

# A review of the CARP and its variants using (simple) network analysis



**Elias J. Willemse**  
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WARP3 - 2019 - Pizzo, Italy

[https://github.com/  
ejwillemse/WARP3](https://github.com/ejwillemse/WARP3)



**Faculty of Engineering,  
Built Environment and  
Information Technology**  
Fakulteit Ingenieurswese, Bou-omgewing en  
Inligtingtegnologie / Lefapha la Boetšenere,  
Tikologo ya Kago le Theknolotši ya Tshedimošo

1956 – 2016

60  
years of  
Engineering  
Education

# Outline

1

Our research focus (starting 2008)

2

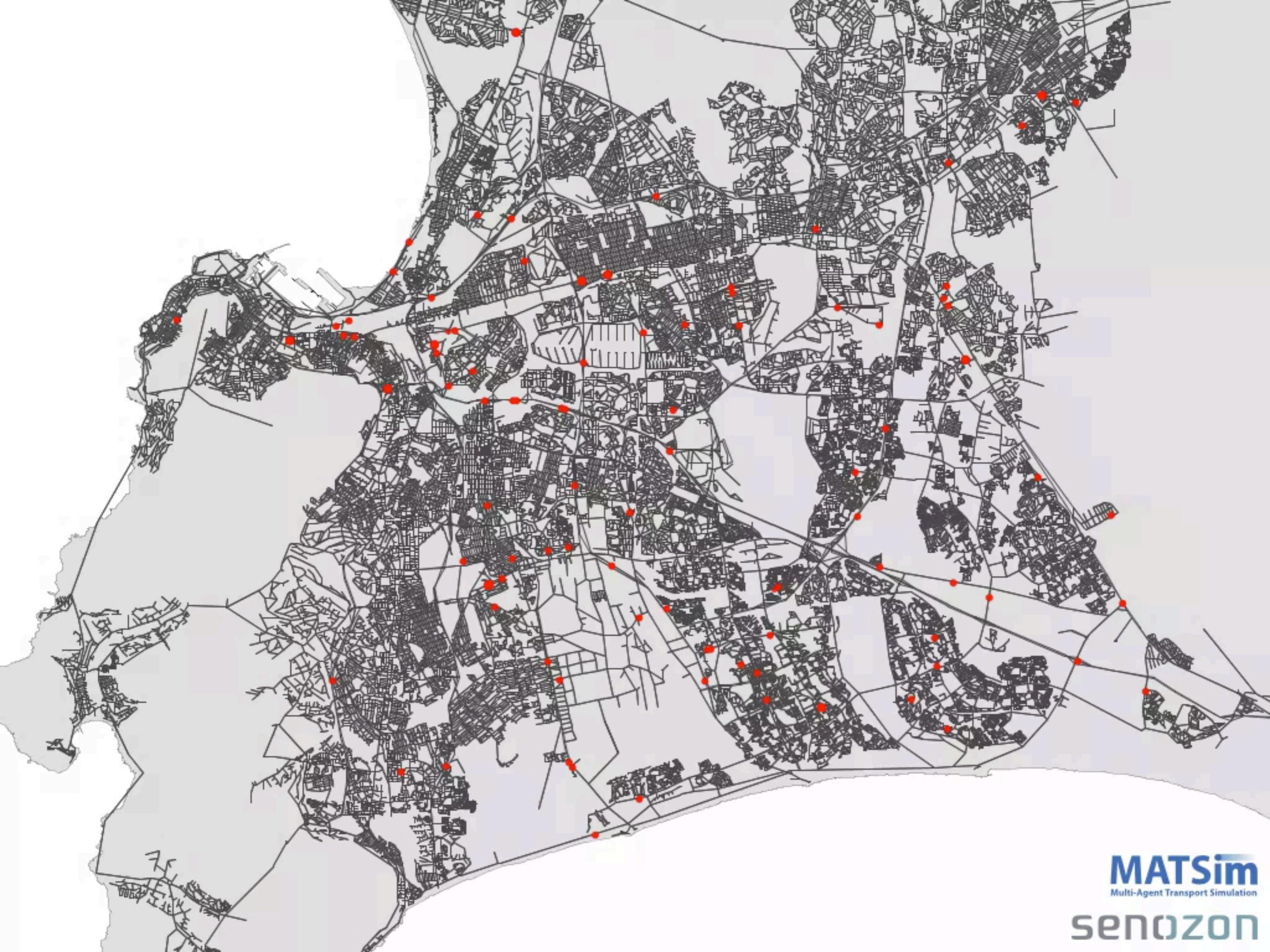
General metrics of CARP publications.

3

Solution approaches and benchmark instances

4

Some ideas on future research opportunities



**MATSim**  
Multi-Agent Transport Simulation

**senozon**

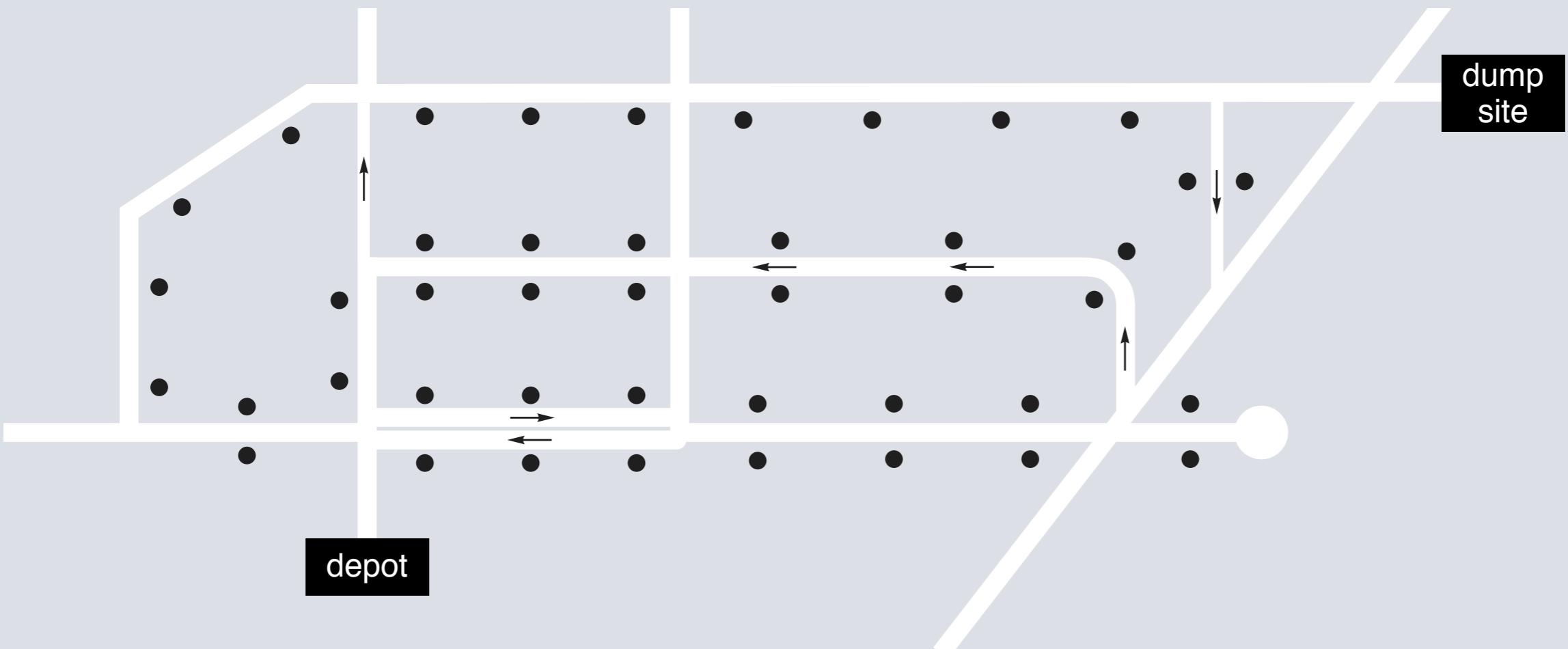
“By the end of 2003, WM had **984 fewer routes, saving \$18 million**. It estimated that its savings for 2004 due to the reduction will be \$44 million.”

Sahoo, S., Kim, S., Kim, B. I., Kraas, B., & Popov Jr, A. (2005). **Routing optimization for waste management.** *Interfaces*, 35(1), 24-36.

**6'282** required arcs and edges



# Waste collection routing optimisation



# Mixed Network

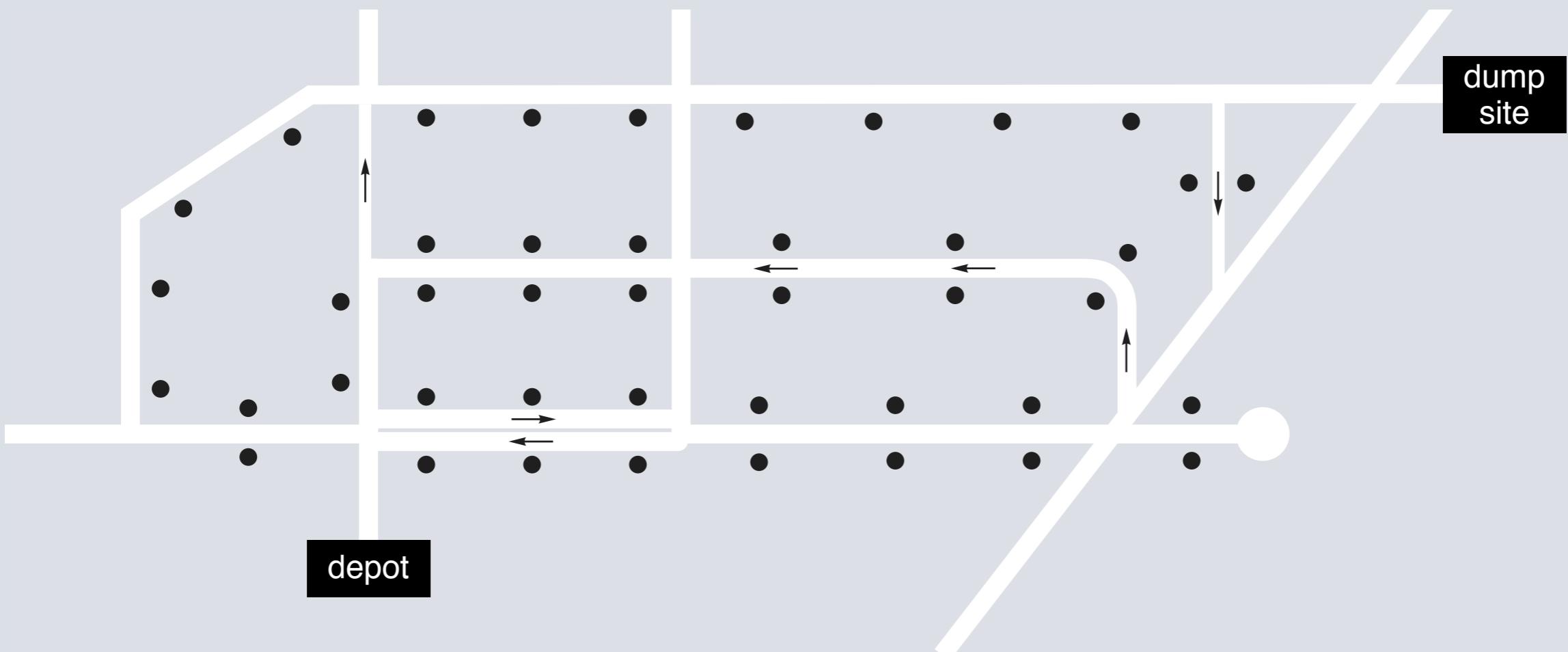
one- and two-way streets

# Capacity

amount of waste a vehicle can carry in one go

# Arc Routing

streets have to be serviced, instead of places



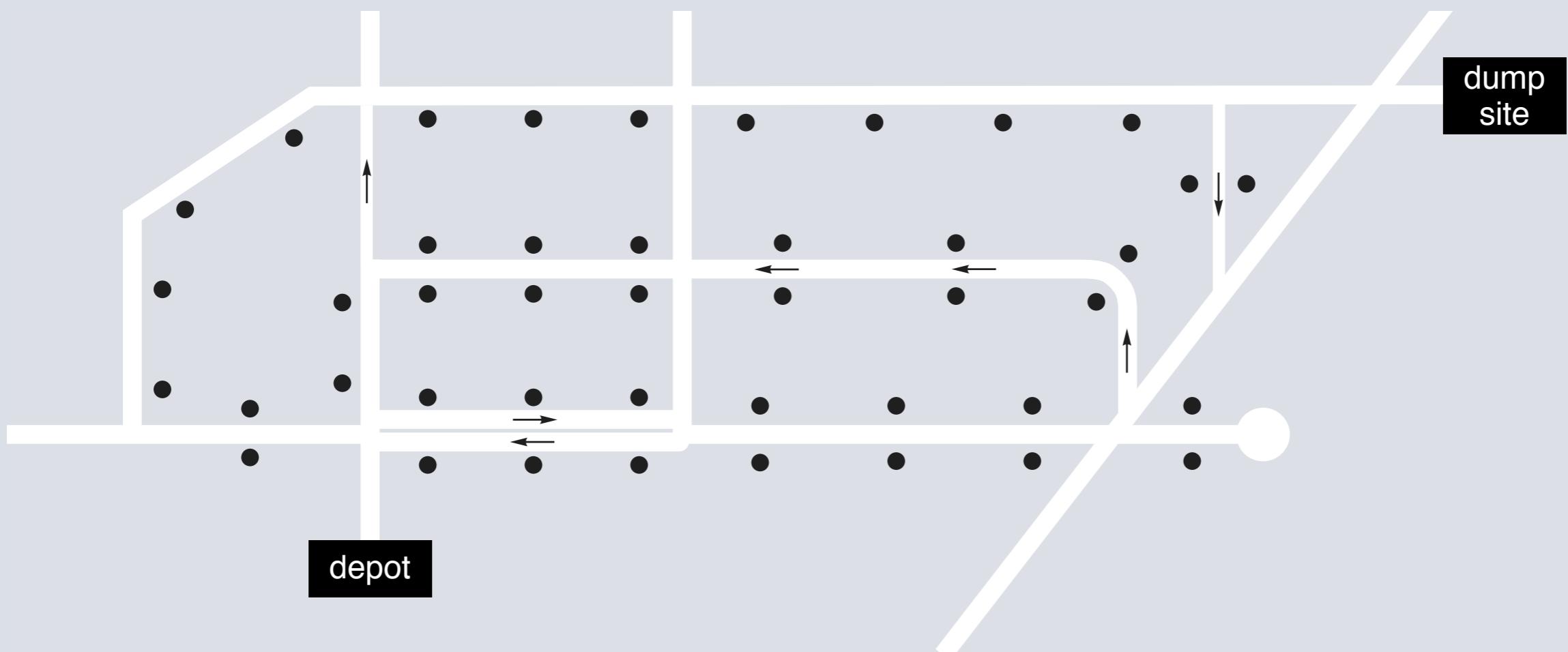
# Time Restrictions

a vehicle crew works 8 hours

# Intermediate Facilities

vehicle can visit a dumpsite, and offload its waste

# The Mixed Capacitate Arc Routing Problem under Time restrictions with Intermediate Facilities (MCARPTIF)





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Computers & Operations Research 33 (2006) 3363–3383

## Lower and upper bounds for the mixed capacitated arc routing problem

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<sup>a</sup>Dept d'Estadística i Investigació Operativa, Universitat de València, C / Dr. Moliner, 50-46100 Burjassot (València), Spain

<sup>b</sup>LIMOS, Université Blaise Pascal, BP 10125, 63173 Aubière Cedex, France

<sup>c</sup>ISTIT, Université de Technologie de Troyes, BP 2060, 10010 Troyes Cedex, France

Available online 30 March 2005

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operations  
research  
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Journal of Mathematical Modelling and Algorithms 3: 209–223, 2004.  
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## Tabu Search Heuristics for the Arc Routing Problem with Intermediate Facilities under Capacity and Length Restrictions

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and ROBERTO MUSMANNO<sup>2</sup>

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73100 Lecce, Italy

<sup>2</sup>Dipartimento di Elettronica, Informatica e Sistemistica, Università della Calabria,  
87036 Rende (CS), Italy

<sup>3</sup>Canada Research Chair in Distribution Management, HEC Montréal, 3000, Chemin de la  
Côte-Sainte-Catherine, Montréal, Canada H3T 2A7

**Abstract.** This paper deals with the *Arc Routing Problem with Intermediate Facilities under Capacity and Length Restrictions* (CLARPIF), a variant of the classical *Capacitated Arc Routing Problem* (CARP), in which vehicles may unload or replenish at intermediate facilities and the length of any route may not exceed a specified upper bound. Three heuristics are developed for the CLARPIF: the first is a constructive procedure based on a partitioning approach while the second and the third are tailored Tabu Search procedures. Computational results on a set of benchmark instances with up to 50 vertices and 92 required edges are presented and analyzed.

**Mathematics Subject Classifications (2000):** 90B60, 9B10, 68T20.

**Key words:** capacitated arc routing problem, intermediate facilities, capacity and distance restrictions.

### Abstract

This paper presents a linear formulation, valid inequalities, and a lower bounding procedure for the mixed capacitated arc routing problem (MCARP). Moreover, three constructive heuristics and a memetic algorithm are described. Lower and upper bounds have been compared on two sets of randomly generated instances. Computational results show that the average gaps between lower and upper bounds are 0.51% and 0.33%, respectively.

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**Keywords:** Capacitated arc routing problem; Mixed graph; Lower bound; Cutting plane; Heuristic; Memetic algorithm; Waste collection

### 1. Introduction and background

The *capacitated arc routing problem* (CARP), introduced by Golden and Wong [1], considers an undirected network in which each edge has a traversal cost. A fleet of identical vehicles of limited capacity is based at a depot node. A subset of required edges with known demands must be serviced by a vehicle. The CARP consists of determining a set of vehicle trips of minimum total cost, such that each trip starts and ends at the depot, each required edge is serviced by one single trip and during one traversal,

### 1. Introduction

The aim of this paper is to describe three heuristics for the *Arc Routing Problem with Intermediate Facilities under Capacity and Length Restrictions* (CLARPIF), a variant of the classical *Capacitated Arc Routing Problem* (CARP) [15]. Both these problems, in their undirected versions, are defined on a graph  $G = (V, E)$  where  $V = \{v_1, \dots, v_n\}$  is a vertex set and  $E$  is a set of edges  $(v_i, v_j)$  ( $i < j$ ), including a subset  $R$  of *required* edges. Each edge of  $R$  must be serviced once but can be traversed several times. Let  $V_R$  be the set of vertices  $v_i$  such that an edge  $(v_i, v_j)$  exists in  $R$ . With each edge  $e = (v_i, v_j)$  are associated a non-negative traversal cost or length  $c_e = c_{ij}$  and a non-negative demand  $q_e = q_{ij}$ . If  $e \notin R$ , then  $q_e = 0$ . Denote by  $d_{ij}$  the length of a shortest chain between two vertices  $v_i$  and  $v_j$ . Vertex  $v_1$  denotes a *depot* at which a fleet of identical *vehicles* is based. In our version of the problem, vehicles bear no fixed costs and their number is a decision variable.

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# The solution techniques

1

## Constructive heuristics

Willemse, E. J., & Joubert, J. W. (2016). **Constructive heuristics for the mixed capacity arc routing problem under time restrictions with intermediate facilities.** *Computers & Operations Research*, 68, 30-62.

2

## Local Search

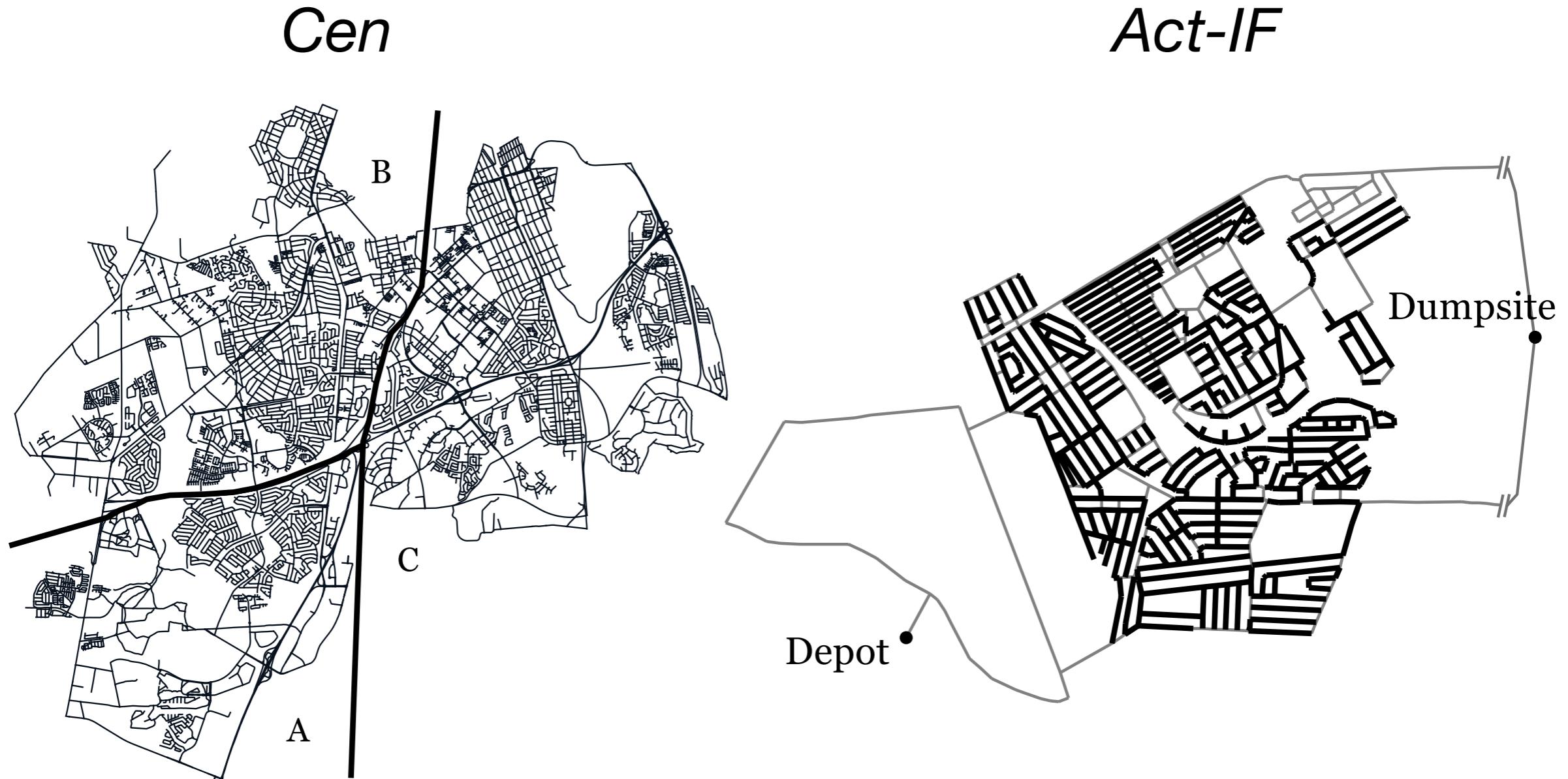
Willemse, E. J., & Joubert, J. W. (2019). **Efficient local search strategies for the Mixed Capacitated Arc Routing Problems under Time Restrictions with Intermediate Facilities.** *Computers & Operations Research*, 105, 203-225.

3

## Very simple tabu-search

Willemse, E. J. (2016). **Heuristics for large-scale Capacitated Arc Routing Problems on mixed networks** (Doctoral dissertation, University of Pretoria).

# New benchmark sets



Willemse, E. J., & Joubert, J. W. (2016). **Benchmark dataset for undirected and mixed capacitated arc routing problems under time restrictions with intermediate facilities.** *Data in brief*, 8, 972-977.

# Outline

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Our research focus (starting 2008)

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General metrics of CARP publications.

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Solution approaches and benchmark instances

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Some ideas on future research opportunities



WEB OF SCIENCE



## WEB OF SCIENCE

SEARCH: '**arc routing**' | '**general routing problem**' |  
**'postman problem'** | '**postmen problem**' in period 2010  
- 2017



## WEB OF SCIENCE

SEARCH: '**arc routing**' | '**general routing problem**' | '**postman problem**' | '**postmen problem**' in period 2010 - 2017

RESULTS: **321**



## WEB OF SCIENCE

SEARCH: '**arc routing**' | '**general routing problem**' | '**postman problem**' | '**postmen problem**' in period 2010 - 2017

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Mourão, M. C., & Pinto, L. S. (2017). **An updated annotated bibliography on arc routing problems.** *Networks*, 70(3), 144-194.

REFERENCED: **210**



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REFERENCED: **210**  
WoS INDEXED: **175**



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SEARCH: '**arc routing**' | '**general routing problem**' | '**postman problem**' | '**postmen problem**' in period 2010 - 2017

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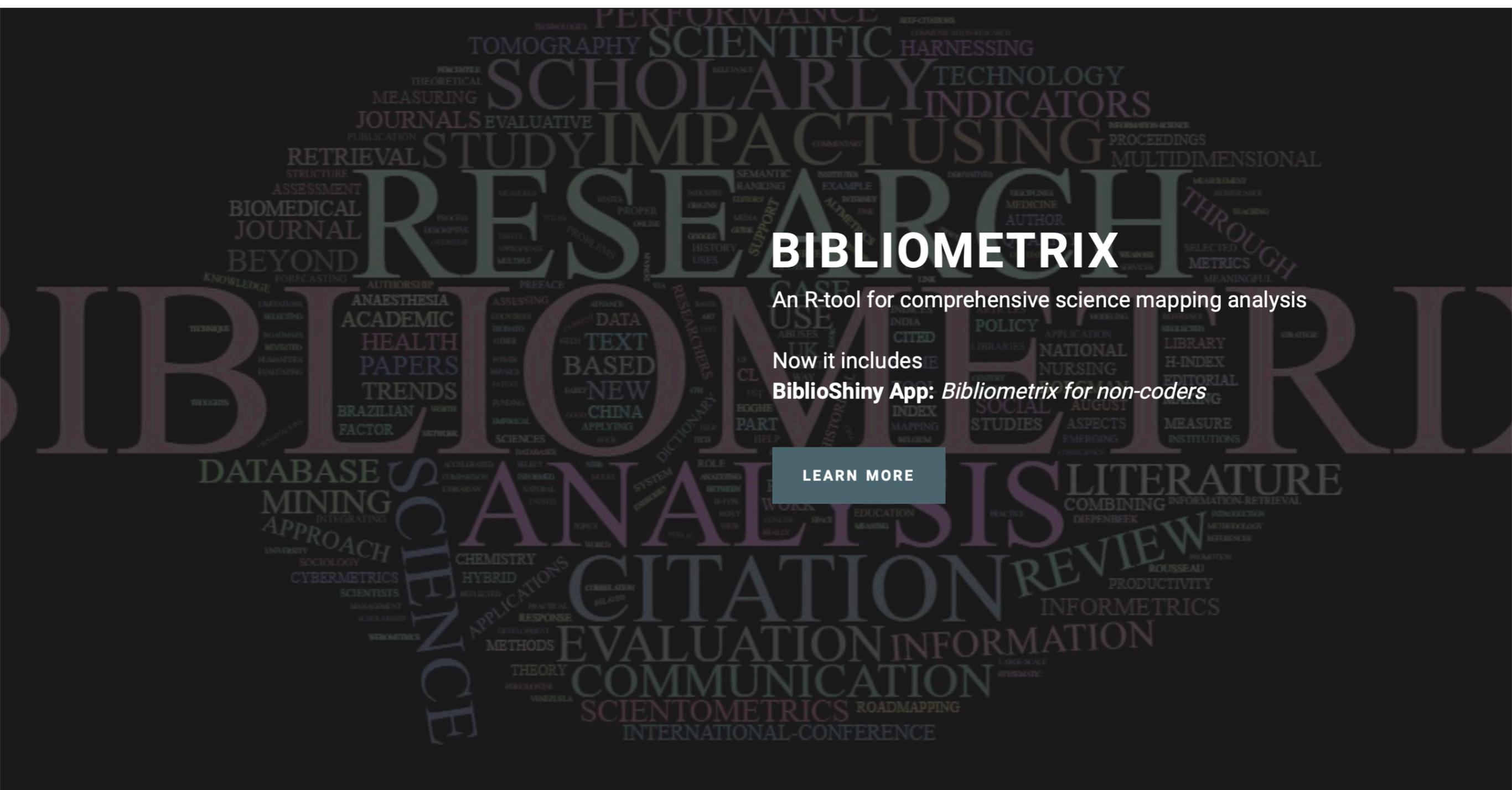
ARTICLES IN BOTH: **153**



WEB OF SCIENCE

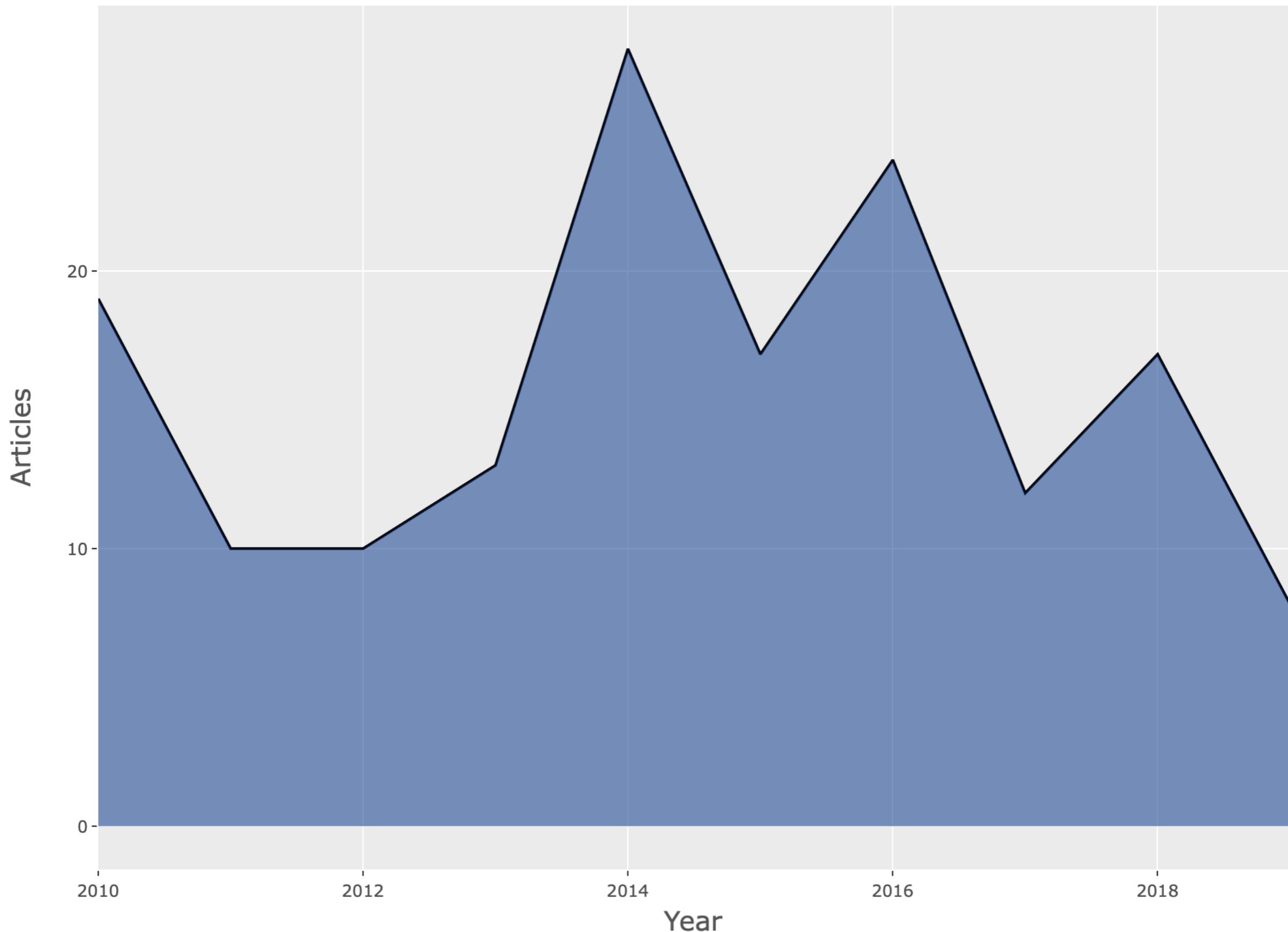
SEARCH: '**capacitated arc routing**' | '**general routing problem**' in period 2010 - 2019

Aria, M., & Cuccurullo, C. (2017). **bibliometrix: An R-tool for comprehensive science mapping analysis.** *Journal of Informetrics*, 11(4), 959-975

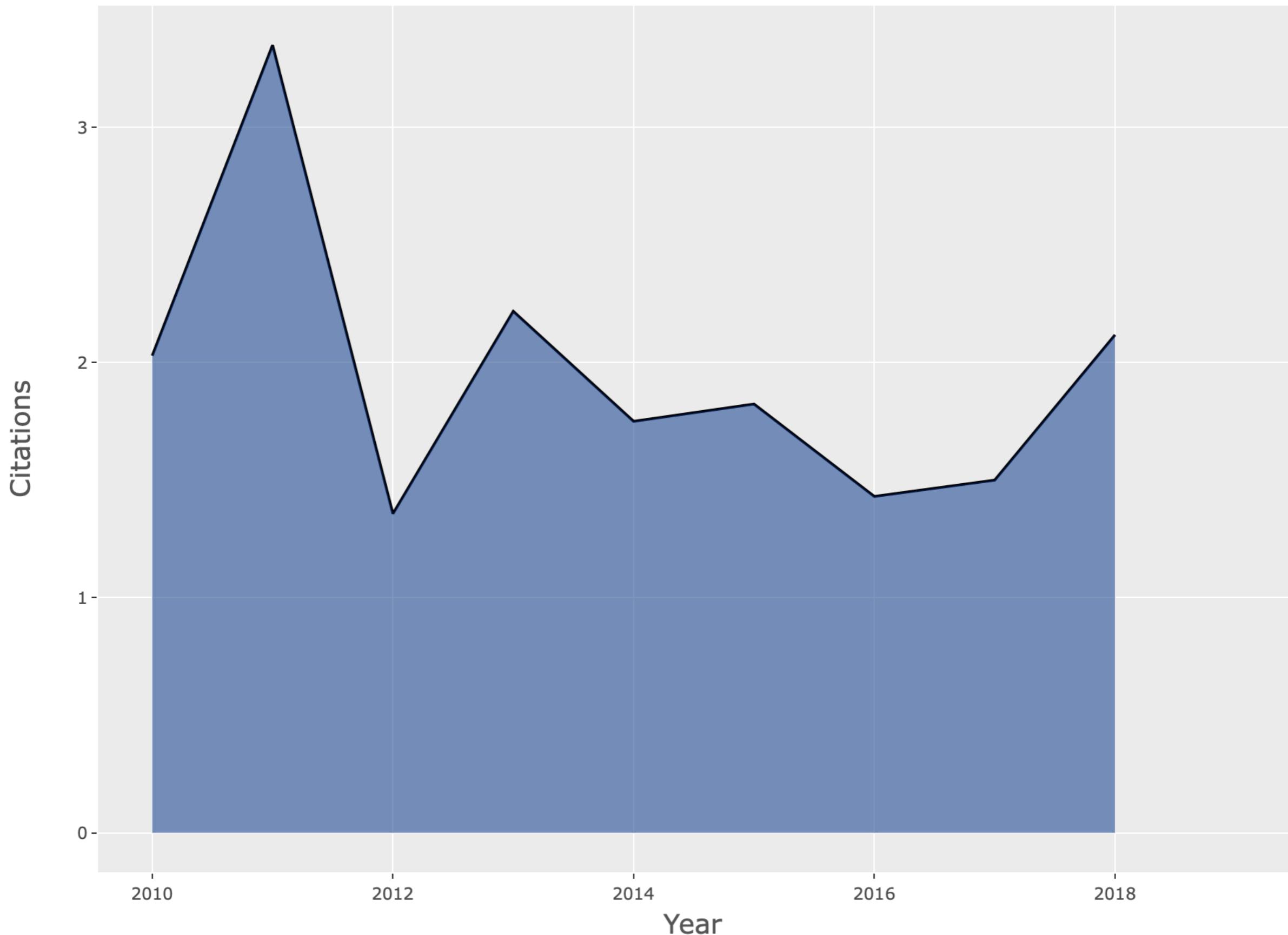


<http://www.bibliometrix.org>

# Annual Scientific Production



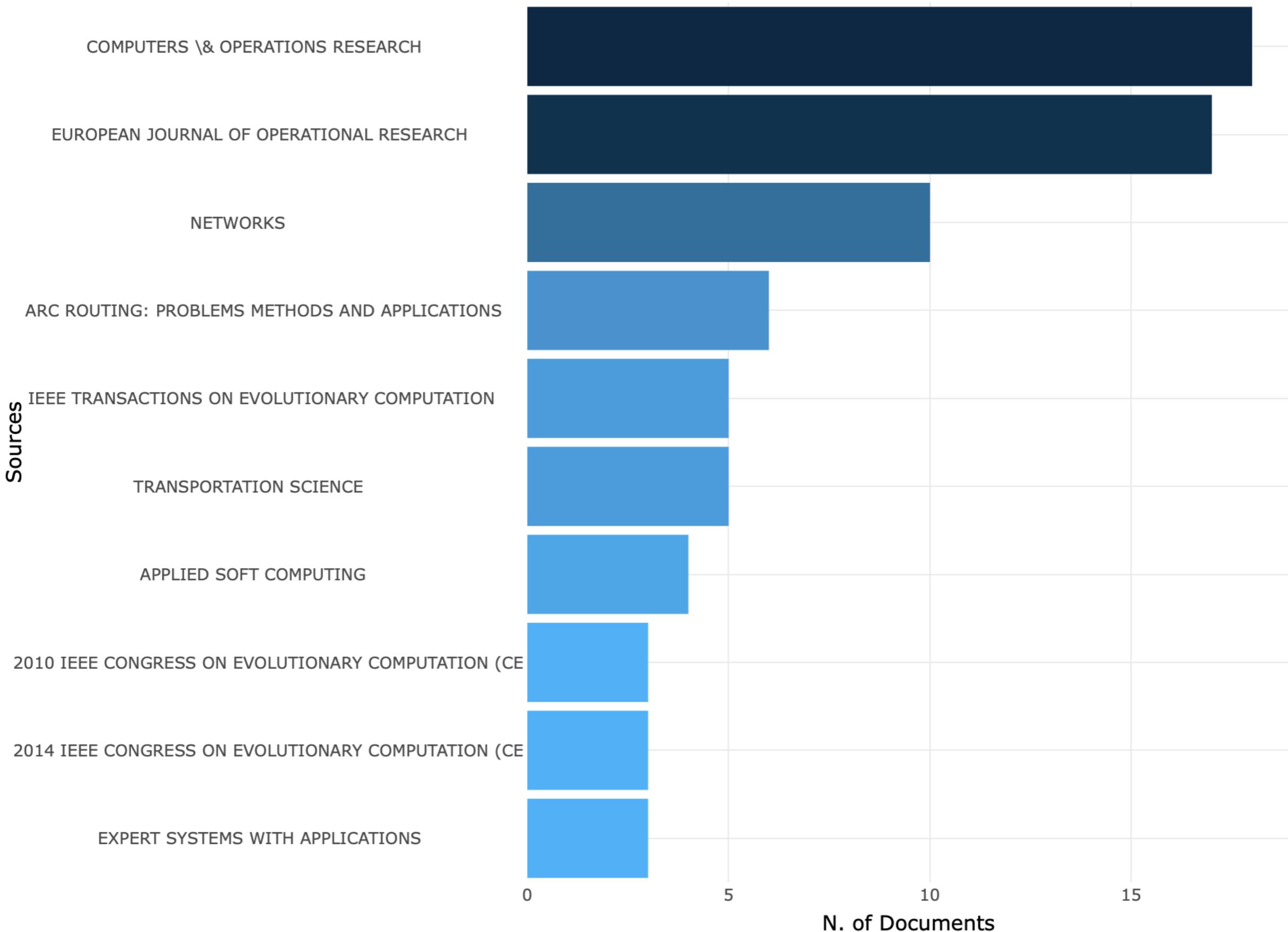
# Average Article Citations per Year



## Most Relevant Sources

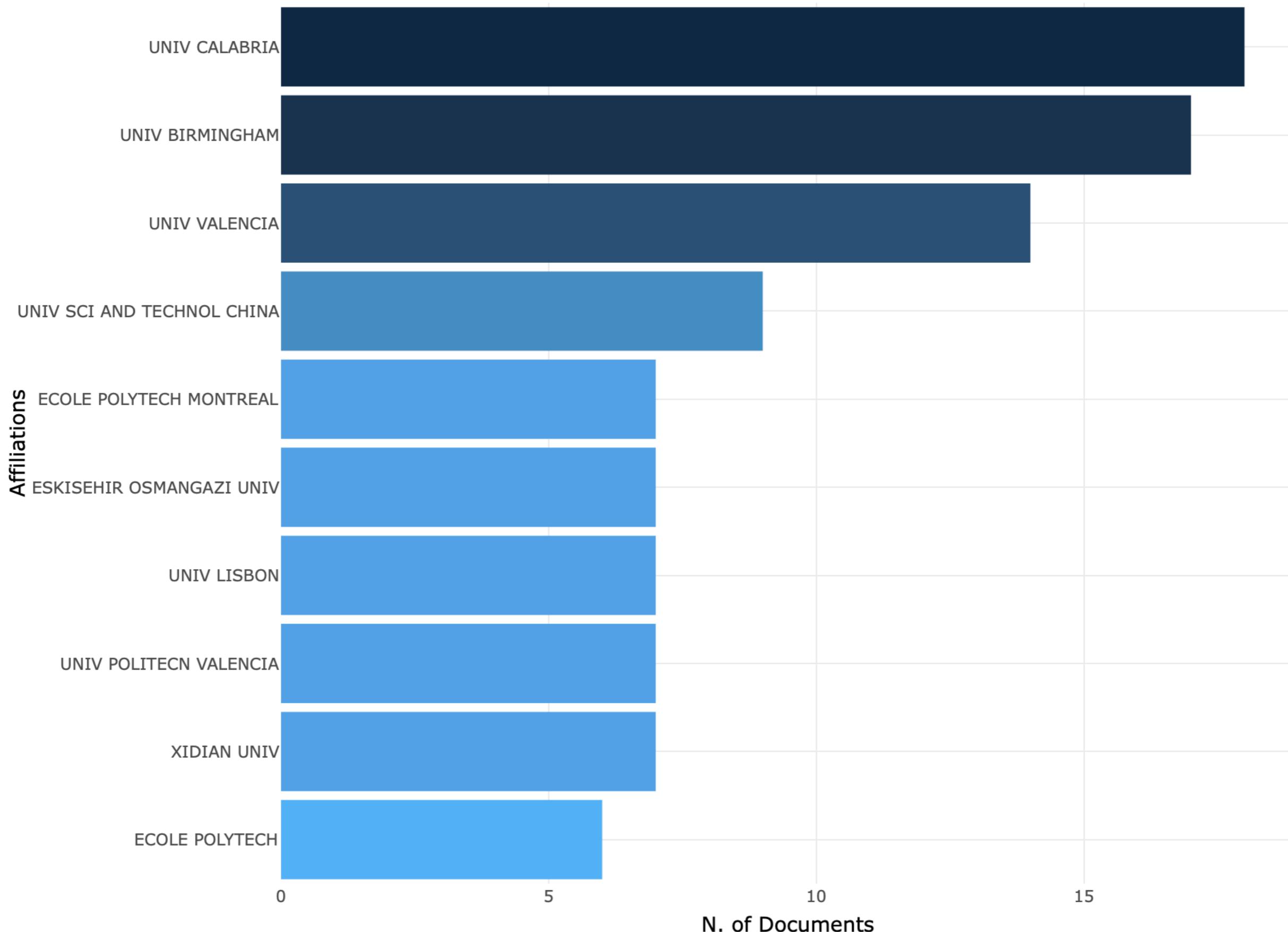
Which Journal has the most CARP publications?

## Most Relevant Sources



Which University has the most CARP publications?

## Most Relevant Affiliations

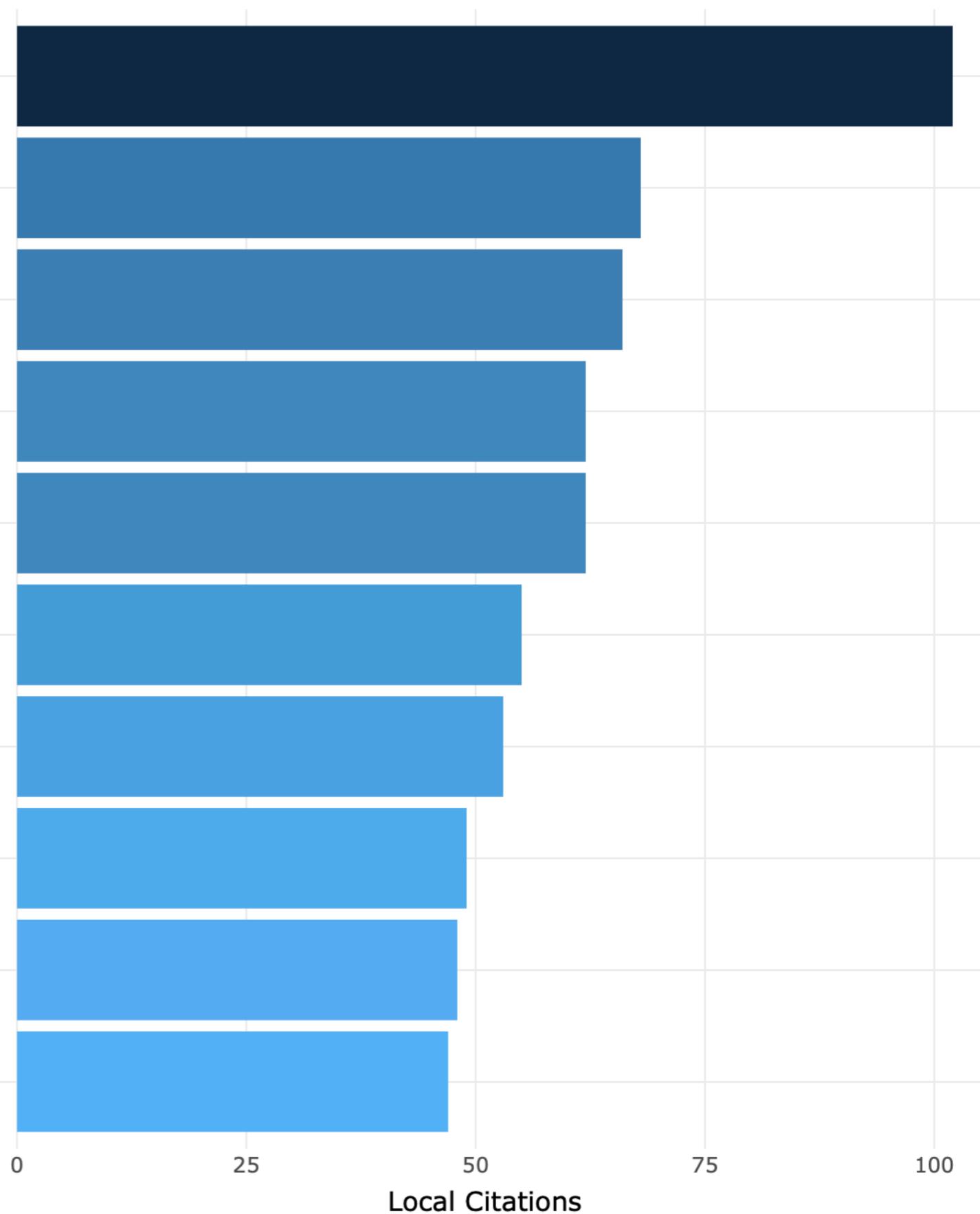


Most cited CARP publications (pre 2019)?

## Most Cited References



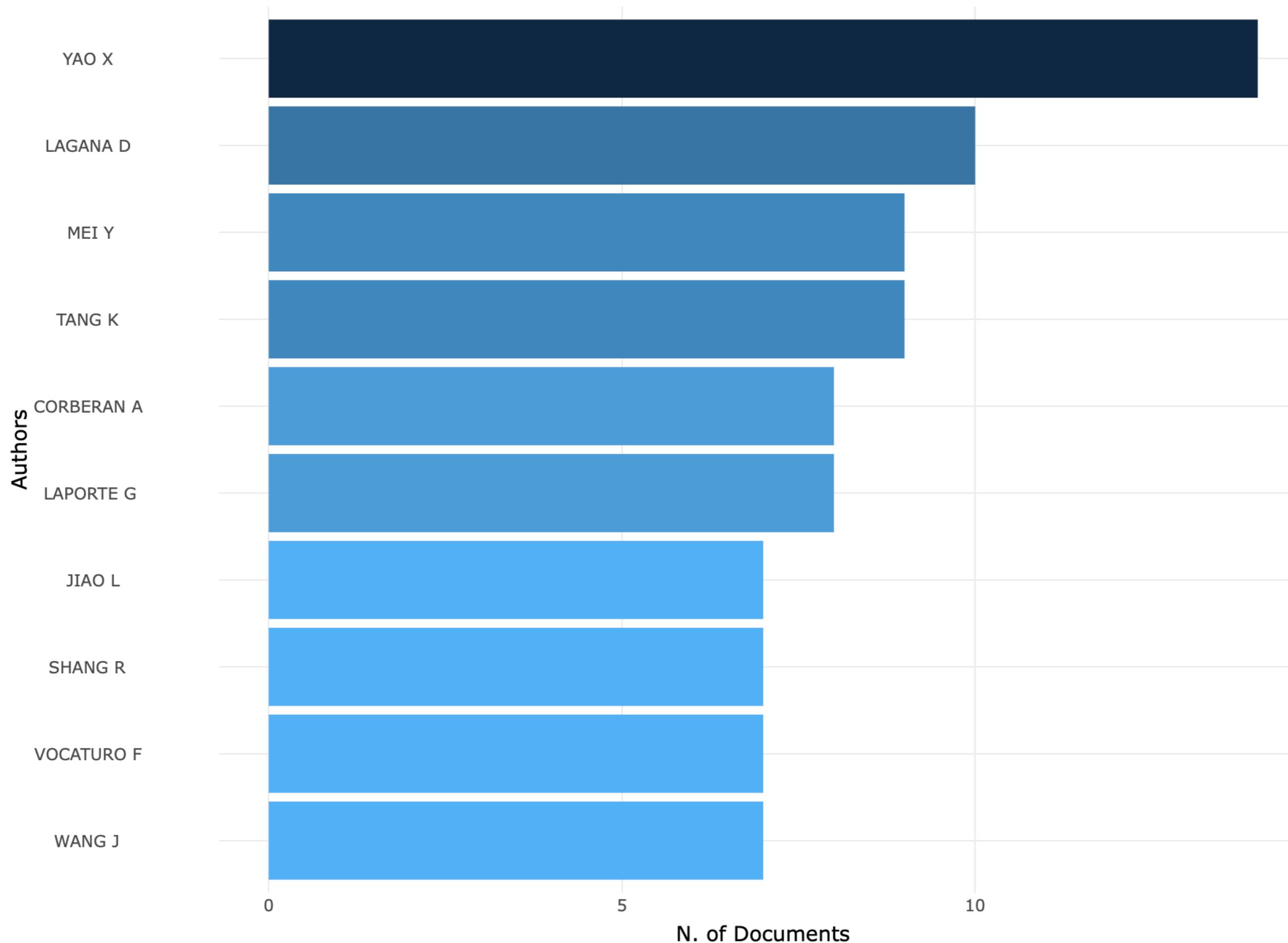
- GOLDEN BL, 1981, NETWORKS, V11, P305, DOI 10.1002/
- GOLDEN BL, 1983, COMPUT OPER RES, V10, P47, DOI 10.
- LACOMME P, 2004, ANN OPER RES, V131, P159, DOI 10.
- BEULLENS P, 2003, EUR J OPER RES, V147, P629, DOI
- BRANDAO J, 2008, COMPUT OPER RES, V35, P1112, DOI
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- TANG K, 2009, IEEE T EVOLUT COMPUT, V13, P1151, DOI 10.1109/TEVC.2009.2021030



## Most Relevant Authors

Most productive CARP authors (2010-2019)?

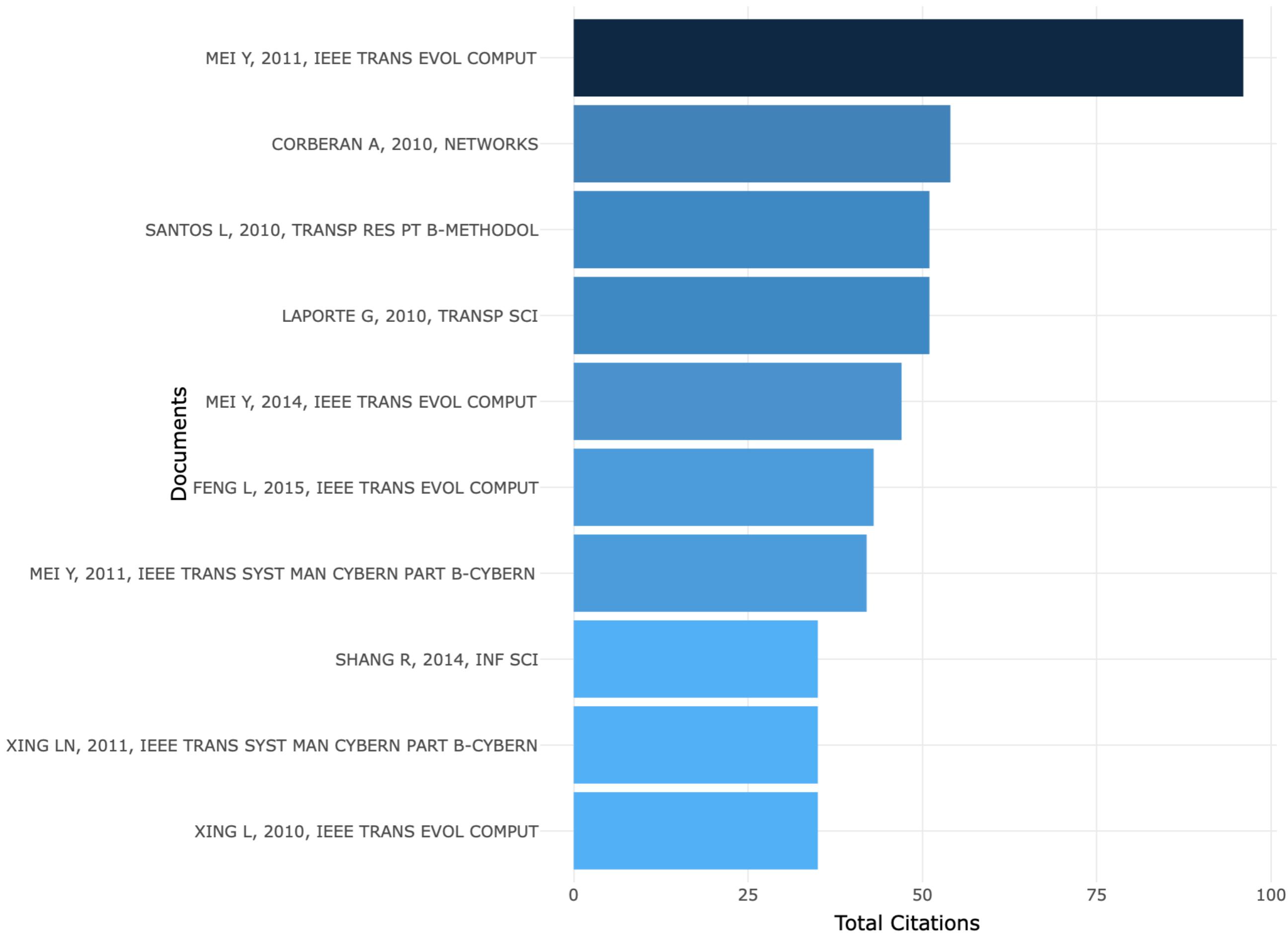
## Most Relevant Authors



## Most Cited Documents

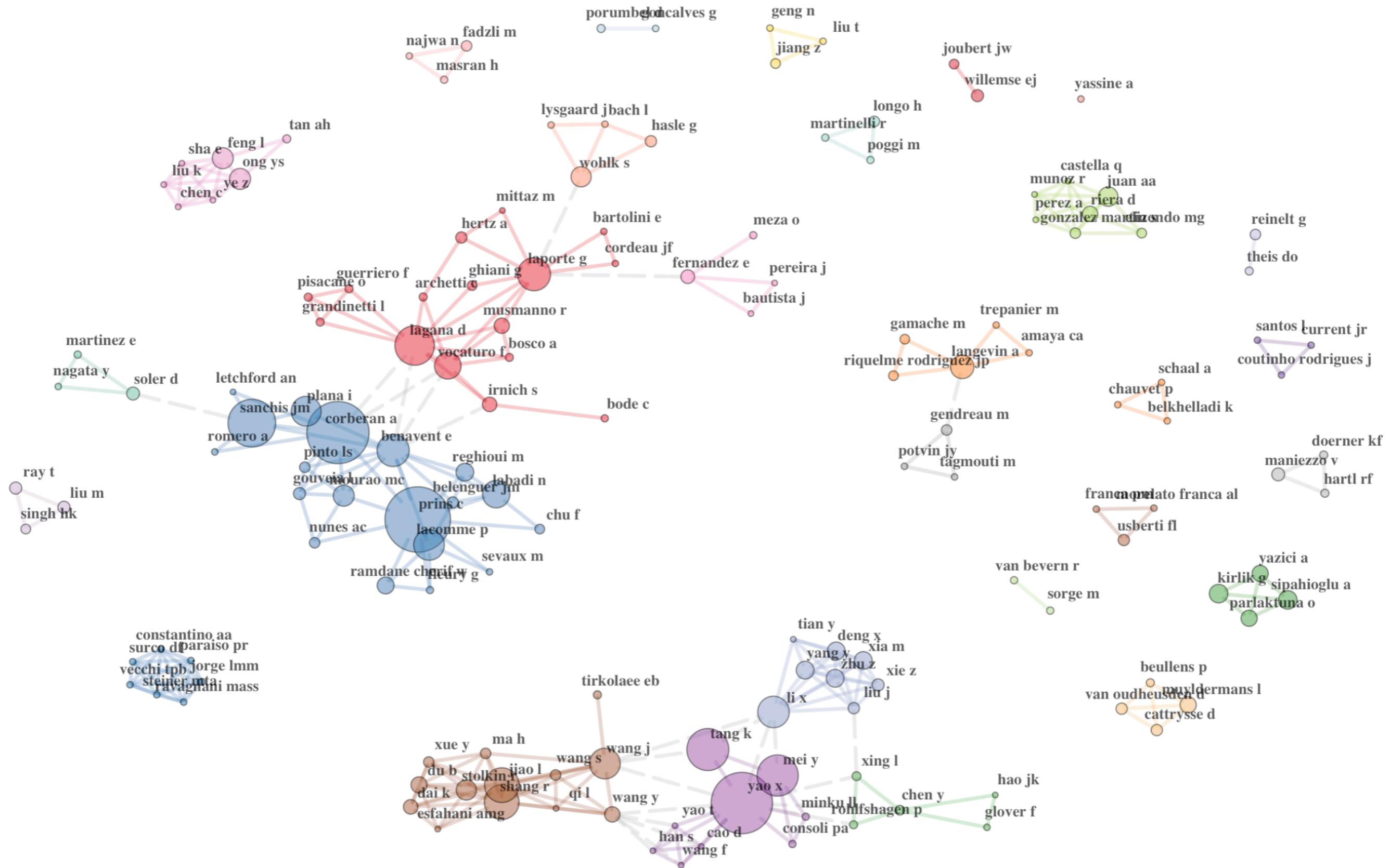
Most cited CARP document (2010-2019)?

## Most Cited Documents

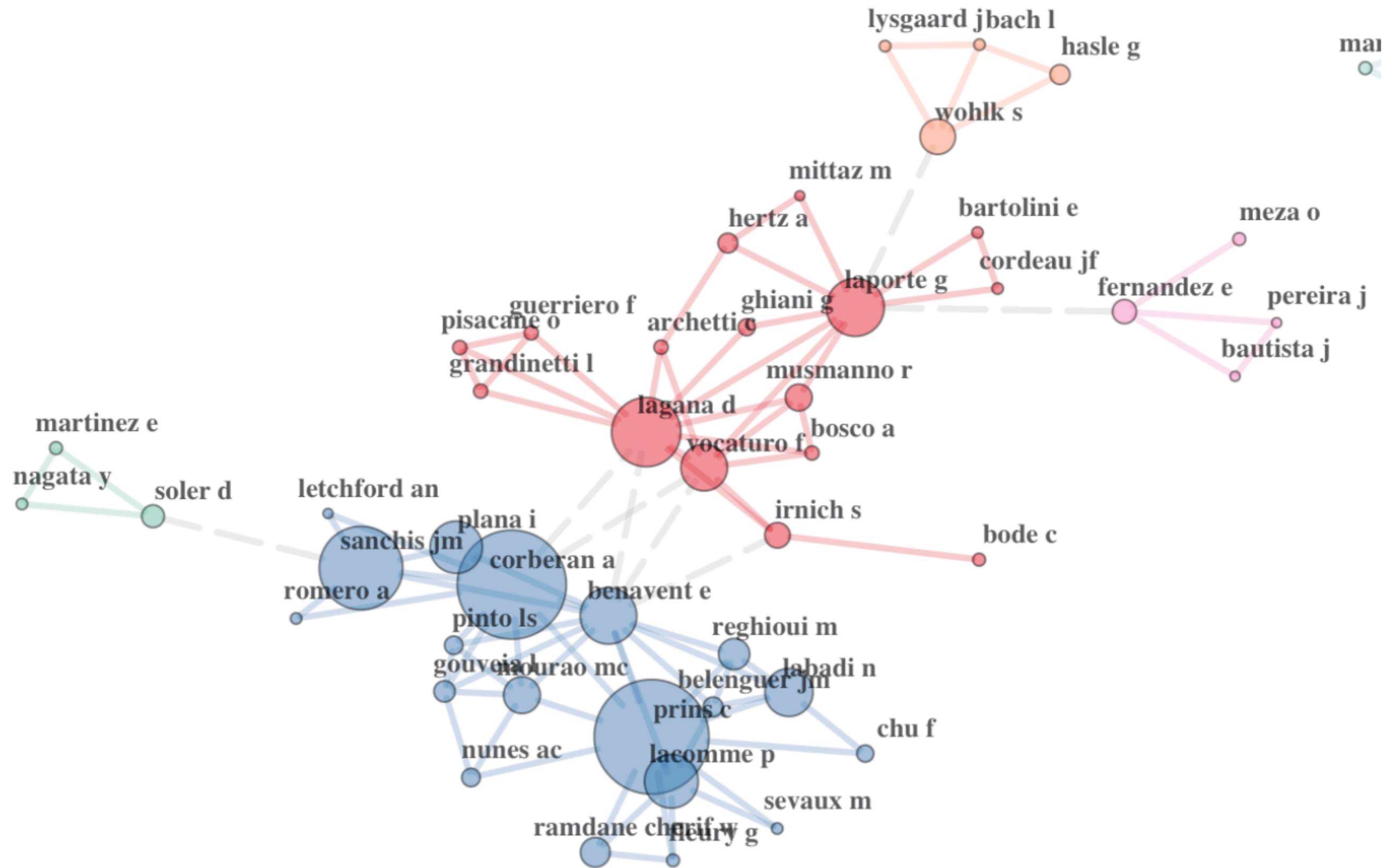


# Co-author network

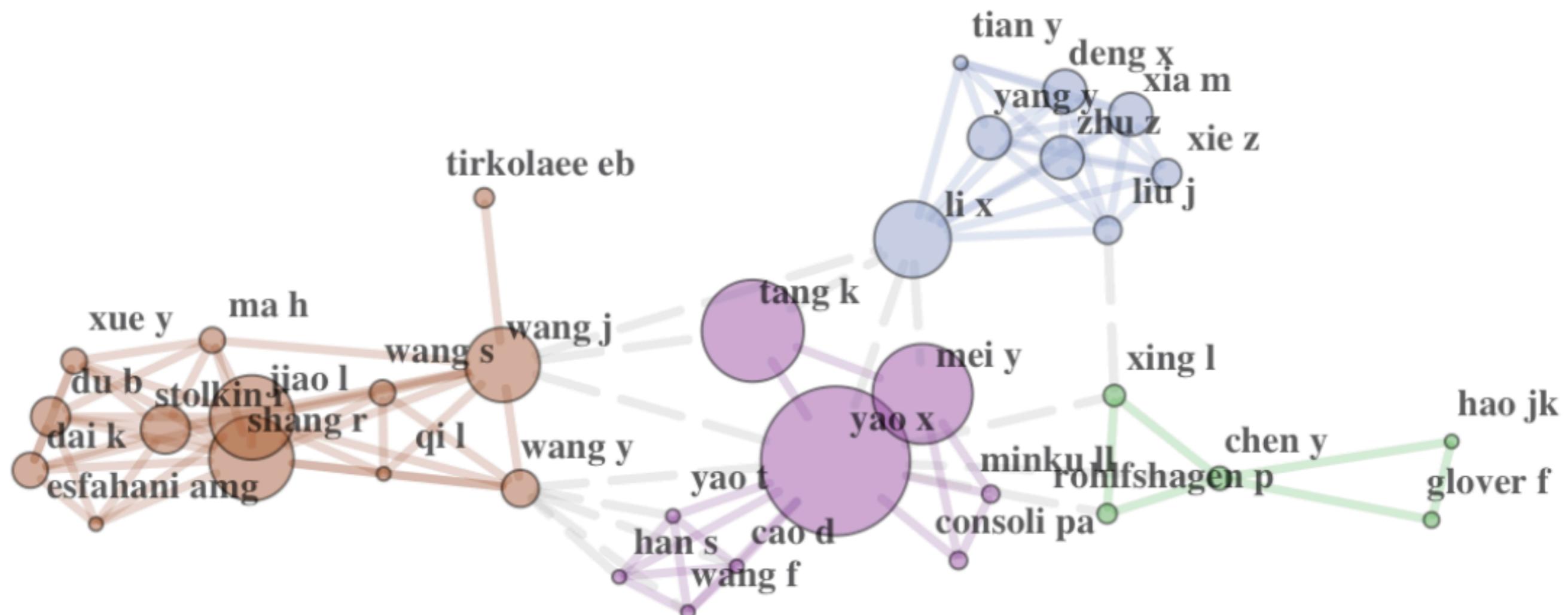
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**Authors**

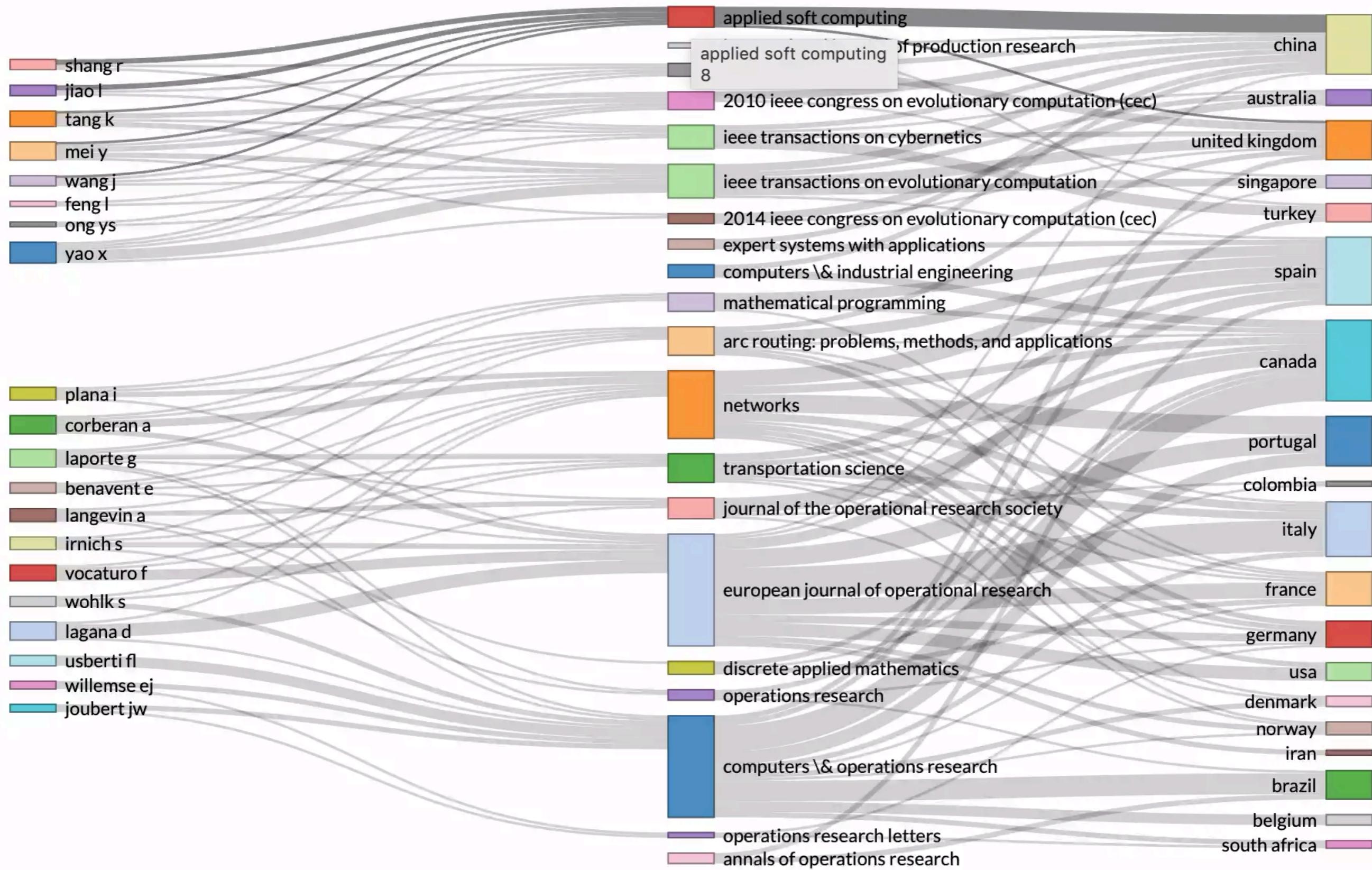
**Journals**

**Countries**

# Authors

# Journals

# Countries



# **General research focus**

# General research focus

1

- New-variants
  - Exact solution methods
  - Adaption of heuristic methods
  - Modification of existing benchmarks

# General research focus

1

New-variants

Exact solution methods

Adaption of heuristic methods

Modification of existing benchmarks

2

Technique  
improvements

Exact solution methods for  
existing variants

Heuristics (mostly MAs) methods  
for existing variants.

# General research focus

1

New-variants

Exact solution methods

Adaption of heuristic methods

Modification of existing benchmarks

2

Technique  
improvements

Exact solution methods for  
existing variants

Heuristics (mostly MAs) methods  
for existing variants.

3

Practical  
improvements

New variants and benchmarks from  
real or near-real problem settings.

Adaption and improvements of  
heuristics.

# Outline

1

Our research focus (starting 2008)

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General metrics of CARP publications.

3

Solution approaches and benchmark instances

4

Some ideas on future research opportunities

# **Second round of analysis**

1

Scanned 158 papers and noted their solution technique, benchmark instances used, and origins of instances.

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Techniques crudely grouped into exact methods, Evolutionary Algorithms, and other heuristics.

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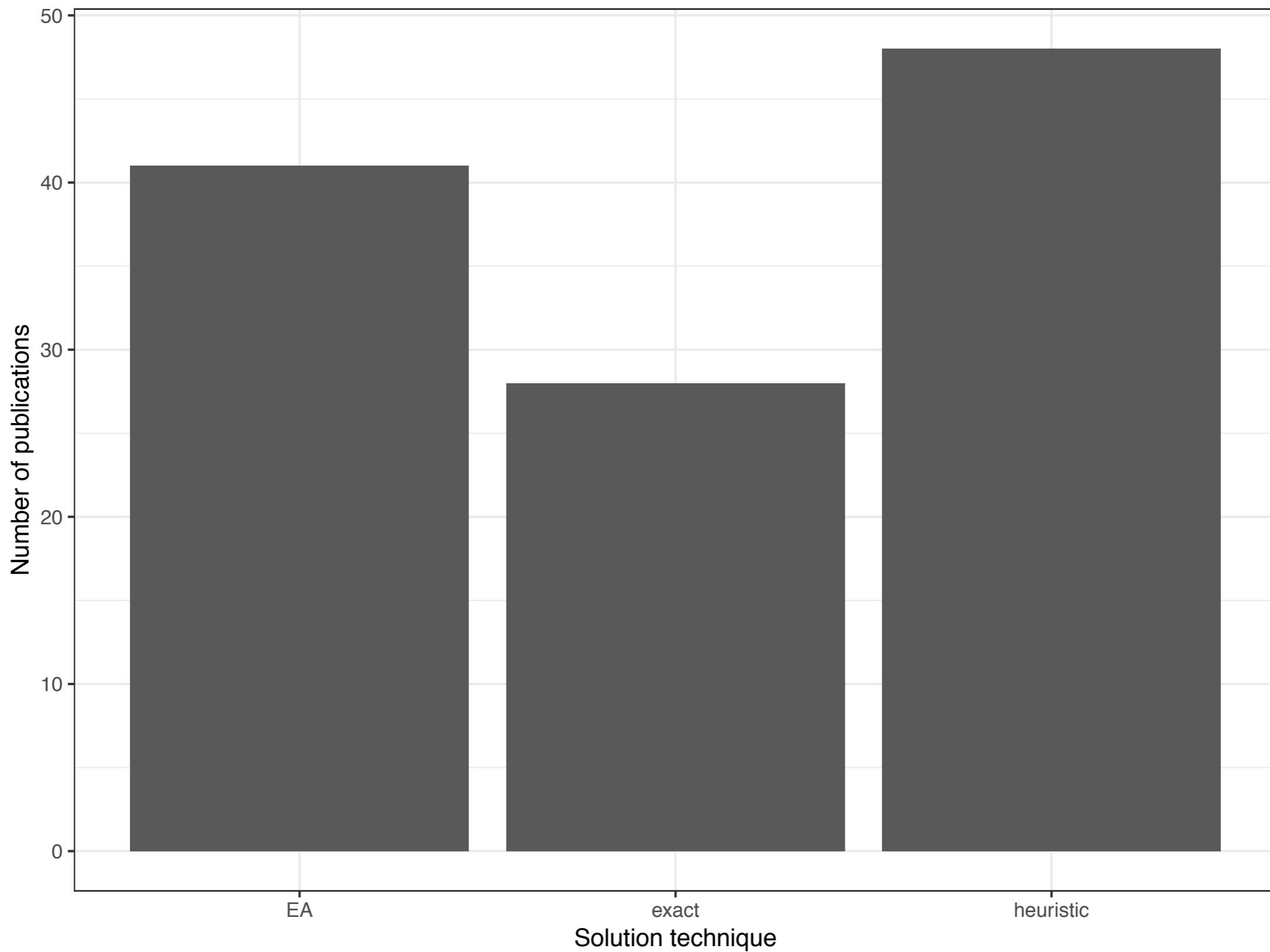
**2**

Techniques crudely grouped into exact, methods, Evolutionary Algorithms, and other heuristics.

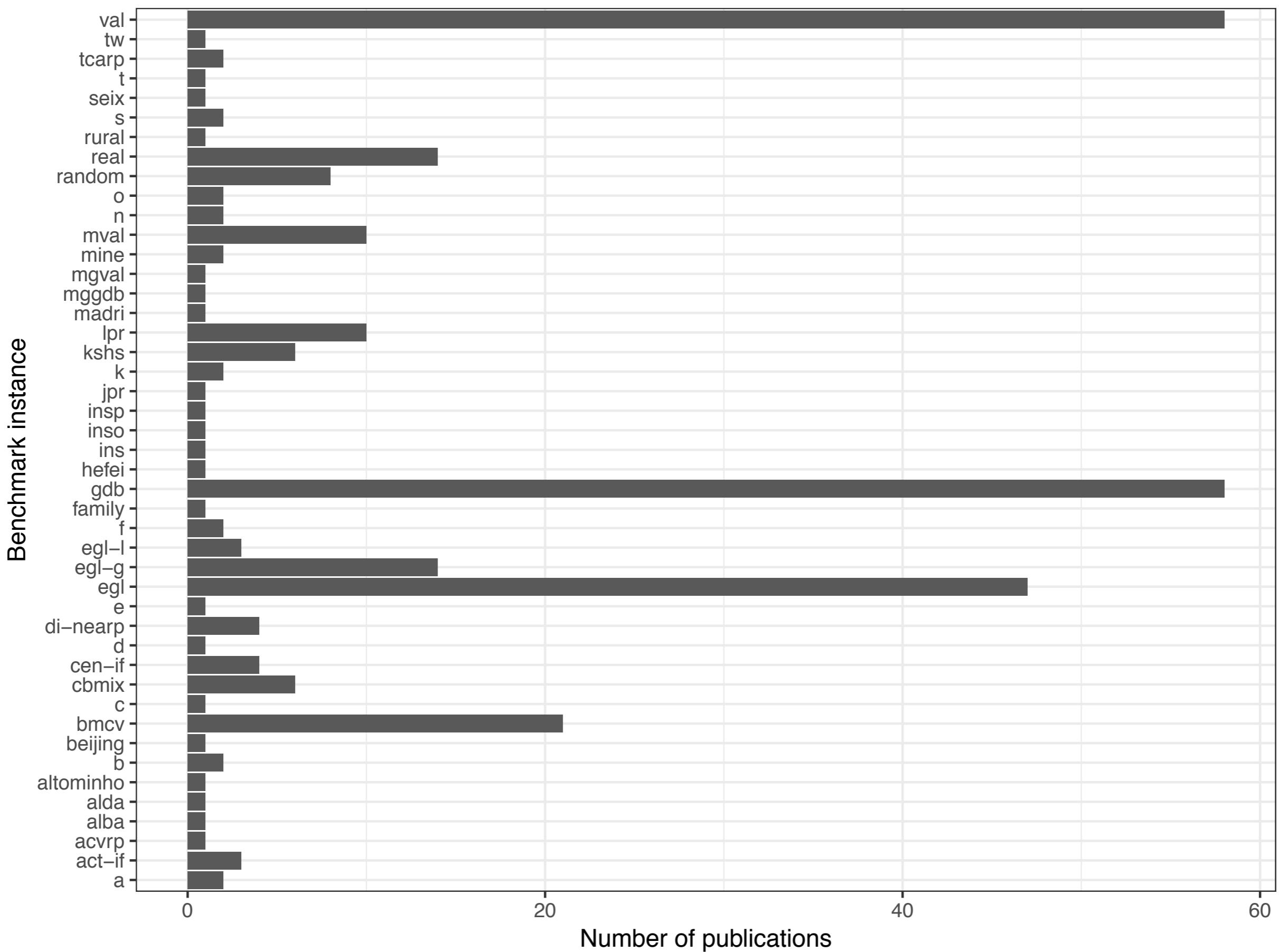
**3**

Analysed which benchmark instances, traced back to their origins, occurred in which technique grouping.

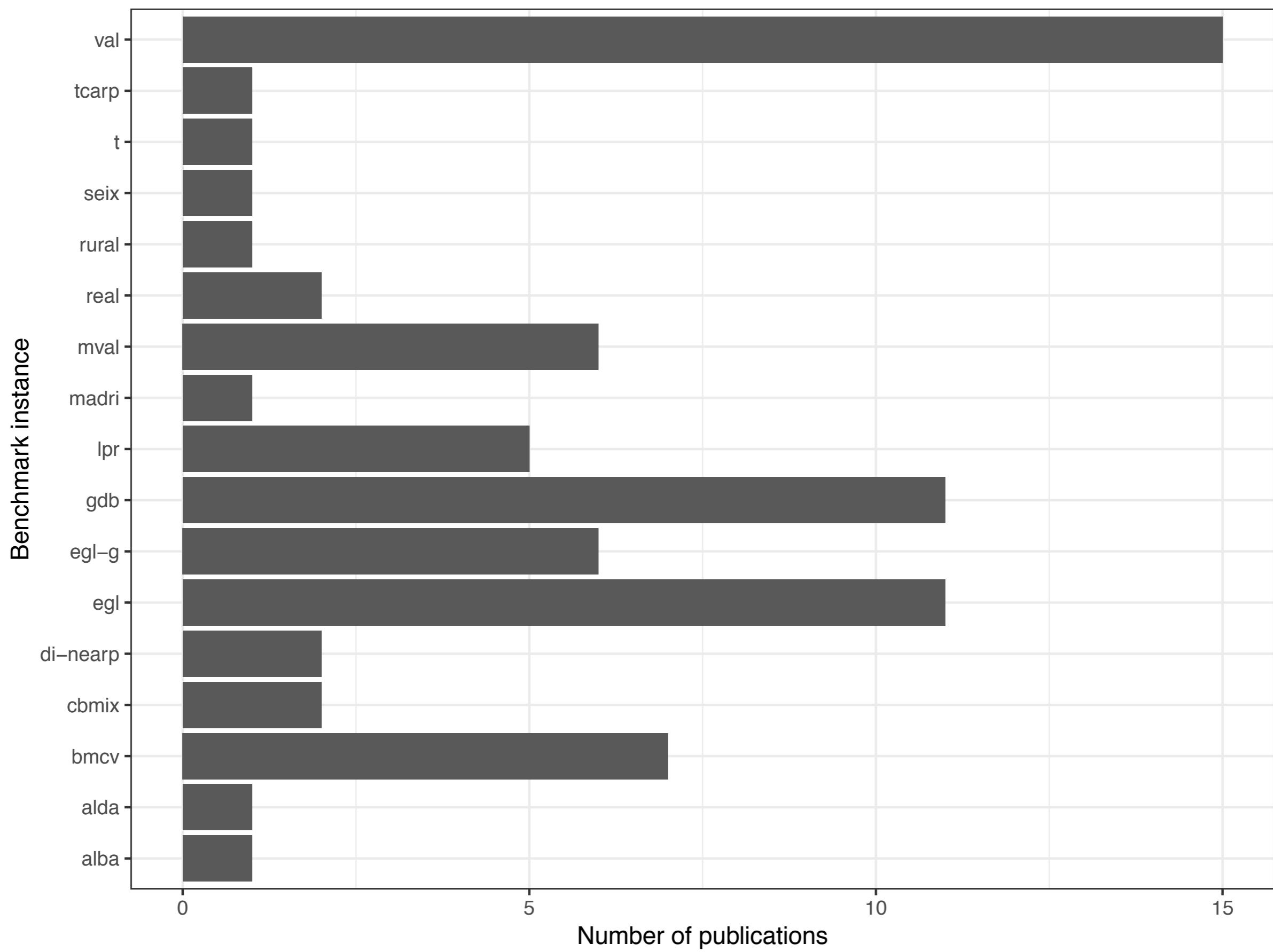
# Technique analysis



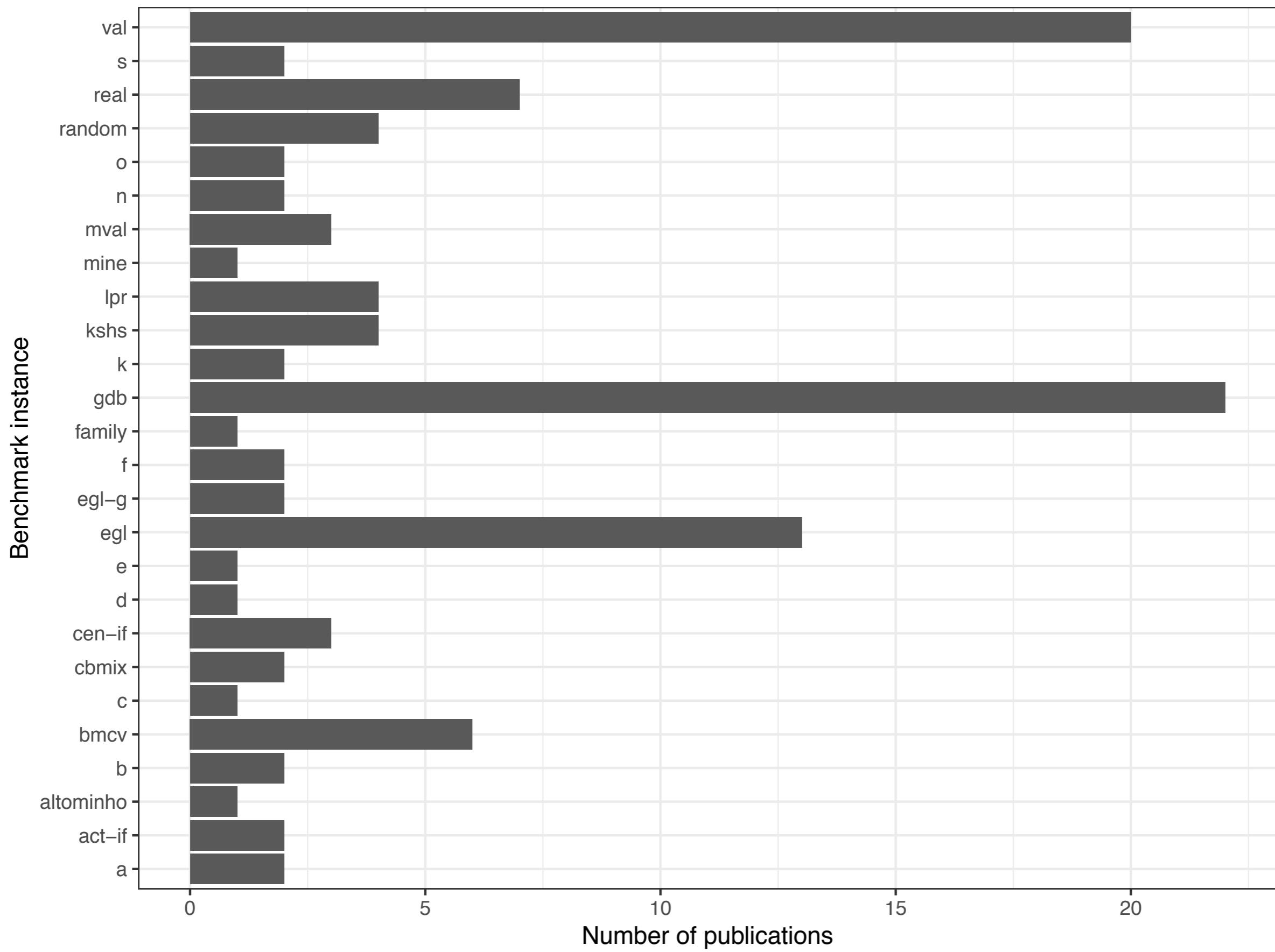
# Benchmarks analysis



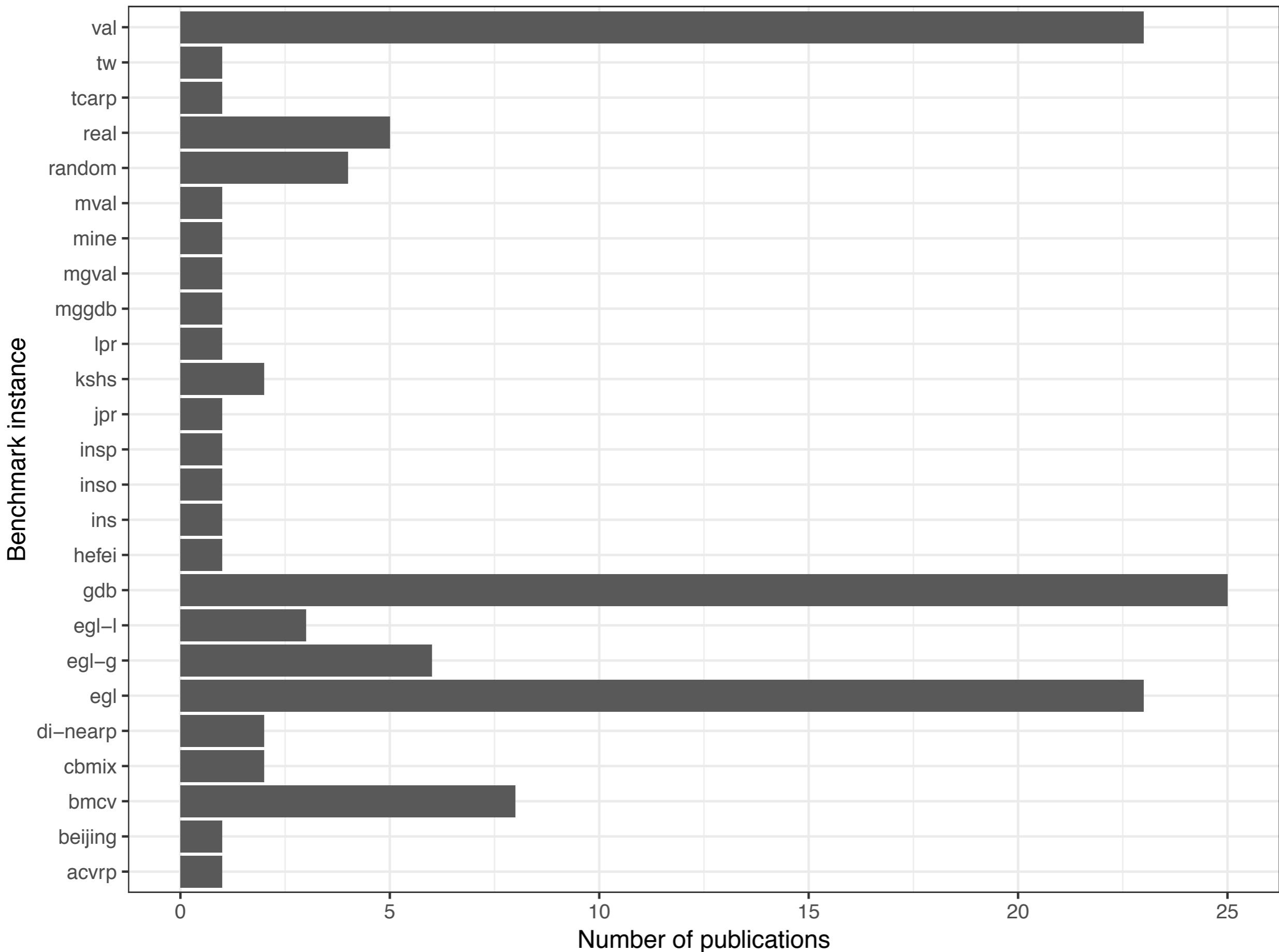
# Exact (19 used)



# Heuristics (26)



# EAs (25 used)



# Benchmarks analysis

Benchmark sets and instances	$ A' \cup E' $	$ A_r \cup E_r $	trips	
			min #	#Arcs/ trip
<i>Waste collection problem instances</i>				
Case study	540	679	5	136
Case study	223	153	3	51
<i>Act-IF-a</i>	259	401	2	76
<i>Act-IF-b</i>	509	151	6	67
<i>Act-IF-c</i>	410	250	4	63
<i>Cen-IF-a</i>	441	1012	17	60
<i>Cen-IF-c</i>	486	2519	37	68
<i>Cen-IF-b</i>	360	2755	39	71
<i>Cen-IF-full</i>	1282	6289	92	68
<i>Lpr-IF-a-05</i>	250	806	18	45
<i>Lpr-IF-b-05</i>	75	801	18	45
<i>Lpr-IF-c-05</i>	38	803	23	35

# Benchmarks analysis

---

Benchmark sets and instances	$ A' \cup E' $	$ A_r \cup E_r $	trips	
			min #	#Arcs/ trip
<i>Mean values over randomly generated benchmark sets</i>				
<i>Lpr</i>	71	352	7	28
<i>mval</i>	0	88	5	21
<i>gdb</i>	0	29	6	5
<i>gdb</i>	0	29	6	5
<i>bccm</i>	0	63	5	15
<i>bccm</i>	0	63	5	15
<i>Mean values over realistic snow-removal benchmark sets</i>				
<i>bmcv</i>	22	66	6	12
<i>eglese</i>	34	110	15	7
<i>egl-large</i>	14	361	31	12

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# Open-research questions



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How can instance characteristics be exploited to improve heuristic performance?

# Open-research questions



What effects do the reuse of benchmark problems have on the development/tuning/choice of heuristics and metaheuristics?



How does instance characteristics influence heuristic performance?



How can instance characteristics be exploited to improve heuristic performance?

# The solution techniques

1

## Constructive heuristics

Willemse, E. J., & Joubert, J. W. (2016). **Constructive heuristics for the mixed capacity arc routing problem under time restrictions with intermediate facilities.** *Computers & Operations Research*, 68, 30-62.

2

## Local Search

Willemse, E. J., & Joubert, J. W. (2019). **Efficient local search strategies for the Mixed Capacitated Arc Routing Problems under Time Restrictions with Intermediate Facilities.** *Computers & Operations Research*, 105, 203-225.

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## Very simple tabu-search

Willemse, E. J. (2016). **Heuristics for large-scale Capacitated Arc Routing Problems on mixed networks** (Doctoral dissertation, University of Pretoria).

# Local Search Operators

1

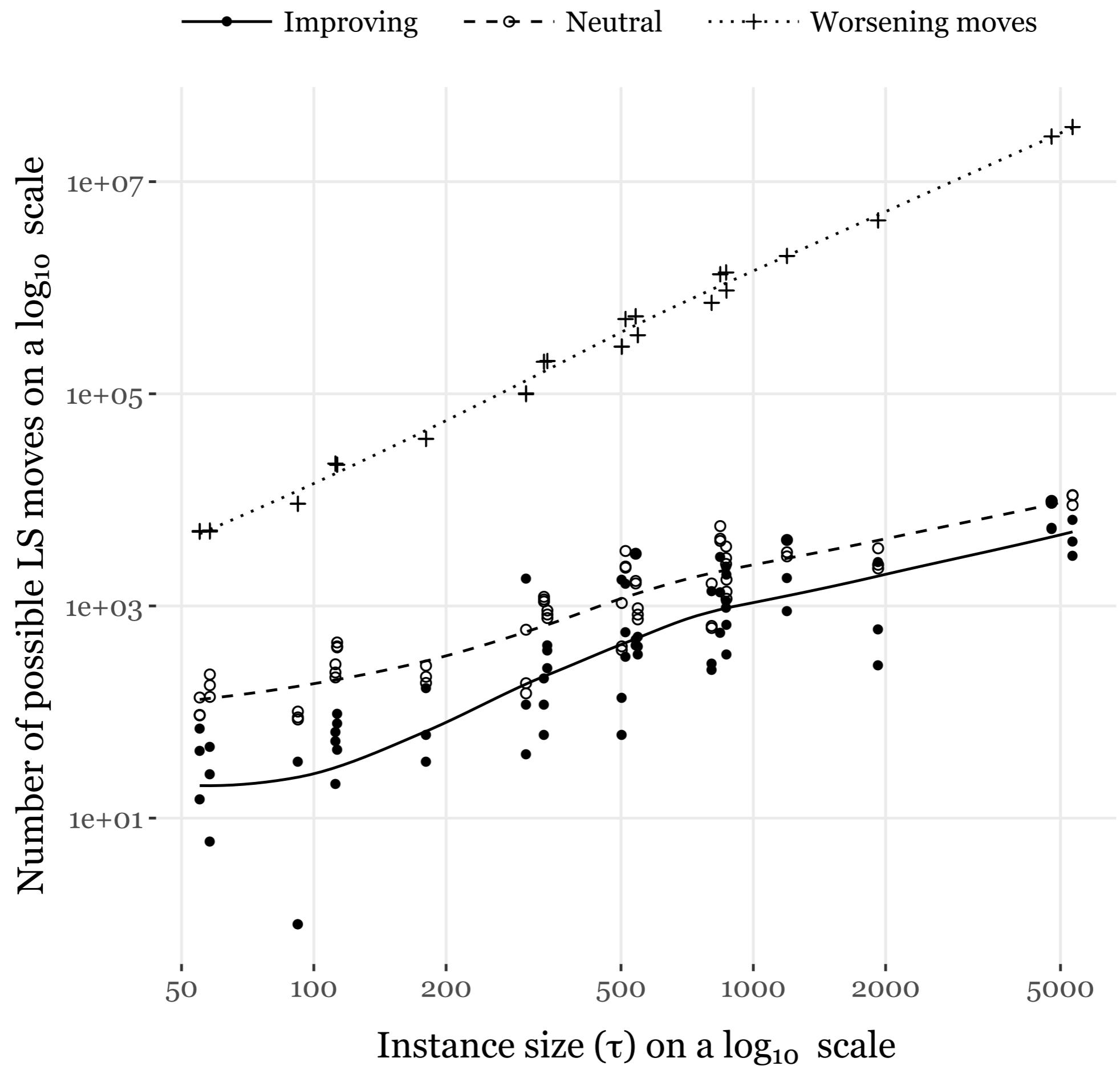
Remove-insert

2

Exchange

3

Cross (swop between end  
segments of routes)



# The solution techniques

1

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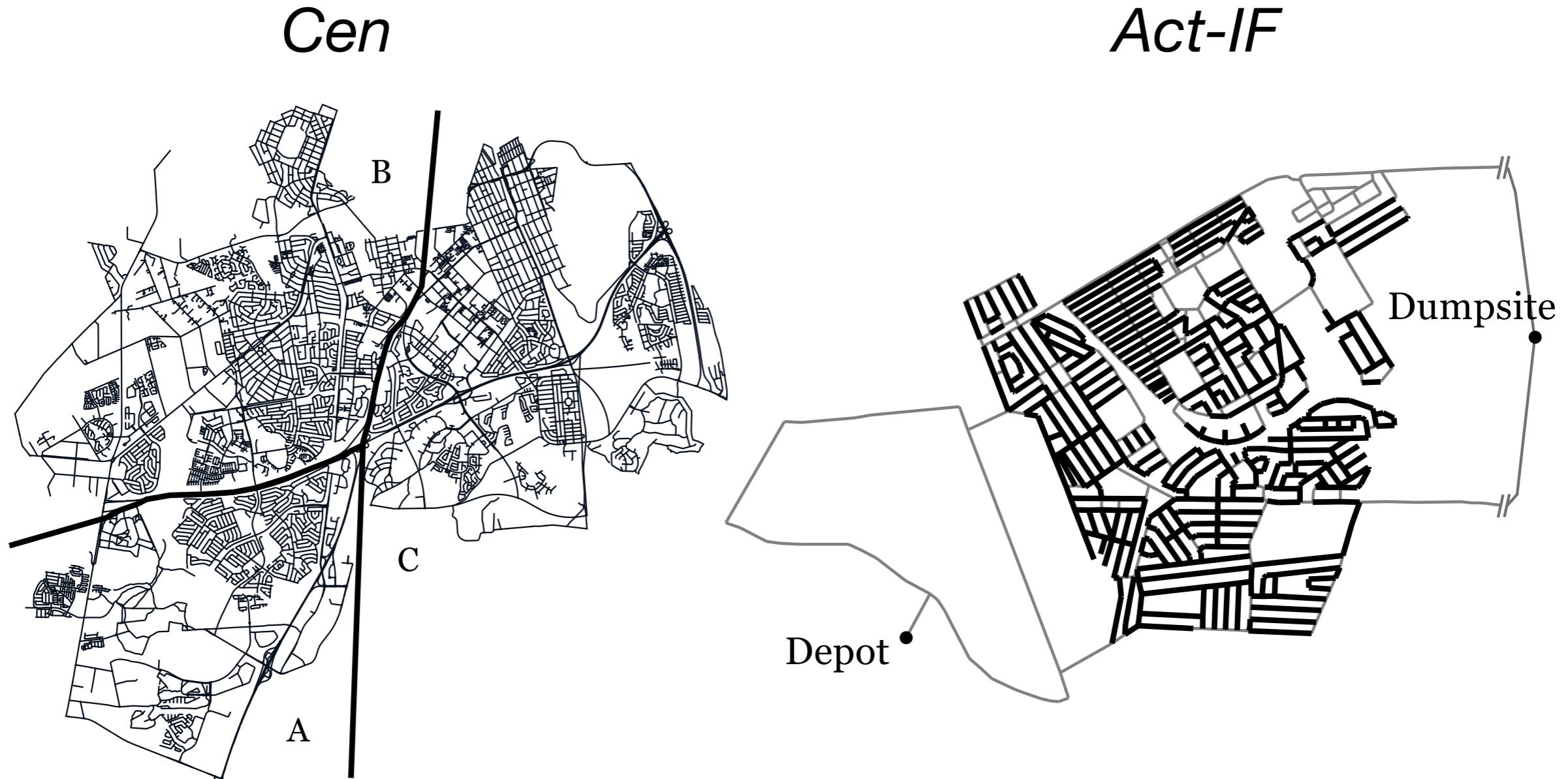
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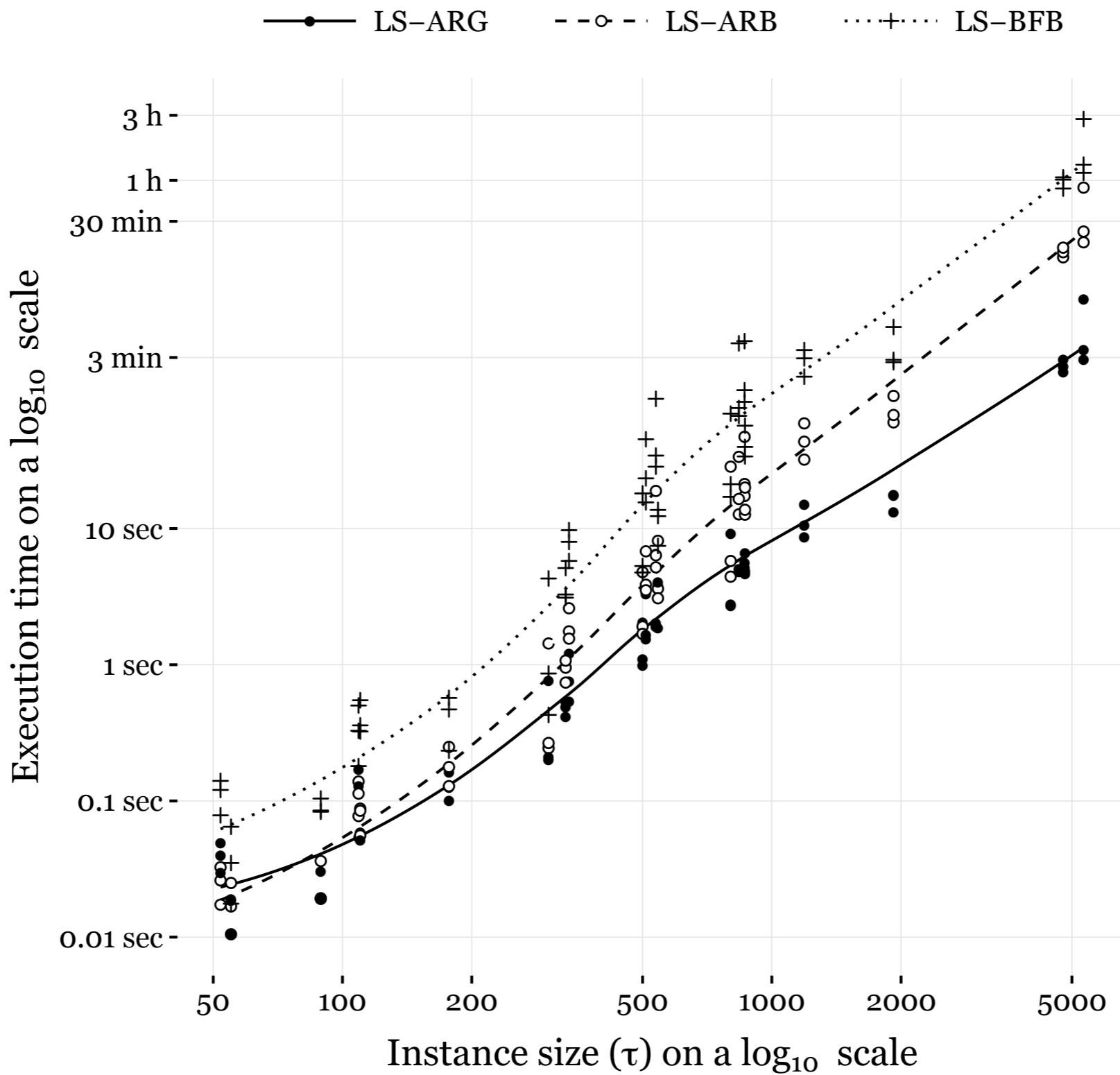
Willemse, E. J. (2016). **Heuristics for large-scale Capacitated Arc Routing Problems on mixed networks** (Doctoral dissertation, University of Pretoria).

# New benchmark sets

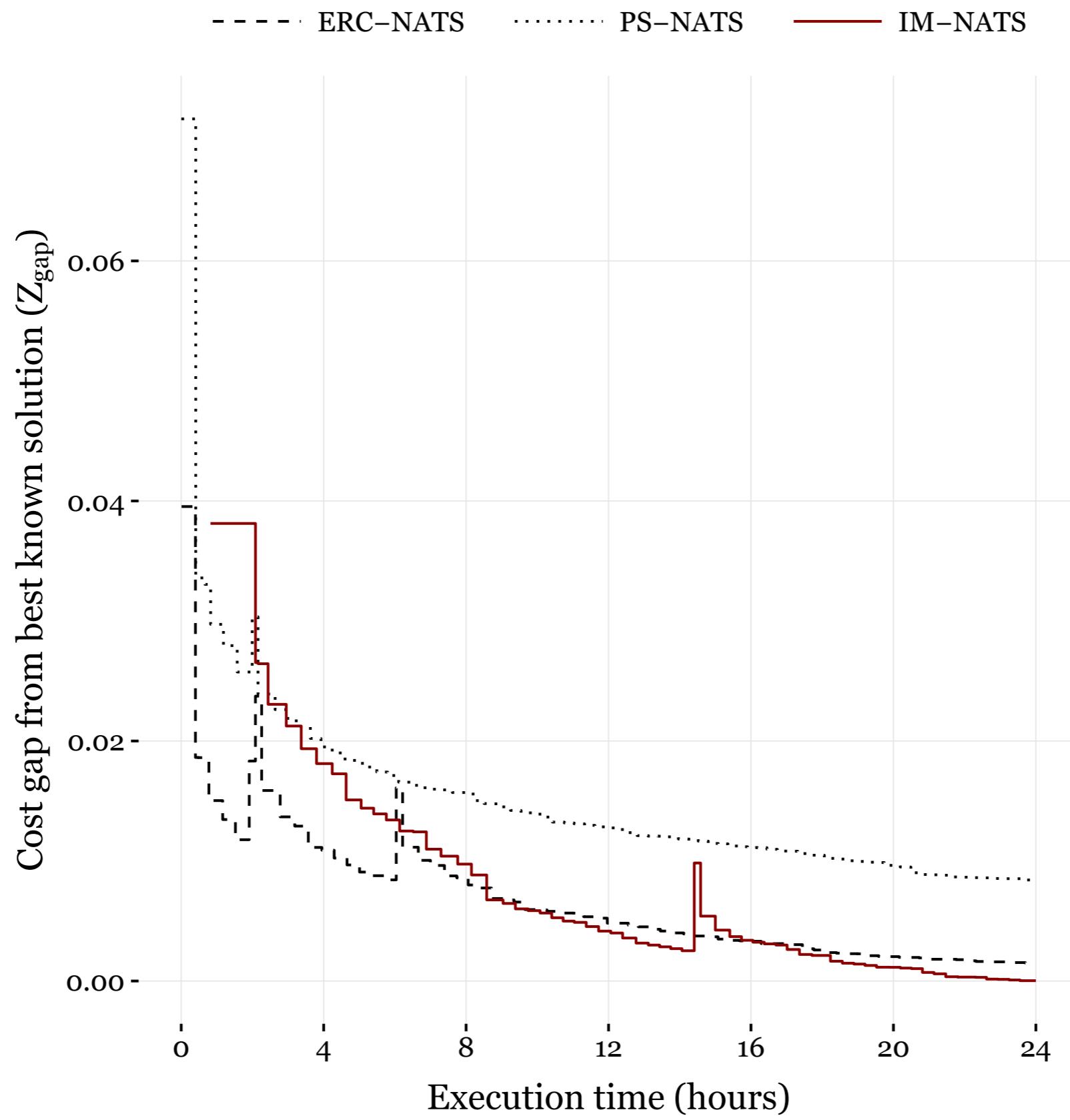


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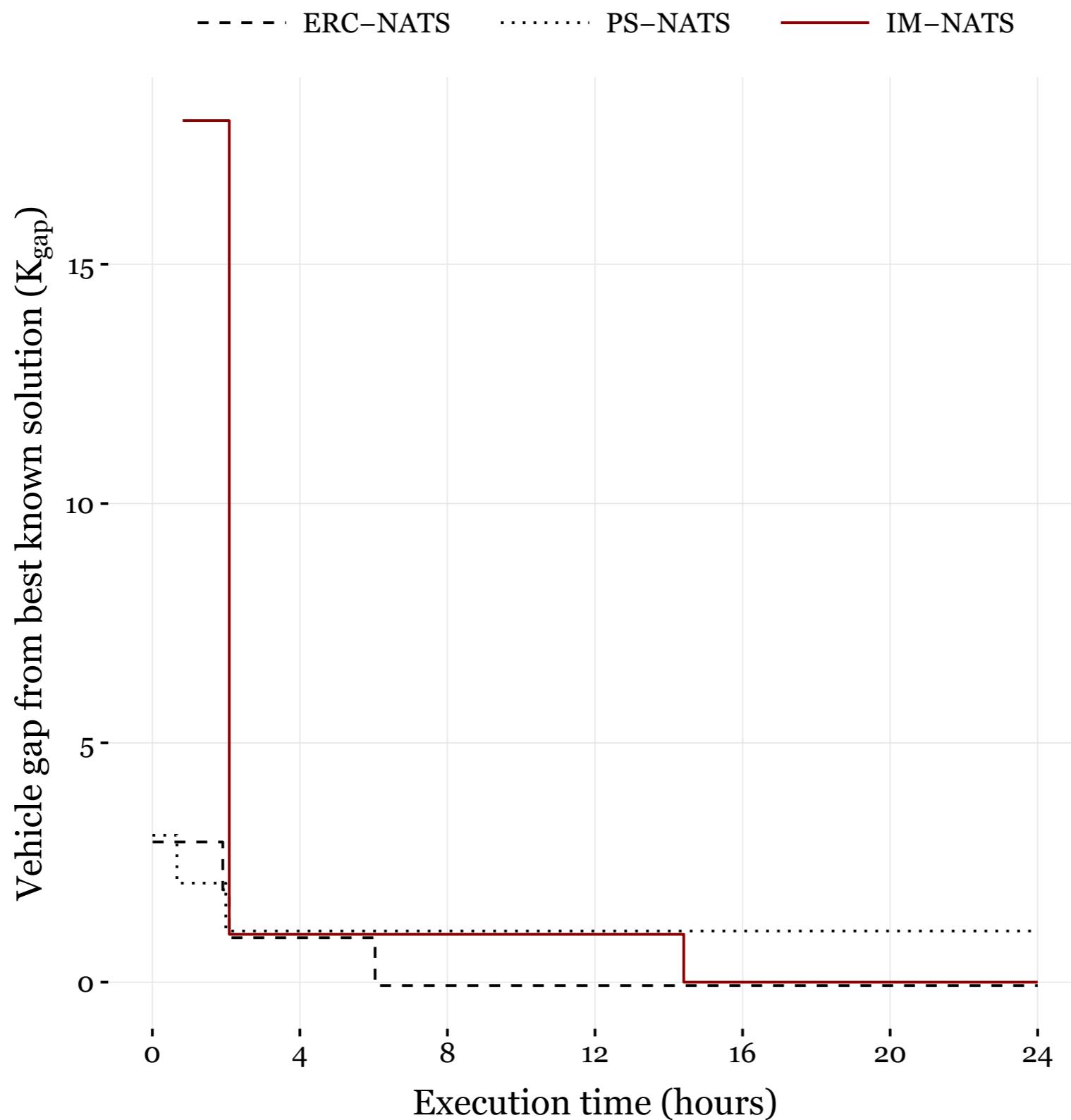
# Compound Moves



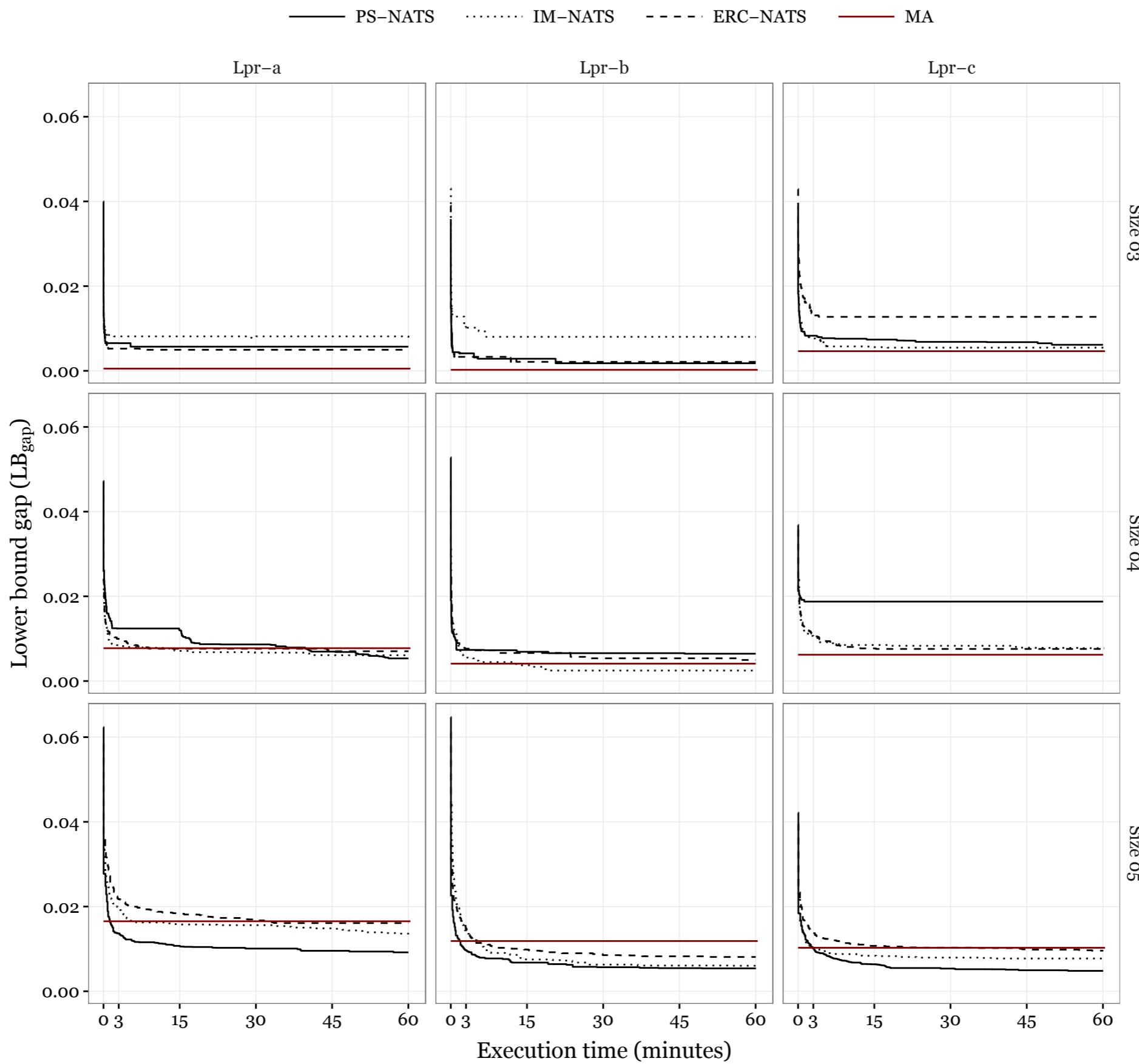
# 6'282 arc MCARPTIF instance



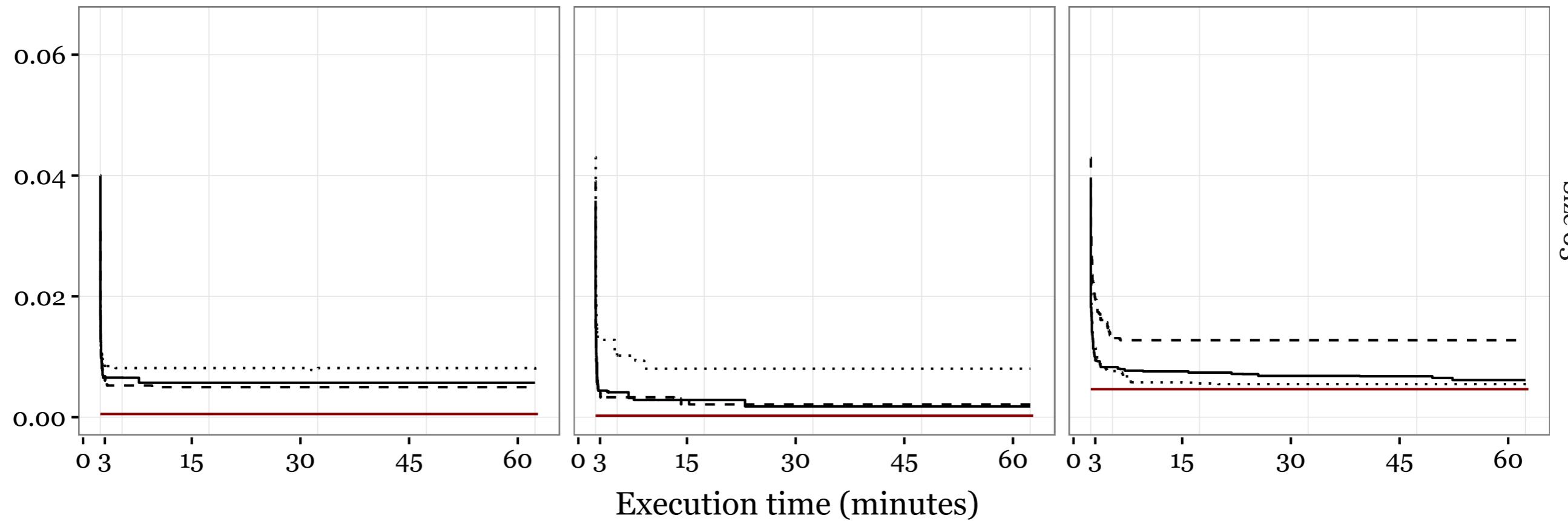
# 6'282 arc MCARPTIF instance



# *Lpr* MCARP instances

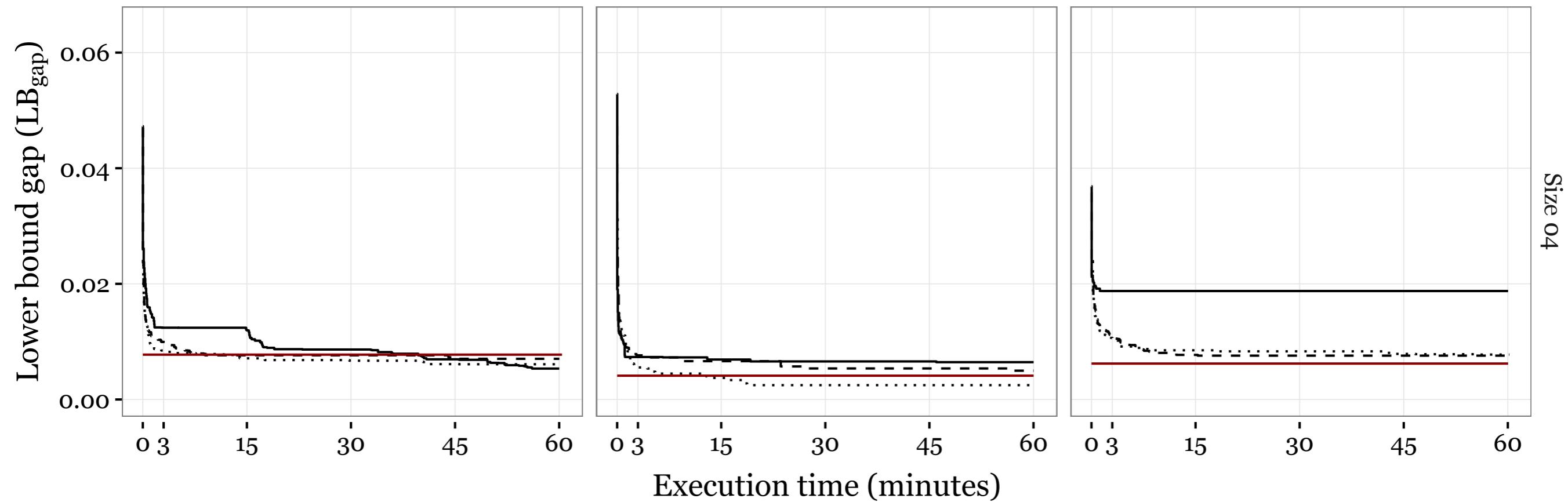


# Solving the MCARP



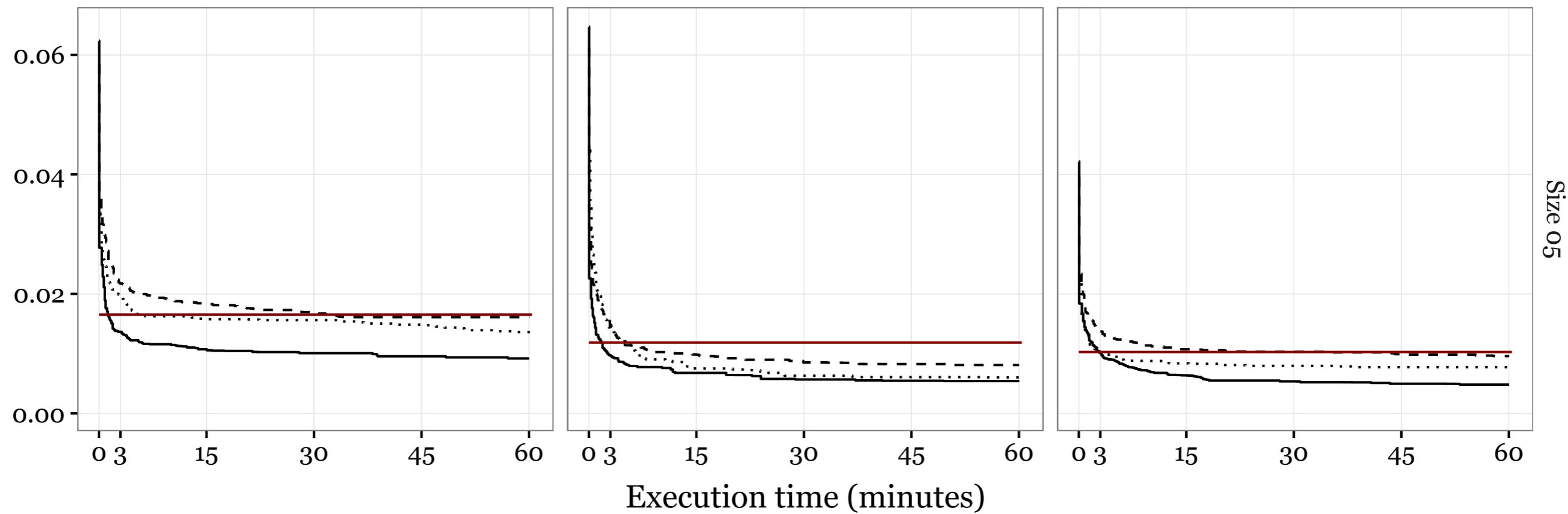
~100 arc instance

# *Lpr* MCARP instances



~250 arc instance

# *Lpr* MCARP instances



~806 arc instance

# Next steps

1

Compare our approach against MCGRP  
Vidal, T. (2017). **Node, edge, arc routing and turn penalties: Multiple problems—one neighborhood extension.** *Operations Research*, 65(4), 992-1010.

2

Make the our implementations available on *github*.

3

Convert new benchmark instances to better evaluate performance of heuristics.  
Kielerich, L., & Wøhlk, S. (2018). **New large-scale data instances for CARP and new variations of CARP.** *INFOR: Information Systems and Operational Research*, 56(1), 1-32.

# THANK YOU!

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WARP3 - 2019 - Pizzo, Italy

<https://github.com/ejwillemse/WARP3>



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Built Environment and  
Information Technology**

Fakulteit Ingenieurswese, Bou-omgewing en  
Inligtingtegnologie / Lefapha la Boetšenere,  
Tikologo ya Kago le Theknolotši ya Tshedimošo

1956 – 2016

**60**  
years of  
Engineering  
Education