Universidad de las Fuerzas Armadas - ESPE

Software

Cálculo Vectorial - 10376

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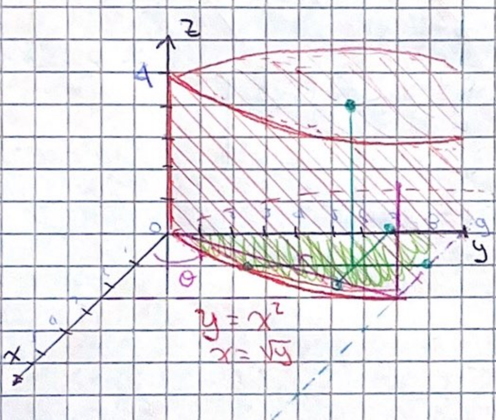
Deber # 2   Parcial # 3

Use una integral triple para hallar el volumen del sólido dado

20. El sólido acotado por el cilindro $y = x^2$ y los planos $z = 0$, $z = 4$, y $y = 9$.



$y = x^2$

$4 \geq z \geq 0$

$0 \leq z \leq 4$

$$\frac{V}{2} = \int_{x=\sqrt{y}}^{x=9} \int_{y=x^2}^{y=9} \int_{z=0}^{z=4} dz\,dy\,dx$$

$$\frac{V}{2} = \int_{x=0}^{x=\sqrt{y}} \int_{y=x^2}^{y=9} \int_{z=0}^{z=4} dz\,dy\,dx$$

$$\frac{V}{2} = \int_{0}^{\sqrt{y}} \int_{x^2}^{9} 4\,dy\,dx$$

$$\frac{V}{2} = 4\int_{0}^{\sqrt{y}} 9 - x^2\,dx = 4\left(9x - \frac{x^3}{3}\right)\Big|_{0}^{\sqrt{y}}$$

$$\frac{V}{2} = 4\left(9\sqrt{y} - \frac{(\sqrt{y})^3}{3}\right) u^3$$

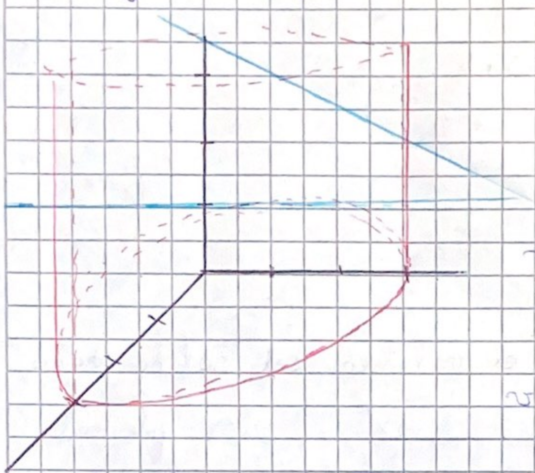
En coordenadas cilíndricas → NO

$$V = 72\sqrt{y} - \frac{y^{3/2}}{3}\,u^3$$

$$4\left[9x - \frac{x^3}{3}\right]_{-3}^{3} = 4\left[27 - 9 + 27 - 9\right]$$

$$\boxed{A = 144\,u^3}$$

21.- El sólido encerrado por el cilindro $x^2+y^2=9$ y los planos $y+z=5$ y $z=1$.



$y^2 = 9-x^2$

$y = \sqrt{9-x^2}$

$y = 5-z$

$$V=\int_{x=-3}^{x=3}\int_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}}\int_{z=1}^{z=5-y} dz\,dy\,dx$$

$$V=\int_{-3}^{3}\int_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}}(4-y)\,dy\,dx$$

$$V=\int_{-3}^{3}\left(4y-\frac{y^2}{2}\right)\Big]_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} = \int_{-3}^{3}\left(4\sqrt{9-x^2}-\cancel{\frac{9-x^2}{2}}+4\sqrt{9-x^2}+\cancel{\frac{9-x^2}{2}}\right)dx$$

$$V=\int_{-3}^{3} 8\sqrt{9-x^2}\,dx = 8\int 9\cos^2(u)$$

$u = \arcsen\left(\frac{x}{3}\right)$

$9\cos^2(u) = 9-x^2$

$x = 3\,\mathrm{sen}\,u$

$dx = 3\cos u\,du$

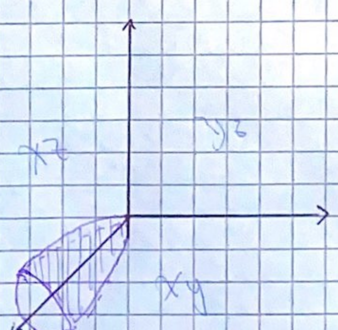
$$V=72\int_{-3}^{3}\frac{\cos(2u)+1}{2}\,du$$

$18\,\mathrm{sen}(2u)+36u$

$$\left[4x\sqrt{9-x^2}+36\arcsen\left(\frac{x}{3}\right)\right]_{-3}^{3}$$

$$\boxed{V=36\pi\ u^3}$$

22.- El sólido encerrado por el paraboloide $x=y^2+z^2$ y el plano $x=16$



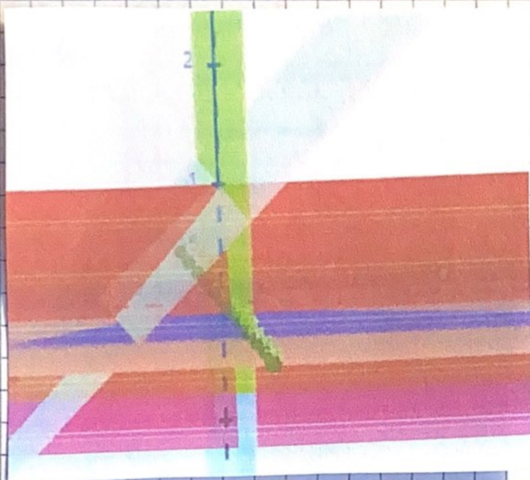
$y^2+z^2=16$

$$V=\int_{\theta=0}^{\theta=2\pi}\int_{r=-4}^{r=4}(16-r^2)\,r\,dr\,d\theta$$

$$V=\int_{\theta=0}^{\theta=2\pi}\int_{r=-4}^{r=4}16r-r^3\,dr\,d\theta = \int_0^{2\pi} 8r^2-\frac{r^4}{4}\Big]_{-4}^{4}d\theta$$

$$V=\int_0^{2\pi}64\,d\theta = 64\theta\Big]_0^{2\pi} = \boxed{128\pi\ u^3}$$

38. E está acotada por el cilindro parabólico $z = 1 - y^2$ y los planos $x + z = 1$, $x = 0$, y $z = 0$, $\rho(x, y, z) = 4$



$$\int_{y=-1}^{y=1} \int_{z=0}^{z=1-y^2} \int_{x=0}^{x=1-z} 4 \, dx \, dz \, dy$$

$$= 4 \int_{-1}^{1} \int_{0}^{1-y^2} (1 - z) \, dz \, dy$$

$$= 4 \int_{-1}^{1} z - \frac{z^2}{2} \Big]_{0}^{1-y^2} dy$$

$$= 2 \int_{-1}^{1} (1 - y^4) \, dy = \frac{16}{5}$$

$$M_{yz} = \int_{-1}^{1} \int_{0}^{1-y^2} \int_{0}^{1-z} 4x \, dx \, dz \, dy$$

$$M_{yz} = 2 \int_{-1}^{1} \int_{0}^{1-y^2} (1 - z)^2 \, dz \, dy = 2 \int_{-1}^{1} \left[-\frac{1}{3} (1 - z)^3 \right]_{0}^{1-y^2} = \frac{2}{3} \int_{-1}^{1} (1 - y^6) \, dy$$

$$M_{yz} = \frac{8}{7}$$

$$M_{xz} = \int_{-1}^{1} \int_{0}^{1-y^2} \int_{0}^{1-z} 4y \, dx \, dz \, dy = \int_{-1}^{1} \int_{0}^{1-y^2} 4y(1 - z) \, dz \, dy$$

$$M_{xz} = \int_{-1}^{1} 4y(1 - y^2) - 2y(1 - y^2)^2 \, dy = \int_{-1}^{1} (2y - 2y^5) \, dy = 0$$

$$M_{xy} = \int_{-1}^{1} \int_{0}^{1-y^2} \int_{0}^{1-z} 4z \, dx \, dz \, dy = \int_{-1}^{1} \int_{0}^{1-y^2} (4z - 4z^2) \, dz \, dy$$

$$M_{xy} = 2 \int_{-1}^{1} (1 - y^2)^2 - \frac{2}{3} (1 - y^2)^3 \, dy = 2 \int_{-1}^{1} \frac{1}{3} - y^4 + \frac{2}{3} y^6 \, dy$$

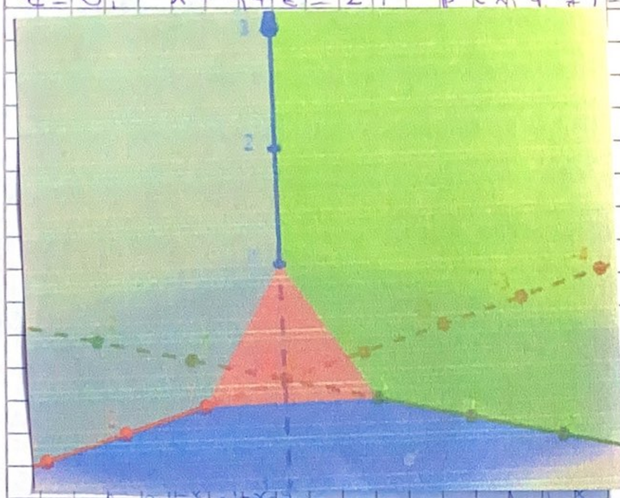
$$M_{xy} = \frac{4}{3} y - \frac{4}{5} y^5 + \frac{8}{21} y^7 \Big]_{0}^{1} = \frac{96}{105} = \frac{32}{35}$$

$$\bar{z} = \frac{\frac{32}{35}}{\frac{16}{5}} = \frac{2}{7}$$

$$\boxed{\left(\frac{5}{14}, 0, \frac{2}{7} \right)}$$

$$\bar{x} = \frac{\frac{8}{7}}{\frac{16}{5}} = \frac{5}{14}$$

40. E es el tetraedro acotado por los planos $x=0$, $y=0$, $z=0$, $x+y+z=1$; $\rho(x,y,z)=y$.



$$m=\int_0^1\int_0^{1-x}\int_0^{1-x-y} y\,dz\,dy\,dx$$

$$\int_0^1\int_0^{1-x}\left[(1-x)y-y^2\right]dy\,dx$$

$$\int_0^1\left[\frac{1}{2}(1-x)^3-\frac{1}{3}(1-x)^3\right]dx$$

$$\frac{1}{6}\int_0^1(1-x)^3\,dx=\frac{1}{24}$$

$$M_{yz}=\int_0^1\int_0^{1-x}\int_0^{1-x-y} xy\,dz\,dy\,dx=\int_0^1\int_0^{1-x}\left[(x-x^2)y-xy^2\right]dy\,dx$$

$$M_{yz}=\int_0^1 \frac{1}{2}x(1-x)^3-\frac{1}{3}x(1-x)^3\,dx=\frac{1}{6}\int_0^1(x-3x^2+3x^3-x^4)\,dx=\frac{1}{6}\left(\frac{1}{2}-1+\frac{3}{4}-\frac{1}{5}\right)$$

$$M_{yz}=\frac{1}{120}$$

$$M_{xz}=\int_0^1\int_0^{1-x}\int_0^{1-x-y} y^2\,dz\,dy\,dx=\int_0^1\int_0^{1-x}\left[(1-x)y^2-y^3\right]dy\,dx$$

$$M_{xz}=\int_0^1\frac{1}{3}(1-x)^4-\frac{1}{4}(1-x)^4\,dx=\frac{1}{12}\left[-\frac{1}{5}(1-x)^5\right]_0^1$$

$$M_{xz}=\frac{1}{60}$$

$$M_{xy}=\int_0^1\int_0^{1-x}\int_0^{1-x-y} yz\,dz\,dy\,dx=\int_0^1\int_0^{1-x}\left[\frac{1}{2}y(1-x-y)^2\right]dy\,dx=\frac{1}{2}\int_0^1\int_0^{1-x}\left[(1-x)^2y-2(1-x)y^2+y^3\right]dy\,dx$$

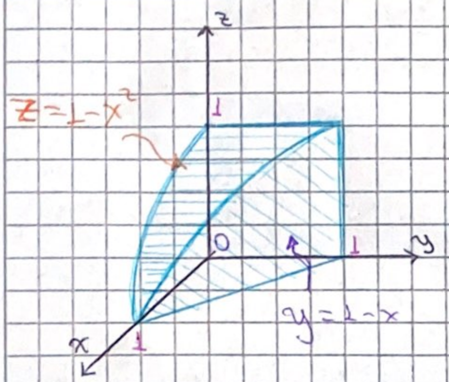
$$M_{xy}=\frac{1}{2}\int_0^1\left[\frac{1}{2}(1-x)^4-\frac{2}{3}(1-x)^4+\frac{1}{4}(1-x)^4\right]dx$$

$$\frac{1}{24}\int_0^1(1-x)^4\,dx=\frac{1}{24}\left[-\frac{1}{5}(1-x)^5\right]_0^1=\frac{1}{120}$$

$$\bar{x}=\frac{\frac{1}{120}}{\frac{1}{24}}=\frac{1}{5}\qquad \bar{y}=\frac{\frac{1}{60}}{\frac{1}{24}}=\frac{2}{5}\qquad \bar{z}=\frac{\frac{1}{120}}{\frac{1}{24}}=\frac{1}{5}$$

$$\boxed{\left(\frac{1}{5},\frac{2}{5},\frac{1}{5}\right)}$$

34- La figura muestra la región de integración para la integral.



$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\, \rho(x,y,z)\, dy\, dz\, dx$$

$$M_{yz} = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} x\, f(x,y,z)\, \rho(x,y,z)\, dy\, dz\, dx$$

$$M_{xz} = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} y\, f\, \rho\, dy\, dz\, dx$$

$$M_{xy} = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} z\, f\, \rho\, dy\, dz\, dx$$

$$\bar{x} = \frac{M_{yz}}{m}, \quad \bar{y} = \frac{M_{xz}}{m}, \quad \bar{z} = \frac{M_{xy}}{m}$$

Calcule la superficie que está sobre el plano XY de los sólidos anteriores

38. El cilindro parabólico $z = 1 - y^2$ y los planos $x + z = 1$, $x = 0$ y $z = 0$, $\rho(x,y,z) = 4$.

$$A = \int_{y=-1}^{y=1} \int_{x=0}^{x=y^2} dx\, dy = \int_{-1}^{1} y^2 - 0\, dy = \int_{-1}^{1} y^2\, dy$$

$$A = \frac{y^3}{3}\Big|_{-1}^{1} = \frac{y^3}{3}\Big|_{-1}^{1} = \frac{1^3}{3} - \frac{(-1)^3}{3} = \frac{1}{3} + \frac{1}{3}$$

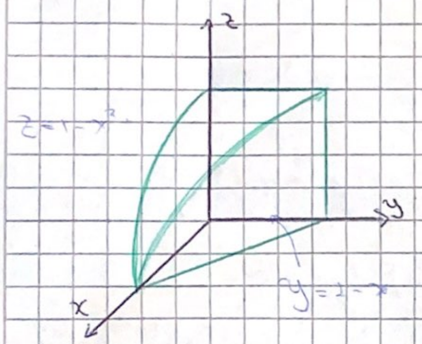
$$\boxed{A = \frac{2}{3}\, u^2}$$

39- El tetraedro acotado por los planos $x=0$, $y=0$, $z=0$, $x+y+z=1$; $\rho(x,y,z) = y$

$$A = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x} dy\, dx = \int_0^1 1 - x - 0\, dx = x - \frac{x^2}{2}\Big|_0^1$$

$$A = 1 - \frac{1}{2} - 0 = 0 = \boxed{\frac{1}{2}\, u^2 = A}$$

34. La Figura que se muestra.



$$A = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x^2} dy\,dx$$

$$A = \int_0^1 1 - x^2 \, dx$$

$$A = x - \frac{x^3}{3} \Big|_0^1$$

$$A = 1 - \frac{1}{3} - 0 - 0$$

$$A = \frac{3-1}{3}$$

$$\boxed{A = \frac{2}{3} \, u^2}$$