5620

Natural Frequencies of Marine Drilling Risers

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Introduction

The effect of wave forces on the life and performance of marine drilling risers is a concern to both contractor and operator because a broken riser means capital loss and expensive down-time. The literature[1] reflects that much effort is being made to improve understanding of riser motion and stresses during operation to upgrade riser design and operating practices. It is hoped that riser technology eventually will lead to reliable predictive techniques, verified by experimental data. While accurate dynamic-stress predictions are desirable, natural-frequency data alone may be directly useful in alleviating dynamic stresses during operations.

Large vibrational stresses are normally associated with resonance that exists when the frequency of the imposed force, from whatever source, is tuned to one of the natural frequencies of the elastic system (marine riser). When this condition exists, a natural mode will dominate the over-all dynamic behavior of the riser. It is possible to excite two or more modes simultaneously; and in the case of the marine drilling riser in the North Sea, the first two modes are the most vulnerable. One of the difficulties in developing a program for predicting the forced vibration response of a marine riser is modeling the wave forces or the source of excitation. In addition, damping is not well defined. A predictive technique, no matter how sophisticated, is no better than the boundary conditions imposed on it. On the other hand, input frequencies (or wave frequencies) can be determined on location and used along with natural-frequency data to determine whether a resonant or a critical dynamic condition exists.

This paper presents data that relate the first five natural frequencies of marine drilling risers to typical riser and drilling parameters. The operational parameters are related through dimensionless numbers that are generated for most drilling risers operating within a water depth of 600 ft. The drilling riser is idealized as a vertical flexible beam with pinned supports. Variable tension and fluid environment make the mathematics different from classical beam theory and lead to a differential equation that is perhaps unique to the oil industry. Eigenvalues and eigenvectors are determined by solving this differential equation. In addition, the exact natural frequencies or eigenvalues are compared with frequencies obtained through an approximate method based on classical, uniformly tensioned beam equations. An example calculation illustrates the practical use of the data.

Differential Equation of Motion

The natural modes and frequencies of marine risers depend on the usual beam parameters. The usual assumptions made in developing beam-column equations also are made here. However, the fact that the riser is surrounded by fluid of one density (sea water) and contains fluid of another density (drilling mud), and has internal tension that varies along its length, makes the riser quite different from an elementary beam. It is, therefore, worthwhile to show how these parameters enter into the differential equation of motion. The following derivation is based on plane motion; the spatial coordinates are illustrated in Fig. 1a.

This paper gives the mathematical basis for determining natural frequencies and corresponding mode shapes for marine drilling risers. Numerical results relate natural frequencies to riser parameters; the data cover risers operating within 600 ft of water. An approximate method is given for establishing marine-riser natural frequencies.

A typical differential element is shown in Fig. 1b. The vertical forces are assumed to be in equilibrium; therefore,

$$\frac{\partial F}{\partial z} = w \quad \text{.............................(1)}$$

Summation of moments gives

$$\frac{\partial M}{\partial z} = F_s \,, \quad \text{.............................(2)}$$

and the Bernoulli-Euler equation is

$$\mathrm{EI}\,\frac{\partial^2 y}{\partial z^2} = M \quad \text{.............................(3)}$$

Before summing forces in the y direction, it is best to convert the inside and outside pressures to statically equivalent forces that can be easily entered into the force summation. Figs. 2 and 3 illustrate the outside pressure conversion and the inside pressure conversion, respectively. The outside pressure conversion, for example, can be achieved by first adding and then subtracting local pressure force vectors on the transverse sections. The compressive force vector adds to the fluid pressures on the curved portion of the differential element to give the buoyant force, B_e. The remaining force vectors on the transverse sections and the buoyant force vector are shown in Fig. 2. Now, summing all forces in the y direction gives

$$m\,\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial z}\left[F\,\theta + (p_e A_e - p_i A_i)\,\theta\right] - \frac{\partial F_s}{\partial z} \quad \text{...(4)}$$

Note that

$$p_e = (L - z)\,\rho_w \,,$$

$$p_i = (L - z)\,\rho_m \,,$$

$$\theta = \frac{\partial y}{\partial z} \,,$$

and

$$F = F_B + wz \,.$$

Substituting into Eq. 4 gives

$$\mathrm{EI}\,\frac{\partial^4 y}{\partial z^4} - \frac{\partial}{\partial z}\left\{\left[(F_B + wz) + (L - z)(A_e\rho_w - A_i\rho_m)\right]\frac{\partial y}{\partial z}\right\} + m\,\frac{\partial^2 y}{\partial t^2} = 0 \quad \text{.........................(5)}$$

This is the equation of motion for undamped free vibration of the marine riser. The solution to this equation defines the mode shapes and natural frequencies. Boundary conditions also must be satisfied at the upper and lower ends of the riser. If we assume ball joints at the ends and no offset, the boundary conditions stated mathematically are

$$y(0, t) = 0 \,,$$

$$\frac{\partial^2 y}{\partial z^2}(0, t) = 0 \,,$$

$$y(L, t) = 0 \,,$$

and

$$\frac{\partial^2 y}{\partial z^2}(L, t) = 0 \,.$$

The independent variables can be separated by letting

$$y(z, t) = Y(z)\sin\omega t \quad \text{....................(6)}$$

Substituting Eq. 6 into Eq. 5 gives

$$\mathrm{EI}\,\frac{d^4 Y}{dz^4} - \frac{d}{dz}\left\{\left[(F_B + wz) + (L - z)(A_e\rho_w - A_i\rho_m)\right]\frac{dY}{dz}\right\} - m\omega^2 Y = 0 \quad \text{.........................(7)}$$

It is convenient to work with dimensionless parameters. Therefore, let

$$\alpha = \frac{(w - A_e\rho_w + A_i\rho_m)\,L^3\,144}{\mathrm{EI}} \,,$$

$$\beta = \frac{(F_B + LA_e\rho_w - LA_i\rho_m)\,L^2\,144}{\mathrm{EI}} \,,$$

$$\lambda^4 = \frac{m\,\omega^2 L^4\,144}{\mathrm{EI}} \,,$$

and

$$\zeta = \frac{z}{L} \,.$$

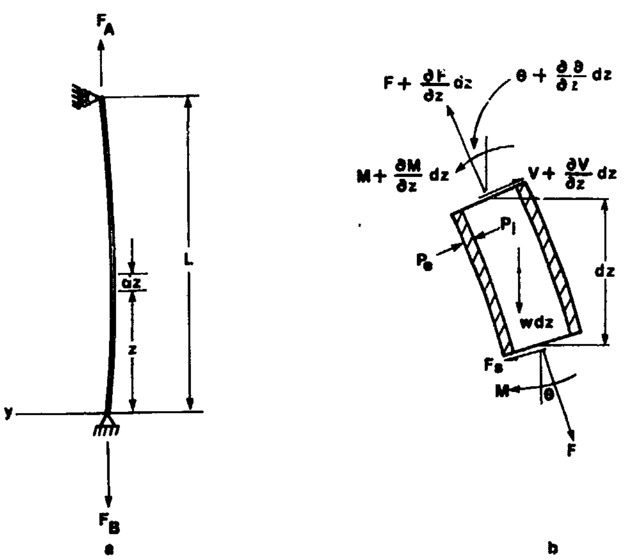

Fig. 1 — Coordinate system and freebody for marine riser.

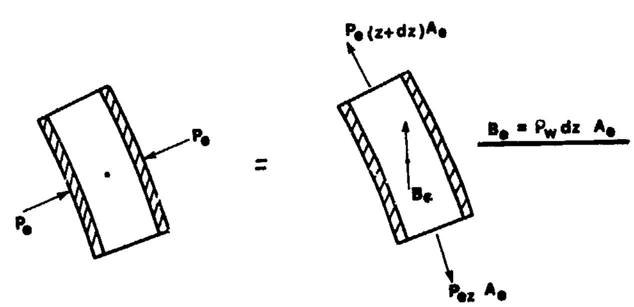

Fig. 2 — Statically equivalent outside pressure forces.

Note that the dimensional unit of EI in these parameters is lb-sq in.; therefore, the factor 144 sq in./sq ft has been added. Eq. 7 now becomes

$$\frac{d^4Y}{d\zeta^4} - \frac{d}{d\zeta}\left[(\beta + \alpha\,\zeta)\frac{dY}{d\zeta}\right] - \lambda^4 Y = 0 \quad \ldots\ldots(8)$$

The second term contains a variable coefficient that is a characteristic of long beams with variable axial-force distribution.

Method of Solution

Because of the variable coefficient, the solution for Y may not be expressed in terms of elementary functions. However, there exists a power series such that

$$Y = a_0 + a_1\zeta + a_2\zeta^2 + \ldots + a_n\zeta^n + \ldots \quad \ldots(9)$$

The series and its derivatives are absolutely uniformly convergent; therefore, the derivatives of Y are achieved by term-wise differentiation. Substituting Eq. 9 into Eq. 8 results in an equation involving a power series in ζ; the coefficient of each term is equal to zero. This equation yields the coefficients in Eq. 9 as well as natural frequencies. The mode shape corresponding to each natural frequency is thus defined by Eq. 9. The details of the method of solution have been documented in other papers[2,3] and is not repeated here.

Discussion of Results

The solution to Eq. 8 leads to eigenvalues, λ_i, that define the natural frequencies of vibration modes corresponding to different α's and β's. λ's for the first and second modes are displayed in Figs. 4 and 5. These data were developed for $0 \leq \alpha \leq 300$ and $0 \leq \beta \leq 400$ and cover most marine risers operating in water depths up to 600 ft. The higher modes may not be of practical interest; nonetheless, λ's for the first five modes are displayed in Table 1 for comparison. When drilling beyond 600-ft water depths, the higher modes may be important and the data may have to be extended beyond α = 300 and β = 400 to cover longer risers. Risers containing buoyant chambers and designed for deeper waters will have different natural-frequency data.

Negative values of β are not covered in this paper because they are of no practical interest in connection with riser vibrations. Negative values of β, however, are covered in Ref. 2 and may be of interest to those concerned with riser buckling. The data in Ref. 2 show that as buckling is approached the natural period of vibration increases until buckling is reached, at which point the natural period of vibration is infinite.

Table 1 gives the eigenvalues to three significant figures beyond the decimal point and represents several hours of computer calculations; the computer calculations required a high degree of precision. These numbers not only define natural frequencies, but also can be used to define the coefficient in Eq. 9. Note that when α = 0, the variable coefficient in Eq. 8 drops out and the eigenvalues, λ, correspond to those from classical solutions.

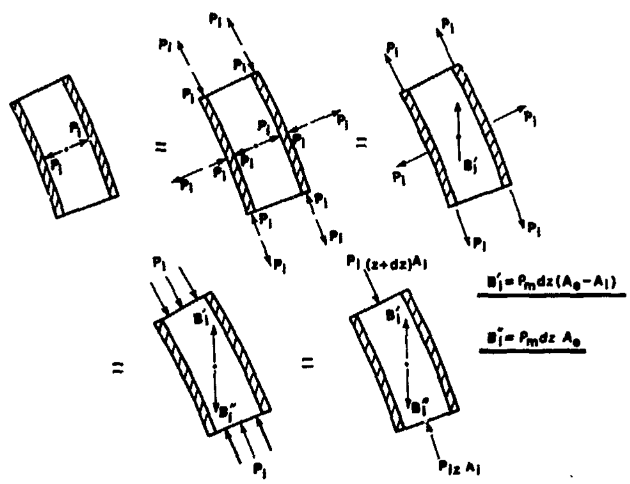

Fig. 3 — Statically equivalent inside pressure forces.

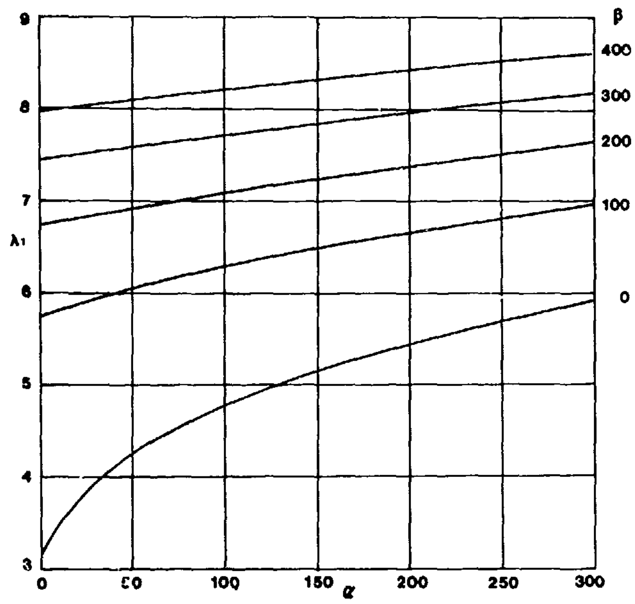

Fig. 4 — Dimensionless parameters for fundamental natural mode.

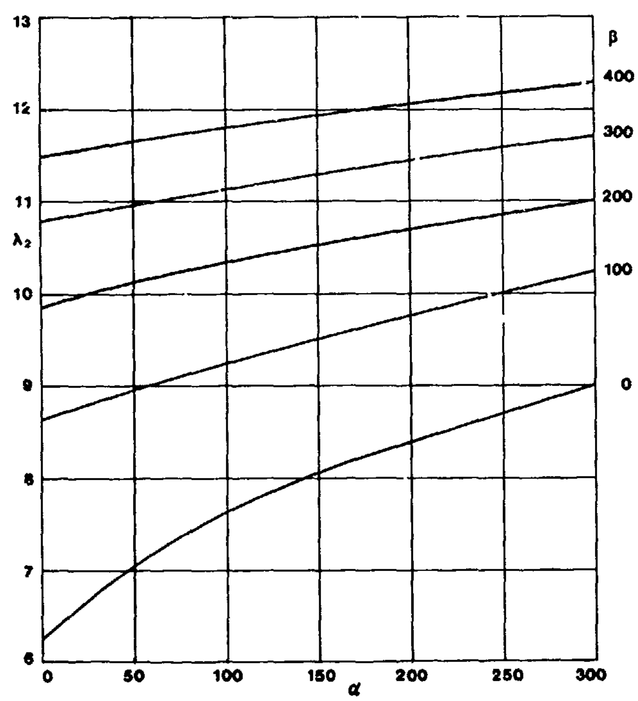

Fig. 5 — Dimensionless parameters for second natural mode.

One method of approximating λ is based on a beam subjected to uniform tension equal to F_B plus one-half the weight of the beam. According to Timoshenko,[4] this approximation leads to

$$\lambda_i = i\pi \sqrt[4]{1 + \frac{\beta + \frac{1}{2}\alpha}{i^2 \pi^2}} \quad \text{.................(10)}$$

The percentage of error from this approximation is defined by

$$\frac{\text{approximate value} - \text{true value}}{\text{true value}} \times 100 ,$$

and is displayed in Table 2. The greatest error occurs when $\beta = 0$. In general, however, the error is very small for the values of α and β covered in this paper. Note that, since $\omega_i \propto \lambda_i^2$, a 1-percent error in λ means a 2-percent error in ω_i. Also, the error increases as α increases, which means that natural frequencies calculated from Eq. 10 may not be realistic for marine drilling risers operating beyond 600-ft depths. However, a few numerical calculations indicate that Eq. 10 is realistic, for engineering purposes, when applied to marine risers operating within 600 ft of water.

Mode shapes corresponding to the various natural frequencies, ω, show how marine risers may respond in space. In general, all modes can participate in forced vibrations of marine risers; however, if the frequency of the source of excitation (wave forces and vessel motion) is tuned to one of the natural frequencies, the riser will respond to that frequency and one mode will be amplified.

Modes corresponding to the first three natural frequencies are illustrated in Fig. 6. These modes correspond to $\alpha = 250$ and $\beta = 100$, which were chosen arbitrarily. The location of the largest amplitudes are not evenly spaced along the riser because internal tension and hydrostatic pressure are not uniformly distributed. The nodal distance is greatest in the upper portion of the riser where the tension is greatest. The modes, then, differ somewhat from trigonometric modes.

There are three practical aspects of the first mode:

1. There is a point of inflection near the top that is not present in trigonometric modes. This point is about $\Delta\zeta = 0.1$ below the top and behaves as a ball joint.
2. The slope at the bottom ball joint is greater than the corresponding slope of trigonometric modes.
3. The maximum bending stress is greater than the

TABLE 1 — NATURAL FREQUENCY PARAMETERS BASED ON SOLUTION TO EQ. 8

α	β	λ_1	λ_2	λ_3	λ_4	λ_5
0	0	3.142	6.283	9.425	12.566	15.708
	100	5.738	8.614	11.380	14.206	17.103
	200	6.746	9.861	12.656	15.419	13.221
	300	7.437	10.760	13.632	16.398	19.165
	400	7.975	11.477	14.435	17.228	19.987
50	0	4.234	7.083	10.020	13.034	16.090
	100	6.029	8.969	11.735	14.536	17.401
	200	6.934	10.105	12.919	15.679	18.470
	300	7.580	10.950	13.845	16.616	19.379
	400	8.092	11.635	14.615	17.417	20.176
100	0	4.778	7.649	10.513	13.449	16.443
	100	6.268	9.275	12.053	14.841	17.683
	200	7.101	10.325	13.161	15.924	18.706
	300	7.711	11.126	14.044	16.823	19.585
	400	8.202	11.784	14.786	17.598	20.359
150	0	5.162	8.093	10.934	13.824	16.772
	100	6.473	9.544	12.342	15.124	17.949
	200	7.251	10.528	13.387	16.156	18.932
	300	7.832	11.291	14.232	17.021	19.783
	400	8.305	11.925	14.948	17.771	20.536
200	0	5.465	8.462	11.303	14.165	17.079
	100	6.554	9.786	12.607	15.388	18.202
	200	7.388	10.715	13.598	16.375	19.149
	300	7.945	11.447	14.410	17.210	19.973
	400	8.402	12.058	15.103	17.938	20.707
250	0	5.719	8.781	11.633	14.479	17.368
	100	6.816	10.006	12.851	15.636	18.442
	200	7.515	10.889	13.797	16.583	19.356
	300	8.051	11.593	14.580	17.390	20.157
	400	8.494	12.186	15.252	18.099	20.872
300	0	5.938	9.062	11.931	14.770	17.641
	100	6.964	10.209	13.079	15.871	18.671
	200	7.633	11.052	13.985	16.781	19.555
	300	8.151	11.732	14.741	17.564	20.334
	400	8.581	12.308	15.395	18.254	21.032

TABLE 2 — ERROR OF APPROXIMATE SOLUTION BASED ON EQ. 10

$$\text{Error} = \frac{\text{approximate value} - \text{true value}}{\text{true value}} \times 100$$

α	β	λ_1	λ_2	λ_3	λ_4	λ_5
0	0	0	0	0	0	0
	100	0	0	0	0	0
	200	0	0	0	0	0
	300	0	0	0	0	0
	400	0	0	0	0	0
50	0	1.73	0.28	0.08	0.02	0.01
	100	0.19	0.09	0.04	0.02	0.01
	200	0.07	0.03	0.02	0.01	0.00
	300	0.03	0.02	0.01	0.01	0.00
	400	0.03	0.01	0.01	0.01	0.00
100	0	3.19	0.79	0.24	0.09	0.04
	100	0.55	0.27	0.13	0.06	0.03
	200	0.22	0.14	0.08	0.04	0.03
	300	0.12	0.09	0.05	0.03	0.02
	400	0.07	0.05	0.03	0.02	0.02
150	0	4.22	1.31	0.45	0.18	0.08
	100	0.97	0.51	0.25	0.12	0.06
	200	0.42	0.26	0.16	0.09	0.05
	300	0.24	0.17	0.11	0.07	0.04
	400	0.15	0.11	0.08	0.05	0.03
200	0	5.00	1.80	0.68	0.29	0.14
	100	1.39	0.76	0.39	0.20	0.10
	200	0.66	0.42	0.25	0.14	0.08
	300	0.38	0.26	0.18	0.11	0.07
	400	0.24	0.19	0.13	0.09	0.05
250	0	5.62	2.23	0.92	0.41	0.20
	100	1.80	1.02	0.55	0.29	0.16
	200	0.89	0.58	0.36	0.21	0.13
	300	0.53	0.38	0.25	0.16	0.10
	400	0.35	0.26	0.19	0.12	0.08
300	0	6.14	2.63	1.15	0.54	0.27
	100	2.19	1.28	0.71	0.38	0.21
	200	1.14	0.76	0.47	0.28	0.17
	300	0.70	0.50	0.34	0.22	0.14
	400	0.48	0.35	0.25	0.17	0.12

corresponding stress within trigonometric modes.

These aspects are characteristic of very long tensioned beams and they would be more pronounced in water depths greater than 600 ft.

Example Calculation

The following example is designed to illustrate the use of the dimensionless parameters α, β, and λ for obtaining the natural period of vibration for a typical marine drilling riser in the northern North Sea. The calculations are made for a 24- × ⅝-in. riser 500 ft long, having a 286,000-lb pull force at the bottom ball joint. Accordingly,

w = 214 lb/ft (includes 38 lb/ft for choke and kill lines),
A_e = 3.14 sq ft,
A_i = 2.99 sq ft,
ρ_w = 64.8 lb/cu ft,
ρ_m = 85 lb/cu ft,
I = 3,136.9 in.,^{4}
$E = 30 \times 10^6$ lb/sq in.,
F_B = 286,000 lb, and
m = 20.8 slugs/ft (includes mass of drilling mud and sea water).

Therefore, $\alpha = 50$ and $\beta = 100$. From Table 1, $\lambda_1 = 6.029$. The dimensionless parameter, λ_1, gives $\omega_1 = 0.815$ rad/sec.

The natural period of the first vibration mode, then, is 7.71 seconds. This natural period is determined from the exact solution. The natural period based on the approximation given by Eq. 1C is 7.68 seconds. The approximate solution is, therefore, quite adequate for engineering purposes when comparing the natural periods of marine risers with the period of deterministic sea waves.

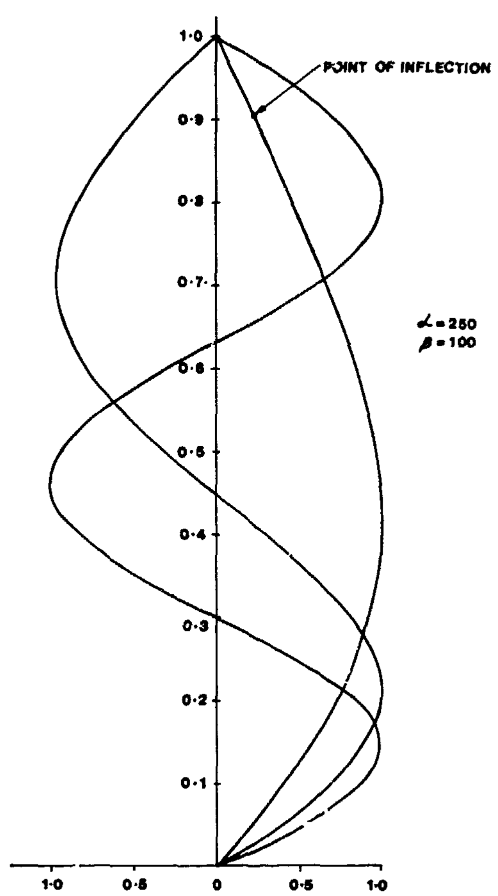

Fig. 6 — Natural mode shapes for marine riser.

Conclusions

Severe vibrational stresses are normally associated with resonance. Large vibrational stresses in marine drilling risers, therefore, may be alleviated by detuning the riser from all input frequencies. This paper gives the mathematical basis and numerical data for determining natural frequencies for marine drilling risers operating in water depths up to 600 ft. An approximate mathematical solution is also given and is compared with the exact mathematical solution based on this analysis. The approximate formulation gives good engineering results and is somewhat easier to use.

Riser frequencies are affected by several parameters; however, not all are easily varied on location. Tensioning and drilling-mud density can be varied on location, and both significantly affect natural frequency. Riser tensioning increases natural frequency and is analogous to the tuning of stringed musical instruments. Like the windings on bass-fiddle strings, drilling mud adds mass but not stiffness and, thus, reduces natural frequency.

Mode shapes are nontrigonometric. As a result, bending is greatest nearer the lower end, where tension is the smallest. Points of inflection are possible near the top where the tension is greatest. These points are dynamically equivalent to ball joints.

Nomenclature

a_i = coefficient in power series
A_e = cross-sectional area of riser exterior, sq ft
A_i = cross-sectional area of riser interior, sq ft
E = modulus of elasticity, psi
EI = exponential integral
F = local tension in riser, lb
F_B = tension in rise at bottom ball joint, lb
F_s = local shear force, lb
I = cross-sectional moment of inertia of riser, in.^{4}
L = length of riser, ft
m = mass (per unit length) participating in riser motion, slugs/ft
M = local bending moment, ft-lb
p_e = local hydrostatic pressure outside riser, lb-sq ft
p_i = local hydrostatic pressure inside riser, lb-sq ft
t = time, seconds
w = weight of riser per unit length, lb/ft
y = transverse displacement, ft
Y = mode displacement, ft
z = vertical coordinate, ft
α = dimensionless parameter =

$$\frac{(w - A_e \rho_w + A_i \rho_m) L^3\ 144}{EI}$$

β = dimensionless parameter =

$$\frac{(F_B + LA_e \rho_w - LA_i \rho_m) L^2\ 144}{EI}$$

λ^4 = dimensionless parameter =

$$\frac{m\,\omega^2 L^4\ 144}{EI}$$

ζ = dimensionless parameter = z/L
ρ_m = density of drilling mud, lb/cu ft
ρ_w = density of sea water, lb/cu ft
ω = natural circular frequency, rad/sec
θ = local riser slope, rad

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References

1. Morgan, G. A. and Peret, J. W.: "Applied Mechanics of Marine Riser Systems," *Pet. Eng.* (Oct. 1974).
2. Huang, T. and Dareing, D. W.: "Buckling and Frequencies of Long Vertical Pipes," *J. Eng. Mech.*, ASCE (Feb. 1969) **95**, No. EMI, 167-181.
3. Huang, T. and Dareing, D. W.: "Buckling and Lateral Vibration of Drill Pipe," *J. Eng. for Ind.* (Nov. 1968) **90**, Series B, No. 4, 613-619; *Trans.*, ASME.
4. Timoshenko, S.: *Vibration Problems in Engineering*, D. Van Nostrand Co., Inc., Princeton, N. J. (1956).

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