Dynamic Programming for Unit Commitment

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***Abstract*—The Unit Commitment (UC) problem is solved in this study using dynamic programming. Reducing the dimensionality of the unit commitment problem is one advantages of dynamic programming. A 4-unit system was used in illustrating the dynamic programming algorithm. The optimal schedule of the generators and the minimum overall cost was successfully obtained. The overall cost includes both the production and start-up costs.**

***Index Terms*—Unit commitment, dynamic programming, feasible state**

I. Introduction

Due to the nature of human activities, customers' electricity demand change on a regular basis. To deal with these cyclic variations, the dispatcher must turn on just enough generating units to match the load demand at each interval of the scheduling horizon. This is the primary goal of Unit Commitment (UC). Another purpose of unit commitment in regulated industry is to reduce operational costs while meeting demand [1].

Optimal unit commitment problem can be stated as "Determine an optimal pattern for the start up and shut down of generators in order to minimize the total operating cost during a period of study called the scheduling horizon without violating any of the operating constraints" [2].

The short-term unit commitment challenge is to calculate the optimum operating level of the available generating units for each hour of the following day and for up to seven days in order to meet the forecasted level of demand with the lowest operating cost while meeting all of the physical and operational restrictions of the power system. Some of the more common constraints incorporated to the unit commitment problem are load balance, spinning reserve, scheduled reserve, offline reserve, must-run units, and fuel consumption, among others. Constraints that are particular to thermal units include minimum and maximum up and down time limits, start-up costs, and minimum and maximum generation limits [1].

To have a complete solution of unit commitment problem, the economic dispatch problem (EDP) must be solved as well. There are two possible ways to do this; the first is to obtain a unit commitment schedule then an economic dispatch is found for this schedule. The other way is to solve both UCP and EDP simultaneously. This makes the problem more difficult to solve. Yet, it is believed that the second method guarantees a more optimal commitment schedule to be found [2]. In this paper the latter is being employed through dynamic programming.

This paper is organized as follows. Section II explains the operation and technical constraints of unit commitment problem. The dynamic programming model used to represent the unit commitment problem is discussed in Section III. In addition, the sample UC problem is presented followed by the resulting schedule of the start-up and shut down of generators. Finally, Section V concludes the paper.

II. Unit Commitment Problem Formulation

A. Notation

$u(i,t)$:	Status of unit i at period t
$p(i,t)$:	Power produced by unit i during period t
$C_i[p(i,t)]$:	Running cost of unit i during period t
$SC_i[u(i,t)]$:	Start-up cost of unit i during period t
$N =$	Number of available generating units
$T =$	Number of periods in the optimization horizon

B. Objective Function

When working on an optimization problem, we work towards maximizing or minimizing an objective function. The objective function of UC problem can be modified as

$$\min_{u(i,t);p(i,t)} \sum_{t=1}^{T}\sum_{i=1}^{N}\{C_i[p(i,t)] + SC_i[u(i,t)]\} \tag{1}$$

C. System Constraints

1) Power balance constraints: The total unit generation output must satisfy the system load demand requirement at each time step t, therefore

$$\sum_{i=1}^{N} u(i,t)p(i,t) = L(t) \tag{2}$$

where is $L(t)$ the load demand.

2) Reserve generation capacity: Spinning reserve is the term used to describe the total amount of generation available from all units synchronized (i.e., spinning) on the system, minus the present load and losses being supplied

$$\sum_{i=1}^{N} u(i,t)\left[P_i^{\max} - p(i,t)\right] \geq R(t) \tag{3}$$

D. Unit Constraints

1) Maximum/Minumum generation limits: For each committed unit, the power generation $p(i,t)$ should be within the generation limits of the unit, i.e. between its minimum and maximum possible generation. This can be expressed as:

$$u(i,t)P_i^{\min} \leq p(i,t) \leq u(i,t)P_i^{\max} \quad \forall i \in N, t \in 1\ldots T \tag{4}$$

2) Minimum up time and down time: Once a plant turns on, it must stay on for a certain number of hours before it can be turned off again. Similarly, once off it must stay off for a certain number of hours before it can be turned on again.

$$\text{If } u(i,t) = 1 \text{ and } t_i^{up} < t_i^{up,\min} \text{ then } u(i,t+1) = 1 \tag{5}$$

$$\text{If } u(i,t) = 0 \text{ and } t_i^{\text{down}} < t_i^{\text{down},\min} \text{ then } u(i,t+1) = 0 \tag{6}$$

3) Maximum ramp rates: To avoid damaging the turbine, the electrical output of a unit cannot change by more than a certain amount over a period of time.

$$\text{Ramp up} \rightarrow \quad p(i,t+1) - p(i,t) \leq \Delta P_i^{up,\max} \tag{7}$$
$$\text{Ramp down} \rightarrow \quad p(i,t) - p(i,t+1) \leq \Delta P_i^{\text{down},\max} \tag{8}$$

4) Unit Hot/Cold Start Constraints: A recently shut-down plant generally is quicker and more efficient to start-up than a cooled one. This difference in cost can be modeled in the costfunction. We assume a step-function for the cost. If a plant is turned off within some time period, it only requires the cold start cost, else the hot-start cost. This is expressed below

$$\text{StartupCost} = \begin{cases} \text{hotstartcost}, & \text{if down-time} \leq \text{cold start T} \\ \text{coldstartcost}, & \text{otherwise} \end{cases} \tag{9}$$

Unit restrictions such as offline time, maintenance schedule, security constraints, and so on are not included but can be represented in addition to those proposed above.

III. Dynamic Programming for Unit Commitment

The dynamic programming approach has distinct advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of the time it has been off-line (i.e., its temperature), then a forward dynamic-program approach is more suitable since the previous history of the unit can be computed at each stage [3]. Also, the initial conditions are easily specified and the computations can go forward in time as long as required. A forward dynamic-programming algorithm is shown by the flowchart in Fig.1.

$$\begin{aligned} F_{\text{cost}}(K,I) = & \min_L [P_{\text{cost}}(K,I) + S_{\text{cost}}(K-1,L:K,I) \\ & + F_{\text{cost}}(K-1,L)] \end{aligned} \tag{10}$$

where

$$\begin{aligned} F_{\text{cosi}}(K,I) &= \text{least total cost to arrive at state } (K,I) \\ P_{\text{cost}}(K,I) &= \text{production cost for state } (K,I) \\ S_{\text{cost}}(K-1,L:K,I) &= \text{transition cost from state } (K-1,L) \\ & \quad\ \text{to state } (K,I) \end{aligned}$$

State (K,I) is the I^{th} combination in hour K. For the forward dynamicprogramming approach, we define a strategy as the transition, or path, from one state at a given hour to a state at the next hour.

Note that two new variables, X and N, have been introduced in Fig. 2.

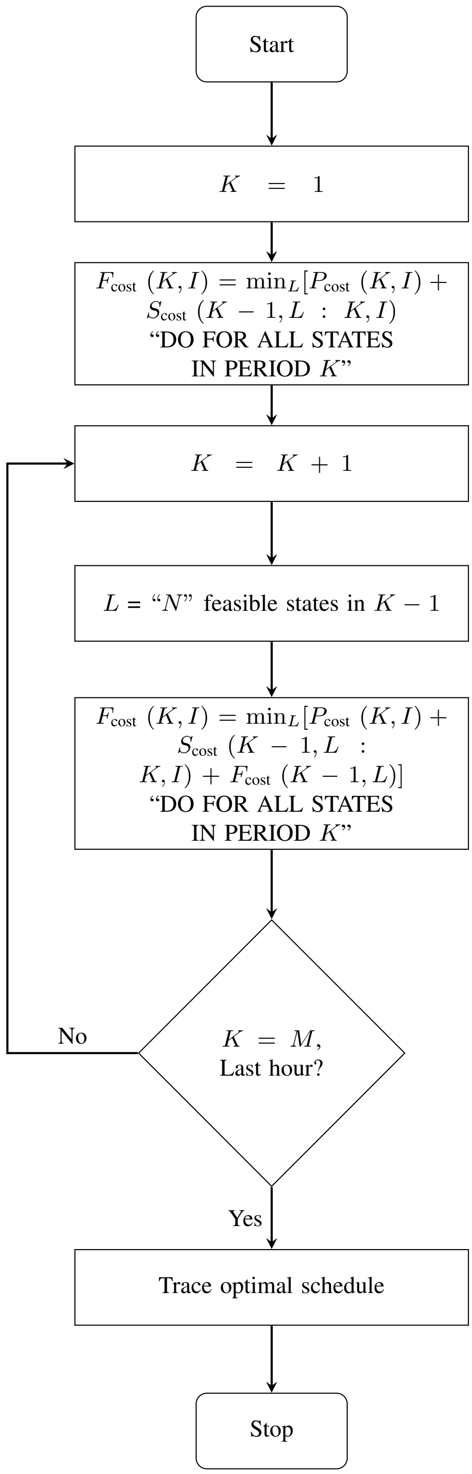

Fig. 1: Unit commitment via forward dynamic programming

X = number of states to search each time period (hour)
N = number of strategies, or paths, to save at each step

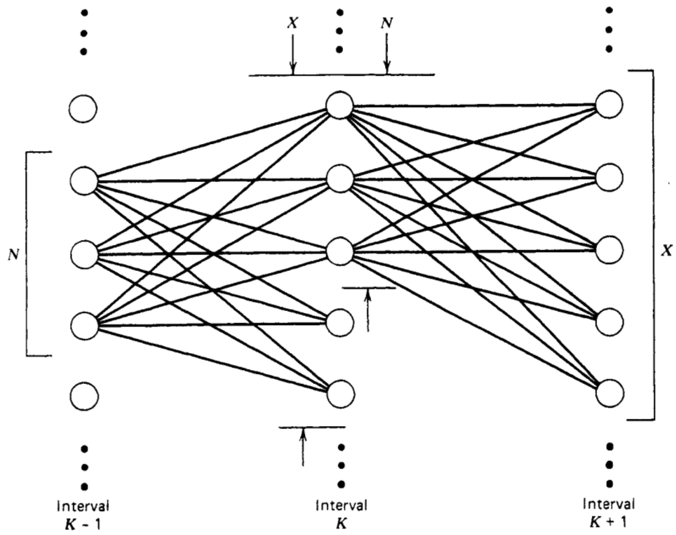

Fig. 2: Restricted search paths in DP algorithm with $N_{K-1} = 3$ and $X = 5$ at interval $K - 1$

In other words, N_{K-1} refers to the number of feasible states at time period $((K - 1))$, while $X = 5$ refers to the paths leading to 5 feasible states in time period K. The feasibility of the states are determined by evaluating the minimum and maximum generating capacities for a certain combination of generators at that state.

Then the feasibility of a path from a feasible state at $K - 1$ to a feasible set at K is evaluated using the minimum up and down time of the generators. This means that if a path does not pass the minimum up and down time constraints, it is considered infeasible and is not saved as a feasible path.

Furthermore, these feasible paths are further evaluated through the running cost incurred by each state through economic dispatch. When performing economic dispatch, the ramp-rate constraints are considered in the economic dispatch since it will alter the lower and upper bounds of the power generation of each unit.

After the cost for each state has been obtained, Start-up costs are then considered which are inherent when transitioning from a state to another. Running costs and transition costs are then added and associated to each state. The solution – path from the 1st state up to the last state, is obtained by finding the path incurring the minimum cost.

In order to illustrate the advantage dynamic programming, we present the commitment of four generating units over an 8-h period. The specific the unit characteristics are presented in TABLEI and TABLEII while the hourly power demand for a duration of 8 hours is given in Fig.3. The simulation is implemented using MATLAB.

A. Unit Commitment sample problem

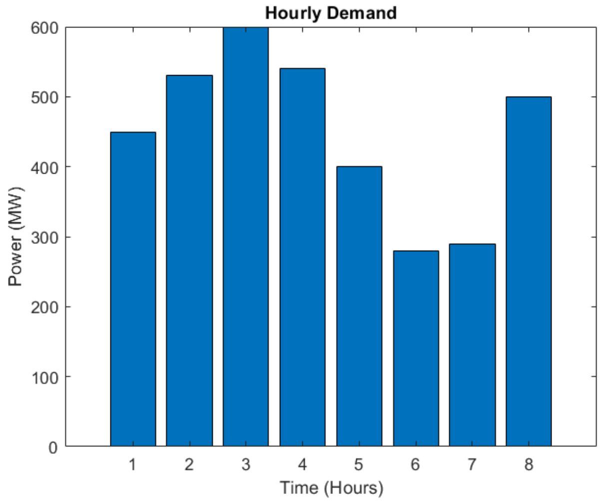

Fig. 3: Hourly power demand for an 8-hour period

TABLE I: Unit Characteristics

Unit	P_min (MW)	P_max (MW)	Inc. heat rate (BTU/kWh)	No load cost ($)	Start cold cost ($)	Fuel cost
1	25	80	10440	213.00	350	2.00
2	60	250	9000	585.62	400	2.00
3	75	300	8730	684.74	1100	2.00
4	20	60	11900	252.00	0.02	2.00

TABLE II: Unit Characteristics

Unit	Min. Up Time (h)	Min. Down Time (h)	In status (h)	Start cost hot	Cold start	Ramp up (MW/$)	Ramp down (MW/$)
1	4	2	-5	150	4	50	75
2	5	3	+8	170	5	80	120
3	5	4	+8	500	5	100	150
4	1	1	-6	0	0	80	120

B. Results

The results for DP are presented and tested on a 4-unit base system with an 8-hour time horizon. The program was written in MATLAB.

Dynamic programming was repeated for an 8-hour period unit commitment. TABLEIII show the schedule of the 4 generating units for 8 hours and their corresponding total costs. The total cost of operating the output schedule of 4 generating units for 8 hours is $74,110.

TABLE III: Hourly Results

Hour	Demand	Total Gen	Prod. Cost	Total Cost	State	Units (On/Off)
0	-	-	0	0	13	0 1 1 0
1	450	450	9208	9208	13	0 1 1 0
2	530	530	10648	19857	13	0 1 1 0
3	600	600	12450	32307	14	0 1 1 1
4	540	540	10828	43135	13	0 1 1 0
5	400	400	8308	51444	13	0 1 1 0
6	280	280	6192	57635	13	0 1 1 0
7	290	290	6366	64002	13	0 1 1 0
8	500	500	10108	74110	13	0 1 1 0

IV. Conclusion

The algorithm employed to solve the unit commitment problem was able to generate the scheduling of the start-up and shutdown of 4 generating units for an 8-hour time period while considering different realistic operational constraints. Among these technical constraints are the different start-up sequences and the different configurations/states that of a generating unit. The minimum and minimum number of intervals that generating unit must remain in a given configuraion/state are also considered. Finally, the dynamic programming approach was able to avoid the dimensionality of the unit commitment problem. Dynamic programming was also able to arrive at the optimal solution.

References

[1] J. Alvarez Lopez, R. Nieva Gomez, and I. Guillen Moya, "Commitment of combined cycle plants using a dual optimization–dynamic programming approach," *IEEE Transactions on Power Systems*, vol. 26, no. 2, pp. 728–737, 2011.

[2] M. Kamh, A. Abdelaziz, S. Mekhamer, and M. Badr, "Modified augmented hopfield neural network for optimal thermal unit commitment," 08 2009, pp. 1 – 8.

[3] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power generation, operation, and control*. John Wiley & Sons, 2013.

[4] P. K. Singhal and R. N. Sharma, "Dynamic programming approach for large scale unit commitment problem," in *2011 International Conference on Communication Systems and Network Technologies*, 2011, pp. 714–717.