PROPRIÉTÉS DE LA TRANSFORMÉE DE FOURIER

Transformée de Fourier:

$$\mathfrak{F}(f)(\alpha) = \hat{f}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx$$

Transformée de Fourier inverse:

$$\mathfrak{F}^{-1}(g)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\alpha) e^{i\alpha x} d\alpha$$

PROPRIÉTÉS:

1) **Linéarité**: $\mathfrak{F}(af + bg) = a\mathfrak{F}(f) + b\mathfrak{F}(g) \quad a, b \in \mathbb{R}$

2) **Parité**: Si f est paire, $\mathfrak{F}(f)(\alpha) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} f(x) \cos(\alpha x) dx$

3) **Imparité**: Si f est impaire, $\mathfrak{F}(f)(\alpha) = -i\sqrt{\frac{2}{\pi}} \int_{0}^{\infty} f(x) \sin(\alpha x) dx$

4) **Transformée de Fourier des dérivées de f**:

$\mathfrak{F}(f')(\alpha) = i\alpha\mathfrak{F}(f)(\alpha)$

$\mathfrak{F}(f'')(\alpha) = -\alpha^2\mathfrak{F}(f)(\alpha)$

$\mathfrak{F}(f^{(n)})(\alpha) = (i\alpha)^n\mathfrak{F}(f)(\alpha)$

5) **Décalage**:

Si $g(x) = e^{i\alpha_0 x} f(x)$, alors $\mathfrak{F}(g)(\alpha) = \mathfrak{F}(f)(\alpha - \alpha_0)$

6) **Convolution**:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t) g(x - t) dt = \int_{-\infty}^{\infty} g(t) f(x - t) dt$$

Alors $\mathfrak{F}(f * g) = \sqrt{2\pi}\mathfrak{F}(f)\mathfrak{F}(g)$

7) **Identité de Plancherel**:

$$\int_{-\infty}^{\infty} |f(y)|^2 dy = \int_{-\infty}^{\infty} |\hat{f}(\alpha)|^2 d\alpha$$

TRANSFORMÉES DE FOURIER

	$f(x)$	$\mathfrak{F}(f)(\alpha) = \hat{f}(\alpha)$
1	$\begin{cases} 1 & \text{si } -b < x < b \\ 0 & \text{sinon} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin(b\alpha)}{\alpha}$
2	$\begin{cases} 1 & \text{si } b < x < c \\ 0 & \text{sinon} \end{cases}$	$\frac{e^{-i\alpha b} - e^{-i\alpha c}}{i\alpha\sqrt{2\pi}}$
3	$\frac{1}{x^2 + a^2} \qquad (a > 0)$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a\lvert\alpha\rvert}}{a}$
4	$\begin{cases} e^{-ax} & \text{si } x > 0 \\ 0 & \text{sinon} \end{cases} \qquad (a > 0)$	$\frac{1}{\sqrt{2\pi}(a + i\alpha)}$
5	$\begin{cases} e^{ax} & \text{si } b < x < c \\ 0 & \text{sinon} \end{cases}$	$\frac{e^{(a-i\alpha)c} - e^{(a-i\alpha)b}}{\sqrt{2\pi}(a - i\alpha)}$
6	$\begin{cases} e^{iax} & \text{si } -b < x < b \\ 0 & \text{sinon} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin[b(\alpha - a)]}{\alpha - a}$
7	$\begin{cases} e^{iax} & \text{si } b < x < c \\ 0 & \text{sinon} \end{cases}$	$\frac{i}{\sqrt{2\pi}} \frac{e^{ib(a-\alpha)} - e^{ic(a-\alpha)}}{a - \alpha}$
8	$e^{-ax^2} \qquad (a > 0)$	$\frac{1}{\sqrt{2a}} e^{-\alpha^2/4a}$
9	$\frac{\sin(ax)}{x} \qquad (a > 0)$	$\sqrt{\frac{\pi}{2}}$ si $\lvert\alpha\rvert < a$; 0 si $\lvert\alpha\rvert > a$
10	$\frac{e^{-\lvert\omega x\rvert}}{\lvert\omega\rvert}$	$\sqrt{\frac{2}{\pi}} \frac{1}{\omega^2 + \alpha^2}$
11	$\frac{4x^2}{(\omega^2 + x^2)^2} \qquad (\omega \neq 0)$	$\sqrt{2\pi} \left(\frac{1}{\lvert\omega\rvert} - \lvert\alpha\rvert \right) e^{-\lvert\omega\alpha\rvert}$
12	$xe^{-\omega^2 x^2} \qquad (\omega \neq 0)$	$\frac{-i\alpha}{2\sqrt{2}\lvert\omega\rvert^3} e^{-\frac{\alpha^2}{4\omega^2}}$