Universidad de las Fuerzas Armadas – ESPE

Cálculo Vectorial - 10376

Taller #1 Parcial #3

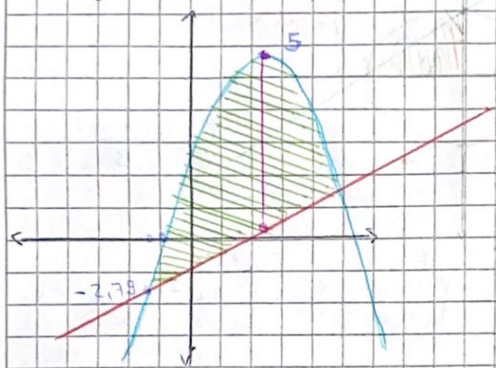
Alejandro Andrade
Josué Merino
Allan Panchi
Álex Trejo
Sebastián Verdugo

01 de agosto de 2023

1- El área limitada por las funciones:

$y = 4x - x^2 + 1$

$y = x - 2.$



$y = y$

$4x - x^2 + 1 = x - 2$

$3x - x^2 + 3 = 0$

$-x^2 + 3x + 3 = 0$

$x^2 - 3x - 3 = 0$

$x = \dfrac{+3 \pm \sqrt{9 - 4(-3)}}{2}$

$x = \dfrac{3 \pm \sqrt{21}}{2}$

$x_1 = \dfrac{3 - \sqrt{21}}{2}$ $\qquad$ $x_2 = \dfrac{3 + \sqrt{21}}{2}$

$x_1 = -0,79$ $\qquad$ $x_2 = 3,79$

$$A = \int_{x=-0,79}^{x=3,79} \int_{y=x-2}^{y=4x-x^2+1} dy\,dx =$$

$$A = \int_{-0,79}^{3,79} 4x - x^2 + 1 - x + 2 . = \int_{-0,79}^{3,79} 3x - x^2 + 3 \quad dx$$

$$A = \left[\frac{3x^2}{2} - \frac{x^3}{3} + 3x\right]_{-0,79}^{3,79} = \frac{3(3,79)^2}{2} - \frac{(3,79)^3}{3} + 3(3,79) - \frac{3(-0,79)^2}{2} + \frac{(-0,79)^3}{3} - 3(-0,79)$$

$$A = 21,546 - 18,147 + 11,37 - 1,195 - 0,164 + 2,37$$

$$\boxed{A = 15,79\ u^2}$$

b. La posición del centroide

$$M_x = \int_{-0,79}^{3,79} \int_{x-2}^{4x-x^2+1} y \, dy \, dx$$

$$M_x = \int_{-0,79}^{3,79} \frac{1}{2}(4x-x^2+1)^2 - \frac{1}{2}(x-2)^2 \, dx$$

$$M_x = \int_{-0,79}^{3,79} \frac{1}{2}x^4 - 4x^3 + 7x^2 + 4x + \frac{1}{2} - \frac{1}{2}x^2 + 2x - 2 \, dx$$

$$M_x = \int_{-0,79}^{3,79} \frac{1}{2}x^4 - 4x^3 + \frac{13}{2}x^2 + 6x - \frac{3}{2} \, dx$$

$$M_x = \frac{x^5}{10} - x^4 + \frac{13x^3}{6} + 3x^2 - \frac{3}{2}x \Big|_{-0,79}^{3,79}$$

$$M_x = \frac{3,79^5}{10} - 3,79^4 + \frac{13(3,79)^3}{6} + 3(3,79)^2 - \frac{3}{2}(3,79) - \left[\frac{-0,79^5}{10} - (-0,79)^4 + \frac{13(-0,79)^3}{6} + 3(-0,79)^2 - \frac{3}{2}(-0,79)\right]$$

$$M_x = 27,23 - 1,56$$

$$M_x = 25,67 \; u^3$$

$$M_y = \int_{x=-0,79}^{3,79} \int_{y=x-2}^{y=4x-x^2+1} x \, dy \, dx$$

$$M_y = \int_{-0,79}^{3,79} \int_{x-2}^{4x-x^2+1} x \, dy \, dx = \int_{0,79}^{3,79} x(-x^2+3x+3) \, dx$$

$$M_y = \int_{-0,79}^{3,79} -x^3 + 3x^2 + 3x \, dx$$

$$M_y = -\frac{x^4}{4} + x^3 + \frac{3x^2}{2} \Big]_{-0,79}^{3,79} = 24,40 - 0,34 = 24,06 \; u^3$$

$$\boxed{\bar{x} = \frac{24,06}{15,79} = 1,52} \qquad \boxed{\bar{y} = \frac{25,67}{15,79} = 1,62}$$

c. El volumen generado por rotación del área alrededor de los ejes x, y y alrededor de la recta que limita cada región

Alrededor de x $= 2\pi \cdot M_x$

$= 2\pi(25,67)$

$\boxed{= 161,289\ u^3}$

Alrededor de y $= 2\pi \cdot M_y$

$= 2\pi(24,06)$

$\boxed{= 151,173\ u^3}$

$$\bar{d} = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}} = \frac{|-(1,5) + (1,6) + 2|}{\sqrt{1^2 + (-1)^2}} = \frac{21\sqrt{2}}{20} = 1,4849$$

Alrededor de L $= A \cdot 2\pi \cdot 1,485$

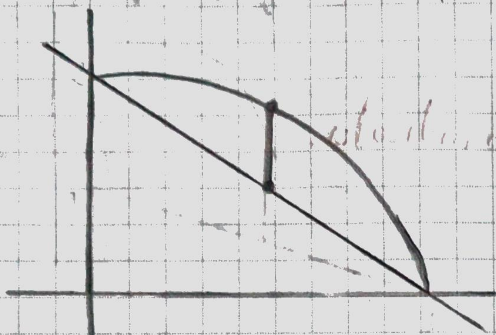
$= (15,79)(2\pi)(1,485)$

$\boxed{= 147,329\ u^3}$

2. El área en el primer cuadrante limitada por:

$$\begin{cases} x = 3\cos t \\ y = 2\operatorname{sen} t \end{cases} \qquad y = \frac{6-2x}{3}$$

a) El area de la región indicada



$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \to y^2 = 4 - \frac{4x^2}{9} \to y = \sqrt{\frac{36-4x^2}{9}} \to \frac{2}{3}\sqrt{9-x^2}$$

$$y^2 = \left(\frac{6-2x}{3}\right)^2 \qquad \to \frac{2}{3}\sqrt{9-x^2}$$

$$y^2 = y^2 \to \frac{36-4x^2}{\cancel{9}} = \frac{36-24x+4x^2}{\cancel{9}}$$

$$\cancel{36} - 4x^2 = \cancel{36} - 24x + 4x^2$$

$$0 = -24x + 8x^2$$

$$0 = -3 + x$$

$$3 = x$$

$$A = \int_0^3 \int_{\frac{6-2x}{3}}^{\frac{2}{3}\sqrt{9-x^2}} dy\,dx$$

$$A = \int_0^3 \frac{2}{3}\sqrt{9-x^2} - \left(\frac{6-2x}{3}\right)dx \to \frac{2}{3}\int_0^3 \left(\sqrt{9-x^2} - 2 + \frac{2}{3}x\right)dx$$

$$= \frac{2}{3}\left(\int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 2\,dx + \int_0^3 \frac{2}{3}x\,dx\right)$$

$$\frac{2}{3}\left(\frac{1}{2}\left[9\,\text{sen}^{-1}\left(\frac{x}{3}\right)+x\sqrt{9-x^2}\right]\Big|_0^3 - 2x\Big|_0^3 + \frac{1}{3}x^2\Big|_0^3\right)$$

$$\frac{2}{3}\left[\frac{9}{2}\,\text{sen}^{-1}\left(\frac{3}{3}\right)+\frac{3}{2}\sqrt{9-9}-\frac{9}{2}\,\text{sen}^{-1}(0)-0\sqrt{9}-2(3)+3\right]$$

$$\frac{\cancel{2}}{\cancel{3}_1}\left[\frac{\cancel{9}^{3}}{\cancel{2}}\cdot\frac{1}{2}\pi - 6 + 3\right] \to \frac{3}{2}\pi - 6 + 3 \to \frac{3}{2}\pi - 3\ u^2$$

b) Posición del centroide de la región indicada.

$$M_x = \int_0^3 \int_{2-\frac{2}{3}x}^{\frac{2}{3}\sqrt{9-x^2}} y\,dy\,dx$$

$$= \frac{1}{2}\int_0^3 \left(\frac{2}{3}\sqrt{9-x^2}\right)^2 - \left(2-\frac{2}{3}x\right)^2 dx$$

$$= \frac{1}{2}\int_0^3 \left(\frac{4}{9}(9-x^2)\right) - \left(4-\frac{8}{3}x+\frac{4}{9}x^2\right) dx$$

$$= \frac{1}{2}\int_0^3 \left(\cancel{4}-\frac{4}{9}x^2-\cancel{4}+\frac{8}{3}x-\frac{4}{9}x^2\right) dx$$

$$= \frac{1}{2}\int_0^3 \left(-\frac{8}{9}x^2+\frac{8}{3}x\right) dx$$

$$= \frac{1}{2}\left[-\frac{8}{27}x^3+\frac{4}{3}x^2\right]\Big|_0^3 dx$$

$$= \frac{1}{2}(-8+12) \to \frac{1}{2}\cdot 4 \to 2$$

$$M_y = \int_0^3 \int_{2-\frac{2}{3}x}^{\frac{2}{3}\sqrt{9-x^2}} x\,dy\,dx$$

$$= \int_0^3 \left(\frac{2}{3}x\sqrt{9-x^2}\right) - \left(2x-\frac{2}{3}x^2\right) dx$$

$$= \int_0^3 \frac{2}{3}x\sqrt{9-x^2}\,dx - \int_0^3 2x + \frac{2}{3}x^2\,dx$$

$$= -\frac{1}{3}\int_0^3 -2x\sqrt{9-x^2}\,dx - \int_0^3 2x + \frac{2}{3}x^2\,dx$$

$$-\frac{1}{3}\left(\frac{2}{3}(9-x^2)\sqrt{9-x^2}\right)\Big|_0^3 - \left(x^2 + \frac{2}{9}x^3\right)\Big|_0^3$$

$$-\frac{1}{3}(-18) - (9+6) \rightarrow 6-3 \rightarrow 3$$

$$\bar{x} = \frac{3}{\frac{3}{2}\pi - 3} \rightarrow \bar{x} = 1{,}75$$

$$C(1{,}75;\ 1{,}17)$$

$$\bar{y} = \frac{2}{\frac{3}{2}\pi - 3} \rightarrow \bar{y} = 1{,}17$$

c) El volumen generado por rotación del area alrededor de los ejes x, y y alrededor de la recta que limita cada región

$$V = 2\pi \cdot A \cdot \bar{d}$$

$$V_x = 2\pi \cdot M_x \qquad V_y = 2\pi\, M_y$$

$$V_x = 2\pi \cdot 2 \qquad = 2\pi \cdot 3$$

$$= 4\pi \qquad = 6\pi$$

$$d = \frac{|Ax + By + C|}{\sqrt{A^2+B^2}} = \frac{|2(1{,}75) + 3(1{,}17) - 6|}{\sqrt{2^2+3^2}} = 0{,}27$$

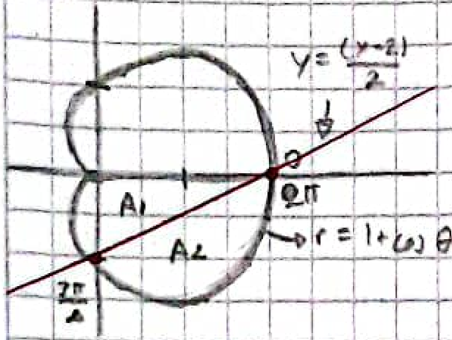
$$V = 2\pi\left(\frac{3\pi}{2} - 3\right)(0{,}27)$$

$$V = 2{,}90\ u^3$$

③ El área en el cuarto cuadrante limitada por:

$$\begin{cases} r = 1 + \cos\theta \\ y = (x-2)/2 \end{cases}$$

a) El área de la región indicada



$$\begin{cases} r = 1 + \cos\theta \\ y = (x-2)/2 \end{cases} \rightarrow \frac{(r\cos\theta - 2)}{2}, \; y = r\,\text{sen}\,\theta$$

$$r\,\text{sen}\,\theta = \frac{r\cos - 2}{2}$$

$$2r\,\text{sen}\,\theta = r\cos\theta - 2$$

$$r = \frac{-2}{2\,\text{sen}\,\theta - \cos\theta}$$

$$A = \int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} \int_{r=\frac{2}{2\text{sen}\,\theta - \cos\theta}}^{r=1+\cos\theta} r\,dr\,d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \frac{r^2}{2} \Big|_{\frac{-2}{2\text{sen}\,\theta - \cos\theta}}^{1+\cos\theta} d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \frac{1}{2}\left((1+\cos\theta)^2 - \left(-\frac{2}{2\text{sen}\,\theta - \cos\theta}\right)^2\right) d\theta$$

$$= \frac{1}{2}\left[\int_{\frac{3\pi}{2}}^{2\pi} (1+\cos\theta)^2 d\theta - \int_{\frac{3\pi}{2}}^{2\pi} \frac{4}{(2\text{sen}\,\theta - \cos\theta)^2} d\theta\right]$$

$$= \frac{1}{2}\left[\int_{\frac{3\pi}{2}}^{2\pi} (1 + 2\cos\theta + \cos^2\theta)\, d\theta - \int_{\frac{3\pi}{2}}^{2\pi} \frac{\frac{4}{\text{sen}^2\theta}}{\frac{2\text{sen}\,\theta}{\text{sen}\,\theta} - \frac{\cos\theta}{\text{sen}\,\theta}} d\theta\right.$$

$$= \frac{1}{2}\left[\int_{\frac{3\pi}{2}}^{2\pi} \left(1 + 2\cos\theta + \frac{1+\cos(2\theta)}{2}\right) d\theta - \int_{\frac{3\pi}{2}}^{2\pi} \frac{4\csc^2\theta}{(2-\text{ctg}\,\theta)^2} d\theta\right.$$

$$u = \cot\theta$$

$$du = -\csc^2\theta\, d\theta \rightarrow -du = \csc^2\theta\, d\theta$$

si $\theta = 2\pi \rightarrow u = \infty$, si $\theta = \frac{3\pi}{2} \rightarrow u = 0$

$$= \frac{1}{2}\left\{\left[\theta\Big|_{\frac{3\pi}{2}}^{2\pi} + 2\,\text{sen}\,\theta\Big|_{\frac{3\pi}{2}}^{2\pi} + \frac{1}{2}(\theta\,\text{sen}(2\theta))\Big|_{\frac{3\pi}{2}}^{2\pi}\right] - \left[\int_0^\infty \frac{4(-du)}{(2-u)^2}\right]\right\}$$

$$= \frac{1}{2}\left\{\left[\left(2\pi - \frac{3\pi}{2}\right) + 2\left(\text{sen}(2\pi) - \text{sen}\left(\frac{3\pi}{2}\right)\right) + \frac{1}{2}\left(\left(2\pi - \frac{3\pi}{2}\right) + \left(\text{sen}(2(2\pi)) - \text{sen}\left(2\left(\frac{3\pi}{2}\right)\right)\right)\right)\right] - \right.$$

$$\left.\left[\lim_{b\to\infty}\left(-4\int_0^b \frac{1}{(2-u)^2} du\right)\right]\right\}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} + 2(1) + \frac{1}{2}\left(\frac{\pi}{2}\right)\right) - \left(-4 \lim_{b\to\infty} \frac{1}{2-u}\Big|_0^b\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} + 2 + \frac{\pi}{4}\right) - \left(-4 \lim_{b\to\infty}\left(\frac{1}{2-b} - \frac{1}{2}\right)\right)\right]$$

$$= \frac{1}{2}\left[\left(2 + \frac{3\pi}{4}\right) - \left(-4\left(-\frac{1}{2}\right)\right)\right] \rightarrow \frac{1}{2}\left(2 + \frac{3\pi}{4} - 2\right) = \frac{3\pi}{8}$$

b) La posición del centroide de la región indicada

$$M_x = \int_{\frac{3\pi}{2}}^{2\pi} \int_{-\frac{2}{2\sin\theta - \cos\theta}}^{1+\cos\theta} \overbrace{r \cdot \sin\theta}^{y} \cdot r \, dr \, d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \int_{-\frac{2}{2\sin\theta-\cos\theta}}^{1+\cos\theta} r^2 \sin\theta \, dr \, d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} r^3/3 \Big|_{-\frac{2}{2\sin\theta-\cos\theta}}^{1+\cos\theta} \sin\theta \, d\theta$$

$$= \frac{1}{3} \int_{\frac{3\pi}{2}}^{2\pi} \sin\theta \left((1+\cos\theta)^3 - \left(-\frac{2}{2\sin\theta - \cos\theta}\right)^3 \right) d\theta$$

$$= \frac{1}{3} \int_{\frac{3\pi}{2}}^{2\pi} \sin\theta (1+\cos\theta)^3 \, d\theta - \int_{\frac{3\pi}{2}}^{2\pi} -\frac{8\sin\theta}{(2\sin\theta - \cos\theta)^3} \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta \to -du = \sin\theta \, d\theta$

si $\theta = 3\pi/2 \to u = 0$

si $\theta = 2\pi \to u = 1$

$$① = \int_0^1 (1+u)^3 \cdot (-du) = -\int_0^1 (1+u)^3 du = -\frac{(1+u)^4}{4}\Big|_0^1 = -\frac{1}{4}\left((1+1)^4 - (1+0)^4\right) = -\frac{1}{4}(16-1) = -\frac{15}{4}$$

$$② = \int_{\frac{3\pi}{2}}^{2\pi} \frac{-8\sin\theta}{\left(\frac{2\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^3} d\theta = -8 \int_{\frac{3\pi}{2}}^{2\pi} \frac{\csc^2\theta}{(2-\operatorname{ctg}\theta)^3} \, d\theta$$

$u = \operatorname{ctg}\theta$

$du = -\csc^2\theta \, d\theta \to -du = \csc^2\theta \, d\theta$

si $\theta = 2\pi \to u = \infty$

si $\theta = 3\pi/2 \to u = 0$

$$= -8 \int_0^\infty \frac{1}{(2-u)^3} (-du) = -8 \lim_{b\to\infty} \int_0^b \frac{1}{(2-u)^3} du = -8 \lim_{b\to\infty} \left(\frac{1}{2(2-u)^2}\right)\Big|_0^b = -8 \lim_{b\to\infty} \left(\frac{1}{\infty} - \frac{1}{8}\right)$$

$$= 8\left(-\frac{1}{8}\right) = -1$$

$$M_x = \frac{1}{3}\left(-\frac{15}{4} + 1\right) = -\frac{11}{12} \,//$$

$$M_y = \int_{\frac{3\pi}{2}}^{2\pi} \int_{-\frac{2}{2\sin\theta - \cos\theta}}^{1+\cos\theta} r^2 \cos\theta \, dr \, d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} r^3/3 \Big|_{-\frac{2}{2\sin\theta-\cos\theta}}^{1+\cos\theta} \sin\theta \, d\theta$$

$$= \frac{1}{3} \int_{\frac{3\pi}{2}}^{2\pi} \cos\theta \left((1+\cos\theta)^3 - \left(-\frac{2}{2\sin\theta - \cos\theta}\right)^3 \right) d\theta$$

$$= \frac{1}{3} \int_{\frac{3\pi}{2}}^{2\pi} \cos\theta (1+\cos\theta)^3 \, d\theta - \int_{\frac{3\pi}{2}}^{2\pi} -\frac{8\sin\theta}{(2\sin\theta - \cos\theta)^3} \, d\theta$$

$$= \frac{1}{3} \int_{\frac{3\pi}{2}}^{2\pi} \cos\theta (1 + 3\cos\theta + 3\cos^2\theta + \cos^3\theta) \, d\theta - \int_{\frac{3\pi}{2}}^{2\pi} -\frac{8\cos\theta/\cos^3\theta}{\left(\frac{2\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}\right)^3} \, d\theta$$

$$① = \int_{\frac{3\pi}{2}}^{2\pi} \cos\theta \, d\theta + \int_{\frac{3\pi}{2}}^{2\pi} 3\cos^2\theta \, d\theta + \int_{\frac{3\pi}{2}}^{2\pi} 3\cos^3\theta \, d\theta + \int_{\frac{3\pi}{2}}^{2\pi} \cos^4\theta \, d\theta$$

$$= -\sin\theta \Big|_{\frac{3\pi}{2}}^{2\pi} + 3\int_{\frac{3\pi}{2}}^{2\pi} \frac{1+\cos 2\theta}{2} \, d\theta + 3\int_{\frac{3\pi}{2}}^{2\pi} \cos^2\theta \cdot \cos\theta \, d\theta + \int_{\frac{3\pi}{2}}^{2\pi} \cos^3\theta \cdot \cos\theta \, d\theta$$

$u = \cos^3\theta$

$du = 3\cos^2\theta(-\sin\theta)\,d\theta$

$dv = \cos\theta \, d\theta \to v = \sin\theta$

$$= \operatorname{sen}(2\pi) - \operatorname{sen}\left(\frac{3\pi}{2}\right) + \frac{3}{2}(\theta + \operatorname{sen} 2\theta)\Big|_{\frac{3\pi}{2}}^{2\pi} + 3\int_{\frac{3\pi}{2}}^{2\pi} (1-\operatorname{sen}^2\theta)\cos\theta\, d\theta + \operatorname{sen}\theta \cdot \cos^3\theta\Big|_{3\pi/2}^{2\pi}$$

$$+ 3\int_{\frac{3\pi}{2}}^{2\pi} \cos^2\theta \cdot \operatorname{sen}^2\theta\, d\theta$$

$u = \operatorname{sen}\theta \to du = \cos\theta\, d\theta$, si $\theta = 2\pi \to u = 0$, si $\theta = \frac{3\pi}{2} \to u = -1$

$$= 0 - (-1) + \frac{3}{2}\left[\left(2\pi - \frac{3\pi}{2}\right) + \left(\operatorname{sen}(2(2\pi)) - \operatorname{sen}\left(2\left(\frac{3\pi}{2}\right)\right)\right)\right] + 3\int_{-1}^{0} (1-u^2)\, du + \left(\operatorname{sen}(2\pi)\cos^3(2\pi)\right.$$

$$\left. - \operatorname{sen}\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)\right) + 3\int_{\frac{3\pi}{2}}^{2\pi}\left(\frac{\operatorname{sen} 2\theta}{2}\right)^2$$

$$= -1 + \frac{3}{2}\left(\frac{\pi}{2}\right) + 3\left((0-(-1)) - \frac{1}{3}(0^3 - (-1)^3)\right) + 0 + \frac{3}{4}\int_{\frac{3\pi}{2}}^{2\pi} \frac{1-\cos(4\theta)}{2}\, d\theta$$

$$= -1 + \frac{3\pi}{4} + 3\left(1 - \frac{1}{3}\right) + \frac{3}{4}\left(\frac{1}{2}\right)\left(\theta - \frac{\operatorname{sen}(4\theta)}{4}\right)\Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= -1 + \frac{3\pi}{4} + 2 + \frac{3}{8}\left(\left(2\pi - \frac{3\pi}{2}\right) - \frac{1}{4}\left(\operatorname{sen}(4(2\pi)) - \operatorname{sen}\left(4\left(\frac{3\pi}{2}\right)\right)\right)\right)$$

$$= -1 + \frac{3\pi}{4} + 2 + \frac{3}{8}\left(\frac{\pi}{2}\right) \to -1 + \frac{3\pi}{4} + 2 + \frac{3\pi}{16} = 3 + \frac{15\pi}{16}$$

② $= \displaystyle\int_{\frac{3\pi}{2}}^{2\pi} \frac{-8\sec^2\theta}{(2\tan\theta - 1)^2}\, d\theta$

$u = \tan\theta$
$du = \sec^2\theta$
si $\theta = 2\pi \to u = 0$
si $\theta = \frac{3\pi}{2} \to u = \infty$

$$= -8\int_{\infty}^{0} \frac{1}{(2u-1)^2}\, du$$

$$= -8 \lim_{a\to\infty} \int_{a}^{0} \frac{1}{(2u-1)^2}\, du$$

$$= -8 \lim_{a\to\infty} \left(-\frac{1}{2}\left(\frac{1}{(2u-1)}\right)\right)\Big|_{a}^{0}$$

$$= -8 \lim_{a\to\infty} -\frac{1}{4}\left(\frac{1}{2(0)-1} - \frac{1}{\infty}\right) = -2$$

$$My = \frac{1}{3}\left(3 + \frac{15}{16}\pi - 2\right) = \frac{1}{3}\left(1 + \frac{15}{16}\pi\right)$$

$\bar{x} = \dfrac{My}{A}$

$= \dfrac{8}{9\pi}\left(1 + \dfrac{15\pi}{16}\right)$ //

$\bar{y} = \dfrac{Mx}{A}$

$= \dfrac{22}{9\pi}$ //

c) Dada la recta $L: y = \dfrac{(x-2)}{2}$

$y - \dfrac{(x-2)}{2} = 0$

$y - \dfrac{x}{2} + 1 = 0$

$$d = \frac{\left|-\frac{22}{9\pi} - \frac{1}{2}\left(\frac{8}{9\pi}\left(1 + \frac{15\pi}{16}\right)\right) + 1\right|}{\sqrt{\left(-\frac{1}{2}\right)^2 + 1^2}}$$

$$d = \frac{\left|-\frac{1}{9\pi}\left(22 - 4 + \frac{15}{4}\pi\right) + 1\right|}{\sqrt{\frac{1}{4} + 1}}$$

$$d = \frac{\left|-\frac{1}{9\pi}\left(26 + \frac{15\pi}{4}\right) + 1\right|}{\sqrt{\frac{5}{4}}} = \frac{|1 - 0.3361|}{\frac{\sqrt{5}}{2}} = 0.30$$

$V_x = |2\pi\, Mx|$

$= \left|2\pi\left(-\frac{11}{12}\right)\right| = \frac{11\pi}{3}\ u^3$

$V_y = 2\pi\left(\frac{1}{3}\left(1 + \frac{15\pi}{16}\right)\right) = \frac{2\pi}{3}\left(1 + \frac{15\pi}{16}\right) u^3$

$V_L = A \cdot 2\pi \cdot d$

$= \frac{9\pi}{8}(2\pi)(0.30)$

$= 0.225\pi^2 u^3$