QUIZ

- Reminder of the honour code that you have committed to uphold:
  "I do hereby undertake that as a student at IIT Delhi:
  (1) I will not give or receive aid in examinations, and
  (2) I will do my share and take an active part in seeing to it that others and myself uphold the spirit and letter of the Honour Code."

Some instructions:

- For each question, name your files as `q<question number>.xyz` where `xyz` is the appropriate extension. For example, for the second question, name the files as `q2.mod`, `q2.dat`, and `q2.run`, whichever are applicable.
- The evaluation of this quiz will be done via an automated script, which will execute only your `.run` file and check for the answers (not manually). So, please run and validate your files before submitting them to Moodle.
- The submission link will expire at 17:45. Please note that late submissions will not be accepted, so plan your time accordingly.
- You can use the AMPL book, but no other resources are allowed.

1. (4 points) Model and solve the following LP using the Cplex solver. Display the objective function value and all the decision variables' names and values.

$$\begin{aligned} \min_{x_1,x_2,x_3} \quad & 2x_1 + x_2 - 3x_3 & \\ \text{s.t.} \quad & 3x_1 + x_2 - 3x_3 & \geq 2, \\ & x_1 - 3x_2 & = 5, \\ & x_1 + x_2 - x_3 & \leq 7, \\ & x_2 & \geq -3, \\ & x_1 & \geq 0, \\ & x_2 & \leq 0. \end{aligned}$$

2. (6 points) A logistics company needs to assign a set of customer delivery zones, say I, to a set of potential Regional Distribution Centers (RDCs), say J. Each RDC, $j \in J$, can serve only a limited number of delivery zones, up to its maximum capacity, K_j, depending on the travel distance and service feasibility. Opening and operating an RDC incurs a fixed cost of c_j. The company must decide which RDCs to open and how to assign delivery zones to them in order to minimize the total operating cost such that (a) every zone is served by at least one RDC that is capable of covering it, and (b) no RDC exceeds its capacity.

   The following mathematical formulation models this problem.

   Notation

I	: set of delivery zones,
J	: set of candidate RDCs,
$\mathcal{A} = \{a_{ij}\}$	: zone–RDC pairs (1, if zone i can be covered by RDC j, 0 otherwise), $i \in I,\ j \in J$
$K_j \in \mathbb{Z}_+$	: capacity (maximum zones) of RDC $j \in J$,
$c_j \geq 0$	: fixed cost to open and operate RDC $j \in J$.

Variables

$$y_j \in \{0,1\} \quad : 1 \text{ if RDC } j \text{ is opened, 0 otherwise, } \; j \in J$$
$$x_{ij} \in \{0,1\} \quad : 1 \text{ if zone } i \text{ is assigned to RDC } j, \;\; i \in I, \; j \in J.$$

Model.

$$\min \quad \sum_{j \in J} c_j \, y_j$$

$$\text{s.t.} \quad \sum_{j \in J} x_{ij} \; \geq \; 1 \qquad \forall \, i \in I \qquad \text{(each zone assigned at least once)}$$

$$\sum_{i \in I} x_{ij} \; \leq \; K_j \, y_j \quad \forall \, j \in J \qquad \text{(assign if opened and within capacity)}$$

$$x_{ij} \; \leq \; a_{ij} \qquad \forall \, i \in I, \; j \in J \qquad \text{(assign if capable)}$$

$$x_{ij} \in \{0,1\} \qquad \forall \, i \in I, \; j \in J, \qquad y_j \in \{0,1\} \quad \forall \, j \in J.$$

The required data is provided in `q2.xlsx` on Moodle. Model and solve this problem using the Gurobi solver. Display the objective function value and all the decision variables' names and values.

3. (10 points) Recall the Diet Problem, where the goal is to select a set of foods that will satisfy a set of daily nutritional requirements at minimum cost. A data instance for this problem is provided in `q3.xlsx` on Moodle. Model and solve this problem using the Gurobi solver. Display the objective function value, all the decision variables' names and values, and the duals of the constraints.