QUIZ 5 (GROUP WORK)

GOOD LUCK

- Show all your work and indicate your final answer clearly. You will be graded not merely on the final answer, but also on the work leading up to it.

1. (3pts) Let $f(x) = 3x^2$, $g = e^x$, $h = x^3$. Find the first derivative of:

   (a) $f + g + h$

   Solution:

$$F_1(x) = f(x) + g(x) + h(x) = 3x^2 + e^x + x^3$$
$$\Longrightarrow F_1'(x) = 6x + e^x + 3x^2.$$

   (b) $\frac{h^2}{g}$

   Solution: Using chain rule, we have

$$\frac{d}{dx}e^{-x} = -e^{-x}.$$

   By product rule,

$$F_2(x) = \frac{h^2(x)}{g(x)} = \frac{(x^3)^2}{e^x} = x^6e^{-x}$$
$$\Longrightarrow F_2'(x) = (6x^5)(e^{-x}) + (x^6)(-e^{-x}) = (6x^5 - x^6)e^{-x}.$$

   (c) fgh

   Solution: Using product rule, we have

$$F_3(x) = f(x)g(x)h(x) = (3x^2)(e^x)(x^3) = 3x^5e^x$$
$$\Longrightarrow F_3'(x) = (15x^4)(e^x) + (3x^5)(e^x) = (15x^4 + 3x^5)e^x.$$

2. (2pts) Find the third derivative of the following functions:

(a) $F(x) = x^3 e^x$

Solution:

$$\begin{aligned} & F(x) = x^3 e^x \\ \implies & F'(x) = (3x^2)(e^x) + (x^3)(e^x) = (3x^2 + x^3)e^x \\ \implies & F''(x) = (6x + 3x^2)(e^x) + (3x^2 + x^3)(e^x) = (6x + 6x^2 + x^3)e^x \\ \implies & F'''(x) = (6 + 12x + 3x^2)e^x + (6x + 6x^2 + x^3)e^x = (6 + 18x + 9x^2 + x^3)e^x. \end{aligned}$$

(b) $F(x) = \frac{x^6}{x^2}$

Solution:

$$\begin{aligned} & F(x) = \frac{x^6}{x^2} = x^4 \\ \implies & F'(x) = 4x^3 \\ \implies & F''(x) = 12x^2 \\ \implies & F'''(x) = 24x. \end{aligned}$$

For x not equal 0.

3. (5pts) At what point(s) on the curve $y = 3x^3$ is the tangent line perpendicular to the line $y = -x$?

Solution:

$$y = 3x^3 \implies \frac{dy}{dx} = 9x^2.$$

The tangent line to the curve $y = 3x^3$ at the point $(a, 3a^3)$ is perpendicular to the line $y = -x$ if and only if

$$(9a^2)(-1) = -1 \iff a^2 = \frac{1}{9} \iff a = \pm\frac{1}{3}.$$

Therefore, the tangent line to the curve $y = 3x^3$ at the points $\left(\frac{1}{3}, \frac{1}{9}\right)$ and $\left(\frac{-1}{3}, \frac{-1}{9}\right)$.