Universidad de las Fuerzas Armadas

Software.

Cálculo Vectorial - 10376.

Deber #1

Fecha: 16 de mayo del 2023

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Determinar si u y v son ortogonales, paralelos o ninguna de las dos cosas.

24) $u = -2\hat{i} + 3\hat{j} - \hat{k}$

$v = 2\hat{i} + \hat{j} - \hat{k}$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -2 & 3 & -1 & -2 & 3 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} = -3\hat{i} - 2\hat{j} - 2\hat{k} - 6\hat{k} + \hat{i} - 2\hat{j}$$

$u \times v = -2\hat{i} - 4\hat{j} - 8\hat{k}$

$u \cdot v = \cancel{(-2+2)} + (3+1) + (-1-1)$

$u \cdot v = 4 - 2$

$u \cdot v = (-2 \cdot 2) + (3 \cdot 1) + (-1)(-1)$

$u \cdot v = -4 + 3 + 1$

$u \cdot v = 0$

R/ Son ortogonales

25) $u = \langle 2, -3, 1 \rangle$

$v = \langle -1, -1, -1 \rangle$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 2 & -3 & 1 & 2 & -3 \\ -1 & -1 & -1 & -1 & -1 \end{vmatrix} = 3\hat{i} - \hat{j} - 2\hat{k} - 3\hat{k} + \hat{i} + 2\hat{j}$$

$u \times v = \langle 4, 1, 5 \rangle$

$u \cdot v = (2)(-1) + (-3)(-1) + (1)(-1)$

$u \cdot v = -2 + 3 - 1$

$u \cdot v = 0$

R/ Son ortogonales

26· $u = \langle \cos\theta, \operatorname{sen}\theta, -1 \rangle$

$v = \langle \operatorname{sen}\theta, -\cos\theta, 0 \rangle$

$$u \times v = \left| \begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ \cos\theta & \operatorname{sen}\theta & -1 & \cos\theta & \operatorname{sen}\theta \\ \operatorname{sen}\theta & -\cos\theta & 0 & \operatorname{sen}\theta & -\cos\theta \end{array} \right| = 0 - \operatorname{sen}\theta\,\hat{j} - \cos^2\theta\,\hat{k} - \operatorname{sen}^2\theta\,\hat{k} - \cos\theta\,\hat{i}$$

$$u \times v = -\cos\theta\,\hat{i} - \operatorname{sen}\theta\,\hat{j} - \cos^2\theta\,\hat{k} - \operatorname{sen}^2\theta\,\hat{k}$$

$$u \cdot v = (\cos\theta)(\operatorname{sen}\theta) + (\operatorname{sen}\theta)(-\cos\theta) + 0$$

$$u \cdot v = \cancel{\cos\theta \operatorname{sen}\theta - \cos\theta \operatorname{sen}\theta + 0}$$

$$u \cdot v = 0$$

R./ Son ortogonales

Dan los vértices de un triángulo. Determinar si el triángulo es agudo, obtuso, recto.

29- $\overset{A}{(2,0,1)}, \overset{B}{(0,1,2)}, \overset{C}{(-0{,}5,\ 1{,}5,\ 0)}$

$$\theta' = \cos^{-1}\left(\frac{A \times B}{\|A\| \cdot \|B\|}\right)$$

$$\theta' = \cos^{-1}\left(\frac{2}{5}\right)$$

$$\theta' = 66^{\circ},42^{\circ}$$

$$\theta'' = \cos^{-1}\left(\frac{A \times C}{\|A\|\,\|C\|}\right)$$

$$\theta'' = \cos^{-1}\left(\frac{-1}{\sqrt{5} \cdot \sqrt{2{,}5}}\right)$$

$$\theta'' = 106{,}42^{\circ}$$

$$\theta''' = -(\theta' + \theta'') + 180$$

$$\theta''' = 7{,}16^{\circ}$$

$A \times B = \hat{i} \quad \hat{j} \quad \hat{k}$

$A \cdot B = (2)(0) + (0)(1) + (2)(1)$

$A \cdot B = 2$

$\|A\| = \sqrt{2^2 + 1^2}$ $\qquad \|B\| = \sqrt{0 + 1^2 + 2^2}$

$\|A\| = \sqrt{5}$ $\qquad \|B\| = \sqrt{5}$

$A \cdot C = (2)(-0{,}5) + (0)(1{,}5) + (0)(1)$

$A \cdot C = -1$

$\|C\| = \sqrt{(-0{,}5)^2 + (1{,}5)^2}$

$\|C\| = \sqrt{0{,}25 + 2{,}25}$

$\|C\| = \sqrt{2{,}50}$

$\|C\| = \sqrt{2{,}5}$

R./ Es un triángulo obtuso

a) Encontrar la proyección de u sobre v

b) Encontrar la componente del vector de u ortogonal a v

49.- $u = 2\hat{i} + \hat{j} + 2\hat{k}$

$v = 3\hat{j} + 4\hat{k}$

a) $\text{proy}_v u = \left(\frac{u \cdot v}{\|v\|^2}\right) v$

$u \cdot v = (2)(0) + (1)(3) + (2)(4)$

$u \cdot v = 0 + 3 + 8$

$u \cdot v = 11$

$\|v\| = \sqrt{0^2 + 3^2 + 4^2}$

$\|v\| = \sqrt{9 + 16}$

$\|v\| = \sqrt{25}$

$\|v\| = 5$

$\text{proy}_v u = \left(\frac{u \cdot v}{\|v\|^2}\right) v$

$\text{proy}_v u = \left(\frac{11}{25}\right)(3\hat{j} + 4\hat{k})$

$\boxed{\text{proy}_v u = \frac{33}{25}\hat{j} + \frac{44}{25}\hat{k}}$

b) $w_2 = u - w_1$

$w_2 = \langle 2-0, 1-3, 2-4 \rangle$

$\boxed{w_2 = \langle 2, -2, -2 \rangle}$

Calcular:

a) $u \times v$

b) $v \times u$

c) $v \times v$

9.- $u = \langle 7, 3, 2 \rangle$

$v = \langle 1, -1, 5 \rangle$

$u \times v = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 7 & 3 & 2 & 7 & 3 \\ 1 & -1 & 5 & 1 & -1 \end{array} \right| = 15\hat{i} + 2\hat{j} - 7\hat{k} - 3\hat{k} + 2\hat{i} - 35\hat{j}$

$= \boxed{17\hat{i} - 33\hat{j} - 10\hat{k}}$

$v \times u \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & -1 & 5 & 1 & -1 \\ 7 & 3 & 2 & 7 & 3 \end{array} \right| = -2\hat{i} + 35\hat{j} + 3\hat{k} + 7\hat{k} - 15\hat{i} - 2\hat{j}$

$= \boxed{-17\hat{i} + 33\hat{j} + 10\hat{k}}$

$$v \times v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ 1 & -1 & 5 & 1 & -1 \\ 1 & -1 & 5 & 1 & -1 \end{vmatrix} = -5\hat{\imath} + 5\hat{\jmath} - \hat{k} + \hat{k} + 5\hat{\imath} - 5\hat{\jmath}$$

$$= \boxed{0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}}$$

Calcular $u \times v$ y probar que es ortogonal tanto a u como a v.

15- $u = \hat{\imath} + \hat{\jmath} + \hat{k}$

$v = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$$u \times v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} = -\hat{\imath} + 2\hat{\jmath} + \hat{k} - 2\hat{k} - \hat{\imath} + \hat{\jmath}$$

$$= \boxed{-2\hat{\imath} + 3\hat{\jmath} - \hat{k} = w}$$

$w \cdot u = (-2)(1) + (3)(1) + (-1)(1)$

$w \cdot u = -2 + 3 - 1$

$w \cdot u = 0$

$\boxed{\text{Es ortogonal a } u}$

$w \cdot v = (-2)(2) + (3)(1) + (-1)(-1)$

$w \cdot v = (-4) + 3 + 1$

$w \cdot v = 0$

$\boxed{\text{Es ortogonal a } v}$

Encontrar $u \times v$ y un vector unitario ortogonal a u y a v

23- $u = -3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$

$v = 0,4\hat{\imath} - 0,8\hat{\jmath} + 0,2\hat{k}$

$$u \times v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ -3 & 2 & -5 & -3 & 2 \\ 0,4 & -0,8 & 0,2 & 0,4 & -0,8 \end{vmatrix} = 0,4\hat{\imath} - 2\hat{\jmath} + 2,4\hat{k} - 0,8\hat{k} - 4\hat{\imath} + 0,6\hat{\jmath}$$

$$= -3,6\hat{\imath} - 1,4\hat{\jmath} + 1,6\hat{k}$$

$0 = (-3,6\hat{\imath})(-3) + (1,4\hat{\jmath})(2) + (1,6)(-5)$

$0 = 10,8\hat{\imath} + 2,8\hat{\jmath} - 8\hat{k}$

$0 = (0,4\hat{\imath})(-3,6) + (-0,8)(-1,4) + (0,2)(1,6)$

$0 = -1,44\hat{\imath} + 1,12\hat{\jmath} + 0,32\hat{k}$

Calcular el área del paralelogramo que tiene los vectores dados como lados adyacentes.

29. $u = \langle 3, 2, -1 \rangle$
$v = \langle 1, 2, 3 \rangle$

$\|u \times v\| = $ Área paralelogramo

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & 2 & -1 & 3 & 2 \\ 1 & 2 & 3 & 1 & 2 \end{vmatrix} = 6\hat{i} - \hat{j} + 6\hat{k} - 2\hat{k} + 2\hat{i} - 9\hat{j}$$
$$= 8\hat{i} - 10\hat{j} + 4\hat{k}$$

$\|u \times v\| = \sqrt{8^2 + 10^2 + 4^2}$

$\|u \times v\| = \sqrt{64 + 100 + 16}$

$\|u \times v\| = \sqrt{180}$

$\boxed{\text{Área} = \sqrt{180}\ u^2}$

Verificar que los puntos son los vértices de un paralelogramo y calcular su área.

31. $A(0, 3, 2)$, $B(1, 5, 5)$, $C(6, 9, 5)$, $D(5, 7, 2)$.

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & 3 & 2 & 0 & 3 \\ 1 & 5 & 5 & 1 & 5 \end{vmatrix} = 15\hat{i} + 2\hat{j} - 3\hat{k} - 10\hat{i}$$
$$= 5\hat{i} + 2\hat{j} - 3\hat{k}$$

A y $B \neq \parallel$.

$$A \times C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & 3 & 2 & 0 & 3 \\ 6 & 9 & 5 & 6 & 9 \end{vmatrix} = 15\hat{i} + 12\hat{j} - 18\hat{k} - 18\hat{i}$$
$$= -3\hat{i} + 12\hat{j} - 18\hat{k}$$

A y C no son $\parallel$.

$$B \times D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 5 & 5 & 1 & 5 \\ 5 & 7 & 2 & 5 & 7 \end{vmatrix} = 10\hat{i} + 25\hat{j} + 7\hat{k} - 25\hat{k} - 35\hat{i} - 2\hat{j}$$
$$= -25\hat{i} + 23\hat{j} - 18\hat{k}$$

B y D no son $\parallel$

$$A \times D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & 3 & 2 & 0 & 3 \\ 5 & 7 & 2 & 5 & 7 \end{vmatrix} = 6\hat{i} + 10\hat{j} + 15\hat{k} - 14\hat{i}$$
$$= -8\hat{i} + 10\hat{j} - 15\hat{k}$$

A y D no son $\parallel$.

$d_{AB} = \sqrt{(0-1)^2 + (3-5)^2 + (2-5)^2}$

$d_{AB} = \sqrt{1 + 4 + 9}$

$d_{AB} = \sqrt{14}$

$d_{AC} = \sqrt{(0-6)^2 + (3-9)^2 + (2-5)^2}$

$d_{AC} = \sqrt{36 + 36 + 9}$

$d_{AC} = \sqrt{81} = 9$

$d_{AD} = \sqrt{(0-5)^2 + (3-7)^2 + (2-2)^2}$

$d_{AD} = \sqrt{25 + 16}$

$d_{AD} = \sqrt{41}$

$d_{BC} = \sqrt{(1-6)^2 + (5-9)^2 + (5-5)^2}$

$d_{BC} = \sqrt{25 + 16} +$

$d_{BC} = \sqrt{41}$

$d_{BD} = \sqrt{(1-5)^2 + (5-7)^2 + (5-2)^2}$

$d_{BD} = \sqrt{16 + 4 + 9}$

$d_{BD} = \sqrt{29}$

$d_{CD} = \sqrt{(6-5)^2 + (9-7)^2 + (5-2)^2}$

$d_{CD} = \sqrt{1 + 4 + 9}$

$d_{CD} = \sqrt{14}$

$\boxed{AB \parallel CD \ \wedge \ AD \parallel BC \ \therefore \text{ Si es paralelogramo}}$

$\vec{AB} = \langle 1, 4, 9 \rangle$

$\vec{AD} = \langle 0, 25, 16 \rangle$

$$AB \times AD = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 4 & 9 & 1 & 4 \\ 0 & 25 & 16 & 0 & 25 \end{vmatrix} = 64\hat{i} + 25\hat{k} - 225\hat{i} - 16\hat{j}$$

$$= -161\hat{i} - 16\hat{j} + 25\hat{k}$$

$\|AB \times AD\| =$ Área del paralelogramo

$\text{Área} = \sqrt{(-161)^2 + (-16)^2 + (25)^2}$

$\text{Área} = \sqrt{25921 + 256 + 625}$

$\text{Área} = \sqrt{26802}$

$\boxed{\text{Área} = 163{,}71 \ u^2}$

Calcular el área del triángulo

$A = (2, -7, 3)$
$B = (-1, 5, 8)$
$C = (4, 6, -1)$

$\vec{AB} = \langle 2+1, -7-5, 3-8 \rangle$
$\vec{AB} = \langle 3, -12, -5 \rangle$

$\vec{AC} = \langle 2-4, -7-6, 3+1 \rangle$
$\vec{AC} = \langle -2, -13, 4 \rangle$

$$\text{Área} = \frac{1}{2} | AB \times AC |$$

$$\text{Área} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & -12 & -5 & 3 & -12 \\ -2 & -13 & 4 & -2 & -13 \end{vmatrix} = -48\hat{i} + 10\hat{j} - 39\hat{k} - 24\hat{k} - 65\hat{i} - 12\hat{j}$$
$$= -113\hat{i} - 2\hat{j} - 63\hat{k}$$

$\sqrt{113^2 + 2^2 + 63^2}$
$\sqrt{12769 + 4 + 3969}$
$\sqrt{16742} \approx 129,39$

$\boxed{\text{Área}\ \Delta = 64,695\ u^2}$

Hallar un conjunto de ecuaciones paramétricas de la recta

18- La recta que pasa por el punto $(-4, 5, 2)$ y es perpendicular al plano dado por: $-x + 2y + z = 5$

Vector normal $= \langle -1, 2, 1 \rangle$

$\langle x+1, y-2, z-1 \rangle \langle a, b, c \rangle = 0$

$a(x+1) + b(y-2) + c(z-1)$

$$\boxed{\begin{aligned} x &= -1 + 4t \\ y &= 2 + 5t \\ z &= 1 + 2t \end{aligned}}$$

19.- La recta pasa por el punto $(5, -3, -4)$ y es paralela a $v = \langle 2, -1, 3 \rangle$

$$\frac{x-5}{2} = \frac{y+3}{-1} = \frac{z+4}{3}$$

$$\boxed{\begin{aligned} x &= 5 + 2t \\ y &= -3 - t \\ z &= -4 + 3t \end{aligned}}$$

Hallar una ecuación del plano.

54.- Pasa por el punto $(2, 2, 1)$ y contiene la recta dada por $\frac{x}{2} = \frac{y-4}{-1} = z$

$\bar{V} = \langle 2, -1, 1 \rangle$

$Q = (0, 4, 0)$

$PQ = (0-2, 4-2, 0-1)$

$\bar{PQ} = (-2, 2, -1)$

$$\begin{vmatrix} x-2 & 2 & -2 \\ y-2 & -1 & 2 \\ z-1 & 1 & -1 \end{vmatrix} = 0$$

$$\left|\begin{matrix} x-2 & 2 & -2 & x-2 & 2 \\ y-2 & -1 & 2 & y-2 & -1 \\ z-1 & 1 & -1 & z+1 & 1 \end{matrix}\right| = x \cancel{-2} + 4z \cancel{-4} \cancel{-2y} \cancel{+4} - 2z \cancel{+2} - 2x \cancel{+4} \cancel{+2y} \cancel{-4}$$

$$= -x + 2z = 0.$$

$$\boxed{x - 2z = 0}$$

59.- El plano pasa por los puntos $(3,2,1)$ y $(3,1,-5)$ y es perpendicular al plano $6x+7y+2z=10$.

$n = \langle 6, 7, 2 \rangle$

$AB = \langle (3-3), (2-1), (1+5) \rangle$

$\overline{AB} = \langle 0, 1, 6 \rangle$

$$n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 6 & 7 & 2 & 6 & 7 \\ 0 & 1 & 6 & 0 & 1 \end{vmatrix} = 42\hat{i} + 6\hat{k} + 2\hat{i} + 36\hat{j}$$

$$= 44\hat{i} + 36\hat{j} + 6\hat{k}$$

$n_2 = \langle 44, 36, 6 \rangle$

$Ec = 44(x-3) + 36(y-2) + 6(z-1) = 0$

$$\boxed{22(x-3) + 18(y-2) + 3(z-1) = 0}$$

Determinar si son paralelos, ortogonales, o ninguna de las dos cosas. Si no hallar el ángulo de intersección

68.- $3x + 2y - z = 7 \rightarrow u$

$x - 4y + 2z = 0 \rightarrow v$

$u = \langle 3, 2, -1 \rangle$

$v = \langle 1, -4, 2 \rangle$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & 2 & -1 & 3 & 2 \\ 1 & -4 & 2 & 1 & -4 \end{vmatrix}$$

$u \cdot v = (3 \cdot 1) + (2 \cdot (-4)) + (-1)(2)$

$u \times v = 4\hat{i} - \hat{j} - 12\hat{k} + 2\hat{k} - 4\hat{i} - 6\hat{j}$

$u \cdot v = 3 + (-8) - 2$

$u \times v = -7\hat{j} - 10\hat{k}$

$u \cdot v = -7$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{14}\sqrt{21}}\right)$

$\|u\| = \sqrt{3^2 + 2^2 + 1^2}$

$\|u\| = \sqrt{14}$

$\theta = \cos^{-1}\left(-\frac{7}{17}\right)$

$\|v\| = \sqrt{1^2 + 4^2 + 2^2}$

$\|v\| = \sqrt{21}$

$$\boxed{\theta = 114{,}09°}$$

Hallar el o los puntos de intersección del plano y la recta. Investigar si la recta se halla en el plano.

96. $5x + 3y = 17, \quad \frac{x-4}{2} = \frac{y+1}{-3} = \frac{z+2}{5}$

$\frac{x-4}{2} = t \qquad \frac{y+1}{-3} = t \qquad \frac{z+2}{5} = t$

$x - 4 = 2t \qquad y + 1 = -3t \qquad z + 2 = 5t$

$x = 2t + 4 \qquad y = -3t - 1 \qquad z = 5t - 2$

$5(2t+4) + 3(-3t-1) - 17 = 0$

$10t + \cancel{20} - 9t - \cancel{3} - \cancel{17} = 0$

$t = 0$

$x = 2(0) + 4 \qquad y = -3(0) - 1$

$x = 4 \qquad y = -1$

$\boxed{R/\ P(4, -1)}$

Hallar la distancia del punto al plano

100. $4x - 4y + 9z = 7$ $\quad (1, 3, -1)$
$4x - 4y + 9z = 18$ $\quad 3x - 4y + 5z = 6$

$$D = \frac{|3(1) + (-4)(3) + (5)(-1) + (-6)|}{\sqrt{3^2 + (-4)^2 + 5^2}}$$

$$D = \frac{|3 - 12 - 5 - 6|}{\sqrt{9 + 16 + 25}} = \frac{|-20|}{\sqrt{50}} = \frac{20}{\sqrt{50}} \approx \boxed{2{,}828\ u}$$

Verifique que los dos planos son paralelos y hallar la distancia entre ellos.

102. $4x - 4y + 9z = 7$

$4x - 4y + 9z = 18$

$n_1 = \langle 4, -4, 9 \rangle$

$n_2 = \langle 4, -4, 9 \rangle$

Paralelos: $n_1 = k n_2$

$\langle 4, -4, 9 \rangle = k \langle 4, -4, 9 \rangle$

$4 = 4k$ $\quad -4 = -4k \quad 9 = 9k$

$k = 1 \quad k = 1 \quad k = 1$

Si son paralelos

$z = \frac{7 - 4x - 4y}{9}$ $\quad$ Con $x \wedge y = 0 \rightarrow z = \frac{7}{9}$

$P(0, 0, \frac{7}{9})$

$4x - 4y + 9z - 18 = 0$

$$d = \frac{|4(0) + (-4)(0) + (9)(\frac{7}{9}) + (-18)|}{\sqrt{4^2 + 4^2 + 9^2}}$$

$$d = \frac{|-11|}{\sqrt{16 + 16 + 81}} = \frac{11}{\sqrt{113}} \approx \boxed{1.034 \ u}$$

Hallar la distancia del punto a la recta dada por medio del conjunto de ecuaciones paramétricas

106. $(1, -2, 4)$; $\quad x = 2t, \quad y = t - 3, \quad z = 2t + 2$

$t = \frac{x}{2} = \frac{y+3}{1} = \frac{z-2}{2}$ $\quad v = \langle 2, 1, 2 \rangle$

$P = (1, -2, 4)$

$$D = \frac{\| PQ \times v \|}{\| v \|}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 2 & 1 & 2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & -2 \\ 2 & 1 \end{matrix} = -4\hat{i} + 8\hat{j} + \hat{k} + 4\hat{k} - 4\hat{i} - 2\hat{j}$$

$$= -8\hat{i} + 6\hat{j} + 5k$$

$$D = \frac{\sqrt{8^2 + 6^2 + 5^2}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{\sqrt{64 + 36 + 25}}{\sqrt{4 + 1 + 4}} = \frac{\sqrt{125}}{\sqrt{9}} = \frac{\sqrt{125}}{3} \approx \boxed{3{,}726 \text{ u}}$$

a) Considerar las siguientes dos rectas en el espacio:

L_1 : $x = 4 + 5t$, $y = 5 + 5t$, $z = 1 - 4t$.

L_2 : $x = 4 + s$, $y = -6 + 8s$, $z = 7 - 3s$

i) Mostrar que estas rectas no son paralelas

$$t = \frac{x-4}{5} = \frac{y-5}{5} = \frac{z-1}{-4} \qquad n_1 = \langle 5, 5, -4 \rangle$$

$$s = \frac{x-4}{1} = \frac{y+6}{8} = \frac{z-7}{-3} \qquad n_2 = \langle 1, 8, -3 \rangle$$

$$n_1 = k n_2$$

$$\langle 5, 5, -4 \rangle = k \langle 1, 8, -3 \rangle$$

$5 = k$ $\qquad$ $5 = 8k$, $k = \frac{5}{8}$ $\qquad$ $-4 = -3k$, $4 = 3k$, $k = \frac{4}{3}$

$\boxed{\text{R./ No son paralelos } L_1 \wedge L_2.}$

ii) Mostrar que estas rectas no se cortan

$4 + 5t = 4 + s$ $\qquad$ $5 + 5t = -6 + 8s$ $\qquad$ $1 - 4t = 7 - 3s$

$5t - s = 0$ $\qquad$ $5t - 8s = -11$ $\qquad$ $-4t + 3s = 6$

$$\begin{cases} 5t - s = 0 \\ 5t - 8s = -11 \\ -4t + 3s = 6 \end{cases}$$

$$\begin{aligned} 5t - s &= 0 \\ - \quad 5t - 8s &= -11 \\ \hline 7s &= 11 \\ s &= \frac{11}{7} \end{aligned}$$

$$\begin{aligned} -5t + 8\left(\frac{11}{7}\right) &= 11 \\ -5t &= 11 - \frac{88}{7} \\ -35t &= -11 \\ t &= \frac{11}{35} \end{aligned}$$

Compruebo

$$-4\left(\frac{11}{35}\right)+3\left(\frac{11}{7}\right)=6$$

$$-\frac{44}{35}+\frac{33}{7}=6$$

$$\frac{-44+165}{35}=6$$

$$\frac{121}{35}\neq 6$$

R/ No se cortan $L_1 \wedge L_2$

iii) Mostrar que las dos rectas están en planos paralelos

$V_1=\langle 5,5,-4\rangle$

$V_2=\langle 1,8,-3\rangle$

$$V_1\times V_2=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\5&5&-4\\1&8&-3\end{vmatrix}\begin{matrix}\hat{\imath}&\hat{\jmath}\\5&5\\1&8\end{matrix}=-15\hat{\imath}-4\hat{\jmath}+40\hat{k}-5\hat{k}+32\hat{\imath}+15\hat{\jmath}$$

$$=17\hat{\imath}+11\hat{\jmath}+35\hat{k}\neq 0.$$

∴ No están en planos paralelos.

iv) Hallar la distancia entre los planos paralelos.

$D=\sqrt{(5-1)^2+(5-8)^2+(-4-3)^2}$

$D=\sqrt{4^2+3^2+7^2}$

$D=\sqrt{16+9+49}$

$D=\sqrt{74}\ u$