Math 151 - Quiz 1 Solutions

September 22, 2017

1. Let $\mathbf{a} = \langle 2, 3\rangle$. Find a unit vector $\mathbf{u}_a$ of $\mathbf{a}$ and a vector with length 4 in the direction of $\mathbf{a}$.

   Solution 1. We can define the unit vector $\mathbf{u}_a$ as

   $$\begin{aligned}\mathbf{u}_a &= \frac{\mathbf{a}}{|\mathbf{a}|} \\ &= \frac{\langle 2,3\rangle}{\sqrt{2^2+3^2}} \\ &= \langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\rangle.\end{aligned}$$

   To find the vector with length 4 in the direction of $\mathbf{a}$, we just multiply $\mathbf{u}_a$ by 4:

   $$4\mathbf{u}_a = \langle \frac{8}{\sqrt{13}}, \frac{12}{\sqrt{13}}\rangle.$$

2. Recall the standard basis vector $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$. Let $\mathbf{v} = -\sqrt{3}\mathbf{i} + \mathbf{j}$. Find the length $|\mathbf{v}|$ and the direction angle θ of $\mathbf{v}$.

   Solution 2. We can write $\mathbf{v}$ in vector notation

   $$\mathbf{v} = \langle -\sqrt{3}, 1\rangle.$$

   Then finding the length

   $$\begin{aligned}|\mathbf{v}| &= \sqrt{\left(\sqrt{3}\right)^2 + 1^2} \\ &= \sqrt{3+1} \\ &= 2\end{aligned}$$

   and angle θ

   $$\begin{aligned}\tan\theta &= \frac{1}{-\sqrt{3}} \\ &\Longrightarrow \\ \theta &= \arctan\left(\frac{1}{-\sqrt{3}}\right) \\ &= -\frac{\pi}{6}.\end{aligned}$$

But notice that the vector $\langle -\sqrt{3}, 1\rangle$ is pointing "north-west" and so the angle $-\frac{\pi}{6}$ doesn't make sense so we need to add π to the angle and thus

$$\boxed{\theta = \frac{5\pi}{6}}$$

3. **Solution 3.** (a)

$$\begin{aligned} \overrightarrow{AB} &= \langle 2, 4\rangle \\ \overrightarrow{BA} &= \langle -2, -4\rangle \\ \overrightarrow{BC} &= \langle -4, 1\rangle \\ \overrightarrow{AC} &= \langle -2, 5\rangle \end{aligned}$$

(b) Consider the dot product $\overrightarrow{AB} \cdot \overrightarrow{BA}$:

$$\overrightarrow{AB} \cdot \overrightarrow{BA} = \langle 2, 4\rangle \cdot \langle -2, -4\rangle = -4 - 16 = -20 \neq 0 \tag{1}$$

so they are not perpendicular. But note that $\overrightarrow{BA} = -1 \cdot \overrightarrow{AB}$ so the vectors are multiples of each other and thus **parallel**.

(c) We see that

$$\overrightarrow{AB} + \overrightarrow{BC} = \langle 2, 4\rangle + \langle -4, 1\rangle = \langle -2, 5\rangle = \overrightarrow{AC}.$$