QUIZ 3 (GROUP WORK)

GOOD LUCK

- Show all your work and indicate your final answer clearly. You will be graded not merely on the final answer, but also on the work leading up to it.

1. (3pts) Let $\mathbf{a} = \langle 2, 1 \rangle$. Find the parametric equation and the Cartesian equation of the line perpendicular to $\mathbf{a}$ and passing through the point $P(-1, 4)$.

Solution. Recall the parametric equation of a line can be written in the form

$$\begin{aligned} \mathbf{r}(t) &= \mathbf{r_0} + t\mathbf{v} \\ &= \langle x_0, y_0 \rangle + t\langle v_1, v_2 \rangle \end{aligned}$$

Let $\langle x_0, y_0 \rangle = \langle -1, 4 \rangle$ (because we want it to pass through this point). Since we want a line perpendicular to $\mathbf{a}$, we want $\mathbf{v}$ to be $\mathbf{v} = \langle -1, 2 \rangle$ so that $\mathbf{a} \cdot \mathbf{v} = 0$. So our parametric equation is

$$\begin{aligned} \mathbf{r}(t) &= \langle -1, 4 \rangle + t\langle -1, 2 \rangle \\ &= \langle -1 - t, 4 + 2t \rangle. \end{aligned}$$

To find the Cartesian equation, we set $x = -1 - t$, $y = 4 + 2t$. Using the equation for x, we can solve for t: $t = -1 - x$. Substituting this into the y equation we see

$$\begin{aligned} y &= 4 + 2t \\ &= 4 + 2(-1 - x) \\ &= 4 - 2 - 2x \\ &= 2 - 2x \end{aligned}$$

2. (3pts) Consider the lines $\mathbf{r}(t) = \langle 1+3t, -1+4t\rangle$ and $\mathbf{s}(w) = \langle 2-4w, 3+3w\rangle$. Determine whether the lines are parallel, perpendicular nor neither. If they are not parallel, find the intersection point.

Solution. Focusing only on "the parts with t and w", we can find out if these two lines are perpendicular/parallel/neither. The t part for $\mathbf{r}$ is $\langle 3, 4\rangle$ and the w part $\mathbf{s}$ is $\langle -4, 3\rangle$. Taking the dot product of these two vectors we see

$$\langle 3, 4\rangle \cdot \langle -4, 3\rangle = -12 + 12 = 0.$$

Thus these lines are perpendicular and thus intersect. To find the intersection point we set the components of each line equation, that is

$$\begin{aligned} 1 + 3t &= 2 - 4w \\ -1 + 4t &= 3 + 3w \end{aligned}$$

Then solving for t and w (this is good practice so I won't do it here :)), we see that $t = 19/25$, $w = -8/25$. Then to find the exact point, we just "plug in" either one of these values into their appropriate equation:

$$\mathbf{r}(t = \frac{19}{25}) = \langle 1 + 3\frac{19}{25}, -1 + 4\frac{19}{25}\rangle = \langle \frac{82}{25}, \frac{51}{25}\rangle$$

3. (4pts) Consider the line $L: 4x - 3y = 0$ and the point $P(2,6)$.
   (a) Find a unit vector $\mathbf{u}$ parallel to the line L.
   (b) Find the vector projection of $\overrightarrow{OP}$ onto $\mathbf{u}$, where O is the origin.

Solution (a). Notice that we can write L as $y = \frac{4}{3}x$ and so our unit vector has to look like $\mathbf{u} = \langle \frac{3}{c}, \frac{4}{c} \rangle$ where the c is the number that will make this into a "unit" vector. All we need to do is find the magnitude of the non-unit vector $\langle 3, 4 \rangle$ which is 5 and so

$$\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$$

Solution (b). First notice that $\overrightarrow{OP} = \langle 2, 6 \rangle$. Then to find the vector projection of $\overrightarrow{OP}$ onto $\mathbf{u}$, we can use our formula

$$\mathrm{proj}_{\mathbf{u}}\overrightarrow{OP} = \frac{\overrightarrow{OP} \cdot \mathbf{u}}{|u|^2}\mathbf{u}$$

NOTE that $\mathbf{u}$ is a **UNIT** vector and has length 1!!! So

$$\begin{aligned}
\mathrm{proj}_{\mathbf{u}}\overrightarrow{OP} &= \frac{\overrightarrow{OP} \cdot \mathbf{u}}{|u|^2}\mathbf{u} \\
&= \frac{\langle 2, 6 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle}{1} \langle \frac{3}{5}, \frac{4}{5} \rangle \\
&= \left(\frac{6}{5} + \frac{24}{5} \right) \langle \frac{3}{5}, \frac{4}{5} \rangle \\
&= \langle \frac{18}{5}, \frac{24}{5} \rangle
\end{aligned}$$