

NAME:

MATH 151, Fall 2017

Section:

QUIZ 10 (INDIVIDUAL WORK)

GOOD LUCK

- Show all your work and indicate your final answer clearly. You will be graded not merely on the final answer, but also on the work leading up to it.

- (5pts) A closed box with a square base is to be constructed. If the surface area of the box is 16 square centimeters, find the dimensions of the box with maximum volume.

- Because box is closed, surface area is given by

$$SA = 16 = 2b^2 + 4bh$$

 \Rightarrow

$$h = \frac{16 - 2b^2}{4b}$$

$$\bullet V'(b) = 4 - \frac{3b^2}{2}$$

$$\bullet V' = 0 \Rightarrow b = \pm \sqrt{\frac{8}{3}}$$

$$\bullet \text{Volume} = b^2 h = b^2 \left(\frac{16 - 2b^2}{4b} \right) = 4b - \frac{b^3}{2}$$

~~2. (5pts) Find $f(x)$ if $f''(x) = x^{-2}$, $x > 0$, $f(1) = 0$, $f(2) = 0$.~~

- But b can't be negative so $V' = 0$ when $b = \sqrt{\frac{8}{3}}$.

- need to check if it's a local max at this point.

Use 2nd derivative test:

$$+ \quad V'' = -\frac{4b}{2} = -3b$$

$$V''\left(\sqrt{\frac{8}{3}}\right) = -3\left(\sqrt{\frac{8}{3}}\right) < 0 \Rightarrow \text{local max.}$$

- So volume is max when $b = \sqrt{\frac{8}{3}}$, $h = \frac{16 - 2 \cdot \frac{8}{3}}{4\sqrt{\frac{8}{3}}}$ $\left. \vphantom{\frac{16 - 2 \cdot \frac{8}{3}}{4\sqrt{\frac{8}{3}}}} \right\} +$

2) Find $f(x)$ if $f''(x) = x^{-2}$, $x > 0$, $f(1) = 0$
 $f(2) = 0$.

Solutions • To find $f'(x)$, get antiderivative of f'' :

$$f' = \frac{x^{-2+1}}{-1} + C$$

$$= -x^{-1} + C$$

$$= -\frac{1}{x} + C + |$$

• Similarly for f :

$$f = -\ln x + Cx + d + |$$

← no absolute value because $x > 0$

← new constant.

• check conditions:

$$\begin{aligned} * f(1) &= -\ln 1 + C + d = 0 \\ \Rightarrow \\ C + d &= 0 \end{aligned}$$

$$\begin{aligned} * f(2) &= -\ln 2 + 2C + d = 0 \\ \Rightarrow \\ 2C + d &= \ln 2 \end{aligned}$$

$$\text{Then } \begin{cases} C + d = 0 \\ 2C + d = \ln 2 \end{cases} \Rightarrow \begin{aligned} C &= \ln 2 + | \\ d &= -\ln 2 + | \end{aligned}$$

$$\text{Thus } \boxed{f(x) = -\ln x + (\ln 2) \cdot x - \ln 2} + |$$