
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##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,16]     [,17]     [,18]     [,19]     [,20]     [,21]     [,22]
##   [1,]  84.33315  75.98455  64.94604  83.09490  91.64268 111.58183  94.62836
##   [2,] 110.36286  94.37706 146.01433  85.61611 105.50817  81.94119  90.19234
##   [3,] 130.18113  87.51778 128.49603 108.88021  83.14443 112.15014 110.46874
##   [4,]  73.46019  95.15252  76.45579  99.29037  95.94368 122.66913  69.76522
##   [5,] 121.64423 103.33620 107.86692  97.33750  73.40859  94.26878 116.33925
##   [6,] 104.14078  93.10525 112.79999  87.09941  95.87654  68.62154  99.03917
##   [7,]  98.15994  88.11533 110.87376  98.54914  73.71839  70.72953 103.91348
##   [8,]  78.07392  66.51789  72.14505 106.63479 118.84706 123.66406  79.87342
##   [9,]  81.15069  89.86067  48.64327 100.18466  78.74107 105.21928  65.65749
##  [10,]  99.18869 103.32609 139.18894  70.59080 110.39482  77.15028  94.33981
##  [11,] 121.49154  81.17317 122.75669 113.31627  64.99982  68.26679 102.53693
##  [12,] 103.06530 133.02159  95.56692  81.08787  74.15857  90.06469  68.93031
##  [13,]  85.79923  85.54084  83.66097  82.32575  69.36236  77.41737  92.68574
##  [14,] 103.61240 122.55543 101.52612  94.03058  75.68953  90.24804  83.79945
##  [15,]  60.15971  96.64138  79.26396  54.58139 106.49459  63.06063  59.44075
##  [16,]        NA  89.19005  51.81495  61.52289 124.23364 103.47348  67.04725
##  [17,]        NA        NA  88.82833 126.84767  97.21978  97.54332 109.95685
##  [18,]        NA        NA        NA  81.26269  87.30547 106.16784  66.00785
##  [19,]        NA        NA        NA        NA 106.90672  82.58651  80.33039
##  [20,]        NA        NA        NA        NA        NA  78.68018 104.54245
##  [21,]        NA        NA        NA        NA        NA        NA  86.28632
##  [22,]        NA        NA        NA        NA        NA        NA        NA
##  [23,]        NA        NA        NA        NA        NA        NA        NA
##  [24,]        NA        NA        NA        NA        NA        NA        NA
##  [25,]        NA        NA        NA        NA        NA        NA        NA
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##  [27,]        NA        NA        NA        NA        NA        NA        NA
##  [28,]        NA        NA        NA        NA        NA        NA        NA
##  [29,]        NA        NA        NA        NA        NA        NA        NA
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##  [31,]        NA        NA        NA        NA        NA        NA        NA
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##  [63,]       NA       NA       NA       NA       NA       NA       NA
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##  [66,]       NA       NA       NA       NA       NA       NA       NA
##  [67,]       NA       NA       NA       NA       NA       NA       NA
##  [68,]       NA       NA       NA       NA       NA       NA       NA
##  [69,]       NA       NA       NA       NA       NA       NA       NA
##  [70,]       NA       NA       NA       NA       NA       NA       NA
##  [71,]       NA       NA       NA       NA       NA       NA       NA
##  [72,]       NA       NA       NA       NA       NA       NA       NA
##  [73,]       NA       NA       NA       NA       NA       NA       NA
##  [74,]       NA       NA       NA       NA       NA       NA       NA
##  [75,]       NA       NA       NA       NA       NA       NA       NA
##  [76,]       NA       NA       NA       NA       NA       NA       NA
##  [77,]       NA       NA       NA       NA       NA       NA       NA
##  [78,]       NA       NA       NA       NA       NA       NA       NA
##  [79,]       NA       NA       NA       NA       NA       NA       NA
##  [80,]       NA       NA       NA       NA       NA       NA       NA
##  [81,]       NA       NA       NA       NA       NA       NA       NA
##  [82,]       NA       NA       NA       NA       NA       NA       NA
##  [83,]       NA       NA       NA       NA       NA       NA       NA
##  [84,]       NA       NA       NA       NA       NA       NA       NA
##  [85,]       NA       NA       NA       NA       NA       NA       NA
```

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##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,23]     [,24]     [,25]     [,26]     [,27]     [,28]     [,29]
##   [1,] 122.85276 101.56291 102.35023  61.75400  73.41749  90.36808 113.81410
##   [2,]  84.07017 130.87537  68.08490  62.16577 108.47564 106.39790 100.08024
##   [3,]  92.07501 116.72511  71.39730  57.16205  76.37497  88.29648 118.12221
##   [4,]  76.71818  91.13501 132.01178 119.33953  88.80763  36.73904  87.75762
##   [5,] 102.92958  93.56941  96.70219  80.13491  63.88054  84.43256 129.04658
##   [6,]  81.56255  87.61453  83.23621  90.05424  95.30662  96.48678  76.39814
##   [7,]  66.80712  82.65601  55.11322  96.73675  97.95391 100.09960  82.59354
##   [8,]  87.31638  75.71160 114.48006 103.23864  92.03147  53.54255  62.46314
##   [9,] 108.10348  62.63458 150.52566 103.61349  99.30113  54.25846  95.45111
##  [10,]  75.92363 122.87672  77.58212  73.86330 102.40646 101.20625  96.44045
##  [11,] 111.87007 128.98749  86.54939  80.54386  91.54732  90.35480 131.06837
##  [12,]  74.27439  81.34642  99.25096  90.91890  95.20826  92.89338  85.73165
##  [13,]  74.10172  72.94920  77.46763  82.39065 104.06206  92.53599  78.16986
##  [14,]  67.80899  77.32589 103.41464 111.75143  85.66707  74.82995  85.53547
##  [15,]  98.58247 101.78175 107.27869  86.64435 126.09017 103.56604  86.44245
##  [16,]  67.52843  85.03607  98.80283 120.89577  91.36082  83.66401  51.29823
##  [17,] 103.95959 102.02367  85.05765  91.33987  89.04577  65.99861  86.81570
##  [18,]  97.13142  62.16515 121.94211 109.76832  85.82748  75.67523  74.43648
##  [19,]  73.43970  91.89491  81.38305  77.13465  88.18234 124.24789  79.88512
##  [20,] 107.53067  80.63895  95.14478  77.07084  78.66855  86.87212 127.03201
##  [21,] 109.98732  85.05805  84.14995  85.16904 123.03131 124.34086  99.77170
##  [22,]  97.57954  81.53924 129.25185 106.21964 133.65833  88.02061  75.59075
##  [23,]        NA  80.37703  67.01728 113.55475  75.07453  81.24677  48.67487
##  [24,]        NA        NA 103.12242 114.33883  95.99894  84.13779  62.95301
##  [25,]        NA        NA        NA  73.22746  69.42158 122.73001  83.02576
##  [26,]        NA        NA        NA        NA  87.01836 118.07002 124.60662
##  [27,]        NA        NA        NA        NA        NA  83.80647  98.55081
##  [28,]        NA        NA        NA        NA        NA        NA  83.57568
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##   [1,] 101.45638 114.81657  84.90382  92.75270 104.08006  74.04085  74.66134
##   [2,]  81.18896 103.99738  87.75486  98.27524  80.22400  97.52583 100.28011
##   [3,]  88.30450 130.72873  84.29446  73.60335  90.10966  74.11358 101.90662
##   [4,]  84.90645  84.46806  73.91598  47.43187  73.44270 101.68055  75.85550
##   [5,] 115.65934 108.80909 115.57635  45.84348 103.06947  94.29506 116.41356
##   [6,]  77.44316  66.85001  94.82366 105.17794  79.63314 103.50956 124.93823
##   [7,]  85.85015  82.32751  96.47004  89.81866  80.42495  81.80205  89.37854
##   [8,]  65.41176  85.42536  74.61276  94.61988  89.61449  69.56313  78.66792
##   [9,]  73.04311 102.45755  84.86255  58.41926  98.68320  79.13887  87.43834
##  [10,]  93.65305  86.01969  94.32320  87.88157  70.31052 117.26686 104.90545
##  [11,]  88.89927  88.37277 109.05995  79.32584  98.71441  87.39995 106.68670
##  [12,]  76.56410 100.84815  64.86213  78.94117  71.05832 122.07632 123.27522
##  [13,]  64.24641  84.57067  69.20243  97.11009  61.61492  82.57005  93.74851
##  [14,]  86.21010  82.25768  83.87210  59.89142  73.15218 119.93328 122.16261
##  [15,]  84.86257  75.22997  89.39898 106.65134  82.15616 113.39835  79.71509
##  [16,] 103.84369  56.15020  72.92179 107.81241  73.27767 110.27168  38.98927
##  [17,]  69.81797  72.13060  95.36341 108.00862  97.75765  37.11296  67.55132
##  [18,]  89.28734  83.43070  71.96754  93.18157  92.11525  89.11763  65.67783
##  [19,] 116.50563  86.15011  76.40846 104.76678  67.54533 140.31815  87.70294
##  [20,]  81.88548 107.48437  89.90661  62.26134  90.75142  81.33132 122.83976
##  [21,]  86.96436  81.61747 129.56645 102.96845 111.03312  95.04519 114.01587
##  [22,]  70.49119  98.42243  78.96990  93.58315  99.15385 104.64854  78.42093
##  [23,]  99.06551  64.28143  64.58093  87.45327  43.02172 115.23993  78.44109
##  [24,]  79.84000  87.89660  94.38001  81.77054  97.11921  83.91195 101.46551
##  [25,] 110.61214  84.13309  93.12411 115.82851  79.82319  88.50497  88.13547
##  [26,]  85.41093 129.13261  81.82720 100.92925  88.96273  83.14838 114.37911
##  [27,] 127.10331  85.50727  78.95789  78.88165  74.74670  95.28526  86.20972
##  [28,]  67.03909  76.72607  73.87325  58.62935  75.09402  72.65489  78.28505
##  [29,]  81.56572  57.24816  73.67798 115.90793  72.70355  96.08419  66.76216
##  [30,]        NA  96.95912  72.41879  94.43909  85.10050  61.55714 103.98593
##  [31,]        NA        NA  94.21994 104.38994  70.69565 101.22555  69.78053
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##  [35,]        NA        NA        NA        NA        NA        NA  84.04977
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##  [74,]       NA       NA       NA       NA       NA       NA       NA
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##  [99,]       NA       NA       NA       NA       NA       NA       NA
```

```
## [100,]        NA        NA        NA        NA        NA        NA        NA
##             [,37]     [,38]     [,39]     [,40]     [,41]     [,42]     [,43]
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##   [2,]  97.80092 103.36650  70.49917  95.30882 111.40084  80.74196  93.65771
##   [3,]  91.54317 113.23715  53.65873 124.91053 102.35222  63.71461 115.34135
##   [4,]  88.84026  56.76333  81.60891 103.56267  65.85389  97.14910  67.29302
##   [5,] 118.26543  92.05939  84.63655 111.01735  95.03719  85.42477  86.03746
##   [6,]  93.83747  75.33265 126.06502  83.36982 106.06928  85.34233  73.06375
##   [7,]  76.90082 107.35301  88.70265  76.00523  92.84967  78.37688 126.40445
##   [8,]  83.62247  63.18303 101.06745  98.41463  65.42465  82.88620  82.63755
##   [9,]  96.65796  70.40772  98.63157 101.19677  74.56301 107.76061  64.21880
##  [10,] 119.70883  85.38620  84.19292  88.86824  96.84579  95.28557  75.57173
##  [11,]  77.18705 123.59960  70.42884 112.02732 132.85009  57.64942  77.93543
##  [12,]  80.43156  62.62815 111.02867  88.66095 110.64945 109.17160  78.23981
##  [13,]  91.77591  99.75114  95.00457  89.12745  80.45838  97.70287  98.18934
##  [14,]  83.85128  54.15269 116.85076  88.65760  99.68855  97.64881  76.64921
##  [15,] 112.11491  84.35234  94.64464  70.85354  89.42728 123.57569  48.07048
##  [16,]  88.24350  63.11576  90.93992  62.65591  48.05546 115.84672  80.76743
##  [17,]  70.99078 119.33844  72.46879 110.53175  81.71156  40.69157  97.68101
##  [18,]  81.10235  72.62968  98.91770  84.17557  66.11503 113.18259  75.78386
##  [19,] 127.74320  71.98479  96.03810  66.94170  74.53372 143.62268  76.23419
##  [20,]  83.91846 113.36368  88.03364 117.63596 114.13296  81.45956  88.40167
##  [21,]  99.91109 104.41285 117.13827  62.33727 125.40199  98.40155  77.29500
##  [22,]  77.85036  62.34536 103.84389  66.31547  98.48041 123.29229  64.70334
##  [23,]  89.23858  57.87314  96.24306  74.40134  52.74903  99.06576 110.88380
##  [24,]  89.82359  60.43120 140.06181  65.62595  75.39003 111.26070  98.57423
##  [25,]  89.14419 114.87078  77.52131  82.70979  89.64972  71.58659 136.97982
##  [26,] 115.40010 125.57414  67.67848 118.92331 111.10396  88.17781  89.56593
##  [27,]  92.03208  95.36710  69.97342 114.29545  68.89573  75.24472 109.19659
##  [28,]  74.92431  73.15369  83.75828 117.45578  65.28425  70.97577  78.22660
##  [29,]  75.99016  51.82472 118.35165  53.48414  57.50881 101.66205 102.62838
##  [30,]  73.04187  90.18890 102.29940 104.19089 102.62411  78.82284  77.46857
##  [31,]  81.49141  75.05556 105.15153  70.68864  66.75033  83.38103  80.90084
##  [32,]  81.23017  76.00859  75.26019 111.17805  69.23176  99.78443  86.94409
##  [33,] 101.85795  77.42123  85.32579 110.95632  84.92770  93.47246  81.43541
##  [34,] 100.54621  73.10581  81.95780 103.68788  59.46928  99.63998  84.39802
##  [35,]  69.00660 122.56736  80.21670 110.40709  95.13419  46.47781 111.21308
##  [36,]  72.01257  89.07705  64.06447  77.70166  56.47806  86.31365 102.05919
##  [37,]        NA  96.63192  85.82389  89.54532 104.43437  58.89185 108.32282
##  [38,]        NA        NA 125.40876  64.52273  63.89580 125.94252  74.62485
##  [39,]        NA        NA        NA 125.94018  85.90800  67.03922 100.82416
##  [40,]        NA        NA        NA        NA  86.46407 119.67540  97.63169
##  [41,]        NA        NA        NA        NA        NA 102.00669  97.90434
##  [42,]        NA        NA        NA        NA        NA        NA 111.85928
##  [43,]        NA        NA        NA        NA        NA        NA        NA
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##  [53,]        NA        NA        NA        NA        NA        NA        NA
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##            [,44]     [,45]     [,46]     [,47]     [,48]     [,49]     [,50]
##   [1,] 105.73050 113.92884 111.58117  96.39518  98.54816  73.52575  74.41058
##   [2,]  45.54329  75.49982 100.52154  76.14748  95.40382  66.05787  70.65760
##   [3,]  73.71853  78.57808 100.17263  80.35680 109.96762  77.35036  89.47475
##   [4,]  94.23941  96.34304  90.12453  81.22217  85.31702 108.76358  67.99347
##   [5,]  92.01469  79.16090  83.64154 114.90082  94.53196  66.34397 101.57512
```

```
##   [6,]  63.79379  99.68483  74.29407 100.17454 104.74705  80.41104 115.04101
##   [7,]  92.02701  40.43026  88.68394  71.52281  71.90699 106.79155  97.03792
##   [8,] 100.83646 115.80989 101.93535  72.24447 114.97785  91.65214 102.77734
##   [9,] 101.90968 108.56467 117.55789  87.42806 106.85709  95.82589  78.38110
##  [10,]  54.38967  72.45047  92.02440  81.43290  81.42918  59.26482  71.60372
##  [11,]  53.54204  86.56292  80.99162 128.35835  95.30045  93.43729  78.26234
##  [12,]  61.26371 108.16077  75.74252  88.23311 107.50058 101.80087  92.45437
##  [13,]  87.89651  74.72489 113.52430  55.95893  88.42067 107.46187  83.62191
##  [14,]  73.81236  92.89606  62.93855  97.96139  96.34973 104.55293 104.24201
##  [15,]  71.97161  97.10710 109.75621  86.67866  73.92570  74.77011  44.94634
##  [16,] 121.08963  96.85528  78.79401  74.04746  50.20975 107.17334  71.07865
##  [17,]  98.01960  91.84564 104.00327  89.68791  99.91593 101.40912  95.87186
##  [18,] 122.51695 121.07661  92.52945  90.11228  86.38187 113.76961  83.92474
##  [19,]  93.48154  92.19672  84.45084  78.18274  61.57134  70.23602  71.45138
##  [20,]  79.92772  91.50262  93.67021 109.75780 107.92441 108.21745  95.04145
##  [21,]  66.33079  73.73176  96.62204 112.06872  87.11425  68.06306  90.93519
##  [22,]  75.35827 112.25554 104.45210  83.12021  96.14748  92.53851  59.96760
##  [23,] 107.79047  66.93130  64.49376  51.94115  60.20753 111.34144 100.20285
##  [24,] 116.22589  85.42768  93.85778  77.82984  97.41270  97.19589 123.57294
##  [25,]  97.75544  59.42290  70.56294  80.00156  70.69707  86.10620 110.82429
##  [26,]  62.80528  99.28185 117.28861  87.92112 114.88533  57.80585  78.78803
##  [27,] 122.05777  92.23419  54.01113 100.84621  76.00503 105.53651 111.68071
##  [28,]  98.73356  98.28195  95.44073  80.18807 100.52143 121.56695  87.55913
##  [29,] 112.65363  87.64380  77.53270  56.82074  75.29687 106.38165 107.26673
##  [30,]  59.40940 105.91015 128.89718  69.25446 135.80318  99.63664  85.68643
##  [31,] 106.13184  82.43144  61.46151  93.53891  56.64136 112.90641  97.80572
##  [32,]  87.54654 117.34868  92.87897  55.73229  97.84524 118.70725  73.88311
##  [33,]  91.18989  76.21139  91.98701  94.46097  91.97470  96.29056  85.07252
##  [34,]  87.70021  88.73593  83.03319  55.64975  75.76225 112.00051  77.87579
##  [35,]  96.35317  88.09206 117.74938  87.53920 120.00148  94.25847 106.51068
##  [36,] 124.08031  87.44555  83.07673  77.02771  55.56265 110.08412  70.88624
##  [37,]  92.04990 102.42667  71.42122  98.28124  98.46158 136.04413 100.29421
##  [38,] 101.08375  97.53306  76.38258  72.71841  82.11162  94.84008  96.09877
##  [39,]  89.54215  88.31619  94.50595  88.01664  80.70596  97.07239  58.49739
##  [40,] 103.81819  71.26411  76.06603  85.69648  61.13649  80.52924  98.63702
##  [41,] 136.27838  80.21515  86.53011  55.04990  61.39305 103.14611  90.85206
##  [42,]  86.66831  84.23904  83.70969 102.95528 104.61059 105.86033 108.40362
##  [43,]  63.23154 122.14767 102.39517 107.87881  99.27604  81.53043  59.46273
##  [44,]        NA 101.78109 108.99103  96.72260 120.33390  73.93588  68.55354
##  [45,]        NA        NA  88.30112  74.84343  59.73435  81.53776  98.40267
##  [46,]        NA        NA        NA 113.58285  59.93725 112.28739 112.69864
##  [47,]        NA        NA        NA        NA  88.02176  93.73087  83.24689
##  [48,]        NA        NA        NA        NA        NA  98.49233  78.90272
##  [49,]        NA        NA        NA        NA        NA        NA  87.87628
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##            [,51]     [,52]     [,53]     [,54]     [,55]     [,56]     [,57]
##   [1,] 103.68206 127.42626  85.90983 108.39749 106.60427  59.40893  90.65702
##   [2,]  98.06480  79.45470  93.83970  95.24017  82.00642 110.30834  86.93311
##   [3,]  92.63737 102.15264 122.53695  85.61972 111.03518  92.51302 100.37241
##   [4,]  94.12034  75.59772  66.41201 112.80814  95.93692 115.39299  93.61789
##   [5,] 120.23090 108.70811  94.46104  82.68299  98.34660 102.16141  96.91576
##   [6,] 103.32878  56.40719  90.44221  53.84100  48.70360 118.84561  82.15264
##   [7,]  64.73948  88.53062 104.96617  81.02861  91.93253  76.26021 108.99888
##   [8,]  76.57817  57.27483  92.37423  92.39451 101.20564  88.83065  88.14349
##   [9,]  96.16981  87.21564  71.95162  98.26529 104.84757  76.21566  76.66561
##  [10,] 117.44408  76.73020  75.93679  99.01459  64.13723 122.47943  87.31860
##  [11,]  93.73316  92.73597  75.35258  71.25000  75.24425 111.41553 110.18455
##  [12,] 109.14343  77.24489 104.59522  68.74487  71.71590 128.11818  53.48520
```

```
##  [13,]  84.08752  89.90525  93.69960  77.38598  70.68767  58.60989  92.51036
##  [14,] 105.38880  67.57212  94.45722  67.22438  71.83132 140.91717  76.45231
##  [15,] 109.21799  85.44387  37.66131 116.06187  62.21838  90.65893  70.99363
##  [16,]  79.82113  87.80537  62.84892 133.51109  86.66011  89.16913  89.76611
##  [17,]  54.25592  77.61879  83.12172  83.13417  95.07046  62.95727 134.61411
##  [18,]  84.39170 100.32335  77.23671 103.28918  97.31838  69.64858  77.99258
##  [19,] 127.62166 110.21017  79.45657 114.58451  66.27872  95.25814  62.20438
##  [20,] 101.30603 112.73507 100.07878  51.32532  81.72513  79.03088  93.61475
##  [21,]  97.15433  82.87902  72.31484  73.97168  70.68403  87.09400  77.97898
##  [22,]  87.11331  70.58429  68.88602 114.33106  99.19804 102.56032  45.95875
##  [23,]  83.14731  76.62373 103.65435  98.99187  79.28679 107.90283  99.16062
##  [24,]  81.30481  76.20808 105.39516  74.68495 100.45958  72.20065  71.27142
##  [25,]  78.68583 104.79140 122.65539  82.23107  87.04616  81.35584 111.82965
##  [26,] 119.74061 118.12749 105.69326  78.22871  80.68962  72.30623  79.65053
##  [27,]  98.01458 124.38365 112.46699  83.15794  90.01710  92.00907 117.82115
##  [28,]  75.52304  66.74924  77.36047  89.25316  94.31472  96.50337 113.54772
##  [29,]  62.93706  55.56494  95.68149  99.89506  88.60686  92.33868  85.17001
##  [30,]  75.92288  51.34807  89.60885  66.41209  82.62615  80.26216  80.22695
##  [31,]  74.92035  67.53190  62.62904  91.88036  61.02318 102.52389 117.44244
##  [32,]  95.35806  92.48116 101.01352  93.15206  75.73070  93.03824  81.54041
##  [33,] 107.56074  95.87283  85.67677  88.76325  99.11062 104.16243  94.83651
##  [34,] 105.73733  87.59496  88.52899  90.77786  52.21411 102.71555  98.53769
##  [35,]  52.43980  82.38271 104.83045  72.64455 117.21550  49.21065 114.80988
##  [36,]  56.51130  94.63072  74.32767 138.85326 111.93392  78.95348 111.38613
##  [37,]  38.64115  74.22884 102.70598  78.68740 107.42844  95.45319 101.97504
##  [38,]  98.87818  61.64525  83.89929 103.14692  85.84728 119.50771  63.24141
##  [39,]  87.12164 122.71382  88.84485 112.68871 102.38508  83.17668 118.77633
##  [40,]  77.01367  73.90932  83.86567 108.72221  96.48494  95.10228  68.47294
##  [41,]  84.97529  90.88360  85.53875 117.75990  92.29670  78.34729 107.86559
##  [42,]  56.92243  82.23582 103.42959  66.39665  98.89185  85.12508 139.07305
##  [43,] 124.88394  79.65364  39.05523  92.60946  56.21379 109.45622  70.31750
##  [44,] 109.85568  72.95348  80.37465  71.59451  65.46788 112.57460  70.19728
##  [45,]  78.07874  89.87484  96.16138  98.14262 100.80601  82.86897 113.03931
##  [46,]  84.83886  95.76407  98.55751  86.95839  83.43745 124.49093 103.29837
##  [47,]  81.11057  75.45214 104.60859 105.34246  93.44307  75.36340  85.66829
##  [48,]  86.76478 105.87351  71.13767 126.88559  86.31622  97.02004 108.77294
##  [49,] 119.48158  94.02038  87.64169  98.00101  92.71445  86.41896  68.28195
##  [50,] 105.98624 101.15892  50.24438 127.43695  82.00662  94.77891  81.46071
##  [51,]        NA  69.15930 101.80792  92.01391 120.82178  74.44057 115.04478
##  [52,]        NA        NA  81.25120  79.89674  82.79712 109.52942  85.14168
##  [53,]        NA        NA        NA 114.06940  68.08056 100.24704  93.31229
##  [54,]        NA        NA        NA        NA  69.60360  88.01526  92.96011
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```

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## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,58]     [,59]     [,60]     [,61]     [,62]     [,63]     [,64]
##   [1,]  94.27535  78.13607  90.73147  97.95196  97.56172  92.97979 104.33637
##   [2,]  52.46254  94.15129  99.13257  92.78286  79.10998 124.77452 111.92465
##   [3,]  78.72796 114.72524 107.59284  74.05038  78.65575 133.06714 105.15652
##   [4,] 122.44393 108.17518  93.38218  73.20493 109.70808  78.36039  75.10397
##   [5,] 116.39026  97.04206  86.83332  52.58668  80.74250  99.15721  89.12178
##   [6,]  82.03028  91.72390 116.17536  87.25247  65.21773  90.68542 101.89613
##   [7,]  76.77780  93.57154  63.73593  85.12732  96.42991  95.98501  67.50292
##   [8,] 107.02368 103.80692 141.28036  91.20704  70.73613  84.06769  79.26990
##   [9,] 112.93496  78.77683  87.38225  65.16347 119.45730  50.49595  60.52772
##  [10,]  75.43371  92.73662  90.33029  89.79175  79.92471 107.82042 103.48650
##  [11,]  65.87861  75.03738  82.90568  63.99593  96.66965 123.02145 116.62445
##  [12,]  85.47242 111.37248 100.78137  86.29677 100.86807  75.68375 102.08581
##  [13,]  73.47661  84.38762  68.12738  92.75689 121.65922  68.43706  67.82750
##  [14,] 108.40029 115.32950  99.25284  68.46540  90.96417  78.34257  87.22661
##  [15,]  69.85597  50.72231  64.33689 107.81477 115.66724  66.52761  92.66723
##  [16,] 109.66165  91.11707  77.30024 133.40225  98.62508  68.72300  82.73485
##  [17,]  80.69367  78.81983 109.45246  86.28223  78.80121 114.40884  88.01670
##  [18,] 111.78069  82.40826  82.62183 105.38969 114.62961  50.11696  78.70309
##  [19,]  93.11065  88.88993  66.30062 128.62197  98.66529  68.25505 100.87567
```

```
##  [20,]  85.85184  84.12662  70.52852  54.38386 123.32800  81.95395  88.85380
##  [21,]  60.21292  41.85828  63.60236  79.06989  91.97760  78.86328  84.52863
##  [22,]  77.72202  73.86629  88.57052  99.36763 110.85158  60.89226  85.12486
##  [23,] 114.75068 137.77523  90.76810 110.59666  81.99246  86.82114  75.82774
##  [24,] 114.19286  91.65836  88.64112  79.48804  89.93652  41.49739  42.27869
##  [25,]  78.28664 107.82564  86.90265 113.04629  62.29065 123.82851 103.01392
##  [26,]  56.14669  81.13558  93.23949  91.35286  93.99617 107.48129 119.76236
##  [27,] 126.66263 127.82962  93.87752  97.11230  78.80436 108.58933 106.89678
##  [28,] 117.46368 107.54730 108.50157  64.80925 102.24905  85.79312  69.72517
##  [29,] 103.46406 108.42102 106.36785 122.25875  71.33859  72.52837  69.29774
##  [30,]  61.38730  79.75823 115.11749  71.54918 104.33436  79.42940  77.59595
##  [31,] 106.86636  89.92497  85.37602 105.01588  82.01364  85.36427  82.24956
##  [32,]  90.83702 123.04660 105.16720 114.13616 112.79314  83.45593 104.26704
##  [33,] 120.94199 103.68599  76.23764  42.25867 111.57626  79.70755  67.84767
##  [34,] 100.55180 123.31192  88.51930 105.77371 108.38040  84.30755  93.10876
##  [35,]  76.59805  78.19002 109.96828  69.80573  80.40114 106.55771  76.06643
##  [36,] 100.97945  93.49606  83.38476 124.45971  90.04559  98.30543  87.47660
##  [37,]  80.37950  99.54352 107.99269  93.77893  86.78992 105.79431  97.37702
##  [38,] 124.64735 111.91333 100.78636  96.99827  85.92800  54.30282  69.11515
##  [39,]  81.31124  99.52663  83.72724  97.62086 102.12453 130.82068 117.01484
##  [40,]  92.02594  77.74798  73.73419 112.99700  74.07620  67.73364  69.85440
##  [41,] 133.67267 115.77114  87.75963 109.53417  89.95374  75.15112  64.93512
##  [42,]  80.08521  97.17186 114.45130  72.87731  69.44039 139.68068 101.01968
##  [43,]  86.81417  62.41979  86.21788  80.39757 116.47519  65.26993  99.68833
##  [44,]  42.90370  73.43677  99.34846  73.59363 100.63054 100.34806 113.50460
##  [45,]  96.06218  93.86098  61.89451  78.00672  80.16023  98.98642  57.81246
##  [46,] 116.36816 118.30871  90.52525 107.03182  68.67185 103.84771 108.24005
##  [47,]  90.34603 113.98208  96.83202 105.88724  94.73884  79.45633  65.62357
##  [48,] 112.31620  95.43040  50.91567 116.93060  90.91680  89.62206  83.95391
##  [49,]  76.04480  67.66101  90.59355  86.16838  68.76047  92.97225  93.42151
##  [50,]  71.02397  71.28645  66.03872  98.63615 131.76642  87.08731 103.24075
##  [51,]  85.51965  93.82778 102.58945  96.00858  74.09815 105.30375  73.66228
##  [52,]  84.62013  90.28723 120.79574  79.64126  73.25232  82.69381  69.59612
##  [53,]  89.43107  53.94124  67.03385  87.42962 114.67612  74.33300  87.66191
##  [54,]  79.07665  91.27093 108.34894  62.30351  84.31119  91.61677  93.27582
##  [55,]  81.04487  84.07356  84.37290  95.70799 104.35774  79.86160 106.20472
##  [56,]  81.09243  69.57121  78.12409  91.88086  99.29916  80.75581  69.81355
##  [57,]  77.37604  79.87517  92.15429  97.67912  98.72927  56.11685  92.61114
##  [58,]        NA  60.25433  92.25605  94.67714  96.06401 106.54540 112.79433
##  [59,]        NA        NA  67.26605  80.28334 104.79025  75.69802  86.36649
##  [60,]        NA        NA        NA  88.89921 124.58751  71.26194  76.09398
##  [61,]        NA        NA        NA        NA  99.95327  86.80127  67.06151
##  [62,]        NA        NA        NA        NA        NA 118.30852  95.62964
##  [63,]        NA        NA        NA        NA        NA        NA  55.94387
##  [64,]        NA        NA        NA        NA        NA        NA        NA
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##  [67,]        NA        NA        NA        NA        NA        NA        NA
##  [68,]        NA        NA        NA        NA        NA        NA        NA
##  [69,]        NA        NA        NA        NA        NA        NA        NA
##  [70,]        NA        NA        NA        NA        NA        NA        NA
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
##  [73,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [74,]        NA        NA        NA        NA        NA        NA        NA
##  [75,]        NA        NA        NA        NA        NA        NA        NA
##  [76,]        NA        NA        NA        NA        NA        NA        NA
##  [77,]        NA        NA        NA        NA        NA        NA        NA
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
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##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
##  [88,]        NA        NA        NA        NA        NA        NA        NA
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##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,65]     [,66]     [,67]     [,68]     [,69]     [,70]     [,71]
##   [1,]  77.41520  96.85546 113.06971  94.05850  90.84005  85.51661 108.92126
##   [2,] 108.40558  75.52185  64.79606 100.68061  72.29404  55.04714  77.05872
##   [3,] 108.57455  78.22720  61.85435 105.68030  85.99942  85.34300  70.90841
##   [4,] 123.65829 131.99202 112.34940  77.31587  78.54002  91.01437 112.66688
##   [5,] 102.30128  95.55001  80.81157  75.32286  80.76685  93.65110  89.13515
##   [6,] 103.78741  97.67358  84.11058  58.66315  72.23187  81.57036  76.87872
##   [7,]  90.19621  52.62861  60.18668 130.11784  96.03583 105.41305  80.40771
##   [8,] 101.71096 119.53071 134.34996  69.47063  65.44728  90.35344  99.82747
##   [9,]  99.64095 104.03742 133.09343  75.62754  86.87470  97.67707 107.94319
##  [10,] 118.78881  89.81185  75.97441  91.12347  58.74938  47.23506  98.42376
##  [11,]  97.24718  93.29279  65.98893  72.28620  85.64588 104.59554  64.46774
##  [12,] 109.45197  92.08705  62.78508  74.30044 116.81519  77.65613  66.59974
##  [13,] 103.68941  59.84207  86.03263 129.71692  92.71458  93.12012 102.92872
##  [14,] 118.02180 107.01675  71.19506  63.13505  97.74617  92.78998  75.25633
##  [15,]  88.86066  90.24130 112.45888  89.30123  77.20747  54.42583 117.97172
##  [16,]  78.33532 115.18647 118.27748  99.25620  92.92067  81.03721 130.63328
##  [17,]  88.02319 100.20294 116.70830  90.41912  59.17746 119.14638  94.04797
##  [18,]  72.72094 107.89515 124.53244  84.95141 112.24437 103.30614 110.65288
##  [19,]  86.03537  85.42883  84.77103 106.18253 100.85311  44.11750 120.36421
##  [20,]  98.74753  74.10874  67.35568  90.82298 113.84481 120.45963  71.91104
##  [21,]  71.63653  57.00175  82.10981  80.77609  85.18933  83.36254  74.53696
##  [22,]  81.62560  94.27262 109.39152  74.58052 103.61381  67.61216  85.85626
##  [23,] 112.46522  98.85257  75.44739 115.89815  89.86926  84.23102 109.49538
##  [24,]  82.00202  75.72628 104.83377  88.10966 100.10185 103.76530  90.04891
##  [25,]  79.02675  64.63193  49.63641 125.71789  94.88865  90.97490  81.31120
##  [26,]  95.51297  63.84559  72.71305 104.69067  89.62358  66.04279  83.59284
```

```
##  [27,]  93.09715 111.43645  71.98083  97.74802 104.58799 112.72775  99.60450
##  [28,] 125.02711 129.14367 119.55486  77.38814  68.94393 115.67549 105.57317
##  [29,]  85.65101  98.59029 106.42943  96.43843  86.12613  85.82730 102.49862
##  [30,] 112.61056  82.71951 108.51454  82.00499  73.92508  92.20465  77.06286
##  [31,]  91.75808 117.78360 104.88327  82.40361  71.81918 104.61292 112.83208
##  [32,] 117.25909 106.78928  88.12948 107.05718 105.60157  80.51784 104.83641
##  [33,] 121.55842  99.52764  86.46580  85.14308  87.75565 101.38445  96.22598
##  [34,] 135.63031 106.09412  81.50738 113.60703  84.89014  78.14631 122.22105
##  [35,]  81.46577  76.77824 107.48224  93.45310  74.42565 122.67835  74.04057
##  [36,]  70.85788 110.08842 111.59928 108.21545  90.61078  96.26697 114.46932
##  [37,]  74.04971  99.83945  82.56000  80.20894 113.24582 127.95222  54.82032
##  [38,] 104.27416 114.04556 106.97256  73.60617  89.08405  72.77862 106.48923
##  [39,]  96.34682  98.48086  78.67635 114.79314  91.71412  92.49266  98.40649
##  [40,]  57.18297  74.39542  92.32256  91.13693  97.82043  75.09629  89.05387
##  [41,] 103.81192 111.87658 115.83496 114.26046  75.86532  89.64692 145.36858
##  [42,]  92.90399  96.94521  82.53269  85.56469  74.35030 129.56476  66.95184
##  [43,] 107.29812 116.36777 117.97179  53.53403  75.35675  71.34878 107.47009
##  [44,] 111.33394  80.26184  75.70067  72.65564  80.19801  66.19331  65.80435
##  [45,]  92.63286  61.14687  71.66127 123.87479  74.95208  92.76014  95.81909
##  [46,]  77.63773 114.93393  63.67016  78.09263 112.69366 107.56949  81.34158
##  [47,] 114.51048  77.77614  96.86579 131.41557  78.18611  69.10309 113.20860
##  [48,]  77.96037  97.92451  83.20259 113.42611  94.25460  87.22238 120.84696
##  [49,]  82.73247  67.37106  93.83230  85.14607  67.15893  48.29637  91.00036
##  [50,] 101.60268  97.49716  99.50978  99.82283  87.63093  62.61092 112.05477
##  [51,]  68.54357  87.48662  97.14421  98.16419  91.36762 124.58317  72.64003
##  [52,] 106.54394 100.20891 111.00708  66.08573  59.38935  87.29746  81.40585
##  [53,]  95.39127 114.02385 123.10429  72.08543  66.88420  80.10556 119.95959
##  [54,] 100.28445  77.92405  72.62640  72.98389  92.84550 115.93851  57.90765
##  [55,] 116.53609 101.01842  85.96291  82.14060  77.77351  77.43571 106.88093
##  [56,]  71.49817  58.98328 106.94008 120.83053  89.26116 103.70981 100.31596
##  [57,]  81.96842  75.09750  90.38993  76.04720 111.36912  51.98910  78.13099
##  [58,]  81.65228  58.82353  77.07968  94.33075  89.82488  73.84292  62.40408
##  [59,]  65.97831  71.66500 112.13384  76.00803  79.29501  85.13171  88.06705
##  [60,]  79.53958  70.13599  79.58954 114.99941 104.16997  88.85487 109.08984
##  [61,] 112.70192  83.84832  91.53588  71.30582  73.37569 111.20630  74.54795
##  [62,]  72.82708  91.07510  85.33087  78.15662  74.27605  89.04774  72.86687
##  [63,]  91.51130  85.91343 115.29097  84.99316  98.71750  81.95195 110.45658
##  [64,]  96.57514  75.00302 112.78249 102.51424  73.93709 101.16709 105.58047
##  [65,]        NA  73.76763  89.65877  86.61432 116.48461  99.11206  70.99421
##  [66,]        NA        NA  67.26778 119.89591  99.45822  80.99147  70.04289
##  [67,]        NA        NA        NA 107.24768 120.53446  90.83864  59.55165
##  [68,]        NA        NA        NA        NA  83.61210  96.22554  70.36857
##  [69,]        NA        NA        NA        NA        NA  79.39658 112.77552
##  [70,]        NA        NA        NA        NA        NA        NA 105.27535
##  [71,]        NA        NA        NA        NA        NA        NA        NA
##  [72,]        NA        NA        NA        NA        NA        NA        NA
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##  [80,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [81,]        NA        NA        NA        NA        NA        NA        NA
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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,72]     [,73]     [,74]     [,75]     [,76]     [,77]     [,78]
##   [1,]  70.31267  62.13140  89.38908  99.77806  89.50411  82.08252 100.68894
##   [2,]  67.37723  57.82024 104.49965  99.46454  93.81925  99.37882  85.76801
##   [3,]  85.61368  35.58712  83.31653 126.24037 107.06963  92.10494  83.29190
##   [4,]  78.24951  84.42577 111.28662 125.42655 126.47036  52.71985  64.60797
##   [5,]  75.43405  77.60143  66.78186 132.44813 117.97674  72.00396  78.11373
##   [6,]  94.50030 117.24370  93.06243  71.04466  76.83052  92.12376 109.74974
##   [7,] 107.55753  93.68973  75.01499  77.71488  97.51528 115.57856  85.47039
##   [8,]  85.48566  98.02567  96.51939  87.61652  96.00184  82.04392 102.87922
##   [9,]  88.49685  96.01627  88.40425 107.61633 112.01745  53.00389  92.12858
##  [10,]  52.63557  76.82369 110.08094 102.64920 108.09675  84.92661  79.61542
##  [11,]  83.25252  68.49081  76.16595  92.91124  80.36204 102.97508 102.64692
##  [12,] 123.19547  97.04694 115.57094 108.74795  85.02026  57.50415  73.56792
##  [13,] 107.71060  97.08862  97.20915  70.67535  99.89749  89.90218 105.79363
##  [14,] 112.30977 108.69990  99.87223 112.95204 101.48472  60.59775  69.98525
##  [15,]  57.64848  98.17615 117.99874  71.87520  85.37880  80.59076  94.89136
##  [16,]  79.49869 109.93410 122.27031  73.28114  86.95170  87.06076  71.58088
##  [17,]  78.84720  81.70421  69.64273  59.19826  83.39732 126.41081 133.05050
##  [18,] 105.31297 109.18480 100.82845  80.70615  80.55449  71.85180  90.77969
##  [19,]  76.43372  98.57271 127.52116  91.47871  90.55462  69.92441  67.66370
##  [20,] 121.69453  82.90278  70.60310 101.81171  93.30292  76.40239 103.29712
##  [21,]  84.02279 113.20302  68.39941  61.34312  71.59651 107.59915 108.36530
##  [22,]  90.45455  98.46414 115.08348  90.81038  77.68651  70.31023  76.19245
##  [23,] 101.22296 104.57848 116.63849  95.49703 112.54031  84.50121  57.78808
##  [24,] 120.65368 136.92314  73.29241  82.29603  96.68960  78.61700  89.73900
##  [25,]  96.88471  82.53320  81.96061  75.37801  80.69310 128.51884  87.73299
##  [26,]  79.98950  51.24707  92.78275  98.29481  86.18198  87.22895 110.26121
##  [27,] 102.47907  79.82973  88.15865 107.85159  97.30846  86.83264  77.75684
##  [28,]  90.22782  88.16862  93.43483 104.40095 119.64071  72.18943  93.57283
##  [29,] 102.24317 128.21760 107.57610  63.20410  84.69336  99.04571  81.81059
##  [30,] 101.55414  90.04569  92.75253  76.24905  89.50008  85.55660 125.74516
##  [31,]  84.52854 126.66299  96.73632  55.78670  86.09118 106.16978  94.80706
##  [32,] 109.21323  74.75606 141.55645 100.28213  95.28341  62.84196  83.90772
##  [33,]  89.38463  80.85948  82.20006 142.61540 137.73035  53.09588  67.06273
```

```
##  [34,]  93.92747  86.96236 135.95452  98.25884 115.47850  66.33317  79.59314
##  [35,]  95.06276  78.41136  50.26791  72.67761  85.22097 120.39969 130.16727
##  [36,]  75.81106  84.67062 103.07657  77.36553  85.28088 109.10705  75.12845
##  [37,] 126.34543  91.29309  82.89244  73.07915  58.35655 114.41592  94.92567
##  [38,]  94.61260 124.26941 116.42954 102.86094 105.97471  56.16480  59.76035
##  [39,]  72.15856  26.67939  99.81017 109.27731  97.45503  97.20149  83.40883
##  [40,]  90.53183 133.03627  88.75575  66.31166  74.41161 105.08670  69.62309
##  [41,]  79.82268 100.64242 108.54130  93.62889 122.23251  79.06668  73.02293
##  [42,]  94.25426  70.47464  60.57882  79.13979  82.15766 128.09669 117.21201
##  [43,]  67.85624  97.23645 112.35481  91.97678  91.33969  55.95053 101.98399
##  [44,]  82.16731  71.73869 101.34877  93.92568  82.14798  81.80319 106.73334
##  [45,]  78.78430  93.48204  67.44497  93.26224 118.62332 110.85100  72.05338
##  [46,] 108.49494 106.94729  93.18638  90.00981  74.16029  99.05985  63.54281
##  [47,]  90.03522  86.08915 115.47673  93.41469 118.02359  81.57727  80.27753
##  [48,]  74.59620 101.55624 101.18478  85.82125  98.18433 100.09180  54.85090
##  [49,]  52.11088  83.65731  79.88196  96.29886  94.38186  90.31917  93.89345
##  [50,]  59.75900  60.53879 124.81545 102.47560  98.22267  71.32828  80.14697
##  [51,] 106.98505  97.22067  69.40756  62.15785  73.42648 133.90864  97.31456
##  [52,]  89.20116 116.76504  91.80752  72.35797  91.38968  95.24625 101.70865
##  [53,]  51.82899  98.31765 103.76905  80.32333  94.97947  80.34786  94.81217
##  [54,] 125.09469 101.79461  65.90097  78.42503  78.58750  94.02420 125.93391
##  [55,]  86.15860 103.70934 115.49652  74.44860  89.05984  76.37246 107.93339
##  [56,]  93.14359  86.55335  66.77499  68.91592  89.26867 107.32501 118.28715
##  [57,]  98.96447 101.28497 109.25611  98.93523  77.49083  59.87320  78.77311
##  [58,]  87.02468  70.26316  90.71048  67.57717  60.55048 111.58802 118.58212
##  [59,]  68.63722  98.54455  71.06038  61.97317  70.80243 102.37314 120.52883
##  [60,]  81.92003  93.88068  87.41288  92.23994 101.06847  85.79213  72.38705
##  [61,]  90.83406  86.63946  55.34417 111.81818 116.30473  77.39062 102.44630
##  [62,]  84.32613 100.58362  69.68735  78.36184  76.61493 122.97805  90.79010
##  [63,] 101.32868 129.37226 100.22592  86.75257  99.92231  54.91508  86.31155
##  [64,]  92.40455 119.49840  69.86827  88.76233 123.42700  85.44999  85.20472
##  [65,]  97.37563 103.35695  66.61033  62.57332  44.10483 124.03956  90.67120
##  [66,]  99.65440  88.12357  66.47231  81.01148  86.11544 106.90589  97.72000
##  [67,] 113.88572  73.55264  88.47269 105.20696  82.61277  97.60284  70.39139
##  [68,]  88.96472 107.87769  82.19295  87.95314  70.58793  80.55103 103.12381
##  [69,]  44.99768  90.92110  83.36731  82.26762 116.69461  97.47312 111.65368
##  [70,]  59.23159  81.62161 125.62591 101.24294  96.92821  70.32853  74.35881
##  [71,] 120.94080  85.20345  63.99562  87.05165  56.28233 110.04170  98.64541
##  [72,]        NA  74.64686  95.77402  94.81704 108.83557  91.75679  89.52118
##  [73,]        NA        NA  98.38070 119.86142  98.80084  88.18710  87.37253
##  [74,]        NA        NA        NA  79.68679  85.37232 121.20314 110.37118
##  [75,]        NA        NA        NA        NA  52.21640 131.38098 127.62736
##  [76,]        NA        NA        NA        NA        NA 119.46516 108.93225
##  [77,]        NA        NA        NA        NA        NA        NA  70.75275
##  [78,]        NA        NA        NA        NA        NA        NA        NA
##  [79,]        NA        NA        NA        NA        NA        NA        NA
##  [80,]        NA        NA        NA        NA        NA        NA        NA
##  [81,]        NA        NA        NA        NA        NA        NA        NA
##  [82,]        NA        NA        NA        NA        NA        NA        NA
##  [83,]        NA        NA        NA        NA        NA        NA        NA
##  [84,]        NA        NA        NA        NA        NA        NA        NA
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
##  [87,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [88,]        NA        NA        NA        NA        NA        NA        NA
##  [89,]        NA        NA        NA        NA        NA        NA        NA
##  [90,]        NA        NA        NA        NA        NA        NA        NA
##  [91,]        NA        NA        NA        NA        NA        NA        NA
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,79]     [,80]     [,81]     [,82]     [,83]     [,84]     [,85]
##   [1,]  91.64972 104.70757  87.53543 103.75632  98.89409 121.03019  71.30016
##   [2,]  96.07449 113.82001 109.97382  57.07680  81.55202 114.42680 127.43512
##   [3,]  90.45939 125.51542 117.10334  75.67555  75.18581 121.36397 105.70072
##   [4,]  92.17327  98.96268  66.58327  64.38061 111.36367  73.19551  62.49917
##   [5,]  77.29712  99.50510  79.23900 106.07147 110.55733  79.79379  63.64091
##   [6,]  83.91841  74.47852  83.90384  93.63014  95.93570  75.29108 106.18323
##   [7,]  71.54468  78.59043 111.92676  82.58306  60.67793  79.49613 105.18279
##   [8,] 102.01722  73.57975  67.06265  65.97458  76.29536 104.62104  95.30596
##   [9,]  92.05153  81.76492  44.99761  91.53626 108.19325  69.89603  67.09095
##  [10,]  86.99662 100.46853  93.80464  61.29427  97.10486  95.44245 104.72649
##  [11,]  63.76726 110.00892  94.87490  99.17955 119.43063  79.68605  87.48415
##  [12,] 113.56513 114.12951  98.38158  96.36662 104.81953  67.70422 102.11918
##  [13,]  68.80638  76.96693 100.00243  88.51134  72.00626  78.39643 100.17834
##  [14,]  94.85646  96.48892  83.50181  93.13195 109.85244  49.41346  83.68758
##  [15,]  92.37006  83.89101  69.60586  79.85873 112.86477  83.01439  91.43868
##  [16,] 103.49900  77.06263  85.47890  75.99998  89.55805  92.19200  72.03014
##  [17,]  58.25401  69.32955  85.36620  77.84204  73.24777 110.61288  92.27423
##  [18,] 101.22756  81.07926  73.11418 107.93661 100.71712  80.38207  63.65796
##  [19,] 108.03945  97.47187 100.56200  96.05145  99.14624  92.77366  83.82929
##  [20,]  63.34240 103.21064  94.23996 130.62738 110.38129  60.05317  74.17674
##  [21,]  79.34083  67.83793  73.54781 105.54412 100.08439  65.47940 106.71320
##  [22,] 132.85982  96.20093  66.38039  73.32815 102.61261  81.94772 105.29501
##  [23,]  92.85608  84.22921 112.96657  66.18265  65.94765  85.33592  87.94718
##  [24,]  98.07110  52.19980  66.79891 102.41913  73.26070  60.62815  89.57997
##  [25,]  81.04511  91.06502 142.13116  91.91765  54.03346 114.93738 106.68701
##  [26,]  86.25010 118.30452 111.27446 101.79120  90.57899 120.25361 107.29268
##  [27,]  75.47668 105.36739 120.53267 115.66625  91.97153  98.37900  51.69497
##  [28,]  71.74445  83.49621  67.79472  69.15392  96.60519  76.43310  67.66683
##  [29,] 108.64394  58.57306  89.65362  65.55417  56.79532  91.28686 104.42660
##  [30,]  85.69710  81.14456  72.23195  70.16894  81.21221  85.69252 121.92746
##  [31,]  67.11314  56.19584  82.70918  83.81271  92.74005  70.82043  72.22041
##  [32,] 100.62207 117.38544 115.03688  79.86234  86.88550 102.71744  89.59570
##  [33,]  76.85964 100.65127  72.42042  90.68744 112.20692  58.46373  59.53314
##  [34,]  75.84032 100.48777 111.96623  75.44269  89.92814  85.39131  78.89451
##  [35,]  71.55507  72.84424  83.01323  85.75237  63.01037 108.19573 104.86763
##  [36,]  98.26807  86.40682  96.96439  67.33691  76.18476 113.74666  78.23270
##  [37,]  98.35105  94.81698 102.76844  88.56367  79.39615  91.57271 101.48139
##  [38,] 118.02427  77.26001  68.45076  73.73858  93.25115  68.02293  83.58586
##  [39,]  79.29584 130.47479 120.52966  80.31139  92.86662 125.54785  78.28772
##  [40,] 117.65369  60.15883  80.00484  83.43761  74.72751  78.55713 105.50089
```

```
##  [41,]  84.02711  71.86870  89.35976  70.98192  72.72560  95.41793  63.66289
##  [42,]  60.09203  89.51434 103.61137  85.83294  76.23954 106.07231  95.32894
##  [43,]  85.87825  92.71708  52.10483  94.07410 145.12422  66.42303  73.93165
##  [44,]  90.78930 112.00755  86.80530  81.21576 109.98249  86.96014 120.52380
##  [45,]  71.76365  70.49951  96.70332  73.02304  63.65398  82.73951  93.90481
##  [46,]  94.47295  95.50569 112.76900 107.41032  97.26258  79.84613  68.91777
##  [47,]  98.54220  83.99669 101.01288  49.03726  49.94403 106.13174 110.62962
##  [48,]  85.23619  82.56204 102.53742  85.39581  90.10804  84.43994  63.85433
##  [49,] 101.09474  85.95963  75.94331  86.19089  89.19574 106.50847 108.78232
##  [50,]  94.08322 118.08897  86.05777  71.48834 117.03773  96.07317  84.13808
##  [51,]  91.10874  69.53163  95.41724  71.81481  52.70856 101.34911 106.68908
##  [52,]  94.89554  60.29062  62.39756  53.86094  77.55187  77.06581 117.75165
##  [53,]  75.69181  79.71841  53.91022  81.03316 129.39322  70.06478  68.03632
##  [54,]  66.91191  82.17163  92.58539 118.82403  90.39003  70.61077 100.05320
##  [55,]  64.67745  87.50963  91.29332  99.92551 115.26251  69.26025  82.39220
##  [56,]  74.73521  70.42461  89.93038 100.64886  63.84491 105.65087  93.49514
##  [57,] 138.29667 100.27487  76.98391  95.71961  99.90322  83.36150 111.92306
##  [58,]  93.42908 102.78522 100.86413  84.48875  85.60445 108.22690 136.75484
##  [59,]  80.68924  71.75677  56.95828 102.10933 109.16913  79.95748  96.27963
##  [60,]  75.54247  88.70273  89.34155 105.76232 106.52639  65.80159  66.57641
##  [61,]  64.09321  83.57636  59.99250  95.04588 105.29363  58.80760  82.94812
##  [62,] 101.77259  72.69167  97.27345  81.16397  59.97472 114.18144 110.01364
##  [63,]  99.20035  66.80383  55.97223 100.87928 101.81131  51.95721  78.76237
##  [64,]  81.13494  45.46633  58.35064  74.71250  68.90247  62.99850  87.46433
##  [65,] 111.89881  79.86319  93.27758 113.22741  80.64216 102.57450  96.84839
##  [66,]  92.38126  82.91094 100.33986  96.80069  62.79229  93.81124 124.48568
##  [67,]  92.35111 121.35269 138.63718 105.53671  85.45822  91.10070 100.33666
##  [68,]  97.74841  85.28290  54.94823 102.40925 124.45884  70.47901  84.91029
##  [69,]  63.15767  61.81774  62.58011  55.59829  84.13269  94.49357  95.11944
##  [70,] 119.08572 103.30622  89.04027  67.82747  92.82379 106.47516 111.55641
##  [71,] 106.12197 102.72425 100.56410 103.43553  83.75820  91.24417 122.83552
##  [72,]  81.65137  85.87743  70.35811  63.45364 101.32090 105.96502  84.58020
##  [73,]  87.99936 140.91466 116.42633  79.31236  93.11421 127.16202  93.50531
##  [74,]  70.04803  63.60080  76.14458 108.90426  77.36979  82.22434  92.16275
##  [75,]  79.25782  52.52617  86.77139  94.82521  73.88537  92.50601 107.69080
##  [76,] 109.34556  93.65880 102.05474 113.41813  91.59613 102.83881 109.62555
##  [77,] 100.23046 109.80526  73.78889  94.10114 122.07788  66.98619  67.89862
##  [78,] 119.63189 109.94760 102.97306  78.10757  91.35209  85.69059  76.24531
##  [79,]        NA  73.46989  86.81145  99.72853  98.15805  73.32762  66.28651
##  [80,]        NA        NA  57.27623  84.99284  69.69602  70.89020  92.54668
##  [81,]        NA        NA        NA  84.30764 110.36415  60.18832  81.89204
##  [82,]        NA        NA        NA        NA  66.19763 107.95250 113.72849
##  [83,]        NA        NA        NA        NA        NA 118.54483 123.91007
##  [84,]        NA        NA        NA        NA        NA        NA  67.74309
##  [85,]        NA        NA        NA        NA        NA        NA        NA
##  [86,]        NA        NA        NA        NA        NA        NA        NA
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##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
```

```
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
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##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,86]     [,87]     [,88]     [,89]     [,90]     [,91]     [,92]
##   [1,]  55.62428 113.47464  90.99716 121.72199 118.94103  85.64716  83.71943
##   [2,] 122.29775  97.87126 127.35265  64.47042  78.35392 101.49821  95.39698
##   [3,]  87.68110  89.20673 117.11261  88.46584  86.24809  96.24537  92.15976
##   [4,]  62.68074 106.10669  94.27811  90.54192  93.84518  78.34056  90.15317
##   [5,]  64.81525  85.50606 110.38225 120.84678 122.22635  71.33692 101.82627
##   [6,] 141.28003  79.87848  81.27865  65.80508  61.04856  83.97363  86.37649
##   [7,]  97.09581  49.10283 102.22366  86.64794  76.90346  95.88816  97.26523
##   [8,]  92.53182 118.92930  50.95776  85.95717  74.51585  61.03893  62.98689
##   [9,]  50.33078 101.08137  69.60896 118.28561 106.26313  75.69725 101.02111
##  [10,] 114.68386  96.16280 133.09599  72.32303  90.47281  85.44379  89.45317
##  [11,] 104.04193  76.37247 117.43876  95.08726  83.53526 102.03636 136.23102
##  [12,] 110.15089  79.54115  91.71189  45.92265  63.63937 128.52215  96.68336
##  [13,]  91.69731  49.62252  92.89157  88.33360  68.20058  99.81891  89.20037
##  [14,] 101.86562  74.39930  91.14139  64.70154  71.94041  98.92637  97.50742
##  [15,]  96.63589 106.65434 103.44181  88.13973 102.74476  94.51146  99.49986
##  [16,]  78.22739 113.02843  79.29271  79.80824  97.78226  85.76623  59.94369
##  [17,]  94.82880  91.26304  72.16066 110.55478  73.07386  61.07118  87.69026
##  [18,]  60.05067 100.96894  56.70339 100.39072  98.25009  93.99387  80.64013
##  [19,]  88.26536  97.62459 111.04851  74.75043 109.54399 105.43977  67.05319
##  [20,]  74.10157  42.47951  97.87352 109.23180  84.68727 110.98792 126.42439
##  [21,] 114.79197  74.80639  94.55225  98.97879  98.95826  91.45626 124.70331
##  [22,]  93.79683 123.93323  75.41448  69.44018  91.77022 110.21324 103.08014
##  [23,]  95.26308  75.83362  96.76529  56.97237  70.55079  87.48183  48.17244
##  [24,]  80.40268  74.77854  48.37671  97.80560  90.46036  76.27412  81.26653
##  [25,] 109.49111  68.11352 106.87616  77.11995  80.18115  95.56997  67.97309
##  [26,]  95.95303  85.44627 116.39391  94.83198  92.20074 107.96741  95.47062
##  [27,]  66.69866  74.76309  98.27163  97.07889  93.84962  88.81192  63.98990
##  [28,]  70.82566  90.47223  76.25393  99.11007  73.59930  66.04850  88.36124
##  [29,] 108.78596  98.40492  56.77042  59.17979  67.51774  78.41640  47.53507
##  [30,] 112.14450  84.69199  68.60170  80.39606  49.01850  90.97511 104.58690
##  [31,] 103.60086  82.03459  79.87214  83.58507  76.53443  68.29992  74.32791
##  [32,]  88.18329  88.06535  91.64086  56.33263  56.78680 118.05627  63.55454
##  [33,]  52.29369  76.09610 107.82104 111.50835 106.87527  80.83658 109.45157
##  [34,]  91.73001  68.86986 111.42246  63.71272  62.75569  98.53657  61.68502
##  [35,]  87.97598  85.97708  63.97917 117.56502  78.56740  68.80901  99.04743
##  [36,]  72.66488 117.96598  83.23016  90.17314  98.78150  82.66981  67.77804
##  [37,] 104.40187  89.75118  61.79341  69.82064  56.12114 108.78514  98.97168
##  [38,]  89.40793 105.81968  72.31594  66.16442  88.57629  81.11698  65.89979
##  [39,]  69.14798  97.74731 125.16835  97.76018  96.39299 102.10938  91.09356
##  [40,] 105.06330 102.09492  74.16487  75.72052 103.74691  86.58907  81.09753
##  [41,]  61.97441  93.94975  84.64602  95.33020  96.17043  61.67835  40.05675
##  [42,] 102.13116  78.45122  85.37546  99.07257  65.69227  76.84806  98.28169
##  [43,]  90.87285 104.49374  94.71568  93.97115  95.75801  90.39431 110.85010
##  [44,] 125.44322  89.62346 110.67275  70.87991  70.20181 112.24077 124.06618
##  [45,]  84.50950  67.69595 112.30067 103.25963 104.95062  68.57233  89.78572
##  [46,]  96.56633  84.04989  89.72204  69.28309  86.02051  98.94987  75.04994
##  [47,]  90.08703  88.28605  90.97581  71.13087  72.48233  84.93399  55.30992
```

```
##  [48,]  74.71086  89.43121 110.88315  90.31877 113.77565  84.64579  72.68057
##  [49,]  98.17291 110.59801 104.67011 103.61702 122.19536  74.29545  92.63512
##  [50,]  76.85903 110.18695 123.02664  88.63906 102.75329 109.09878 106.99453
##  [51,]  98.44667  92.75322  57.96470  86.31016  69.96783  80.64546  85.59606
##  [52,] 126.24801  98.86675  63.22894  68.48538  58.62625  68.73288  88.77361
##  [53,]  83.06752 106.48254 100.52680 104.80810 107.28536  77.34952 109.64274
##  [54,] 113.63849  48.34212  75.02158  89.70021  56.73792  94.37939 105.56504
##  [55,] 113.54316  66.77960 106.07199  75.50654  68.60883  97.02732  89.81540
##  [56,]  70.46857  78.19519  75.25155 124.71660  98.84774  78.89441  86.40360
##  [57,]  99.94550 108.78787  81.08343  68.69134  96.68290 116.29106  95.63526
##  [58,] 126.86974  91.04744  99.31053  76.45773  71.90529 116.45508 116.34745
##  [59,]  94.15490  97.53614  86.11276 118.77261 110.92843  84.96560 128.71517
##  [60,]  62.70852  70.81474 118.78136 112.09595 123.86958  99.04904 109.70101
##  [61,]  78.21492  69.33837  92.62537 121.10136  94.10617  71.90476 130.46772
##  [62,] 116.71906 105.76299  73.55919  80.92647  87.93598  65.05652  64.42737
##  [63,]  74.61024  84.68519  69.34560  95.77678  99.59170  88.46113  90.62085
##  [64,]  71.04118  76.78072  71.26951 111.96095  99.40156  55.34586  88.16455
##  [65,]  91.17476 107.13775  67.48973  98.30455 112.37653  97.64689  93.59685
##  [66,]  98.44378  68.67855  96.62655  97.30631  96.61889 100.45618 104.17349
##  [67,] 103.20371  61.77677 120.32794  67.80989  80.93128 126.48856  97.01036
##  [68,] 105.46794 109.57118  66.90719  89.27467  87.69881  83.28824 110.61621
##  [69,] 100.25769  97.80409  94.84943 106.46207  89.33474  37.17153  85.17693
##  [70,] 103.64247 117.15735 113.63911  68.64706 102.80594  98.59357  78.08004
##  [71,] 118.56601  85.37700  76.68131  75.17978  72.11187 114.09072 118.04809
##  [72,]  84.32074 120.15405 116.74607 109.51273 121.25422  60.44449  90.22443
##  [73,]  77.53980  99.13272 126.26264  93.33272  92.93641 107.86620  99.03070
##  [74,]  85.12531  74.31898  73.61138 130.51501 104.26932  64.49731 113.03080
##  [75,] 121.65091  85.31212  61.08923  87.19726  71.23937  82.65452  86.10251
##  [76,] 121.07685 102.14366  66.27749  72.21342  77.30219 117.37985  98.10796
##  [77,]  65.96242  91.25371 100.44385  86.38458  97.18604 100.90707  90.77363
##  [78,]  69.47245 102.35727 110.29454  72.54709 112.27000 101.84533  75.80687
##  [79,]  82.15317  49.12885 104.45726 122.56277  83.37499  66.49312 102.27344
##  [80,]  97.38966  81.80124  56.37204 108.89248  91.38340  43.16513  81.08451
##  [81,]  81.48660 105.91248  68.00441 116.84870 107.75314  56.72506 110.84240
##  [82,] 101.36963 113.90719  96.20264  71.88515  78.60962  71.75007  74.88780
##  [83,] 102.30728  85.96444  72.57897  80.19377  74.60421  76.45079  57.81026
##  [84,]  83.28439  60.92066  85.30354  98.55983  91.17574  86.96711 116.06857
##  [85,]  43.09234  81.49855  99.59349 118.30030 114.36183  79.20735  88.01071
##  [86,]        NA  89.71818  98.43494 129.14248 128.13911  81.13975  90.04799
##  [87,]        NA        NA  99.66123  97.58600  71.01920  95.54351  99.41496
##  [88,]        NA        NA        NA  89.23159  74.06256  76.43350  79.86204
##  [89,]        NA        NA        NA        NA  52.07743 122.74654  70.59780
##  [90,]        NA        NA        NA        NA        NA 104.25291  82.02084
##  [91,]        NA        NA        NA        NA        NA        NA  76.50986
##  [92,]        NA        NA        NA        NA        NA        NA        NA
##  [93,]        NA        NA        NA        NA        NA        NA        NA
##  [94,]        NA        NA        NA        NA        NA        NA        NA
##  [95,]        NA        NA        NA        NA        NA        NA        NA
##  [96,]        NA        NA        NA        NA        NA        NA        NA
##  [97,]        NA        NA        NA        NA        NA        NA        NA
##  [98,]        NA        NA        NA        NA        NA        NA        NA
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##            [,93]     [,94]     [,95]     [,96]     [,97]     [,98]     [,99]
```

```
##   [1,]  83.60920  66.65142  93.49692  79.41308  96.69626  60.27605  79.90213
##   [2,]  65.25086 131.27959  70.72010 117.86990 103.45446 124.01544  95.40306
##   [3,]  79.45930 126.43543  76.61688 102.06273  85.78520 110.64073  92.95825
##   [4,]  78.84663 113.43439  62.02356  64.26588  92.00565  65.03972  77.60213
##   [5,]  98.49002  96.89917  99.35056  96.74880  89.80772  74.14874  61.58210
##   [6,] 105.15299  98.87970 119.76121 104.13558  64.13279 100.36931  96.88247
##   [7,]  88.39333  93.86689  77.49493 104.38589 105.09429 127.36080 108.32169
##   [8,]  94.26720  94.02370  79.23558  46.71659  68.74055  81.09390  77.46789
##   [9,] 112.47717  82.35778  83.93359  65.88484  91.74779  56.50509  61.48354
##  [10,]  67.92717 124.11068  77.99880 117.87129 105.66459 101.67934  85.33490
##  [11,]  73.44730  95.98736  95.02799 104.51631 103.13622 112.42471 100.53298
##  [12,] 116.62054 130.37148 110.60532 119.08647  52.68126  85.92735 103.95366
##  [13,] 105.30632  85.80483  94.62428 108.39831  93.25080  98.39527 116.31487
##  [14,] 111.14472 124.13428 104.23582 101.60157  59.91350  84.14575  91.78728
##  [15,]  76.93383  79.55924  83.18009 102.67016 123.39595  77.56927  83.04362
##  [16,]  66.85946  72.23146  72.04305  65.52978 101.15340  64.33207  95.27919
##  [17,]  69.62751  64.73385  77.78105  52.21006  99.69945 110.42533  99.92582
##  [18,] 103.85988  58.87375  98.01315  64.35448  81.29550  49.36028  89.75446
##  [19,]  87.71412  89.06940 101.79783 121.20840  94.01517  59.58949  92.69437
##  [20,] 121.49082  85.52018 121.45217 112.52259  78.34237  86.32929 104.19483
##  [21,] 103.23039  72.25720 104.60030 117.08590 114.85053 111.28390  76.83867
##  [22,]  93.69364 102.00630  71.14761  84.68564  98.43803  83.67811  73.56576
##  [23,]  81.07004 114.18537  77.81217  90.34858  74.87926  89.02291 111.93775
##  [24,] 141.11340  77.40691 102.40410  79.91801  72.04043  80.07208  72.49122
##  [25,]  75.05545  89.41779  94.89069 109.44226  87.83086 118.76791 117.86987
##  [26,]  94.78344  96.71023 109.53300 129.17764  88.18252  93.11778  98.40527
##  [27,]  84.47549  82.61211 111.11897  85.59465  62.98896  65.89979 112.13995
##  [28,]  86.11665  99.57291  71.30565  48.77195  81.52542  79.07741  89.82497
##  [29,]  89.76010  89.94942  77.78756  67.30661  76.05644  93.88731  97.59897
##  [30,] 109.07127 104.33945  85.89517  84.90224  80.61879 110.87352  95.33853
##  [31,]  71.63666  69.24241  89.44996  68.96616  93.94530  88.16922 105.23711
##  [32,]  89.25659 113.37643  93.42353  94.36308  58.61355  71.13189 130.20789
##  [33,] 101.81711 111.05626  82.18205  89.97771  91.99169  72.04120  70.65431
##  [34,]  82.22451 113.21563  93.27248 104.21854  72.76586  73.86307 128.68857
##  [35,]  94.16459  72.03635  80.50749  61.28794  94.00757 117.97684  86.38108
##  [36,]  45.31109  72.75360  51.22970  50.33515 112.40228  85.88695  96.39978
##  [37,]  83.35178  89.83597  82.06837  64.36647  75.12780 116.05477 114.19941
##  [38,] 103.87357 109.31680  84.19931  78.46023  71.30517  64.70563  73.90796
##  [39,]  45.84827  96.49447  68.81573  88.02466 107.01960  91.25538 109.65196
##  [40,]  92.38869  79.43526  79.52642  89.21270 105.12806  98.72019  71.23895
##  [41,]  77.72981  83.04417  72.56333  62.69862  89.08947  61.60016  92.56865
##  [42,]  72.02469  86.26311  84.66589  68.65860  86.34362 125.37709 107.64347
##  [43,]  94.16577  87.99635 101.96650  94.93948  95.95824  60.30063  77.96767
##  [44,]  92.76149 120.73411  95.81236 123.40460  92.10323 111.33952  93.50110
##  [45,]  79.32016  94.48134  64.67790  97.07600 120.83789 116.05154  79.37472
##  [46,]  78.35769  88.11189 103.86268  85.99224  69.24321  84.01794 108.95976
##  [47,]  88.63885 114.27059  64.19367  89.55664  87.67943  95.08154  98.96309
##  [48,]  56.87634  76.39602  72.88591  87.15569 116.57407  79.22971  95.23502
##  [49,]  93.75795  91.95078  90.75511 110.96738 109.16156  92.87437  50.33392
##  [50,]  60.89165 100.17023  65.72873  99.77069 123.97350  77.53483  91.37065
##  [51,]  75.87266  78.44705  62.07172  49.39444  95.19265 126.43305  98.58092
##  [52,]  94.74366 107.55749  73.29385  69.91027  84.46245 114.33857  79.78341
##  [53,]  68.55045  74.11027  77.09966  80.20322 126.91459  72.67817  76.20443
##  [54,] 129.65076  89.12092 134.69872 105.42973  54.55007 105.53584 106.55297
```

```
##  [55,]  93.77508  90.21155 121.68110 117.66885  79.71994  78.17989 116.48964
##  [56,] 101.80326  53.75030  91.60250  82.57071 102.70940  92.88487  89.22929
##  [57,] 121.46242 105.89873 102.39379 114.72982  78.44338  76.80077  70.88796
##  [58,]  85.26658  95.60634  90.54989 116.15021 103.72470 127.97359 102.08688
##  [59,]  94.02814  55.94631  93.62237  94.60021 127.98940  95.97216  66.99622
##  [60,]  85.59040  71.78567  87.16556 110.95223 127.93020  78.33910  86.36788
##  [61,] 113.57485  99.13585  91.48286  89.67389  94.31921  98.22385  65.75653
##  [62,]  82.66342  89.93922  88.18088  75.62911  78.58259 112.84847  78.21676
##  [63,] 128.99502  78.81090 102.27055  92.48866  86.20406  57.80380  72.95035
##  [64,] 110.95341  84.19097  69.78159  71.41864 103.95846  92.16148  62.25838
##  [65,]  88.35350  50.96443  94.24605  80.73494 101.73470  97.85668  79.37083
##  [66,] 111.59289  88.44177  93.21607 123.04361 104.48014 119.69621  82.98057
##  [67,]  90.46320 113.57183 104.47485 131.98251  77.45553 109.75363 115.75416
##  [68,] 102.48307  86.99090 107.21502  75.90565  75.24330  82.11811  70.01973
##  [69,]  74.74974  91.88013  70.13999  74.39913 110.79008 100.89387  70.51026
##  [70,]  83.28051 114.85994  81.43342 119.04472 101.10512  81.95409  75.42745
##  [71,] 106.33681 102.06273 101.28371  99.83714  73.54766 128.33727  91.74711
##  [72,]  54.65406  87.59729  63.03501  85.22413 134.21433  85.19316  62.26788
##  [73,]  63.30853 111.88424  74.13153 100.47623  99.51216  96.57636 101.26986
##  [74,] 105.87341  64.18847  94.80919  78.05186 100.54404 114.78609  66.65328
##  [75,]  88.11077  51.18508  99.70117  79.19941  95.96262 110.92844 104.13888
##  [76,]  89.83159  67.45025 111.14668  90.80777  79.70155 104.70672 105.37971
##  [77,] 112.80688 114.03282 100.66169 104.06631  73.08007  42.84434  81.43408
##  [78,]  76.50589 116.20891  67.61048  94.42844  93.89160  74.82404  81.51501
##  [79,]  85.57291  70.69520 100.42352  89.19607 100.90887  93.79515 104.97663
##  [80,] 104.23695  59.16346  87.94283  66.05599 100.14131  98.61427  69.96086
##  [81,] 107.42851  77.13508  83.08067  68.33679 106.61179  77.52496  41.99728
##  [82,]  60.46611 121.98420  32.57743  72.33665 108.69273 110.91974  81.58924
##  [83,]  89.43041  93.57770  70.65551  78.67238  85.93157 122.98191  94.98346
##  [84,] 119.30040  86.41111 106.10924  96.50265  88.14314  77.09571  81.23850
##  [85,]  83.89739  68.80463 100.11459  76.28648  91.21109  41.76286  91.16656
##  [86,]  90.07473  76.60132  80.34123  75.67368 102.55287  51.77501  75.51181
##  [87,] 114.12875  86.54699 117.38698 114.18049  77.25521  98.22003 117.46715
##  [88,] 116.20832  67.59473  98.01182  52.43143  66.48597  91.71608  81.66539
##  [89,]  84.20038 126.65926  89.56381 103.75213  62.62795 101.49836 118.30026
##  [90,]  96.04433 109.30021  99.38067  89.21341  54.87680 113.36103 133.19325
##  [91,]  85.67484  73.65793  73.54345  55.82123 105.46624  90.55177  58.41725
##  [92,]  80.71644  92.01369  86.75894  76.37698  68.13920  75.23803 103.48148
##  [93,]        NA  92.16461  52.55535  74.46518 120.50454  99.58914 101.15144
##  [94,]        NA        NA 106.95957  71.52948 104.00812  78.77728  86.55559
##  [95,]        NA        NA        NA  63.44327 126.92148 108.38838  72.74785
##  [96,]        NA        NA        NA        NA  92.62956  87.71014  78.21730
##  [97,]        NA        NA        NA        NA        NA  78.04346 114.46914
##  [98,]        NA        NA        NA        NA        NA        NA  84.95130
##  [99,]        NA        NA        NA        NA        NA        NA        NA
## [100,]        NA        NA        NA        NA        NA        NA        NA
##          [,100]
##   [1,]  95.62664
##   [2,]  76.76081
##   [3,]  95.63125
##   [4,]  95.14605
##   [5,] 106.00484
##   [6,]  82.45503
##   [7,]  68.28842
```

```
##   [8,]  88.81780
##   [9,] 102.48302
##  [10,]  68.25687
##  [11,]  92.86030
##  [12,] 117.11886
##  [13,]  55.46769
##  [14,] 109.70220
##  [15,]  69.55171
##  [16,]  77.34363
##  [17,]  60.92406
##  [18,] 102.91348
##  [19,]  82.73770
##  [20,] 102.51913
##  [21,]  83.40106
##  [22,] 112.57719
##  [23,]  70.22150
##  [24,]  99.11411
##  [25,]  70.60644
##  [26,]  84.92738
##  [27,]  95.47005
##  [28,]  82.54160
##  [29,]  75.97179
##  [30,]  78.95907
##  [31,]  60.16274
##  [32,]  84.92246
##  [33,] 102.60463
##  [34,]  58.50382
##  [35,]  81.70444
##  [36,]  79.28986
##  [37,] 109.05178
##  [38,] 100.34881
##  [39,]  82.76652
##  [40,]  93.75880
##  [41,]  63.08807
##  [42,]  80.65681
##  [43,]  90.21248
##  [44,]  91.04439
##  [45,]  65.52782
##  [46,] 107.07422
##  [47,]  60.69643
##  [48,]  73.72449
##  [49,]  88.49717
##  [50,]  81.69284
##  [51,]  86.51502
##  [52,]  79.05214
##  [53,]  73.52907
##  [54,]  93.37081
##  [55,]  62.48617
##  [56,]  71.80980
##  [57,] 120.66652
##  [58,]  83.57048
##  [59,]  84.98109
##  [60,]  81.57846
##  [61,]  96.84153
```

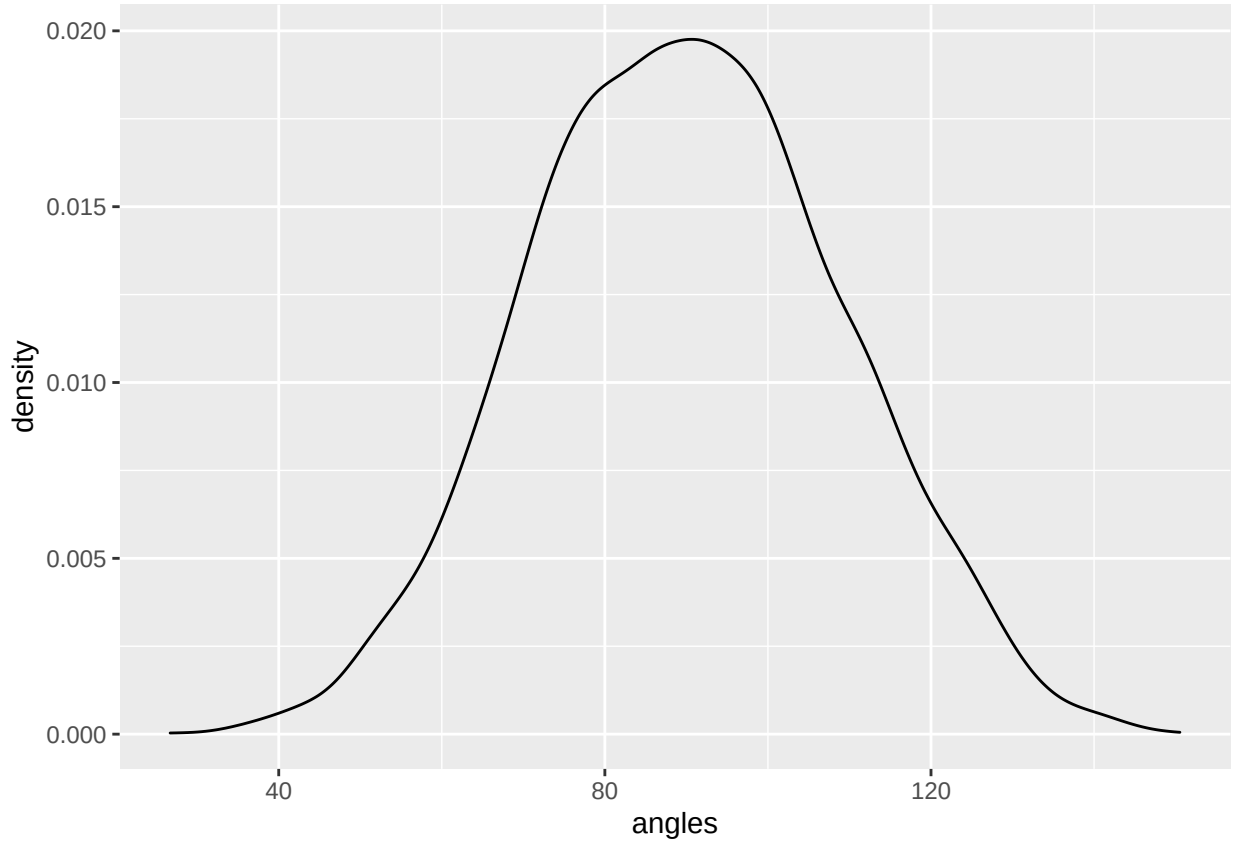
```
##  [62,]  92.89661
##  [63,]  93.71362
##  [64,]  75.61820
##  [65,] 113.38597
##  [66,]  85.89212
##  [67,] 103.37282
##  [68,] 118.80168
##  [69,]  47.68457
##  [70,]  83.92035
##  [71,] 126.76642
##  [72,]  65.11934
##  [73,]  91.85058
##  [74,]  97.14911
##  [75,]  66.77304
##  [76,] 111.87244
##  [77,] 104.22647
##  [78,] 110.12409
##  [79,]  50.73834
##  [80,]  64.88533
##  [81,]  92.02910
##  [82,]  65.76458
##  [83,]  72.91147
##  [84,]  95.42880
##  [85,]  91.59064
##  [86,]  97.81563
##  [87,]  72.49552
##  [88,] 104.47224
##  [89,]  97.31654
##  [90,]  78.03091
##  [91,]  63.07849
##  [92,]  73.13799
##  [93,]  71.02038
##  [94,]  82.15779
##  [95,]  78.31877
##  [96,]  89.74471
##  [97,] 110.05796
##  [98,]  99.71834
##  [99,] 104.87770
## [100,]        NA
```

Plot the density of these angles.

```
pacman::p_load(ggplot2)

ggplot(data.frame(angles = c(all_angles(X)))) + aes(x = angles) + geom_density()
```

```
## Warning: Removed 5050 rows containing non-finite values (stat_density).
```



Write an Rcpp function `all_angles_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
pacman::p_load(Rcpp)

cppFunction('
    NumericMatrix all_angles_cpp(NumericMatrix X) {
      int n = X.nrow();
      int p = X.ncol();

      NumericMatrix A(n, n);

      std::fill(A.begin(), A.end(), NA_REAL);

    for (int i_1 = 0; i_1 < (n - 1); i_1++){
      for (int i_2 = i_1 + 1; i_2 < n; i_2++){

        double sum_sqd_u = 0;
        double sum_sqd_v = 0;
        double sum_u_v = 0;

        for (int j = 0; j < p; j++){
          sum_sqd_u += pow(X(i_1, j), 2);
          sum_sqd_v += pow(X(i_2, j), 2);
          sum_u_v = X(i_1, j) * X(i_2, j);
          acos(sum_u_v/sqrt(sum_sqd_u * sum_sqd_v)) * (180/M_PI);
```

```
        }
      A(i_1, i_2) = acos(sum_u_v/sqrt(sum_sqd_u * sum_sqd_v)) * (180/M_PI);
    }
  }
  return A;
}
')
```

Test the time difference between these functions for `n = 1000` and `Nvec = 100, 500, 1000, 5000`. Store the results in a matrix with rows representing 'Nvec' and two columns for base R and Rcpp.

```
pacman::p_load(microbenchmark)

n = 1000
Nvec = c(100, 200, 300, 400)
time_r = c()
time_rcpp = c()
for (i in 1:length(Nvec)){
  X = c()

  for (j in 1:n){
    x = rnorm(Nvec[i])
    X  = cbind(X, x)
}
time_r = c(time_r, mean(microbenchmark(angles_r = all_angles(X), times = 3, unit = "s")$time))
time_rcpp = c(time_rcpp, mean(microbenchmark(angles_rcpp = all_angles_cpp(X), times = 3, unit = "s")$ti

A = as.matrix(cbind(time_r, time_rcpp))
}

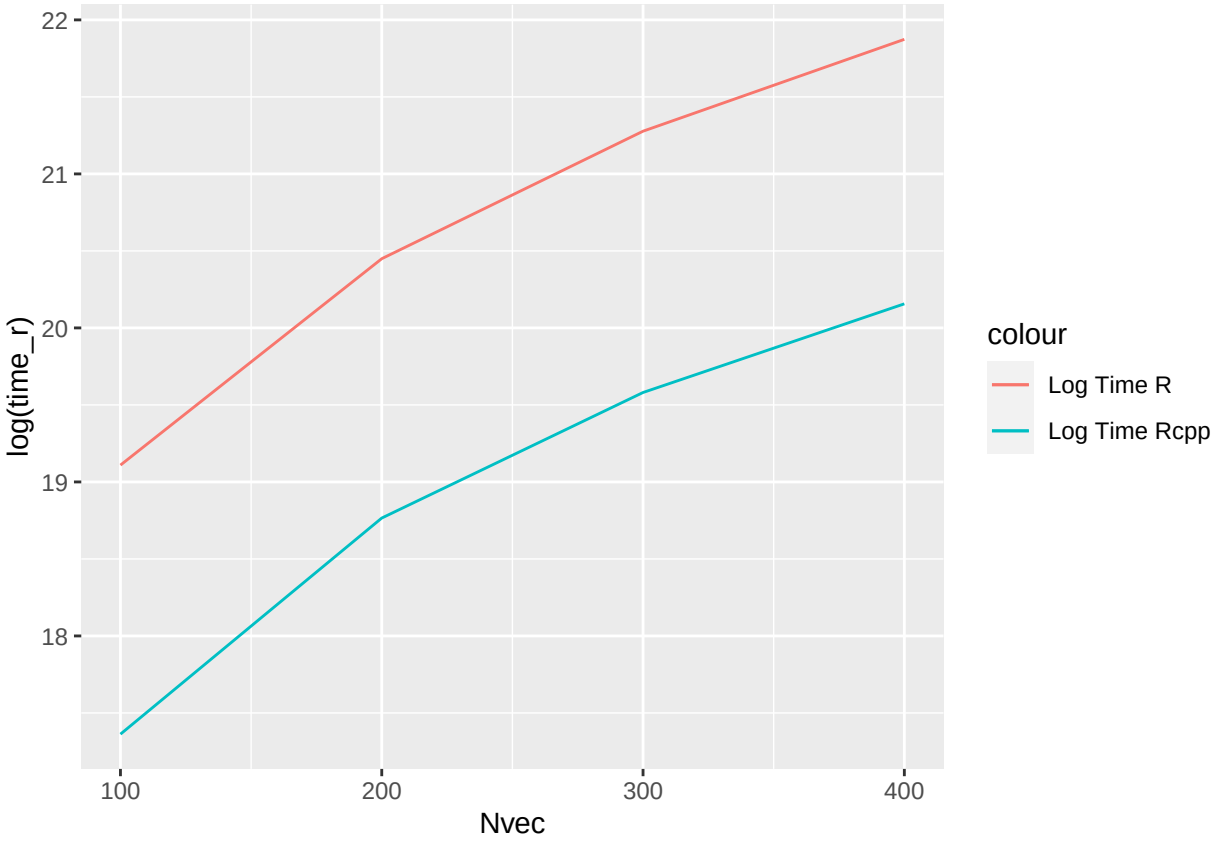
A
```

```
##          time_r time_rcpp
## [1,]  199073068  34703501
## [2,]  760445268 141178101
## [3,] 1740440134 319018101
## [4,] 3159582568 567267401
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
pacman::p_load(ggplot2)

ggplot()+
  geom_line(aes(x = Nvec, y = log(time_r), col = "Log Time R")) +
  geom_line(aes(x = Nvec, y = log(time_rcpp), col = "Log Time Rcpp"))
```



Let `Nvec = 10000` and vary `n` to be 10, 100, 1000. Plot the density of angles for all three values of `n` on one plot using color to signify `n`. Make sure you have a color legend. This is not easy.

```
Nvec = 1000
n = c(10, 100, 1000)
X = c()

for (i in 1:n[1]){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_matrix_1 = all_angles(X)

for (i in 1:n[2]){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_matrix_2 = all_angles(X)

for (i in 1:n[3]){
  x = rnorm(Nvec)
  X = cbind(X, x)
}
ang_matrix_3 = all_angles(X)

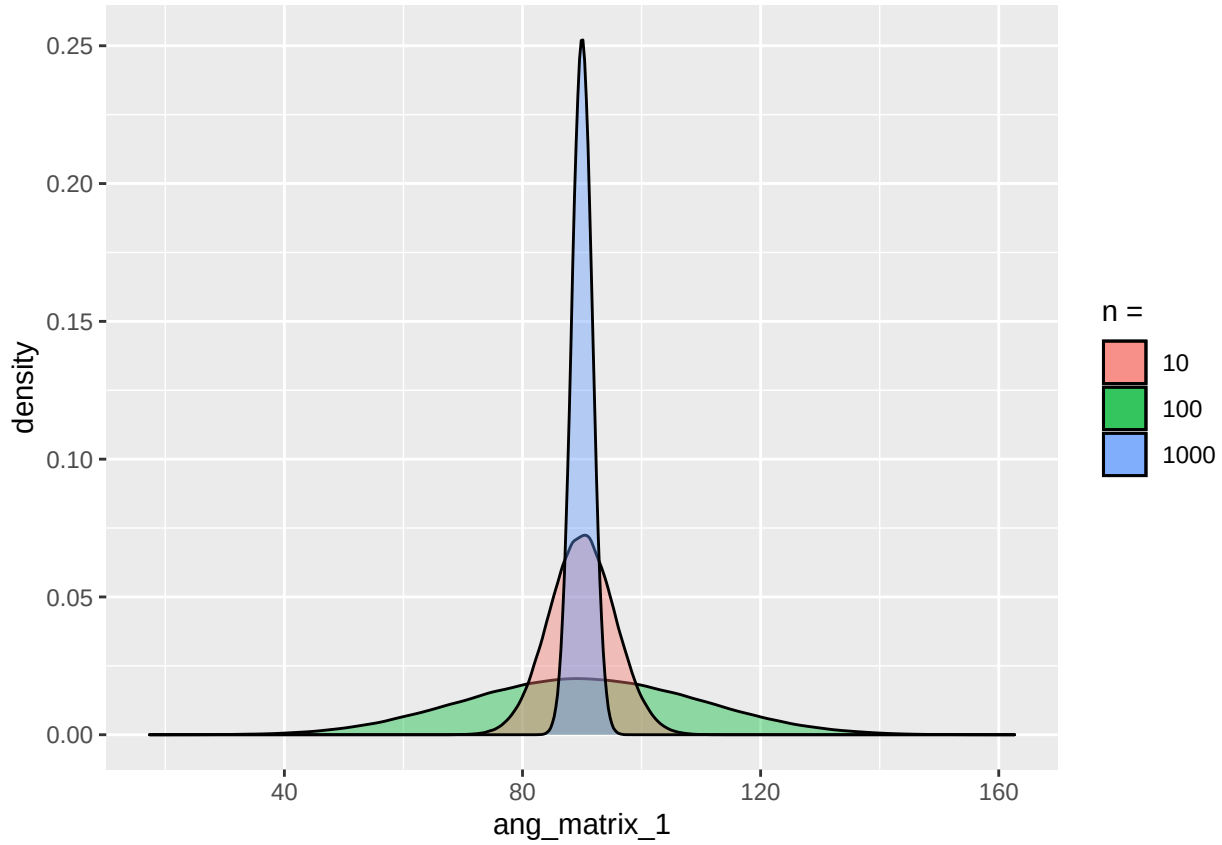
ggplot() +
```

```
  geom_density(aes(x = ang_matrix_1, fill = "green"), alpha = .4) +
  geom_density(aes(x = ang_matrix_2, fill = "dark green"), alpha = .4) +
  geom_density(aes(x = ang_matrix_3, fill = "sky blue"), alpha = .4) +
  scale_fill_discrete(name = "n =", labels = c("10", "100", "1000"))
```

```
## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).

## Warning: Removed 500500 rows containing non-finite values (stat_density).
```



Write an R function `nth_fibonacci` that finds the nth Fibonacci number via recursion but allows you to specify the starting number. For instance, if the sequence started at 1, you get the familiar 1, 1, 2, 3, 5, etc. But if it started at 0.01, you would get 0.01, 0.01, 0.02, 0.03, 0.05, etc.

```
nth_fibonacci = function(start_num = 1, n = 5){
  fib = array(0, dim = n)
    fib[1] = start_num
    fib[2] = start_num

    if (n != 1 & n != 2){
      for (i in 3:n){
        fib[i] = fib[i - 2] + fib[i - 1]
      }
    }
```

```
  fib[n]
}

nth_fibonacci(1.4, 6)
```

```
## [1] 11.2
```

Write an Rcpp function `nth_fibonacci_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
pacman::p_load(Rcpp)

cppFunction('

// Find nth Fibonacci number starting with a given number
double nth_fibonacci_cpp(double start_num, int n)
{
  if (n == 1 || n == 2) return start_num;
    else
    {
      return nth_fibonacci_cpp(start_num, n - 2) + nth_fibonacci_cpp(start_num, n -1);
      }

 }

')

nth_fibonacci_cpp(1.4, 6)
```

```
## [1] 11.2
```

Time the difference in these functions for n = 100, 200, ..., 1500 while starting the sequence at the smallest possible floating point value in R. Store the results in a matrix.

```
pacman::p_load(microbenchmark)

Nvec = c(10, 20, 30, 40, 50)
time_r = c()
time_rcpp = c()

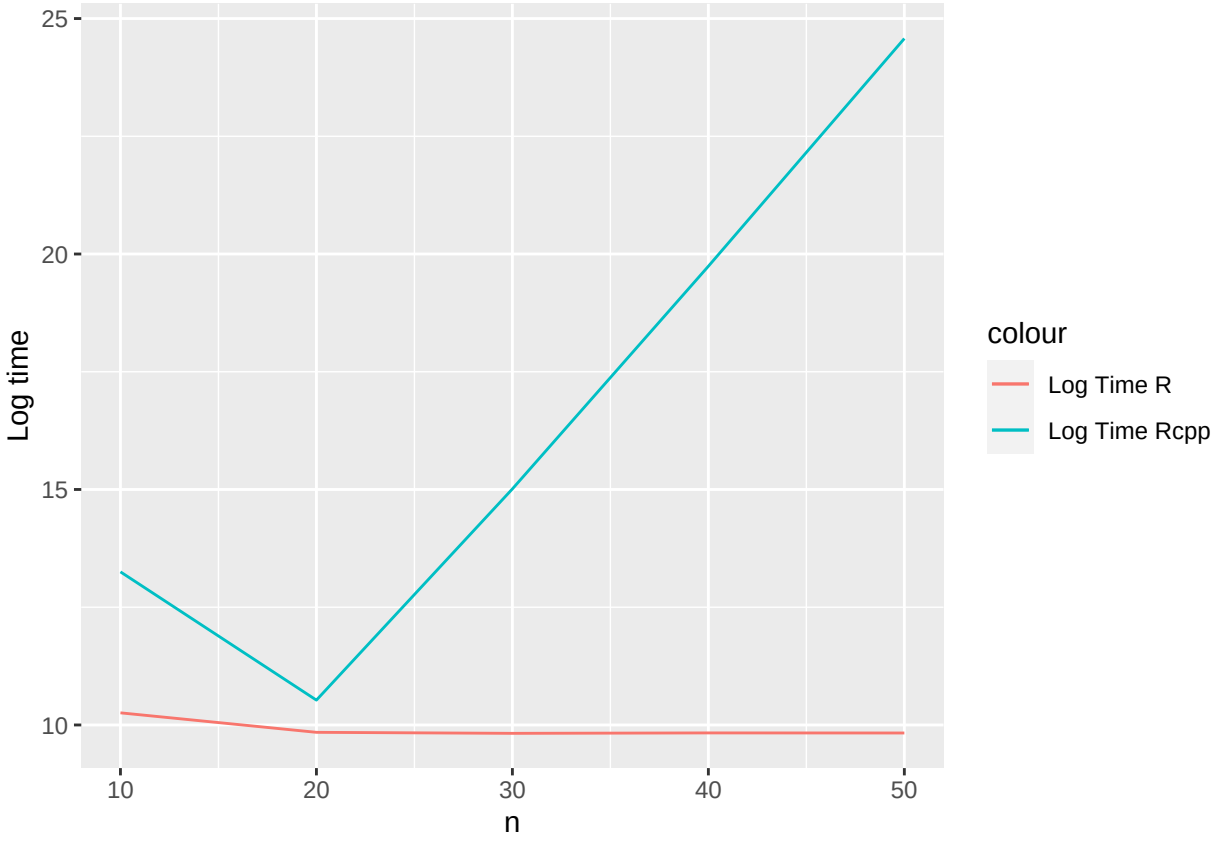
for (i in 1:length(Nvec)){
  time_r = c(time_r, mean(microbenchmark(nth_fibonacci(.Machine$double.xmin, Nvec[i]), times = 3, unit =
  time_rcpp = c(time_rcpp, mean(microbenchmark(nth_fibonacci_cpp(.Machine$double.xmin, Nvec[i]), times =
  A = as.matrix(cbind(time_r, time_rcpp))
}
A
```

```
##        time_r    time_rcpp
## [1,] 28500.67 5.687673e+05
## [2,] 18834.33 3.733433e+04
## [3,] 18434.00 3.308401e+06
## [4,] 18601.33 3.741466e+08
## [5,] 18567.67 4.708978e+10
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
pacman::p_load(ggplot2)

ggplot()+
  geom_line(aes(x = Nvec, y = log(time_r), col = "Log Time R")) +
  geom_line(aes(x = Nvec, y = log(time_rcpp), col = "Log Time Rcpp")) +
  labs(x = "n", y = "Log time")
```



Data Wrangling / Munging / Carpentry

Throughout this assignment you can use either the `tidyverse` package suite or `data.table` to answer but not base R. You can mix `data.table` with `magrittr` piping if you wish but don't go back and forth between `tbl_df`'s and `data.table` objects.

```
pacman::p_load(dplyr, tidyverse, magrittr, data.table)
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below.

```
data(storms)
str(storms)
```

```
## tibble[,13] [10,010 x 13] (S3: tbl_df/tbl/data.frame)
##  $ name       : chr [1:10010] "Amy" "Amy" "Amy" "Amy" ...
##  $ year       : num [1:10010] 1975 1975 1975 1975 1975 ...
##  $ month      : num [1:10010] 6 6 6 6 6 6 6 6 6 6 ...
##  $ day        : int [1:10010] 27 27 27 27 28 28 28 28 29 29 ...
##  $ hour       : num [1:10010] 0 6 12 18 0 6 12 18 0 6 ...
##  $ lat        : num [1:10010] 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4 34 ...
##  $ long       : num [1:10010] -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -74.8 ...
##  $ status     : chr [1:10010] "tropical depression" "tropical depression" "tropical depression" "trop
##  $ category   : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<..: 1 1 1 1 1 1 1 1 2 2 ...
##  $ wind       : int [1:10010] 25 25 25 25 25 25 25 30 35 40 ...
##  $ pressure   : int [1:10010] 1013 1013 1013 1013 1012 1012 1011 1006 1004 1002 ...
##  $ ts_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
##  $ hu_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
```

```
summary(storms)
```

```
##      name                year          month             day
##  Length:10010       Min.   :1975   Min.   : 1.000   Min.   : 1.00
##  Class :character   1st Qu.:1990   1st Qu.: 8.000   1st Qu.: 8.00
##  Mode  :character   Median :1999   Median : 9.000   Median :16.00
##                     Mean   :1998   Mean   : 8.779   Mean   :15.86
##                     3rd Qu.:2006   3rd Qu.: 9.000   3rd Qu.:24.00
##                     Max.   :2015   Max.   :12.000   Max.   :31.00
##
##       hour             lat             long            status
##  Min.   : 0.000   Min.   : 7.20   Min.   :-109.30   Length:10010
##  1st Qu.: 6.000   1st Qu.:17.50   1st Qu.: -80.70   Class :character
##  Median :12.000   Median :24.40   Median : -64.50   Mode  :character
##  Mean   : 9.114   Mean   :24.76   Mean   : -64.23
##  3rd Qu.:18.000   3rd Qu.:31.30   3rd Qu.: -48.60
##  Max.   :23.000   Max.   :51.90   Max.   :  -6.00
##
##  category       wind           pressure       ts_diameter       hu_diameter
##  -1:2545   Min.   : 10.00   Min.   : 882.0   Min.   :   0.00   Min.   :  0.00
##  0 :4373   1st Qu.: 30.00   1st Qu.: 985.0   1st Qu.:  69.05   1st Qu.:  0.00
##  1 :1685   Median : 45.00   Median : 999.0   Median : 138.09   Median :  0.00
##  2 : 628   Mean   : 53.49   Mean   : 992.1   Mean   : 166.76   Mean   : 21.41
##  3 : 363   3rd Qu.: 65.00   3rd Qu.:1006.0   3rd Qu.: 241.66   3rd Qu.: 28.77
##  4 : 348   Max.   :160.00   Max.   :1022.0   Max.   :1001.18   Max.   :345.23
##  5 :  68                                     NA's   :6528      NA's   :6528
```

```
head(storms)
```

```
## # A tibble: 6 x 13
##   name   year month   day  hour   lat  long status       category  wind pressure
##   <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>        <ord>    <int>    <int>
## 1 Amy    1975     6    27     0  27.5 -79   tropical de~ -1          25     1013
## 2 Amy    1975     6    27     6  28.5 -79   tropical de~ -1          25     1013
## 3 Amy    1975     6    27    12  29.5 -79   tropical de~ -1          25     1013
## 4 Amy    1975     6    27    18  30.5 -79   tropical de~ -1          25     1013
## 5 Amy    1975     6    28     0  31.5 -78.8 tropical de~ -1          25     1012
## 6 Amy    1975     6    28     6  32.4 -78.7 tropical de~ -1          25     1012
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

Reorder the columns so name is first, status is second, category is third and the rest are the same.

```
storms%>%
  select(name, status, category, everything())
```

```
## # A tibble: 10,010 x 13
##    name  status      category  year month   day  hour   lat  long  wind pressure
##    <chr> <chr>       <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>    <int>
##  1 Amy   tropical d~ -1        1975     6    27     0  27.5 -79      25     1013
##  2 Amy   tropical d~ -1        1975     6    27     6  28.5 -79      25     1013
##  3 Amy   tropical d~ -1        1975     6    27    12  29.5 -79      25     1013
##  4 Amy   tropical d~ -1        1975     6    27    18  30.5 -79      25     1013
##  5 Amy   tropical d~ -1        1975     6    28     0  31.5 -78.8    25     1012
##  6 Amy   tropical d~ -1        1975     6    28     6  32.4 -78.7    25     1012
##  7 Amy   tropical d~ -1        1975     6    28    12  33.3 -78      25     1011
##  8 Amy   tropical d~ -1        1975     6    28    18  34   -77      30     1006
##  9 Amy   tropical s~ 0         1975     6    29     0  34.4 -75.8    35     1004
## 10 Amy   tropical s~ 0         1975     6    29     6  34   -74.8    40     1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storms only in the 1970's.

```
storms%>%
  filter(year >= 1970 && year < 1980)
```

```
## # A tibble: 10,010 x 13
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storm observations only with category 4 and above and wind speed 100MPH and above.

```
storms%>%
  filter(category >=4 & wind >= 100)
```

```
## # A tibble: 416 x 13
##    name   year month   day  hour   lat  long status    category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>    <int>
##  1 Anita  1977     9     2     0  24.6 -96.2 hurricane 5          140      931
```

```
##  2 Anita  1977     9     2     6  24.2 -97.1 hurricane 5          150      926
##  3 Anita  1977     9     2    12  23.7 -98   hurricane 4          120      940
##  4 David  1979     8    28     0  12.2 -52.9 hurricane 4          115      947
##  5 David  1979     8    28     6  12.5 -54.4 hurricane 4          125      941
##  6 David  1979     8    28    12  12.8 -55.7 hurricane 4          130      938
##  7 David  1979     8    28    18  13.2 -56.9 hurricane 4          125      941
##  8 David  1979     8    29     0  13.7 -58   hurricane 4          120      944
##  9 David  1979     8    29     6  14.2 -59.2 hurricane 4          120      942
## 10 David  1979     8    29    12  14.8 -60.3 hurricane 4          125      938
## # ... with 406 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Create a new feature `wind_speed_per_unit_pressure`.

```
storms%>%
  mutate(wind_speed_per_unit_pressure = wind / pressure)
```

```
## # A tibble: 10,010 x 14
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Create a new feature: `average_diameter` which averages the two diameter metrics. If one is missing, then use the value of the one that is present. If both are missing, leave missing.

```
storms%>%
  rowwise()%>%
    arrange(desc(year))%>%
      mutate(average_diameter = mean(c(ts_diameter, hu_diameter), na.rm = TRUE))
```

```
## # A tibble: 10,010 x 14
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50      998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50     1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45     1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45     1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40     1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40     1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35     1002
```

```
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30     1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30     1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30     1010
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>
```

For each storm, summarize the maximum wind speed. "Summarize" means create a new dataframe with only the summary metrics you care about.

```
storms%>%
  group_by(name)%>%
    summarize(max_wind_speed = max(wind, na.rm = TRUE))
```

```
## # A tibble: 198 x 2
##    name     max_wind_speed
##    <chr>             <int>
##  1 AL011993             30
##  2 AL012000             25
##  3 AL021992             30
##  4 AL021994             30
##  5 AL021999             30
##  6 AL022000             30
##  7 AL022001             25
##  8 AL022003             30
##  9 AL022006             45
## 10 AL031987             40
## # ... with 188 more rows
```

Order your dataset by maximum wind speed storm but within the rows of storm show the observations in time order from early to late.

```
storms %>%
  group_by(name) %>%
    mutate(max_wind_by_storm = max(wind, na.rm = TRUE)) %>%
      select(name, max_wind_by_storm, everything()) %>%
        arrange(desc(max_wind_by_storm, year), month, day, hour)
```

```
## # A tibble: 10,010 x 14
## # Groups:   name [198]
##    name   max_wind_by_sto~  year month   day  hour   lat  long status   category
##    <chr>             <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>    <ord>
##  1 Gilbe~              160  1988     9     8    18  12   -54   tropica~ -1
##  2 Gilbe~              160  1988     9     9     0  12.7 -55.6 tropica~ -1
##  3 Gilbe~              160  1988     9     9     6  13.3 -57.1 tropica~ -1
##  4 Gilbe~              160  1988     9     9    12  14   -58.6 tropica~ -1
##  5 Gilbe~              160  1988     9     9    18  14.5 -60.1 tropica~ 0
##  6 Gilbe~              160  1988     9    10     0  14.8 -61.5 tropica~ 0
##  7 Gilbe~              160  1988     9    10     6  15   -62.8 tropica~ 0
##  8 Gilbe~              160  1988     9    10    12  15.3 -64.1 tropica~ 0
##  9 Gilbe~              160  1988     9    10    18  15.7 -65.4 tropica~ 0
## 10 Gilbe~              160  1988     9    11     0  15.9 -66.8 hurrica~ 1
## # ... with 10,000 more rows, and 4 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
    arrange(year, desc(wind)) %>%
      slice(1) %>%
        select(name, year)
```

```
## # A tibble: 41 x 2
## # Groups:   year [41]
##    name      year
##    <chr>    <dbl>
##  1 Caroline  1975
##  2 Belle     1976
##  3 Anita     1977
##  4 Cora      1978
##  5 David     1979
##  6 Ivan      1980
##  7 Harvey    1981
##  8 Debby     1982
##  9 Alicia    1983
## 10 Diana     1984
## # ... with 31 more rows
```

For each named storm, find its maximum category, wind speed, pressure and diameters. Do not allow the max to be NA (unless all the measurements for that storm were NA).

```
storms %>%
  group_by(name) %>%
    summarize(max_category = max(category),
           max_wind_sp = max(wind),
      max_pressure = max(pressure),
      max_ts_diam = max(ts_diameter),
      max_hu_diam = max(hu_diameter))
```

```
## # A tibble: 198 x 6
##    name     max_category max_wind_sp max_pressure max_ts_diam max_hu_diam
##    <chr>    <ord>              <int>        <int>       <dbl>       <dbl>
##  1 AL011993 -1                    30         1003        NA            NA
##  2 AL012000 -1                    25         1010        NA            NA
##  3 AL021992 -1                    30         1009        NA            NA
##  4 AL021994 -1                    30         1017        NA            NA
##  5 AL021999 -1                    30         1006        NA            NA
##  6 AL022000 -1                    30         1010        NA            NA
##  7 AL022001 -1                    25         1012        NA            NA
##  8 AL022003 -1                    30         1010        NA            NA
##  9 AL022006 0                     45         1008        69.0           0
## 10 AL031987 0                     40         1015        NA            NA
## # ... with 188 more rows
```

For each year in the dataset, tally the number of storms. "Tally" is a fancy word for "count the number of". Plot the number of storms by year. Any pattern? Storms per year seems to be increasing.

```
storms %>%
  group_by(year) %>%
  tally()
```

```
## # A tibble: 41 x 2
##     year     n
##    <dbl> <int>
##  1  1975    86
##  2  1976    52
##  3  1977    53
##  4  1978    54
##  5  1979   301
##  6  1980   161
##  7  1981   164
##  8  1982   105
##  9  1983    79
## 10  1984   236
## # ... with 31 more rows
```

For each year in the dataset, tally the storms by category.

```
storms %>%
  group_by(year, category) %>%
    tally()
```

```
## # A tibble: 233 x 3
## # Groups:   year [41]
##     year category     n
##    <dbl> <ord>    <int>
##  1  1975 -1          30
##  2  1975 0           33
##  3  1975 1           12
##  4  1975 2            9
##  5  1975 3            2
##  6  1976 -1          10
##  7  1976 0           20
##  8  1976 1           10
##  9  1976 2            9
## 10  1976 3            3
## # ... with 223 more rows
```

For each year in the dataset, find the maximum wind speed per status level.

```
storms %>%
  group_by(year, status) %>%
    tally()
```

```
## # A tibble: 123 x 3
## # Groups:   year [41]
##     year status                 n
##    <dbl> <chr>              <int>
```

```
##  1  1975 hurricane              23
##  2  1975 tropical depression    30
##  3  1975 tropical storm         33
##  4  1976 hurricane              22
##  5  1976 tropical depression    10
##  6  1976 tropical storm         20
##  7  1977 hurricane              20
##  8  1977 tropical depression    16
##  9  1977 tropical storm         17
## 10  1978 hurricane               5
## # ... with 113 more rows
```

For each storm, summarize its average location in latitude / longitude coordinates.

```
storms %>%
  group_by(name) %>%
    summarize(mean(lat), mean(long))
```

```
## # A tibble: 198 x 3
##    name     `mean(lat)` `mean(long)`
##    <chr>          <dbl>        <dbl>
##  1 AL011993       24.7         -78.0
##  2 AL012000       20.8         -93.1
##  3 AL021992       26.7         -84.5
##  4 AL021994       33.6         -79.7
##  5 AL021999       20.4         -96.4
##  6 AL022000        9.9         -28.5
##  7 AL022001       11.9         -45.3
##  8 AL022003        9.62        -43.4
##  9 AL022006       41.3         -63.5
## 10 AL031987       30.8         -88.7
## # ... with 188 more rows
```

For each storm, summarize its duration in number of hours (to the nearest 6hr increment).

```
storms %>%
  group_by(name) %>%
    tally() %>%
      mutate(duration = (n -1) * 6)
```

```
## # A tibble: 198 x 3
##    name         n duration
##    <chr>    <int>    <dbl>
##  1 AL011993     8       42
##  2 AL012000     4       18
##  3 AL021992     5       24
##  4 AL021994     6       30
##  5 AL021999     4       18
##  6 AL022000    12       66
##  7 AL022001     5       24
##  8 AL022003     4       18
##  9 AL022006     5       24
## 10 AL031987    32      186
## # ... with 188 more rows
```

For storm in a category, create a variable `storm_number` that enumerates the storms 1, 2, ... (in date order).

```
storms %>%
  group_by(category, name, year, month, day) %>%
    slice(1) %>%
      arrange(category, year, month, day) %>%
        group_by(category) %>%
        mutate(storm_number = dense_rank(paste(year, as.numeric(month), day)))
```

```
## # A tibble: 3,945 x 14
## # Groups:   category [7]
##    name    year month   day  hour   lat  long status     category  wind pressure
##    <chr>  <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>      <ord>    <int>    <int>
##  1 Amy     1975     6    27     0  27.5 -79   tropical ~ -1          25     1013
##  2 Amy     1975     6    28     0  31.5 -78.8 tropical ~ -1          25     1012
##  3 Carol~  1975     8    24    12  22.4 -69.8 tropical ~ -1          25     1011
##  4 Carol~  1975     8    25     0  21.6 -72.5 tropical ~ -1          25     1010
##  5 Carol~  1975     8    26     0  20.4 -77.7 tropical ~ -1          25     1011
##  6 Carol~  1975     8    27     0  20.4 -82.8 tropical ~ -1          25     1013
##  7 Carol~  1975     8    28     0  22   -87.5 tropical ~ -1          25     1014
##  8 Carol~  1975     8    29     0  23   -91.9 tropical ~ -1          30     1007
##  9 Carol~  1975     9     1     0  25.1 -98.3 tropical ~ -1          30     1000
## 10 Belle   1976     8     6     6  26   -72.8 tropical ~ -1          25     1012
## # ... with 3,935 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, storm_number <int>
```

Convert year, month, day, hour into the variable `timestamp` using the `lubridate` package. Although the new package `clock` just came out, `lubridate` still seems to be standard. Next year I'll probably switch the class to be using `clock`.

```
pacman::p_load(lubridate)

storms %>%
  mutate(timestamp = make_datetime(year, month, day, hour)) %>%
  select(name, timestamp, everything())
```

```
## # A tibble: 10,010 x 14
##    name  timestamp            year month   day  hour   lat  long status category
##    <chr> <dttm>              <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>  <ord>
##  1 Amy   1975-06-27 00:00:00  1975     6    27     0  27.5 -79   tropi~ -1
##  2 Amy   1975-06-27 06:00:00  1975     6    27     6  28.5 -79   tropi~ -1
##  3 Amy   1975-06-27 12:00:00  1975     6    27    12  29.5 -79   tropi~ -1
##  4 Amy   1975-06-27 18:00:00  1975     6    27    18  30.5 -79   tropi~ -1
##  5 Amy   1975-06-28 00:00:00  1975     6    28     0  31.5 -78.8 tropi~ -1
##  6 Amy   1975-06-28 06:00:00  1975     6    28     6  32.4 -78.7 tropi~ -1
##  7 Amy   1975-06-28 12:00:00  1975     6    28    12  33.3 -78   tropi~ -1
##  8 Amy   1975-06-28 18:00:00  1975     6    28    18  34   -77   tropi~ -1
##  9 Amy   1975-06-29 00:00:00  1975     6    29     0  34.4 -75.8 tropi~ 0
## 10 Amy   1975-06-29 06:00:00  1975     6    29     6  34   -74.8 tropi~ 0
## # ... with 10,000 more rows, and 4 more variables: wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

Using the `lubridate` package, create new variables `day_of_week` which is a factor with levels "Sunday", "Monday", ... "Saturday" and `week_of_year` which is integer 1, 2, ..., 52.

```
storms %>%
  mutate(timestamp = make_datetime(year, month, day, hour)) %>%
    mutate(day_of_week = weekdays(timestamp, abbreviate = TRUE)) %>%
      mutate(week_of_year = week(timestamp)) %>%
        select(name, timestamp, day_of_week, week_of_year, everything())
```

```
## # A tibble: 10,010 x 16
##    name  timestamp           day_of_week week_of_year  year month   day  hour
##    <chr> <dttm>              <chr>              <dbl> <dbl> <dbl> <int> <dbl>
##  1 Amy   1975-06-27 00:00:00 Fri                   26  1975     6    27     0
##  2 Amy   1975-06-27 06:00:00 Fri                   26  1975     6    27     6
##  3 Amy   1975-06-27 12:00:00 Fri                   26  1975     6    27    12
##  4 Amy   1975-06-27 18:00:00 Fri                   26  1975     6    27    18
##  5 Amy   1975-06-28 00:00:00 Sat                   26  1975     6    28     0
##  6 Amy   1975-06-28 06:00:00 Sat                   26  1975     6    28     6
##  7 Amy   1975-06-28 12:00:00 Sat                   26  1975     6    28    12
##  8 Amy   1975-06-28 18:00:00 Sat                   26  1975     6    28    18
##  9 Amy   1975-06-29 00:00:00 Sun                   26  1975     6    29     0
## 10 Amy   1975-06-29 06:00:00 Sun                   26  1975     6    29     6
## # ... with 10,000 more rows, and 8 more variables: lat <dbl>, long <dbl>,
## #   status <chr>, category <ord>, wind <int>, pressure <int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

For each storm, summarize the day in which it started in the following format "Friday, June 27, 1975".

```
storms %>%
  mutate(timestamp = make_datetime(year, month, day, hour)) %>%
    mutate(day_of_week = weekdays(timestamp, abbreviate = TRUE)) %>%
      mutate(week_of_year = week(timestamp)) %>%
        mutate(month_of_year = month(timestamp)) %>%
        group_by(name) %>%
          slice(1) %>%
           select(name, day_of_week, month_of_year, day, year)

data(storms)
storms %>%
  mutate(timestamp = make_datetime(year, month, day, hour)) %>%
    summarize(date(timestamp)) %>%
```

Create a new factor variable `decile_windspeed` by binning wind speed into 10 bins.

```
storms %>%
  mutate(decile_windspeed = (bin = (ntile(wind, 10))))
```

```
## # A tibble: 10,010 x 14
##    name   year month   day  hour   lat  long status      category  wind pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>    <int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25     1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25     1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25     1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25     1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25     1012
```

```
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25     1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25     1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30     1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35     1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40     1002
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, decile_windspeed <int>
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
serious_storms = storms %>%
  group_by(name, category) %>%
    filter(category >= 3)

head(serious_storms)
```

```
## # A tibble: 6 x 13
## # Groups:   name, category [3]
##   name      year month   day  hour   lat  long status    category  wind pressure
##   <chr>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>    <int>
## 1 Caroline  1975     8    31     0  24   -97   hurricane 3          100      973
## 2 Caroline  1975     8    31     6  24.1 -97.5 hurricane 3          100      963
## 3 Belle     1976     8     8    18  29.5 -75.3 hurricane 3          100      958
## 4 Belle     1976     8     9     0  30.9 -75.3 hurricane 3          105      957
## 5 Belle     1976     8     9     6  32.5 -75.2 hurricane 3          105      959
## 6 Anita     1977     9     1    18  25.2 -95.5 hurricane 3          110      945
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

In `serious_storms`, merge the variables lat and long together into `lat_long` with values `lat / long` as a string.

```
serious_storms %>%
  mutate(lat_long = paste(lat, long, separate = " ")) %>%
    summarize(name, year, month, day, hour, lat_long, status, category, wind, pressure)
```

```
## `summarise()` has grouped output by 'name', 'category'. You can override using the `.groups` argumen
```

```
## # A tibble: 779 x 10
## # Groups:   name, category [134]
##    name    category  year month   day  hour lat_long     status    wind pressure
##    <chr>   <ord>    <dbl> <dbl> <int> <dbl> <chr>        <chr>    <int>    <int>
##  1 Alberto 3         2000     8    12     6 "35.1 -56.7~ hurrica~   100      960
##  2 Alberto 3         2000     8    12    12 "35.9 -55.3~ hurrica~   110      950
##  3 Alberto 3         2000     8    12    18 "36.8 -53.8~ hurrica~   110      954
##  4 Alberto 3         2000     8    13     0 "37.4 -52  " hurrica~   105      958
##  5 Alex    3         2004     8     5     0 "38.5 -66  " hurrica~   105      957
##  6 Alex    3         2004     8     5     6 "39.5 -63.1~ hurrica~   105      957
##  7 Alex    3         2004     8     5    12 "40.8 -59.6~ hurrica~   100      962
##  8 Alicia  3         1983     8    18     6 "28.9 -95  " hurrica~   100      963
##  9 Alicia  3         1983     8    18     7 "29.1 -95.1~ hurrica~   100      962
## 10 Andrew  3         1992     8    23     0 "25.6 -71.1~ hurrica~   110      961
## # ... with 769 more rows
```

Let's return now to the original storms data frame. For each category, find the average wind speed, pressure and diameters (do not count the NA's in your averaging).

```
data(storms)

storms %>%
  group_by(category) %>%
    summarize(avg_wind = mean(wind),
           avg_pressure = mean(pressure),
           avg_hu_diam = mean(hu_diameter, na.rm = TRUE),
           avg_ts_diam = mean(ts_diameter, na.rm = TRUE))
```

```
## # A tibble: 7 x 5
##   category avg_wind avg_pressure avg_hu_diam avg_ts_diam
##   <ord>       <dbl>        <dbl>       <dbl>       <dbl>
## 1 -1           27.3        1008.         0            0
## 2 0            45.8         999.         0          160.
## 3 1            70.9         982.        57.3        278.
## 4 2            89.4         967.        78.8        282.
## 5 3           105.          954.        91.4        307.
## 6 4           122.          940.       102.         315.
## 7 5           145.          916.       120.         317.
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
storms %>%
  group_by(name) %>%
    summarize(max_category = max(category),
              max_wind = max(wind),
              max_pressure = max(pressure),
              max_hu_diam = max(hu_diameter, na.rm = TRUE),
              max_ts_diam = max(ts_diameter, na.rm = TRUE))
```

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## returning -Inf

## # A tibble: 198 x 6
```

```
##    name     max_category max_wind max_pressure max_hu_diam max_ts_diam
##    <chr>    <ord>           <int>        <int>       <dbl>       <dbl>
##  1 AL011993 -1                 30         1003        -Inf      -Inf
##  2 AL012000 -1                 25         1010        -Inf      -Inf
##  3 AL021992 -1                 30         1009        -Inf      -Inf
##  4 AL021994 -1                 30         1017        -Inf      -Inf
##  5 AL021999 -1                 30         1006        -Inf      -Inf
##  6 AL022000 -1                 30         1010        -Inf      -Inf
##  7 AL022001 -1                 25         1012        -Inf      -Inf
##  8 AL022003 -1                 30         1010        -Inf      -Inf
##  9 AL022006 0                  45         1008           0        69.0
## 10 AL031987 0                  40         1015        -Inf      -Inf
## # ... with 188 more rows
```

Calculate the distance from each storm observation to Miami in a new variable `distance_to_miami`. This is very challenging. You will need a function that computes distances from two sets of latitude / longitude coordinates.

```
MIAMI_LAT_LONG_COORDS = c(25.7617, -80.1918)
```

For each storm observation, use the function from the previous question to calculate the distance it moved since the previous observation.

```
#TO-DO
```

For each storm, find the total distance it moved over its observations and its total displacement. "Distance" is a scalar quantity that refers to "how much ground an object has covered" during its motion. "Displacement" is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

```
#TO-DO
```

For each storm observation, calculate the average speed the storm moved in location.

```
#TO-DO
```

For each storm, calculate its average ground speed (how fast its eye is moving which is different from windspeed around the eye).

```
#TO-DO
```

Is there a relationship between average ground speed and maximum category attained? Use a dataframe summary (not a regression).

```
#TO-DO
```

Now we want to transition to building real design matrices for prediction. This is more in tune with what happens in the real world. Large data dump and you convert it into X and y how you see fit.

Suppose we wish to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the `y` and identify which features you need $x_1, ...x_p$ and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to "featurize" as creatively as you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

```
data(storms)

storms_cleaned_frame = storms %>%
  group_by(name) %>%
    slice(1, 2, 3) %>%
      mutate(max_wind = max(wind),
           max_pressure = max(pressure),
           max_hu_diam = max(hu_diameter),
           max_ts_diam = max(ts_diameter),
           max_category = max(category),
           avg_wind = mean(wind),
           avg_pressure = mean(pressure)) %>%
              replace(is.na(.), 0) %>%
                slice(1) %>% ungroup

y = storms_cleaned_frame$max_wind
X = storms_cleaned_frame %>%
      select(max_category, max_pressure, max_hu_diam, max_ts_diam, avg_wind, avg_pressure)
```

Fit your model. Validate it.

```
n = nrow(X)
K = 6
test_indices = sample(1:n, n/ K)
train_indices = setdiff(1:n, test_indices)

X_test = X[test_indices, ]
y_test = y[test_indices]

X_train = X[train_indices, ]
y_train = y[train_indices]

#Train model
mod_storm = lm(y_train ~ ., data.frame(X_train))

#in-sample metrics
Rsq = summary(mod_storm)$r.squared
RMSE = summary(mod_storm)$sigma

Rsq
```

```
## [1] 0.9599379
```

```
RMSE
```

```
## [1] 1.736671
```

```
#out-of-sample metrics
y_bar = sum(y)/n
y_hat = predict(mod_storm, data.frame(X_test))
```

```
## Warning in predict.lm(mod_storm, data.frame(X_test)): prediction from a rank-
## deficient fit may be misleading
```

```
e = y_hat - y_test
SSE = sum(e^2)
SST = sum((y_test - y_bar)^2)
MSE = SSE / (n-2)
RMSE_oos = sqrt(MSE)
Rsq_oos = 1 - (SSE/SST)

Rsq_oos
```

```
## [1] 0.9751455
```

```
RMSE_oos
```

```
## [1] 0.8847367
```

Assess your level of success at this endeavor.

It appears that this model is a good fit, though with such a high R Squared (both in sample and out-of-sample) and small RMSE, I suspect I probably did something odd in the code (aka "the model probably isn't what I was going for").

The Forward Stepwise Procedure for Probability Estimation Models

Set a seed and load the `adult` dataset and remove missingness and randomize the order.

```
set.seed(1)
pacman::p_load_gh("coatless/ucidata")
data(adult)
adult = na.omit(adult)
adult = adult[sample(1 : nrow(adult)), ]
```

Copy from the previous lab all cleanups you did to this dataset.

```
adult$income = ifelse(adult$income == ">50K", 1, 0)

adult$marital_status = as.character(adult$marital_status)
adult$marital_status = ifelse(adult$marital_status == "Married-AF-spouse" | adult$marital_status == "Ma
adult$marital_status = as.factor(adult$marital_status)

adult$education = as.character(adult$education)
adult$education = ifelse(adult$education == "1st-4th" | adult$education == "Preschool", "<=4th", adult$
adult$education = as.factor(adult$education)


adult$native_country = as.character(adult$native_country)
tab = sort(table(adult$native_country))
adult$native_country = ifelse(adult$native_country %in% names(tab[tab < 50]), "Other", adult$native_cou

adult$native_country = as.factor(adult$native_country)
```

```
adult$worktype = paste(adult$occupation, adult$workclass, sep = " : ")
tab = sort(table(adult$worktype))
adult$worktype = ifelse(adult$worktype %in% names(tab[tab < 100]), "Other", adult$worktype)

adult$worktype = paste(adult$relationship, adult$marital_status, sep = " : ")
```

We will be doing model selection. We will split the dataset into 3 distinct subsets. Set the size of our splits here. For simplicitiy, all three splits will be identically sized. We are making it small so the stepwise algorithm can compute quickly. If you have a faster machine, feel free to increase this.

```
Nsplitsize = 1000
```

Now create the following variables: `Xtrain`, `ytrain`, `Xselect`, `yselect`, `Xtest`, `ytest` with `Nsplitsize` observations. Binarize the y values.

```
Xtrain = adult[1 : Nsplitsize, ]
Xtrain$income = NULL
ytrain = ifelse(adult[1 : Nsplitsize, "income"] == ">50K", 1, 0)

Xselect = adult[(Nsplitsize + 1) : (2 * Nsplitsize), ]
Xselect$income = NULL
yselect = ifelse(adult[(Nsplitsize + 1) : (2 * Nsplitsize), "income"] ==">50K", 1, 0)

Xtest = adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), ]
Xtest$income = NULL
ytest = ifelse(adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), "income"] == ">50K", 1, 0)
```

Fit a vanilla logistic regression on the training set.

```
logistic_mod = glm(ytrain ~ ., Xtrain, family = "binomial")
```

```
## Warning: glm.fit: algorithm did not converge
```

and report the log scoring rule, the Brier scoring rule.

```
p_hat_train = predict(logistic_mod, Xtrain, type = 'response')
```

```
## Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
## prediction from a rank-deficient fit may be misleading
```

```
log_score = mean(ytrain * log(p_hat_train) + (1 - ytrain) * log(1 - p_hat_train))

brier_score = mean(-(ytrain - p_hat_train)^2)

log_score
```

```
## [1] -2.90068e-12
```

```
brier_score
```

```
## [1] -8.414069e-24
```

We will be doing model selection using a basis of linear features consisting of all first-order interactions of the 14 raw features (this will include square terms as squares are interactions with oneself).

Create a model matrix from the training data containing all these features. Make sure it has an intercept column too (the one vector is usually an important feature). Cast it as a data frame so we can use it more easily for modeling later on. We're going to need those model matrices (as data frames) for both the select and test sets. So make them here too (copy-paste). Make sure their dimensions are sensible.

```
Xmm_train = data.frame(model.matrix( ~ . , Xtrain))
Xmm_select = data.frame(model.matrix( ~ . , Xselect))
Xmm_test = data.frame(model.matrix( ~ . , Xtest))


dim(Xmm_train)
```

```
## [1] 1000   96
```

```
dim(Xmm_select)
```

```
## [1] 1000   99
```

```
dim(Xmm_test)
```

```
## [1] 1000   97
```

```
#These dimensions don't make sense. The dimensions should be the same for each set. Hmm...
```

Write code that will fit a model stepwise. You can refer to the chunk in the practice lecture. Use the negative Brier score to do the selection. The negative of the Brier score is always positive and lower means better making this metric kind of like s_e so the picture will be the same as the canonical U-shape for oos performance.

Run the code and hit "stop" when you begin to the see the Brier score degrade appreciably oos. Be patient as it will wobble.

```
pacman::p_load(Matrix)
p_plus_one = ncol(Xmm_train)
predictor_by_iteration = c() #keep a growing list of predictors by iteration
in_sample_brier_by_iteration = c() #keep a growing list of briers by iteration
oos_brier_by_iteration = c() #keep a growing list of briers by iteration
i = 1
repeat {
  #TO-DO
  #wrap glm and predict calls with use suppressWarnings() so the console is clean during run

  if (i > Nsplitsize || i > p_plus_one){
    break
  }
}
```

Plot the in-sample and oos (select set) Brier score by p. Does this look like what's expected?

```
#TO-DO

ggplot() +
  aes() +
  geom_line()
```