PHYS 416 Project Proposal
The Physics of Flying Discs

Errol Jordheim

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1 Introduction

If you have ever thrown a Frisbee or a disc golf disc, it is apparent that there are three main forces at play in the motion of the disc, rotation, drag, and lift. As the disc flies through the air, one can watch it slowly rise, like an airplane wing, as it travels away from the thrower and slowly curve off to one direction while maintaining its stability. In this project, I will seek to mathematically model the forces at play in a flying disc to simulate the unique motion of a rotating Frisbee disc.

2 Overview of Physical Processes

The forces at play in a flying disc are gravity, lift, and drag. Of course, gravity is acting on the disc essentially uniformly, depending only on the mass of the disc and always acting in the -z direction. Drag is caused by the motion of the disc through the air, and, as will be discussed later, is dependent on both the velocity of the disc through the air and the angle at which the disc is facing relative to its velocity vector. This is called the angle of attack. Similar to drag, the lift force is also dependent on the velocity of the disc and the angle of attack, but acts perpendicular to the velocity vector.

The lift and drag forces do not necessarily act on the center of mass of the disc. This offset from the center results in a torque, which without stabilization would flip the disc over and severely decrease the flight distance. The disc is stabilized through the addition of spin, with the disc rotating very rapidly around vector normal to the face. This rotation gives a very large angular momentum, meaning that the torque caused by the lift and drag results in a small change in the orientation of the disc. This stabilization keeps the disc relatively flat, only allowing for a slight curve near the end of the trajectory dependent on the rotation of the disc. This curve in the flight path is caused by gyroscopic precession of the disc.

3 Equations and Theory

The lift and drag equations are very similar in form, while acting perpendicular to each-other. The lift equation is given by

$$L = \frac{1}{2} C_L A \rho V^2 \tag{1}$$

where L is the lift force, C_L is the lift coefficient, A is the cross-sectional area in the direction of the velocity, ρ is the density of the air, and V is the velocity of the air flowing over the disc. The lift coefficient can be approximated to vary linearly with the angle of attack of the disc as can be derived using the Bernoulli principle. The force acts perpendicular to the velocity vector in the direction of the normal vector to the face of the disc.

The drag equation is given by

$$D = \frac{1}{2} C_D A \rho V^2 \tag{2}$$

where D is the drag force, C_D is the drag coefficient, A is the cross-sectional area in the direction of the velocity, ρ is the density of the air, and V is the velocity of the air flowing over the disc. The drag coefficient has been experimentally found to vary quadratically with the angle of attack of the disc. The force acts parallel to the velocity vector of the disc, directly opposing the motion.

The rotational inertia of the disc tells us how much torque results in an angular acceleration of the disc. The angular momentum of a solid object rotating around an axis can be given by $\vec{L} = \vec{I} \cdot \vec{\omega}$, where $\vec{L}$ is the angular velocity vector, $\vec{I}$ is the inertia tensor, and $\vec{\omega}$ is the angular velocity vector. The inertia tensor is dependent on the geometry and orientation of the disc. Assuming that the angular velocity is sufficiently large, we can approximate the angular velocity to be given by $\vec{L} = I\omega\hat{\omega}$, where $\hat{\omega}$ is the unit vector defining the axis or rotation. This means that the inertia of the disc is constant, with the only changing vector being the axis of rotation. Defining torque as the rate of change of the angular momentum, we can approximate the gyroscopic precession rate of the disc to be given by $\omega_p = \frac{\tau}{L}$, where τ is the torque, and L is the angular momentum of the disc. We can then use this to calculate the moment applied to the disc using the equation

$$M = I\frac{d\omega}{dt} + \omega_p \times I\omega \tag{3}$$

4 Computational Methods

To a zeroth order approximation, this system is almost identical to the ballistic trajectory models that were simulated earlier in this course. As detailed above, the forces acting on the system are far more complex, but the basic motion and system of equations are very similar. Per this comparison, I intend to use a 4th order Runge-Kutta method to solve for the motion of the disc. This will allow for the numerical error to be minimized, which is very important considering how dependent the forces are on the instantaneous conditions of the disc. I will also have to incorporate very small time steps, due to the state dependence of the system, meaning that less accurate schemes may run into issues with error propagation.

As more forces are added into the model, the effects of the additions will become much more subtle. The fourth order scheme will allow for the effects of these forces to be accurately analyzed with minimal noise resulting from the propagation of numerical error.

5 Validation of Results

In order to validate my results, I will compare my simulated flight trajectories to actual documented Frisbee trajectories as provided in academic papers regarding the topic [Schroeder 23][Morrison 7].

Potts and Crowther wrote a paper regarding testing of a Frisbee in a wing tunnel, measuring the lift and drag coefficients as functions of attack angle, as well as pitch and roll coefficients, giving numerical values that I can use to make my model match the actual flight profile of a Frisbee as closely as possible.

The basics of the model will first be developed using a 2-dimensional simulation, with only the drag and lift forces affecting the ballistic trajectory. While this model will not account for any of the gyroscopic effects, it will give a sufficient approximation to validate the ratios of drag and lift, and verify that they are implemented correctly.

From this 2-dimensional model, I intend to expand it into 3-dimensions, accounting for the torque caused by the lift and drag forces and the rotation of the disc. This will have a noticeable effect on the trajectory of the disc, as well as allowing for analysis of the curve of the disc resulting from the initial rotation and possible instability of the system.

6 Potential Further Additions

Once all of the main forces affecting the flight of the Frisbee have been implemented and validated, smaller forces and effects can be incorporated into the simulation to see how the approximations made in the derivations effect the model.

A couple of the forces that may be incorporated into the simulation is the Magnus force arising from the rotation of the disc and the change in Inertia resulting from a low initial rotational velocity. These effects are not expected to substantially alter the trajectory of the flight, but will serve to further refine the model.

Another potential addition to the model is the incorporation of air flow across the disc. The addition of wind into the model would allow for a more realistic simulation of flight conditions, as well as allowing for analysis of flight trajectories and unexpected instability resulting from differing wind conditions.

7 References

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