Universidad de las Fuerzas Armadas - ESPE

Software

Cálculo Vectorial 10376.

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Fecha: 23 de mayo del 2023

Deber #2.

Hallar el dominio de la Función vectorial

1. $r(t) = \frac{1}{t+1}\hat{\imath} + \frac{t}{2}\hat{\jmath} - 3t\hat{k}$

$\text{Dom } r(t) = \text{Dom } F(t) \cap \text{Dom } g(t) \cap \text{Dom } h(t)$

$\text{Dom}\left(\frac{1}{t+1}\right) = \mathbb{R} - \{-1\}$

$\text{Dom}\left(\frac{t}{2}\right) = \mathbb{R}$

$\text{Dom}\left(-3t\right) = \mathbb{R}$

$\text{Dom } r(t) = \mathbb{R} - \{-1\} \cap \mathbb{R} \cap \mathbb{R}$

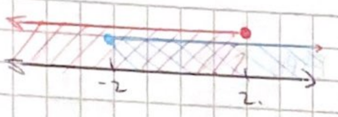
$\boxed{\text{Dom } r(t) = \mathbb{R} - \{-1\}}$

2. $r(t) = \sqrt{4-t^2}\,\hat{\imath} + t^2\hat{\jmath} - 6b$

$\text{Dom } r(t) = \text{Dom } F(t) \cap \text{Dom } g(t) \cap \text{Dom } h(t)$

$\text{Dom}\left(\sqrt{4-t^2}\right) = 4 - t^2 \geq 0$ $\quad 2 - t \geq 0 \quad 2 \geq -t$

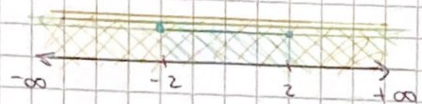
$(2-t)(2+t) \geq 0 \quad 2 \geq t \quad t \geq -2$



Dom $g(t) = [-2, 2]$

Dom $(t^2) = \mathbb{R}$

Dom $(-6t) = \mathbb{R}$



$\boxed{\text{Dom } r(t) = [-2, 2]}$

3. $r(t) = \ln t\,\hat{\imath} - e^t\,\hat{\jmath} - t\,\hat{k}$

Dom $f(t) = t > 0$ ∴ Dom $(\ln t) =]0, +\infty[$

Dom $g(t) = t \geq 0$ Dom $(e^t) = [0, +\infty[$

Dom $h(t) = \mathbb{R}$

$\boxed{\text{Dom } r(t) =]0, +\infty[}$

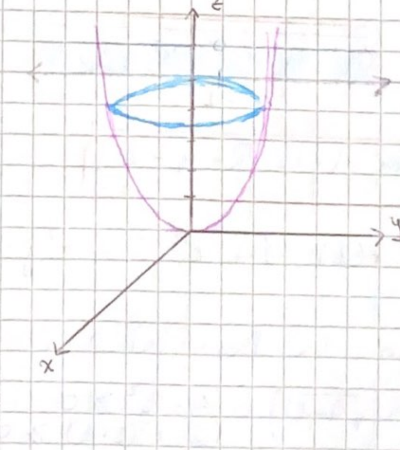
Resuelva y grafique

60. $z = x^2 + y^2$, $z = 4$ Parámetro $x = 2\cos t$

$x^2 + y^2 = 4$

$x = 2\cos t$
$y = 2\sin t$
$z = 4$

$r(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath} + 4\hat{k}$



62. $4x^2 + 4y^2 + z^2 = 16, \quad x = z^2$

Parámetro
$z = t$

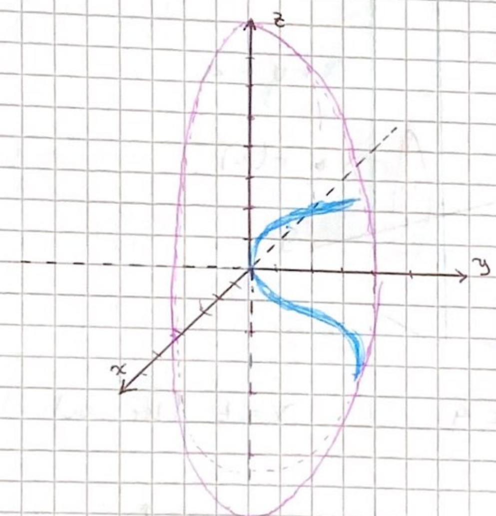
$x = t^2$

$4t^4 + 4y^2 + t = 16$

$y = \sqrt{\dfrac{16 - t - 4t^4}{4}}$

$y = \dfrac{1}{2}\sqrt{16 - t - 4t^4}$

$r(t) = t^2\,\hat{\imath} + \dfrac{1}{2}\sqrt{16 - t - 4t^4}\,\hat{\jmath} + t\,\hat{k}$



64- $x^2 + y^2 + z^2 = 10 \qquad x + y = 4$

Parámetro
$x = 2 + \operatorname{sen} t$

$2 + \operatorname{sen} t - 4 = -y$
$-2 + \operatorname{sen} t = -y$
$y = 2 - \operatorname{sen} t$

$(2 + \operatorname{sen} t)^2 + (2 - \operatorname{sen} t)^2 + z^2 = 10$

$z^2 = 10 - (2 + \operatorname{sen} t)^2 - (2 - \operatorname{sen} t)^2$

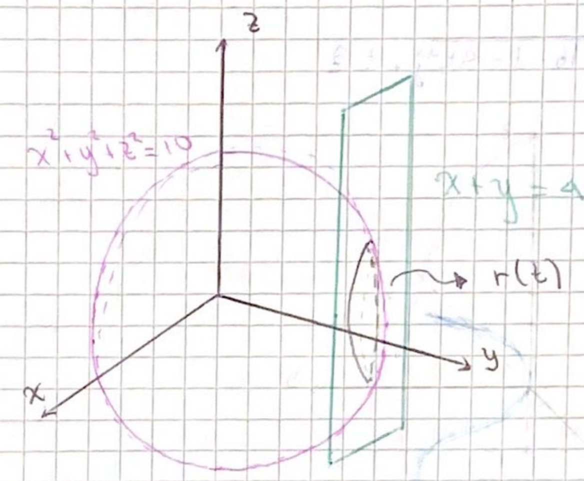
$z = \sqrt{10 - (2 + \operatorname{sen})^2 - (2 - \operatorname{sen} t)^2}$

$z = \sqrt{\cancel{10} - 4 - \cancel{4\operatorname{sen}^2 t} - \operatorname{sen}^2 t - \cancel{4} + \cancel{4\operatorname{sen}^2 t} - \operatorname{sen}^2 t}$

$z = \sqrt{2 - 2\operatorname{sen}^2 t} \approx \sqrt{2(1 - \operatorname{sen}^2 t)} \approx \sqrt{2\cos^2 t} \approx \sqrt{2}\cos t$

$r(t) = (2+\sin t)\hat{\imath} + (2-\sin t)\hat{\jmath} + \sqrt{2}\cos t\,\hat{k}$

t	x	y	z
$-\frac{\pi}{2}$	1	3	0
0	2	2	$\sqrt{2}$
$\frac{\pi}{2}$	3	1	0



66) $x^2 + y^2 + z^2 = 16$ $\quad xy = 4 \quad$ $x = t$ (1er octante.

$x = t$

$y = \frac{4}{t}$

$t^2 + \frac{16}{t^2} + z^2 = 16$

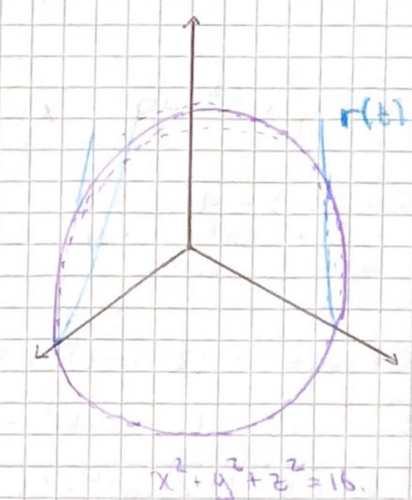
$z^2 = 16 - t^2 - \frac{16}{t^2}$

$z = \sqrt{16 - t^2 - \frac{16}{t^2}} = \sqrt{16\left(1 - \frac{1}{t^2}\right) - t^2}$

$z = \sqrt{16t^2 - t^4 - 16}$

$t^2\hat{\imath} + \frac{16}{t^2}\hat{\jmath} + z$

$t\hat{\imath} + \frac{4}{t}\hat{\jmath} + \sqrt{16t^2 - t^4 - 16}\,\hat{k}$



Evaluar el límite

71. $\lim_{t \to 0} \left(t^2 \hat{\imath} + 3t\hat{\jmath} + \frac{1-\cos t}{t} \hat{k} \right)$

$\hat{\imath}\left(\lim_{t \to 0} t^2\right) + \hat{\jmath}\left(\lim_{t \to 0} 3t\right) + \hat{k}\left(\lim_{t \to 0} \frac{1-\cos(t)}{t}\right)$

$0\hat{\imath} + 0\hat{\jmath} + \frac{0}{0}\hat{k}$

Levanto indeterminación por L'hopital

$\frac{0 + \operatorname{sen}(t)}{1} = \operatorname{sen}(t) \quad \lim_{t \to 0} \operatorname{sen}(0) = 0$

$\boxed{0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} \quad v \quad \langle 0, 0, 0 \rangle}$

72. $\lim_{t \to 1} \left(\sqrt{t}\, \hat{\imath} + \frac{\ln t}{t^2 - 1} \hat{\jmath} + \frac{1}{t-1} \hat{k} \right)$

$\hat{\imath}\left(\lim_{t \to 1} \sqrt{t}\right) + \hat{\jmath}\left(\lim_{t \to 1} \frac{\ln(t)}{t^2-1}\right) + \hat{k}\left(\lim_{t \to 1} \frac{1}{t-1}\right)$

$\hat{\imath} + \boxed{\frac{0}{0}}\hat{\jmath} + \frac{1}{0}\hat{k}$

Por l'hopital

$\frac{\frac{1}{t}}{2t} = \frac{1}{2t^2} = \lim_{t \to 1} = \frac{1}{2} . \hat{\jmath}$

$\hat{\imath} + \frac{1}{2}\hat{\jmath} + \boxed{\frac{1}{0}\hat{k}}$ No existe el límite

74. $\lim_{t \to \infty} \left(e^{-t} \hat{\imath} + \frac{1}{t} \hat{\jmath} + \frac{t}{t^2+1} \hat{k} \right)$

$\hat{\imath}\left(\lim_{t \to \infty} e^{-t}\right) + \hat{\jmath}\left(\lim_{t \to \infty} \frac{1}{t}\right) + \hat{k}\left(\lim_{t \to \infty} \frac{t}{t^2+1}\right)$

$\boxed{0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}}$

ESTILO

Hallar $r'(t)$

16- $r(t) = \frac{1}{t}\hat{\imath} + 16t\,\hat{\jmath} + \frac{t^2}{2}$

$$\boxed{r'(t) = -\frac{1}{t^2}\hat{\imath} + 16\hat{\jmath} + t\hat{k}}$$

12- $r(t) = \sqrt{t}\,\hat{\imath} + (1 - t^3)\hat{\jmath}$

$r'(t) = t^{\frac{1}{2}}\hat{\imath} + (1 - t^3)\hat{\jmath}$

$r'(t) = \frac{1}{2\sqrt{t}}\hat{\imath} + (0 - 3t^2)\hat{\jmath}$

$$\boxed{r'(t) = \frac{1}{2\sqrt{t}}\hat{\imath} - 3t^2\hat{\jmath}}$$

14- $r(t) = \langle t\cos(t), -2\,\text{sen}(t)\rangle$

$r'(t) = t\cos(t)\,\hat{\imath} - 2\,\text{sen}(t)\,\hat{\jmath}$

$$\boxed{r'(t) = (-t\,\text{sen}(t) + \cos(t))\hat{\imath} - 2\cos(t)\hat{\jmath}}$$

18- $r(t) = 4\sqrt{t}\,\hat{\imath} + t^2\sqrt{t}\,\hat{\jmath} + \ln t^2$

$r'(t) = \frac{2}{\sqrt{t}}\hat{\imath} + \left(\frac{1}{2\sqrt{t}}\cdot t^2 + 2t\sqrt{t}\right)\hat{\jmath} + \frac{2t}{t^2}\hat{k}$

$$\boxed{r'(t) = \frac{2}{\sqrt{t}}\hat{\imath} + \left(\frac{t^2}{2\sqrt{t}} + 2t\sqrt{t}\right)\hat{\jmath} + \frac{2}{t}\hat{k}}$$

20- $r(t) = \langle t^3, \cos(3t), \text{sen}(3t)\rangle$

$r'(t) = 3t^2\,\hat{\imath} + -3\,\text{sen}(3t)\hat{\jmath} + 3\cos(3t)\hat{k}$

$$\boxed{r'(t) = 3t^2\hat{\imath} - 3\,\text{sen}(3t)\hat{\jmath} + 3\cos(3t)\hat{k}}$$

22- $r(t) = \langle \text{arcsen}(t), \text{arccos}(t), 0\rangle$

$$\boxed{r'(t) = \left\langle \frac{1}{\sqrt{1-x^2}}, -\frac{1}{\sqrt{1-x^2}}, 0 \right\rangle}$$

Hallar a) $r'(t)$

b) $r''(t)$

c) $r'(t) \cdot r''(t)$

24- $r(t) = (t^2 + t)\hat{\imath} + (t^2 - t)\hat{\jmath}$

a) $r'(t) = (2t+1)\hat{\imath} + (2t-1)\hat{\jmath}$

b) $r''(t) = 2\hat{\imath} + 2\hat{\jmath}$

c) $r'(t) \cdot r''(t) = ((2t+1)(2))\hat{\imath} + (2t-1)(2)$

$r'(t) \cdot r''(t) = 4t + \cancel{2} + 4t - \cancel{2}$

$r'(t) \cdot r''(t) = 8t$

26- $r(t) = 8\cos t\,\hat{\imath} + 3\,\text{sen}\,t\,\hat{\jmath}$

a) $r'(t) = -8\,\text{sen}\,t\,\hat{\imath} + 3\cos t\,\hat{\jmath}$

b) $r''(t) = -8\cos t\,\hat{\imath} - 3\,\text{sen}\,t\,\hat{\jmath}$

c) $r'(t) \cdot r''(t) = (-8\,\text{sen}\,t)(-8\cos t) + (-3\,\text{sen}\,t)(3\cos t)$

$r'(t) \cdot r''(t) = 64\,\text{sen}\,t\cos t - 9\,\text{sen}\,t\cos t$

$r'(t) \cdot r''(t) = 55\,\text{sen}\,t\cos t$

28- $r(t) = t\,\hat{\imath} + (2t+3)\hat{\jmath} + (3t-5)\hat{k}$

a) $r'(t) = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

b) $r''(t) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

c) $r'(t) \cdot r''(t) = (1)(0) + (2)(0) + (3)(0)$

$r'(t) \cdot r''(t) = 0$

30- $r(t) = \langle e^{-t}, t^2, \tan t \rangle$

a) $r'(t) = \langle -e^{-t}, 2t, \sec^2 x \rangle$

b) $r''(t) = \langle e^{t}, 2, 2\sec^2 x \tan x$

c) $r'(t) \cdot r''(t) = (-e^{-t})(e^{-t}) + (2t)(2) + (2\sec^2 x \tan x)(\sec^2 x)$

$r'(t) \cdot r''(t) = -e^{-2t} + 4t + 2\sec^4 x \tan x$

ESTILO

Determinar el/los intervalo(s) en que la función vectorial es continua.

75.- $r(t) = t\hat{i} + \frac{1}{t}\hat{j}$

Dom $r(t) = \mathbb{R} \cap \mathbb{R} - \{0\}$

Dom $r(t) = \mathbb{R} - \{0\} \vee \,]-\infty, 0[\cup]0, +\infty[$

$\lim_{t \to 0} [t]\hat{i} + \lim_{t \to 0} \left[\frac{1}{t}\right]\hat{j}$

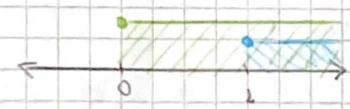
$0\hat{i} + \infty\hat{j}$

La función no es continua en 0 $\therefore$ continua en $]-\infty, 0[\cup]0, +\infty[$

76.- $r(t) = \sqrt{t}\,\hat{i} + \sqrt{t-1}\,\hat{j}$

Dom $r(t) = \mathbb{R} \cap$ $t - 1 \geq 0$
$t \geq 1$

Dom $r(t) = t \geq 0 \cap t \geq 1$



Dom $r(t) = [1, +\infty[$

$\left[\lim_{t \to 1} \sqrt{t}\right]\hat{i} + \left[\lim_{t \to 1} \sqrt{1-1}\right]\hat{j}$

$1\hat{i} + 0\hat{j}$

$r(t)$ es continua en $[1, +\infty[$

77.- $r(t) = t\hat{i} + \operatorname{arcsen} t\,\hat{j} + (t-1)\hat{k}$

Dom $r(t) = \mathbb{R} \cap [-1, 1] \cap \mathbb{R}$

Dom $r(t) = [-1, 1]$

$\left[\lim_{t \to 1} t\right]\hat{i} + \left[\lim_{t \to 1} \operatorname{arcsen}(t)\right]\hat{j} + \left[\lim_{t \to 1} t - 1\right]\hat{k}$

$\hat{i} + \frac{\pi}{2}\hat{j} + 0\hat{k}$

$$\left[\lim_{t\to -1} t\right]\hat{i} + \left[\lim_{t\to 1} \arcsin(-1)\right]\hat{j} + \left[\lim_{t\to -1} t-1\right]\hat{k}$$

$$-\hat{i} - \frac{\pi}{2}\hat{j} - 2\hat{k}$$

$r(1)$ es continua en $[-1, 1]$

78- $r(t) = 2e^{-t}\hat{i} + e^{-t}\hat{j} + \ln(t-1)\hat{k}$

$\text{Dom } r(t) = \mathbb{R} \cap \mathbb{R} \cap t-1>0 = t>1$

$\text{Dom } r(t) =]1, +\infty[$

$$\left[\lim_{t\to 1} 2e^{-t}\right]\hat{i} + \left[\lim_{t\to 1} e^{-t}\right]\hat{j} + \ln\left[t-1\right]\hat{k}$$

$$\frac{2}{e}\hat{i} + \frac{1}{e}\hat{j} + \infty$$

$r(t)$ No es continua en $]1, +\infty[$.

79- $r(t) = \langle e^{-t}, t^2, \tan t\rangle$

$\text{Dom } r(t) = \mathbb{R} \cap \mathbb{R} \cap \mathbb{R} - \left\{\frac{\pi}{2} + \pi\right\}$

$\text{Dom } r(t) = \mathbb{R} - \left\{\frac{\pi}{2} + \pi\right\}$

$$\left[\lim_{t\to \frac{\pi}{2}} e^{-t}\right]\hat{i} + \left[\lim_{t\to \frac{\pi}{2}} t^2\right]\hat{j} + \left[\lim_{t\to \frac{\pi}{2}} \tan(t)\right]\hat{k}$$

$$\frac{1}{e^{\pi/2}}\hat{i} + \frac{\pi^2}{4}\hat{j} + \infty\hat{k}$$

Discontinua en $\frac{\pi}{2}$.

80. $r(t) = \langle 8, \sqrt{t}, \sqrt[3]{t} \rangle$

Dom $r(t) = \mathbb{R}, \mathbb{R} \geq 0, \mathbb{R}$

Dom $r(t) = [0, +\infty[$

$$\left[\lim_{t \to 0} 8\right]\hat{i} + \left[\lim_{t \to 0} \sqrt{t}\right]\hat{j} + \left[\lim_{t \to 0} \sqrt[3]{t}\right]\hat{k}$$

$$8\hat{i} + 0\hat{j} + 0\hat{k}$$

$r(t)$ es continua en $[0, +\infty[$