UNIVERSIDAD DE LAS FUERZAS ARMADAS – ESPE

CÁLCULO VECTORIAL

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TALLER 1 – PARCIAL 2

Día | Mes | Año

1. Analizar el dominio y la continuidad de la función. Representar el dominio de la siguiente función

$$f(x,y) = \frac{1}{12}\sqrt{144-16x^2-9y^2} \qquad \pm 3 \quad , \pm 4$$

$$\text{Dom } z = \left\{ (x,y) \;/\; 144-16x^2-9y^2 \geq 0 \right\}$$

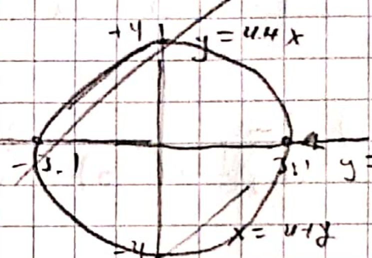
$x=0 \rightarrow 9y^2 \geq 144$ $\quad y=0 \mapsto 16x^2 \geq 144$

$y \geq \sqrt{16}$ $\quad x^2 \geq 9$

$-4 \leq y \leq 4 \longleftarrow y \geq \pm 4$ $\quad x \geq \sqrt{9}$

$-3 \leq x \leq 3 \quad x \geq 3 \pm$

$(3,4)$



$$\lim_{(x,y)\mapsto(3,4)} \frac{1}{12}\sqrt{144-16x^2-9y^2} = \frac{\sqrt{-144}}{12} = \nexists$$

i) $\lim\limits_{(x,y)\mapsto(x,0)} \frac{1}{12}\sqrt{144-16x^2} = \frac{1}{3}\sqrt{-x^2+9}$

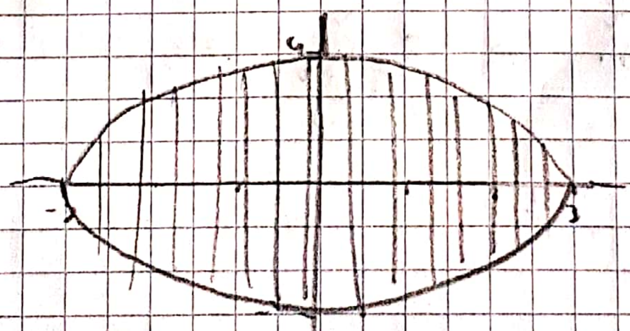
$$\lim_{x\to 3} \frac{\sqrt{-3^2+9}}{3} = \frac{\frac{-x}{\sqrt{9-x^2}}}{\frac{0}{1}} = -\frac{x}{\sqrt{9-x^2}\,(0)} = \infty$$

ii) $\lim\limits_{(x,y)\mapsto(4+y,y)} \frac{1}{12}\sqrt{144-16x^2-9y^2} = \frac{1}{12}\sqrt{-25y^2-128y-112}$

$$\lim_{y\to 4} \frac{\sqrt{-25y^2-128y-112}}{12} = \frac{\frac{-25y+64}{\sqrt{-25y^2-128y-112}}}{\frac{0}{1}} = \infty$$

$$\nexists \lim_{(x,y)\mapsto(3,4)} \frac{\sqrt{144-16x^2-9y^2}}{12} \Rightarrow z \neq \text{cont en } (3,4)$$

$z = \text{cont en Dom } z$



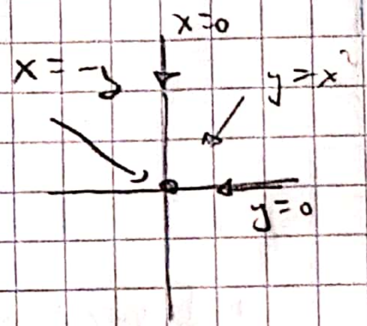
2. Analizar el dominio y la continuidad de la función

$$z = \frac{xy}{\sqrt{x^2+y^2}}$$

$$x^2+y^2>0$$

$$\text{Dom } z = \left\{ (x,y) \;\middle/\; x^2+y^2>0 \right\}$$

$(0,0)$



$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \frac{0}{0}$$

i) $$\lim_{(x,y)\to(x,0)} \frac{xy}{\sqrt{x^2+y^2}} = \frac{0}{\sqrt{x^2}} = 0$$

$$\lim_{x\to 0} 0 = 0 //$$

ii) $$\lim_{(x,y)\to(x,x)} \frac{xy}{\sqrt{x^2+y^2}} = \frac{x^2}{\sqrt{2x^2}} = \frac{x}{\sqrt{2}} =$$

$$\lim_{x\to 0} \frac{0}{\sqrt{2}} = 0 //$$

$$\nexists \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} \Rightarrow z \nexists \text{ cont en } (0,0)$$

z cont Dom z

3-

$$z = \operatorname{arctg}\sqrt{\frac{x^2-y^2}{x^2+y^2}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-\frac{x^2-y^2}{x^2+y^2}}} \cdot \frac{1}{2\sqrt{\frac{x^2-y^2}{x^2+y^2}}} \cdot \frac{2x(x^2+y^2)-2x(x^2-y^2)}{(x^2+y^2)^2} =$$

$$= \frac{1}{\frac{\sqrt{\cancel{x^2}+y^2-\cancel{x^2}+y^2}}{\sqrt{x^2+y^2}}} \cdot \frac{1}{2\frac{\sqrt{x^2-y^2}}{\cancel{\sqrt{x^2+y^2}}}} \cdot \frac{4xy^2}{\cancel{(x^2+y^2)^2}} =$$

$$= \frac{\cancel{4}xy^2}{\cancel{2}\sqrt{2y^2}\sqrt{x^2-y^2}(x^2+y^2)} = \frac{xy^2}{\sqrt{2y^2(x^2-y^2)}(x^2+y^2)} =$$

$$= \frac{xy^2}{\sqrt{2(x^2y^2-y^4)}(x^2+y^2)} = \frac{xy^2\sqrt{2(x^2y^2-y^4)}}{2(x^2y^2-y^4)(x^2+y^2)} \quad //$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{2y^2}} \frac{\cancel{4}x^2y}{\cancel{2}\sqrt{x^2-y^2}(x^2+y^2)} = \frac{x^2y}{\sqrt{2(x^2y^2-y^4)}(x^2+y^2)} =$$

$$= \frac{x^2y\sqrt{2(x^2y^2-y^4)}}{2(x^2y^2-y^4)(x^2+y^2)} \quad //$$

$$z = \ln\left(\frac{\sqrt{x^2+y^2}-x}{\sqrt{x^2+y^2}+x}\right)$$

$$\frac{\partial z}{\partial x} = \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x} \cdot \frac{\left(\frac{x}{\sqrt{x^2+y^2}}-1\right)\left(\sqrt{x^2+y^2}+x\right)-\left(\sqrt{x^2+y^2}-x\right)\left(\frac{x}{\sqrt{x^2+y^2}}+1\right)}{(\sqrt{x^2+y^2}+x)^2}$$

$$= \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x} \cdot \frac{\cancel{x}+x^2(x^2+y^2)^{-1/2}-\sqrt{x^2+y^2}-\cancel{x}+x^2(x^2+y^2)^{-1/2}-\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}+x^2)^2} =$$

$$= \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x} \cdot \frac{2x^2(x^2+y^2)^{-1/2}-2\sqrt{x^2+y^2}}{x^2+y^2+2x\sqrt{x^2+y^2}+x^4} =$$

$$= \frac{2x^2-2(x^2+y^2)+2x^3(x^2+y^2)^{-1/2}-2x\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}-x)(\sqrt{x^2+y^2}+x)^2}$$

$$= \frac{-2y^2+2x^3(x^2+y^2)^{-1/2}-2x\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}-x)(\sqrt{x^2+y^2}+x)^2} \quad //$$

$$\frac{\partial z}{\partial y} = \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x} \cdot \frac{\left(\frac{y}{\sqrt{y^2+x^2}}-1\right)\left(\sqrt{x^2+y^2}+x\right)-\left(\sqrt{x^2+y^2}-x\right)\left(\frac{y}{\sqrt{x^2+y^2}}+1\right)}{(\sqrt{x^2+y^2}+x)^2} =$$

$$= \frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x} \cdot \frac{xy(x^2+y^2)^{-1/2}-\sqrt{x^2+y^2}-\sqrt{x^2+y^2}+xy(x^2+y^2)^{-1/2}}{(\sqrt{x^2+y^2}+x)^2}$$

$$= \frac{2xy-2(x^2+y^2)+x^2y(x^2+y^2)^{-1/2}-2x\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2}-x)(\sqrt{x^2+y^2}+x)^2} \quad //$$

ESTILO

$$z = \ln\left[xy^2 + x^2y + \sqrt{1 + (xy^2 + x^2y)^2}\right] \qquad x^2y^4 + 2x^2y^2 + x^4y^2$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy^2 + x^2y + \sqrt{1+(xy^2+x^2y)^2}} \cdot \left(y^2 + 2xy + \frac{(1)(\cancel{2}xy^4 + \cancelto{2}{4}xy^2 + \cancelto{2}{4}x^3y^2)}{\cancel{2}\sqrt{1+(xy^2+x^2y)^2}} \right) =$$

$$= \frac{y^2 + 2xy + (xy^4 + 2xy^2 + 2x^3y^2)(1 + (xy^2+x^2y)^2)^{-1/2}}{xy^2 + x^2y + \sqrt{1+(xy^2+x^2y)^2}} \quad //$$

$$\frac{\partial z}{\partial y} = \frac{1}{x^2y + xy^2 + \sqrt{1+(xy^2+x^2y)^2}} \cdot \left(2xy + x^2 + \frac{(1)\cancelto{2}{4}x^2y^3 + \cancelto{2}{4}x^2y + \cancel{2}x^4y^2}{\cancel{2}\sqrt{1+(xy^2+x^2y)^2}} \right) =$$

$$= \frac{2xy + x^2 + (2x^2y^3 + 2x^2y + x^4y^2)(1 + (xy^2+x^2y)^2)^{-1/2}}{x^2y + y^2x + \sqrt{1+(xy^2+x^2y)^2}} \quad //$$

4- Calcular y Simplificar $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

$$x^3 + y^3 + z^3 = 3axyz$$

- Para $\frac{\partial z}{\partial x}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$F_x = 3x^2 + 0 + 0 - 3ayz \Rightarrow 3x^2 - 3ayz$$

$$F_z = 0 + 0 + 3z^2 - 3axy \Rightarrow 3z^2 - 3axy$$

$$\frac{\partial z}{\partial x} = -\frac{3x^2 - 3ayz}{3z^2 - 3axy}$$

$$\frac{\partial z}{\partial x} = -\frac{\cancel{3}(x^2 + ayz)}{\cancel{3}(z^2 - axy)}$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{x^2 + ayz}{z^2 - axy}}$$

- Para $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$F_y = 0 + 3y^2 + 0 - 3axz \Rightarrow 3y^2 - 3axz$$

$$F_z = 3z^2 - 3axy$$

$$\frac{\partial z}{\partial y} = -\frac{3y^2 - 3axz}{3z^2 - 3axy}$$

$$\frac{\partial z}{\partial y} = -\frac{\cancel{3}(y^2 + axz)}{\cancel{3}(z^2 - axy)}$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{y^2 + axz}{z^2 - axy}}$$

$$(x^2 + y^2 + z^2)^3 - 27xyz = 0$$

- Para $\frac{\partial z}{\partial x}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$F_x = 3(x^2+y^2+z^2)^2 \cdot 2x - 27yz$$

$$F_x = 6x(x^2+y^2+z^2)^2 - 27yz$$

$$F_z = 3(x^2+y^2+z^2)^2 \cdot 2z - 27xy$$

$$F_z = 6z(x^2+y^2+z^2)^2 - 27xy$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{6x(x^2+y^2+z^2)^2 - 27yz}{6z(x^2+y^2+z^2)^2 - 27xy}}$$

- Para $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$F_y = 3(x^2+y^2+z^2)^2 \cdot 2y - 27xy$$

$$F_y = 6y(x^2+y^2+z^2)^2 - 27xy$$

$$F_z = 6z(x^2+y^2+z^2)^2 - 27xy$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{6y(x^2+y^2+z^2)^2 - 27xy}{6z(x^2+y^2+z^2)^2 - 27xy}}$$

5. Para las siguientes funciones calcular la derivada respecto de t.

$$w = \ln\sqrt{x^2+y^2+z^2}, \quad x = \operatorname{sen} t, \quad y = \cos t, \quad z = \tan t$$

$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$\frac{dw}{dt} = \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}}\cancel{2}x \cdot \cos t + \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}}\cancel{2}y \cdot (-\operatorname{sen} t)$$

$$+ \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}}\cancel{2}z \; \sec^2(t)$$

$$\frac{dw}{dt} = \frac{x\cos(t)}{x^2+y^2+z^2} - \frac{y\,\mathrm{sen}(t)}{x^2+y^2+z^2} + \frac{z\sec^2(t)}{x^2+y^2+z^2}$$

$$\boxed{\frac{dw}{dt} = \frac{1}{x^2+y^2+z^2}\left(x\cos(t) - y\,\mathrm{sen}(t) + z\sec^2(t)\right)}$$

$$z = \ln\left[\mathrm{sen}\left(\frac{x}{\sqrt{y}}\right)\right],\quad x = 3t^2,\quad y = \sqrt{t^2+1}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{\mathrm{sen}\left(\frac{x}{\sqrt{y}}\right)}\cdot\cos\left(\frac{x}{\sqrt{y}}\right)\left(\frac{1}{\sqrt{y}}\right)\cdot 6t + \frac{1}{\mathrm{sen}\left(\frac{x}{\sqrt{y}}\right)}\cos\left(\frac{x}{\sqrt{y}}\right)\left(-\frac{x}{2y^{3/2}}\right)\cdot\frac{1}{\cancel{2}\sqrt{t^2+1}}\cdot \cancel{2}t$$

$$\frac{dz}{dt} = \overset{ctg}{\cancel{\frac{\cos\left(\frac{x}{\sqrt{y}}\right)}{\mathrm{sen}\left(\frac{x}{\sqrt{y}}\right)}}}\cdot\frac{1}{\sqrt{y}}\cdot 6t + \overset{ctg}{\cancel{\frac{\cos\left(\frac{x}{\sqrt{y}}\right)}{\mathrm{sen}\left(\frac{x}{\sqrt{y}}\right)}}}\left(-\frac{x}{2y^{3/2}}\right)\cdot\frac{\cancel{2}t}{\cancel{2}\sqrt{t^2+1}}$$

$$\frac{dz}{dt} = ctg\left(\frac{x}{\sqrt{y}}\right)\frac{6t}{\sqrt{y}} + \left(-\frac{ctg\left(\frac{x}{\sqrt{y}}\right)x}{2y^{3/2}}\right)\frac{t}{\sqrt{t^2+1}}$$

$$\frac{dz}{dt} = \frac{6t\,ctg\left(\frac{x}{\sqrt{y}}\right)}{\sqrt{y}} - \frac{xt\,ctg\left(\frac{x}{\sqrt{y}}\right)}{2y^{3/2}\sqrt{t^2+1}}$$

$$\boxed{\frac{dz}{dt} = t\,ctg\left(\frac{x}{\sqrt{y}}\right)\left[\frac{6}{\sqrt{y}} - \frac{x}{2y^{3/2}\sqrt{t^2+1}}\right]}$$

6) Para las siguientes funciones calcular las derivadas parciales respecto a las variables independientes correspondientes.

→ $z = f(x,y) = \arctan\left(\frac{y}{x}\right)$, $x = u + v$, $y = u - v$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= \frac{y}{x^2+y^2} \cdot 1 + \left(-\frac{x}{x^2+y^2}\right) \cdot 1 = \frac{y-x}{x^2+y^2}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= \frac{y}{x^2+y^2} \cdot 1 + \left(-\frac{x}{x^2+y^2}\right) \cdot -1 = \frac{y+x}{x^2+y^2}$$

→ $z = u^v$, $u = \frac{x}{y}$, $v = xy$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= (u^{v-1} \cdot v) \cdot \left(\frac{1}{y}\right) + (\ln|u|) u^v \cdot y = \frac{u^{v-1} \cdot v}{y} + \ln|u| \cdot u^v y = \frac{u^{v-1} v + u^v y^2 \ln|u|}{y}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$= (u^{v-1} \cdot v) \cdot -\frac{x}{y^2} + (\ln|u|) \cdot x = -\frac{(u^{v-1} \cdot v)x}{y^2} + \ln|u| \cdot x = -\frac{(u^{v-1} \cdot v)x + xy^2 \ln|u|}{y^2}$$

→ $w = f(x,y)$, $x = e^u \cdot \cos(v)$, $y = e^u \cdot \operatorname{sen}(v)$

$$\frac{\partial x}{\partial u} = e^u \cos(v)$$

$$\frac{\partial y}{\partial u} = \operatorname{sen}(v) \cdot e^u$$

$$\frac{\partial x}{\partial v} = -e^u \operatorname{sen}(v)$$

$$\frac{\partial y}{\partial v} = e^u \cos(v)$$

7- Dada la superficie $x^2 + 2y^2 + 3z^2 = 21$, encontrar los puntos donde los planos tangentes son paralelos al plano $x + 4y + 6z = 0$

$S: x^2 + 2y^2 + 3z^2 = 21$

$\varphi: x + 4y + 6z = 0 \qquad n = \langle 1, 4, 6 \rangle$

$F(x,y,z) = x^2 + 2y^2 + 3z^2 - 21$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 2x \\ \frac{\partial F}{\partial y} = 4y \\ \frac{\partial F}{\partial z} = 6z \end{array}\right\} \nabla F = \langle 2x, 4y, 6z \rangle$$

$$\frac{2x}{1} = \frac{4y}{4} = \frac{6z}{6} = k \Rightarrow 2x = y = z = k$$

$x^2 + 8x^2 + 12x^2 = 21 \qquad 2x = z \qquad 2x = y$

$21x^2 = 21 \qquad 2(-1) = z = -2 \qquad 2(-1) = y = -2$

$x^2 = \pm 1 \qquad 2(1) = z = 2 \qquad 2(1) = y = 2$

$P(1,2,2) \quad , \quad Q(-1,-2,-2)$

$\nabla F|_P = \langle 2, 8, 12 \rangle$

$= \langle -2, -8, -12 \rangle$

con $P(1,2,2)$

$\varphi = 1(x-1) + 4(y-2) + 6(z-2) = 0$

$$\boxed{\varphi: x + 4y + 6z = 21}$$

con $Q(-1,-2,-2)$

$\varphi: 1(x+1) + 4(y+2) + 6(z+4) = 0$

$$\boxed{\varphi: x + 4y + 6z = -21}$$