QUIZ 4 (INDIVIDUAL WORK)

GOOD LUCK

- Show all your work and indicate your final answer clearly. You will be graded not merely on the final answer, but also on the work leading up to it.

1. (6pts) Find the limits.

(a) $\lim_{x\to -1} \frac{x^2-x-2}{x^2+x-6}$ Note that we can factor top/bottom:

$$\lim_{x\to -1} \frac{x^2-x-2}{x^2+x-6} = \lim_{x\to -1} \frac{\cancel{(x-2)}(x+1)}{(x+3)\cancel{(x-2)}} = \lim_{x\to -1} \frac{(x+1)}{(x+3)}$$

$$= \frac{-1+1}{-1+3} = 0.$$

(b) $\lim_{x\to 2} \frac{x^2-x-2}{x^2+x-6}$ Similarly as part a:

$$\lim_{x\to 2} \frac{x^2-x-2}{x^2+x-6} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{(x+3)\cancel{(x-2)}}$$

$$= \lim_{x\to 2} \frac{x+1}{x+3} = \frac{2+1}{2+3} = \frac{3}{5}.$$

(c) $\lim_{x\to-\infty} \frac{x^2-x-2}{x^2+x-6}$ $= \lim_{x\to-\infty} \frac{1 - x/x^2 - 2/x^2}{1 + x/x^2 - 6/x^2} = \lim_{x\to-\infty} \frac{1 - 1/x - 2/x^2}{1 + 1/x - 6/x^2}$

all go to 0.

$= \frac{1}{1} = 1.$

2. (4pts) Let

$$g(x) = \begin{cases} \dfrac{x-1}{\sqrt{x^2+8}-3} & \text{if } x < 1, \\ \left(k\sin\left(\frac{\pi x}{3}\right)\right)^2 & \text{if } x \geq 1. \end{cases}$$

For what value(s) of k makes(s) $g(x)$ continuous?

1st step: Want to find limit as $x \to 1$ of top function:

$$\lim_{x\to1} \frac{x-1}{\sqrt{x^2+8}-3} = \lim_{x\to1} \frac{x-1}{\sqrt{x^2+8}-3} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3} = \lim_{x\to1} \frac{\sqrt{x^2+8}+3}{x+1} = 3.$$

just algebra at this step.

2nd step: Since $x \geq 1$, let $x=1$ for bottom function:

$$\left(k \cdot \sin\left(\frac{\pi}{3}\right)\right)^2 = k^2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3k^2}{4}.$$

3rd step: Let $3 = \frac{3k^2}{4} \Rightarrow 4 = k^2 \Rightarrow \boxed{k = \pm 2}$