UNIVERSIDAD DE LAS FUERZAS ARMADAS ESPE

TALLER 2 PARCIAL 3

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CÁLCULO VECTORIAL – NRC: 10376

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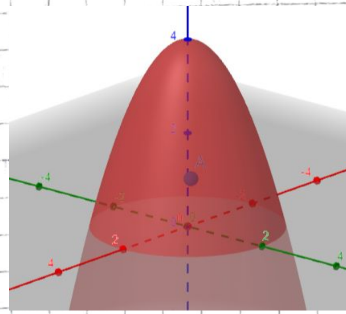
AGOSTO, 2022

1: Encuentre el centroide del sólido definido en el octante por el paraboloide $z = 4 - x^2 - y^2$. Considere que el material del sólido es homogeneo

$$V = \int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=2} \int_{z=0}^{z=4-r^2} r \, dz\, dr\, d\theta =$$



$$\int_0^{\pi/2} \int_0^2 r(4-r^2)\, dr\, d\theta = \int_0^{\pi/2} 8 - \frac{16}{4}\, d\theta =$$

$$4\theta \Big|_0^{\pi/2} = 2\pi_{//}$$

$$M_{xy} = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} r\, z\, dz\, dr\, d\theta = \int_0^{\pi/2} \int_0^2 \frac{r(4-r^2)^2}{2}\, dr\, d\theta = \int_0^{\pi/2} \int_0^2 \frac{r(16 - 8r^2 + r^4)}{2}$$

$$\frac{1}{2} \int_0^{\pi/2} \int_0^2 16r - 8r^3 + r^5 = \frac{1}{2} \int_0^{\pi/2} 8r^2 - 2r^4 + \frac{r^6}{16} \Big|_0^2 d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} 32 - \cancel{32} - \frac{16}{3}$$

$$\frac{16}{3} \theta \Big|_0^{2\pi} = \frac{8\pi}{3}$$

$$M_{xz} = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} r^2 \operatorname{sen}\theta \, dz\, dr\, d\theta = \int_0^{\pi/2} \int_0^2 r^2(4-r^2) \operatorname{sen}\theta \, dr\, d\theta =$$

$$\int_0^{\pi/2} \operatorname{sen}\theta \left(\frac{4r^3}{3} - \frac{r^5}{5} \right)\Big|_0^2 d\theta = \frac{32}{3} - \frac{32}{5} \int_0^{\pi/2} \operatorname{sen}\theta\, d\theta = \frac{64}{15}\left(-\cos\theta\right)\Big|_0^{\pi/2} = \frac{64}{15}$$

$$M_{yz} = \int_0^{\pi/2} \int_0^2 \int_0^{4-r^2} r^2 \cos\theta \, dz\, dr\, d\theta = \int_0^{\pi/2} \int_0^2 r^2(4-r^2) \cos\theta \, dr\, d\theta =$$

$$\int_0^{\pi/2} \cos\theta \left(\frac{4r^3}{3} - \frac{r^5}{5} \right|_0^2 d\theta = \frac{32}{3} - \frac{32}{5} \int_0^{\pi/2} \cos\theta\, d\theta = \frac{64}{15}(\operatorname{sen}\theta)\Big|_0^{\pi/2} = \frac{64}{15}$$

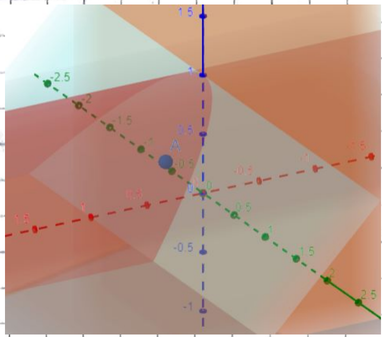
$$\bar{x} = \frac{64/15}{2\pi} = \frac{32}{15\pi}$$

$$\bar{y} = \frac{64/15}{2\pi} = \frac{32}{15\pi}$$

$$CG\left(\frac{32}{15\pi}, \frac{32}{15\pi}, \frac{4}{3} \right)_{//}$$

$$\bar{z} = \frac{8\pi/3}{2\pi} = \frac{4}{3}$$

2.- Encuentre el centro de masa del sólido que se encuentra acotado por el cilindro parabólico $z = 1 - y^2$ y los planos $x + z = 1$, $x = 0$ y $z = 0$, la función de densidad del sólido es $\rho = 2$



$$F = \int_{y=-1}^{y=1}\int_{z=0}^{z=1}\int_{x=0}^{x=1-z} 2\,dx\,dz\,dy =$$

$$2\int_{-1}^{1}\int_{0}^{1}(1-z)\,dz\,dy = 2\int_{-1}^{1}\frac{1}{2}\,dy = 2$$

$$M_{xy} = \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-z} 2z\,dx\,dz\,dy =$$

$$2\int_{-1}^{1}\int_{0}^{1} z - z^2\,dz\,dy = 2\int_{-1}^{1}\frac{1}{2}-\frac{1}{3}\,dy = \frac{1}{3}y\Big|_{-1}^{1} = \frac{2}{3}$$

$$M_{xz} = \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-z} 2y\,dx\,dz\,dy = 2\int_{-1}^{1}\int_{0}^{1} y(1-z)\,dz\,dy = 2\int_{-1}^{1} y - \frac{y}{2}\,dy =$$

$$2\left(\frac{y^2}{2}\left(1-\frac{1}{2}\right)\right)\Big|_{-1}^{1} = 0$$

$$M_{yz} = \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-z} 2x\,dx\,dz\,dy = \int_{-1}^{1}\int_{0}^{1}(1-z)^2\,dz\,dy = \int_{-1}^{1}\int_{0}^{1} 1 - 2z + z^2\,dz\,dy$$

$$\int_{-1}^{1}\frac{1}{3}\,dy = \frac{2}{3}$$

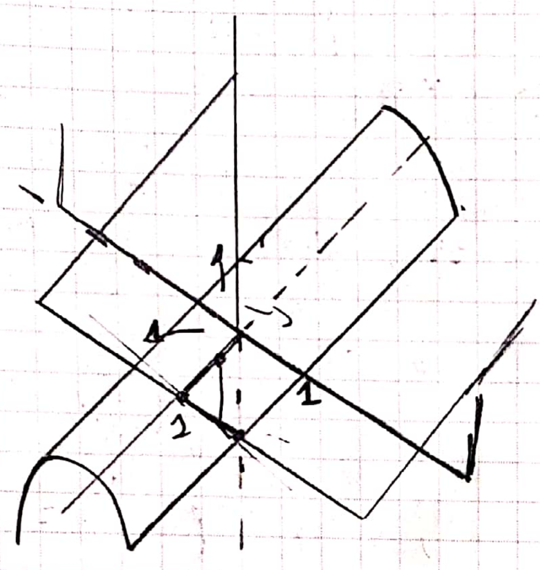
$$\bar{x} = \frac{2/3}{2} = \frac{1}{3}$$

$$\bar{y} = \frac{0}{2} = 0 \qquad CG\left(\frac{1}{3}, 0, \frac{1}{3}\right)$$

$$\bar{z} = \frac{2/3}{2} = \frac{1}{3}$$

3. Encuentre el centro de masa del sólido que se encuentra acotado por el cilindro parabólico $z = 1-y^2$ y los planos $x+z=1$, $x=0$ y $z=0$, la función de densidad del sólido es $\rho = 2$

$\bar{x} = \frac{M_{yz}}{m}$ $\bar{y} = \frac{M_{xz}}{m}$ $\bar{z} = \frac{M_{xy}}{m}$



$\frac{V}{2} = \int_{x=0}^{x=1} \int_{y=0}^{y=1} \int_{z=0}^{z=1-x} dz\,dy\,dx \cdot S$

$\frac{V}{2} = \int_{x=0}^{x=1} \int_{y=0}^{y=1} 1-x \; dy\,dx$

$\frac{V}{2} = \int_{x=0}^{x=1} 1-x \; dx$

$\frac{V}{2} = 1 - \frac{1}{2} = \frac{1}{2}$

$V = 1u^3$ //

$M_{xy} = \int_0^1 \int_0^1 \int_0^{1-x} z \, dz\,dy\,dx \; \rho S$

$M_{xy} = \int_0^1 \int_0^1 \frac{1}{2}(1-x)^2 \, dz\,dy\,dx \; (2)$

$M_{xy} = \frac{1}{2}\int_0^1 1-2x+x^2 \; dy\,dx \; (2)$

$M_{xy} = \frac{1}{2}\int_0^1 1-2x+x^2 \; dx \, (2)$

$M_{xy} = \frac{1}{2}\left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1 \; (2)$

$M_{xy} = \frac{1}{2}\left(\frac{1}{3}\right)(2)$

$\boxed{M_{xy} = \frac{1}{3} u^4}$

$M_{xz} = \int_0^1 \int_0^1 \int_0^{1-x} y \, dz\,dy\,dx \; (2)$

$M_{xz} = (2)\int_0^1 \int_0^1 y - xy \; dy\,dx$

$M_{xz} = 2\int_0^1 \left(\frac{1}{2}y^2 - \frac{1}{2}xy^2\right)\Big|_0^1 dx$,

$M_{xz} = 2\int_0^1 \frac{1}{2} - \frac{1}{2}x \; dx$

$M_{xz} = 2\left(\frac{1}{2}x - \frac{1}{4}x^2\right)\Big|_0^1$

$\boxed{M_{xz} = \frac{1}{2} u^4}$

$M_{yz} = \delta \cdot \int_0^1 \int_0^1 \int_0^{1-x} x \, dz\,dy\,dx$

$M_{yz} = 2\int_0^1 \int_0^1 x - x^2 \; dy\,dx$

$M_{yz} = 2\int_0^1 x - x^2 \; dx$

$M_{yz} = 2\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$

$\boxed{M_{yz} = \frac{1}{3} u^4}$

$\bar{x} = \frac{M_{yz}}{V} = \frac{\frac{1}{3}}{\frac{1}{1}} = \frac{1}{3} u$ //

$\bar{y} = \frac{M_{xz}}{V} = \frac{\frac{1}{2}}{1} \quad \frac{1}{2} u$ //

$\bar{z} = \frac{M_{xy}}{V} = \frac{\frac{1}{3}}{1} \quad \frac{1}{3} u$ //

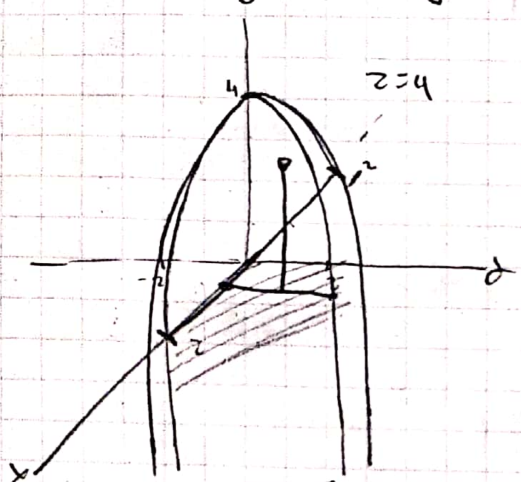
4. Calcular el Área de la Superficie del paraboloide $z=4-x^2-y^2$ para valores de $z\geq 0$

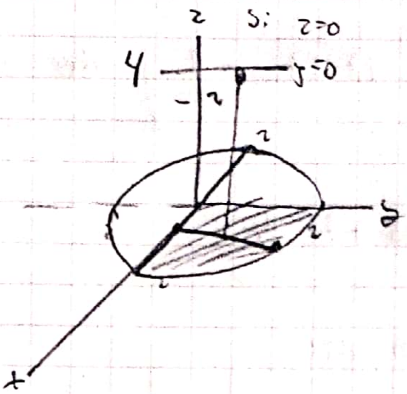
$z=f(x,y)$

Si $z\geq 0$

$y=\sqrt{4-x^2}$

Si $x=0$, $y=0$ $\rightarrow$ $\underline{z=4}$





$x=\pm 2$

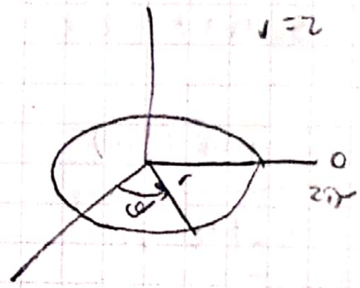
$\underline{r=2}$

$4-x^2-y^2=0$

$f(x)=-2x$

$f(y)=-2y$

$r=2$



$$S=\int_{x=-2}^{x=2}\int_{y=-2}^{y=\sqrt{4-x^2}}\sqrt{1+(f_x)^2+f_{(y)}^2}\,dy\,dx$$

$$S=\int_{x=2}^{x=2}\int_{y=0}^{y=\sqrt{4-x^2}}\sqrt{1+(-2x)^2+(-2y)^2}\,dy\,dx$$

$$S=\int_{x=-2}^{x=2}\int_{y=0}^{y=\sqrt{4-x^2}}\sqrt{1+4x^2+4y^2}\,dy\,dx$$

$x=r\cos\theta$

$y=r\,sen\,\theta$

$$S=\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\sqrt{1+f(r)^2}\,r\,dr\,d\theta$$

$4-x^2-y^2=0$

$4-(r\cos\theta)^2-(r\,sen\,\theta)^2=0$

$f(r)\rightarrow 4-r^2=0$

$$S=\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\sqrt{1+(-2r)^2}\,r\,dr\,d\theta$$

$$S=\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\sqrt{1+4r^2}\,\cancel{r}\,dr\,d\theta$$

$u=1+4r^2$

$\frac{du}{dr}=8r$

$dr=\frac{1}{8\cancel{r}}\,du$

$$S=\int_{\theta=0}^{\theta=2\pi}\int_{u=0}^{u=2}\sqrt{u}\ \frac{1}{8}\,du$$

$u^{1/2}$

$$S=\int_{\theta=0}^{\theta=2\pi}\frac{(1+4r^2)^{3/2}}{\frac{3}{2}}\ \frac{1}{8}\,du$$

$\frac{1}{\sqrt{u}}\ ^{1/2}$

$u^{1/2}$

$$S=\int_{\theta=0}^{\theta=2\pi}\frac{\sqrt[2]{(1+4r^2)^3}}{\frac{3}{2}}\ \frac{1}{8}\,du$$

$$S = \int_{\theta=0}^{\theta=2\pi} \frac{2\sqrt{(1+4r^2)^3}}{12} \Bigg|_{r=0}^{r=2} d\theta$$

$$S = \int_{\theta=0}^{\theta=2\pi} \frac{2\sqrt{(1+4(2)^2)^3}}{12} - \frac{2\sqrt{(1+4(0)^2)^3}}{12} \, d\theta$$

$$S = \int_{\theta=0}^{\theta=2\pi} \frac{1}{12}\left(17\sqrt{17} - 1\right) d\theta$$

$$S = \int_{\theta=0}^{\theta=2\pi} 5.7577 \, d\theta$$

$$S = 5.7577\,\theta \Big|_{\theta=0}^{\theta=2\pi}$$

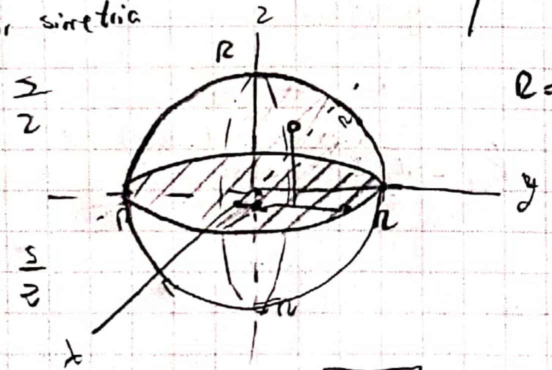
$$S = 5.7577\,(2\pi - 0)$$

$$S = 5.7577\,(2\pi)$$

$$\boxed{S = 36.176 \ u^2}$$

5. Calcular el área de la superficie de una esfera con centro en el origen y radio R

Por simetría



$\rho = r$

$x^2+y^2+z^2 = R$

$z = f(x,y)$

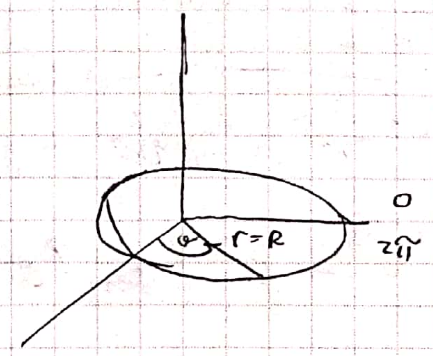
$z = \sqrt{R^2 - x^2 - y^2}$

$$f(x) = \frac{1}{2}\left(R^2 - x^2 - y^2\right)^{-1/2}(-2x)$$

$$f(x) = \frac{-2x}{2\sqrt{R^2-x^2-y^2}} = \frac{-x}{\sqrt{R^2-x^2-y^2}}$$

$$\frac{S}{2} = \int_{x=0}^{x=R}\int_{y=0}^{y=\sqrt{R^2-x^2}} \sqrt{1 + \left(\frac{-x}{\sqrt{R^2-x^2-y^2}}\right)^2 + \left(\frac{-y}{\sqrt{R^2-x^2-y^2}}\right)^2} \, dy\,dx$$

$$\frac{S}{2} = \int_{x=0}^{x=R}\int_{y=0}^{y=\sqrt{R^2-x^2}} 1 + \frac{x^2+y^2}{(R^2-x^2-y^2)} \, dy\,dx$$



$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=R} \sqrt{1 + \frac{r^2}{R^2-r^2}}\, dr\, dr\, d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=R} \sqrt{\frac{R^2-r^2 \quad +r^2}{R^2-r^2}}\; r\, dr\, d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=R} \frac{R.r}{\sqrt{R^2-r^2}}\, dr\, d\theta$$

$$u = R^2 - r^2 \qquad dr = \frac{1}{-2r}\, du$$

$$\frac{du}{dr} = -2r$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} -\frac{R}{2} \int_{r=0}^{r=R} \frac{1}{\sqrt{u}}\, du\, d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} -\frac{R}{2} \left(\frac{(R^2-r^2)^{-1/2+1}}{\frac{1}{2}} \right) \Bigg|_{r=0}^{r=R} d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} -\frac{R}{\cancel{2}} \left(\cancel{2}\sqrt{R^2-r^2} \right) \Bigg|_{r=0}^{r=R} d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} -R\sqrt{R^2-r^2} \Bigg|_{r=0}^{r=R}$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} -R \left(\sqrt{\cancel{R^2-R^2}}^{\,0} - \sqrt{R^2-0} \right) d\theta$$

$$\frac{S}{2} = \int_{\theta=0}^{\theta=2\pi} +R^2 \quad d\theta$$

$$\frac{S}{2} = R^2\theta \Bigg|_{\theta=0}^{\theta=2\pi}$$

$$\frac{S}{2} = 2R^2\pi$$

$$\boxed{S = 4\pi R^2}$$